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Centre Number

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Student Number

2020

Year 12

Mathematics Advanced

Task 2
02/03/20

**General
Instructions:**

- Reading time – 5 minutes
- Working time – 50 minutes
- Write using blue or black pen
- NESA approved calculators may be used
- Show relevant mathematical reasoning and/or calculations

Total Marks:
42

Section I - 3 marks

Allow about 5 minutes for this section

Section II - 38 marks

Allow about 45 minutes for this section

Q	Marks
1	/1
2	/1
3	/1
4	/1
5	/2
6	/2
7	/4
8	/4
9	/3
10	/4
11	/6
12	/4
13	/4
14	/5
Total	/42

This question paper must not be removed from the examination room.

This assessment task constitutes 20% of the course.

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Section I

3 marks

Allow about 5 minutes for this section

Use the multiple-choice sheet for Question 1–3

1 How many solutions of the equation $(\sin x + 1)(\tan x - 2) = 0$ lie between 0 and 2π ?

(A) 1

(B) 2

(C) 3

(D) 4

2 The probability of winning a game is $\frac{4}{9}$. Which expression represents the probability of winning two consecutive games?

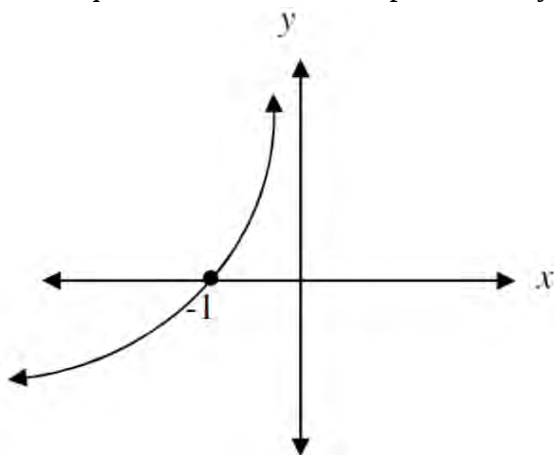
(A) $\frac{4}{9} \times \frac{1}{2}$

(B) $\frac{4}{9} \times \frac{3}{8}$

(C) $\frac{4}{9} \times \frac{1}{3}$

(D) $\frac{4}{9} \times \frac{4}{9}$

- 3 Which equation could best be represented by the graph below?



- (A) $y = -\ln(-x)$
- (B) $y = \ln(-x)$
- (C) $y = -\ln(x)$
- (D) $y = -e^x$

End of Section I

Section II

38 marks

Allow about 45 minutes for this section

In questions 4-14, your response should include relevant mathematical reasoning and/or calculations.

4. Simplify $5e^{\log_e 2}$

1

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5. Solve the equation: $\log(5^x) = \log 5 - \log \sqrt{5}$

2

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6. Let $\log_x 4 = m$ and $\log_x 7 = n$.
Find an expression for $\log_x(112x)$ in terms of m and n .

2

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7. Differentiate with respect to x and simplify:

(a)

$$(2 + 3e^x)^4$$

2

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(b)

$$\frac{e^{bx}}{bx + 1}$$

2

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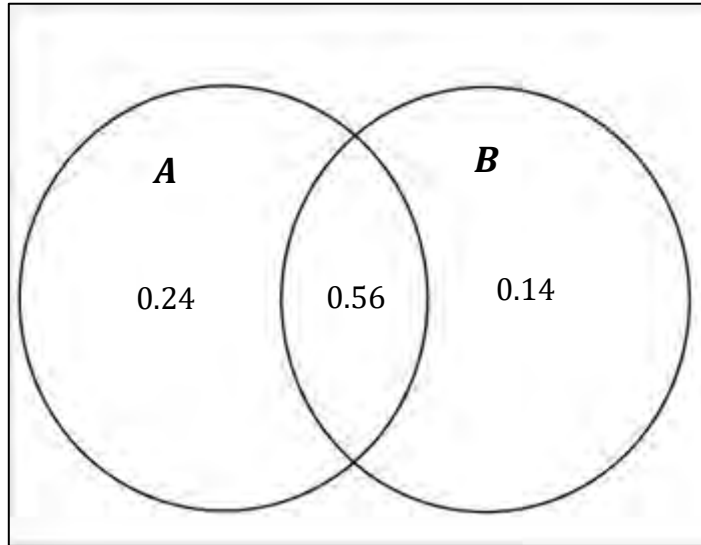
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8. Consider the following probability Venn diagram for the events A and B :



- (a) Find $P(A \cup B)$

1

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- (b) Find $P(\bar{A} \cap \bar{B})$

1

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(c) Find $P(A|\bar{B})$

1

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(d) Hence, demonstrate whether A and $\bar{B}$ are independent events.

1

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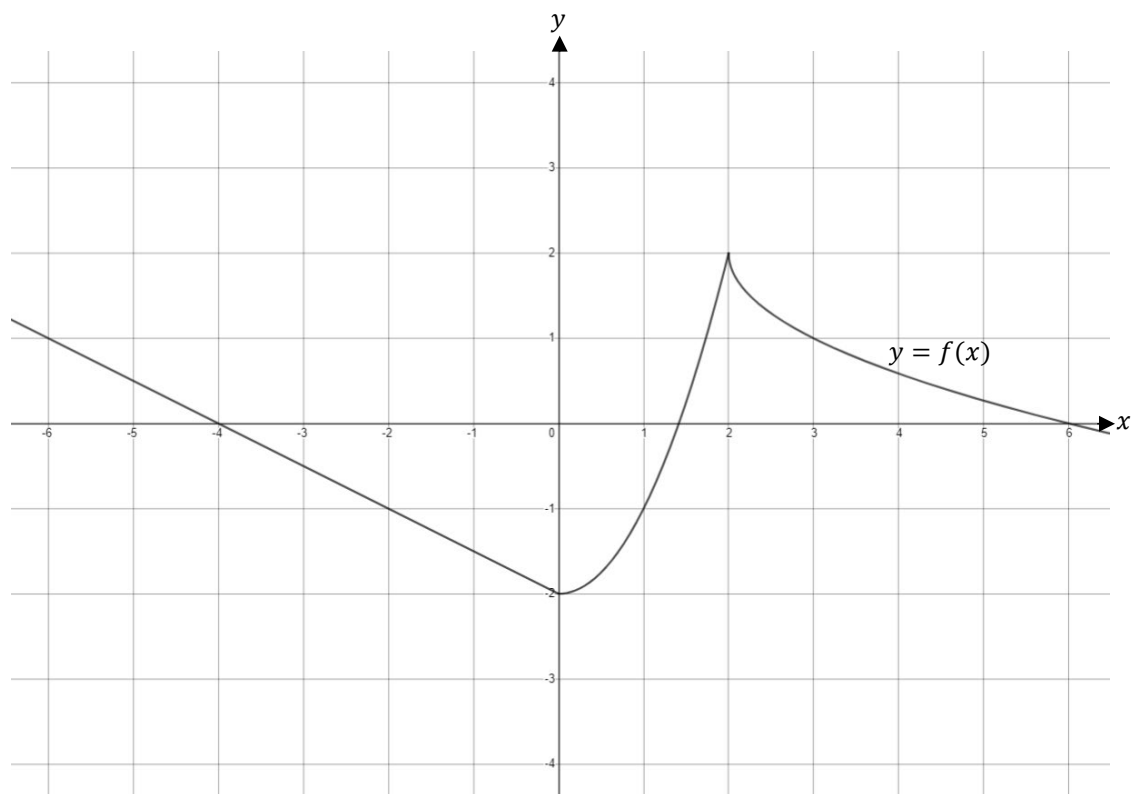
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9. Consider the graph of $y = f(x)$ shown below:



- (a) Explain how the above graph can be transformed to produce the graph of: $y = 2f(x + 1)$ 1

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- (b) Sketch the graph of $y = 2f(x + 1)$ on the same set of axes. 2

10. Consider the probability distribution for the random variable X shown below.

x	-1	0	1	2
$P(X = x)$	p	$1 - 5p$	$\frac{3p}{2}$	m

- (a) Show that $m = \frac{5p}{2}$

1

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- (b) Hence, find an expression for the variance of X in terms of p .

3

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11. Zachary has a biased coin that lands on heads 95% of the time.

- (a) What is the chance that he gets 1 head and 1 tail (in any order) when he flips the coin twice? **1**

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- (b) What is the chance that he gets at least one tail when he flips the coin three times? **2**

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- (c) What is the least number of times Zachary must flip the coin to be 90% certain that he will get at least one tail? **3**

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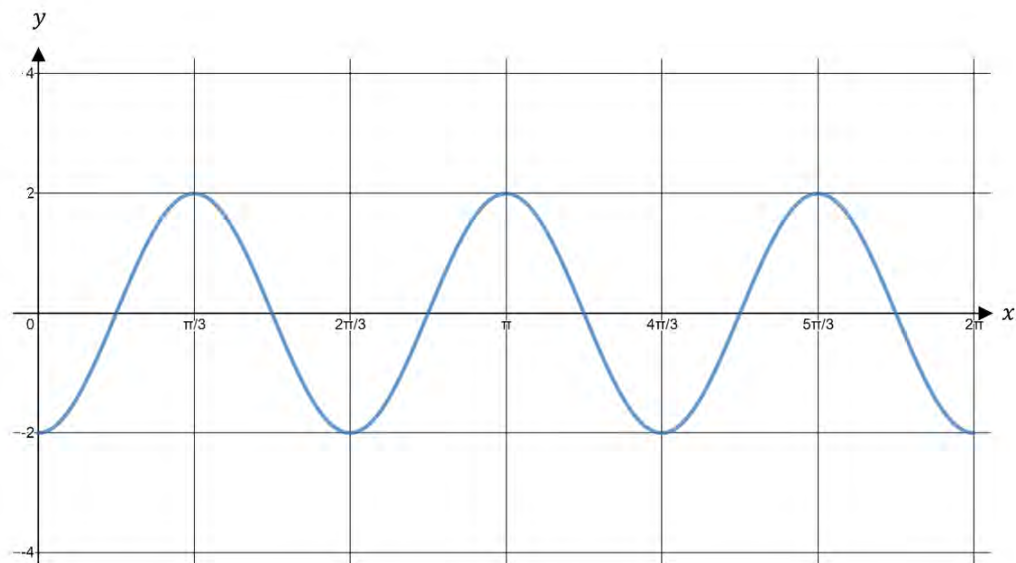
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12. The graph of the curve $y = k \sin(a(x + b))$ is given below for the domain $0 \leq x \leq 2\pi$.



- (a) State the amplitude and the period of the curve.

2

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.....

- (b) Hence, find the equation of the curve.

2

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13. The population of a mob of kangaroos is given by $P(t) = 600 + 80 \cos(\frac{\pi}{6}t)$, where t is in months.

- (i) Find all times during the first 12 months when the population equals 560. 2

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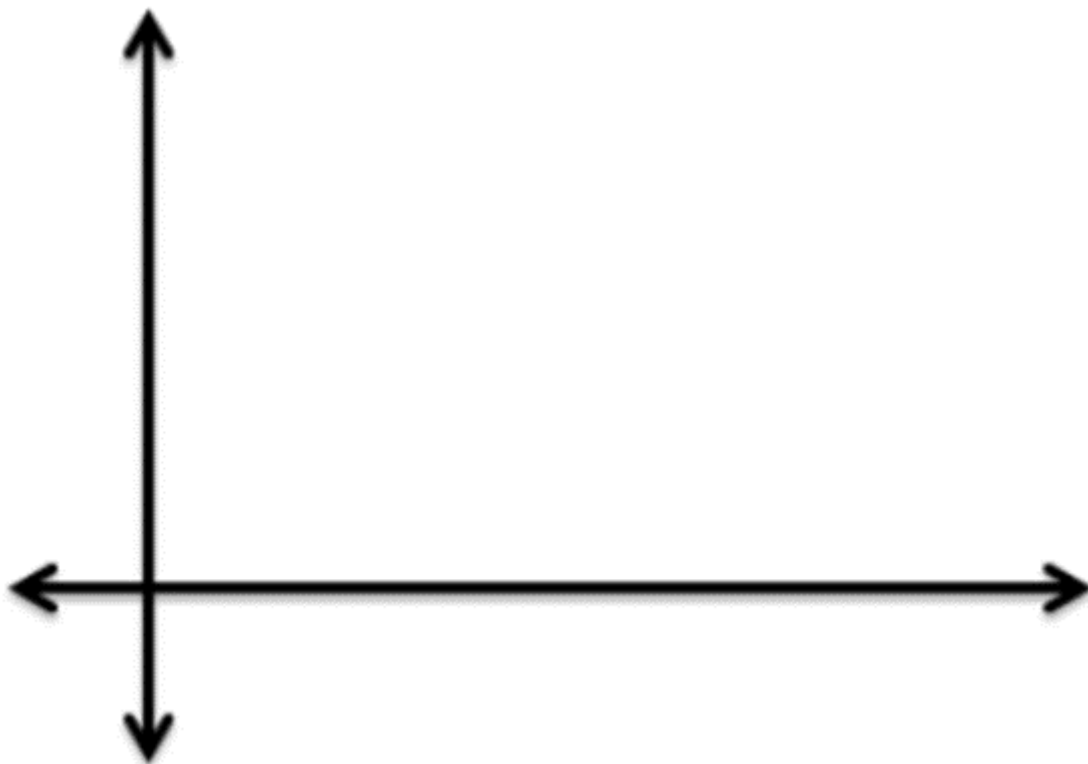
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- (ii) Sketch the graph of $P(t)$ for $0 \leq t \leq 12$. 2



14. (a) Find the horizontal and vertical asymptote of the graph $y = \frac{x}{x^2-4}$

2

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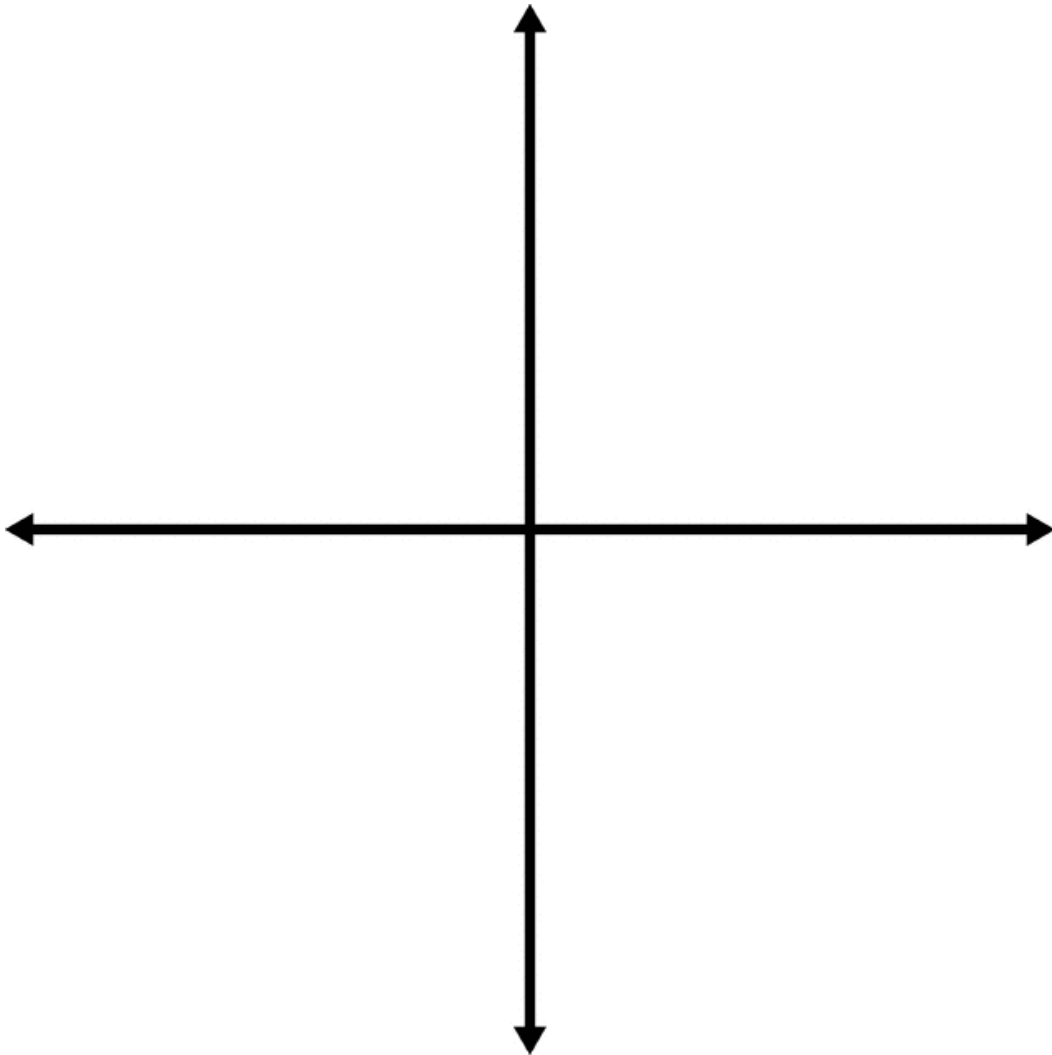
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- (b) Without the use of calculus, sketch the graph $y = \frac{x}{x^2-4}$, showing all asymptotes found in part (i).

3



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Student Number

Mathematics Advanced – Year 12

Section I – Multiple Choice Answer Sheet

Use this multiple-choice answer sheet for questions 1 – 3. Detach this sheet.

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample: $2 + 4 =$ (A) 2 (B) 6 (C) 8 (D) 9
 A ☐ B ☒ C ☐ D ☐

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

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A ☒ B ☒ C ☐ D ☐
 correct
 A ☒ B ☒ C ☐ D ☐

Start Here → A ☐ B ☐ C ☐ D ☐

2. A ☐ B ☐ C ☐ D ☐

3. A ☐ B ☐ C ☐ D ☐

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☒ (C) 3

(D) 4

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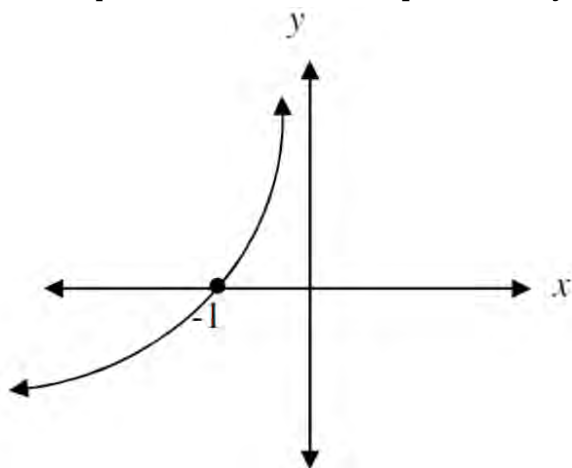
(A) $\frac{4}{9} \times \frac{1}{2}$

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- (B) $y = \ln(-x)$
- (C) $y = -\ln(x)$
- (D) $y = -e^x$

End of Section I

Section II

38 marks

Allow about 45 minutes for this section

In questions 4-14, your response should include relevant mathematical reasoning and/or calculations.

4. Simplify $5e^{\log_e 2}$

1

$$5 \times 2 = 10$$

5. Solve the equation: $\log(5^x) = \log 5 - \log \sqrt{5}$

2

$$\log 5^x = \log \frac{5}{\sqrt{5}} \quad (1 \text{ mark})$$

$$\log 5^x = \log 5^{\frac{1}{2}} \quad (1 \text{ mark})$$

$$x = \frac{1}{2}$$

6. Let $\log_x 4 = m$ and $\log_x 7 = n$.
Find an expression for $\log_x(112x)$ in terms of m and n .

2

$$\begin{aligned} & \log_x 112x \\ &= \log_x 16 + \log_x 7 + \log_x x \quad (1 \text{ mark}) \end{aligned}$$

$$= \log_x 4^2 + \log_x 7 + \log_x x$$

$$= 2m + n + 1 \quad (1 \text{ mark})$$

7. Differentiate with respect to x and simplify:

(a)

$$(2 + 3e^x)^4$$

2

$$\begin{aligned} y' &= 4(2+3e^x)^3 \cdot 3e^x \\ &= 12e^x(2+3e^x)^3 \end{aligned}$$

(b)

$$\frac{e^{bx}}{bx+1}$$

2

$$\begin{aligned} u &= e^{bx} & u' &= be^{bx} \\ v &= bx+1 & v' &= b \end{aligned}$$

$$y' = \frac{be^{bx}(bx+1) - be^{bx}}{(bx+1)^2}$$

$$= \frac{b^2xe^{bx}}{(bx+1)^2}$$

8. Consider the probability distribution for the random variable X shown below.

$x^2 p(x)$	p	0	$\frac{3p}{2}$	$10p$
x^2	1	0	1	4
x	-1	0	1	2
$P(X = x)$	p	$1 - 5p$	$\frac{3p}{2}$	m
				$\frac{5p}{2}$
$x p(x)$	$-p$	0	$\frac{3p}{2}$	$5p$

- (a) Show that $m = \frac{5p}{2}$

1

$$p + 1 - 5p + \frac{3}{2}p + m = 1$$

$$m = \frac{5p}{2}$$

- (b) Hence, find an expression for the variance of X in terms of p .

3

$$u = E(X)$$

$$= \sum x p(x)$$

$$= -p + 0 + \frac{3p}{2} + 5p$$

$$= \frac{11p}{2} \quad (1 \text{ mark})$$

$$E(X^2) = \sum x^2 p(x)$$

$$= p + \frac{3p}{2} + 10p$$

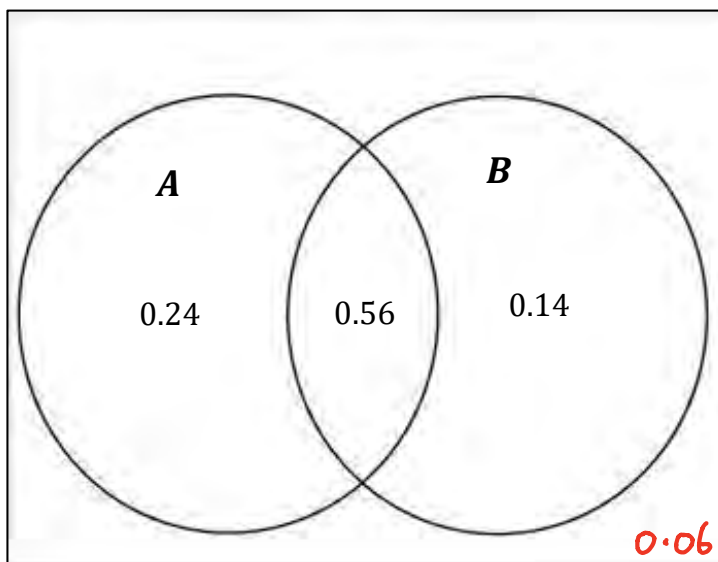
$$= \frac{25p}{2} \quad (1 \text{ mark})$$

$$\text{Var} = E(X^2) - u^2$$

$$= \frac{25p}{2} - \left(\frac{11p}{2}\right)^2$$

$$= \frac{50p - 121p^2}{4} \quad (1 \text{ mark})$$

9. Consider the following probability Venn diagram for the events A and B :



- (a) Find $P(A \cup B)$

1

$$P(A \cup B) = 0.24 + 0.56 + 0.14$$
$$= 0.94$$

- (b) Find $P(\bar{A} \cap \bar{B})$

1

$$P(\bar{A} \cap \bar{B}) = 1 - 0.94$$
$$= 0.06$$

(c) Find $P(A|\bar{B})$

1

$$P(A|\bar{B}) = \frac{P(A \cap \bar{B})}{P(\bar{B})}$$

$$= \frac{0.24}{0.3}$$

$$= 0.8$$

(d) Hence, demonstrate whether A and $\bar{B}$ are independent events.

1

$$P(A) = 0.24 + 0.56$$

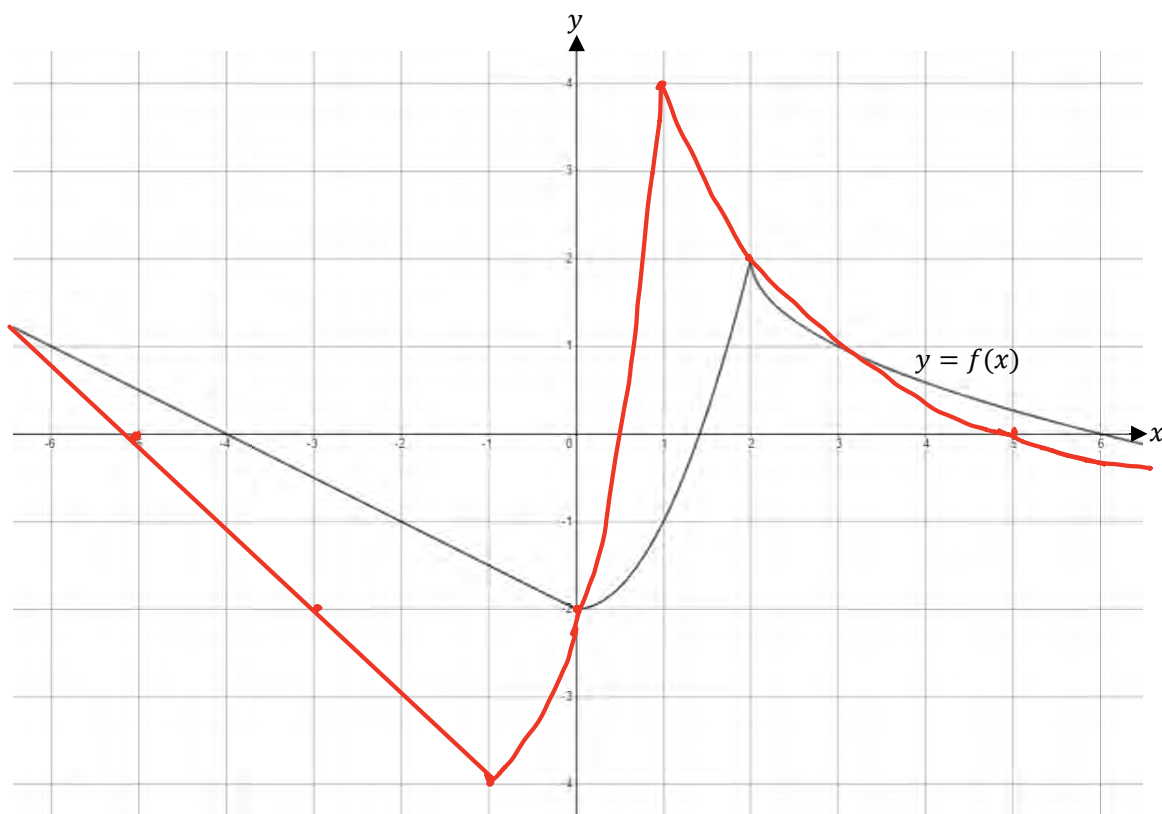
$$= 0.8$$

$$P(A|\bar{B}) = 0.8$$

$$P(A|\bar{B}) = P(A)$$

$\therefore A$ and $\bar{B}$ are independent events

10. Consider the graph of $y = f(x)$ shown below:



- (a) Explain how the above graph can be transformed to produce the graph of:
 $y = 2f(x + 1)$

1

vertical dilation by scale factor 2

translate 1 unit left

- (b) Sketch the graph of $y = 2f(x + 1)$ on the same set of axes.

2

11. Zachary has a biased coin that lands on heads 95% of the time.

- (a) What is the chance that he gets 1 head and 1 tail (in any order) when he flips the coin twice? 1

$$5\% \times 95\% \times 2 = 0.095$$

- (b) What is the chance that he gets at least one tail when he flips the coin three times? 2

↓ 1 mark

$$1 - (95\%)^3 = 0.142625$$

1 mark for using complementary events

* Alternative method $5\% \times 95\%^2 \times 3 + 5\%^2 \times 95\% \times 3 + 5\%^3 = 0.142625$

working towards the correct answer 1 mark

- (c) What is the least number of times Zachary must flip the coin to be 90% certain that he will get at least one tail? 3

$$1 - (95\%)^n \geq 90\%$$

$$(95\%)^n \leq 10\% \quad (1 \text{ mark})$$

$$n \ln 0.95 \leq \ln 0.1$$

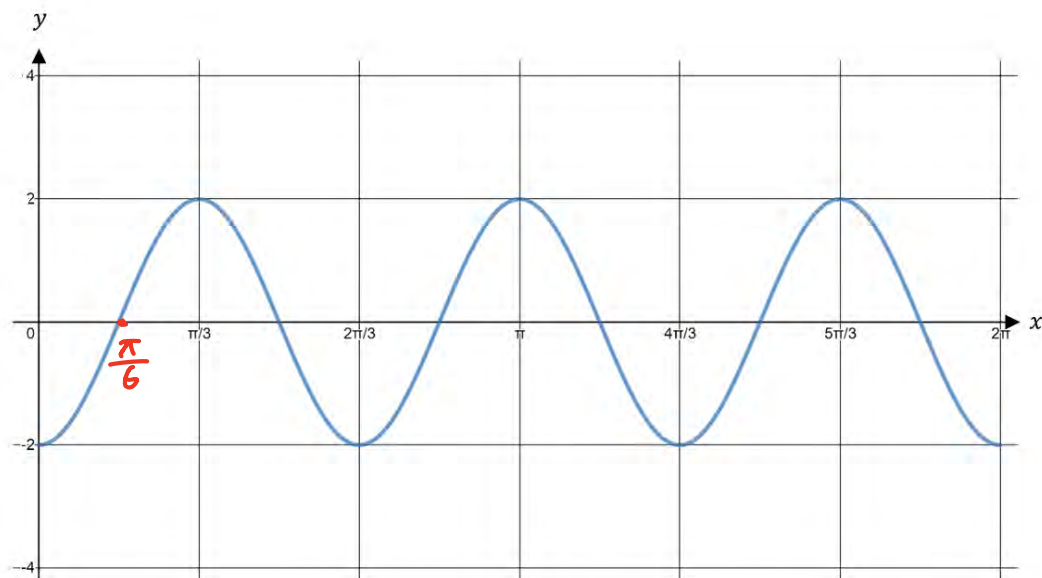
$$n \geq \frac{\ln 0.1}{\ln 0.95} \quad (1 \text{ mark})$$

$$n \geq 44.89$$

at least 45 times (1 mark)

* guess and check 1 mark

12. The graph of the curve $y = k \sin(a(x + b))$ is given below for the domain $0 \leq x \leq 2\pi$.



- (a) State the amplitude and the period of the curve.

2

2 $\frac{2\pi}{3}$

.....

.....

.....

- (b) Hence, find the equation of the curve.

2

$k=2, a=3$

$y = 2 \sin 3(x - \frac{\pi}{6})$

$b = -\frac{\pi}{6}$

.....

$y = 2 \sin 3(x - \frac{\pi}{6})$

$y = 2 \sin (3x - \frac{\pi}{2})$

13. The population of a mob of kangaroos is given by $P(t) = 600 + 80 \cos\left(\frac{\pi}{6}t\right)$, where t is in months.

$$\text{period} = \frac{2\pi}{\frac{\pi}{6}} = 12$$

- (i) Find all times during the first 12 months when the population equals 560. 2

$$P(t) = 600 + 80 \cos\left(\frac{\pi}{6}t\right) = 560$$

$$80 \cos\left(\frac{\pi}{6}t\right) = -40$$

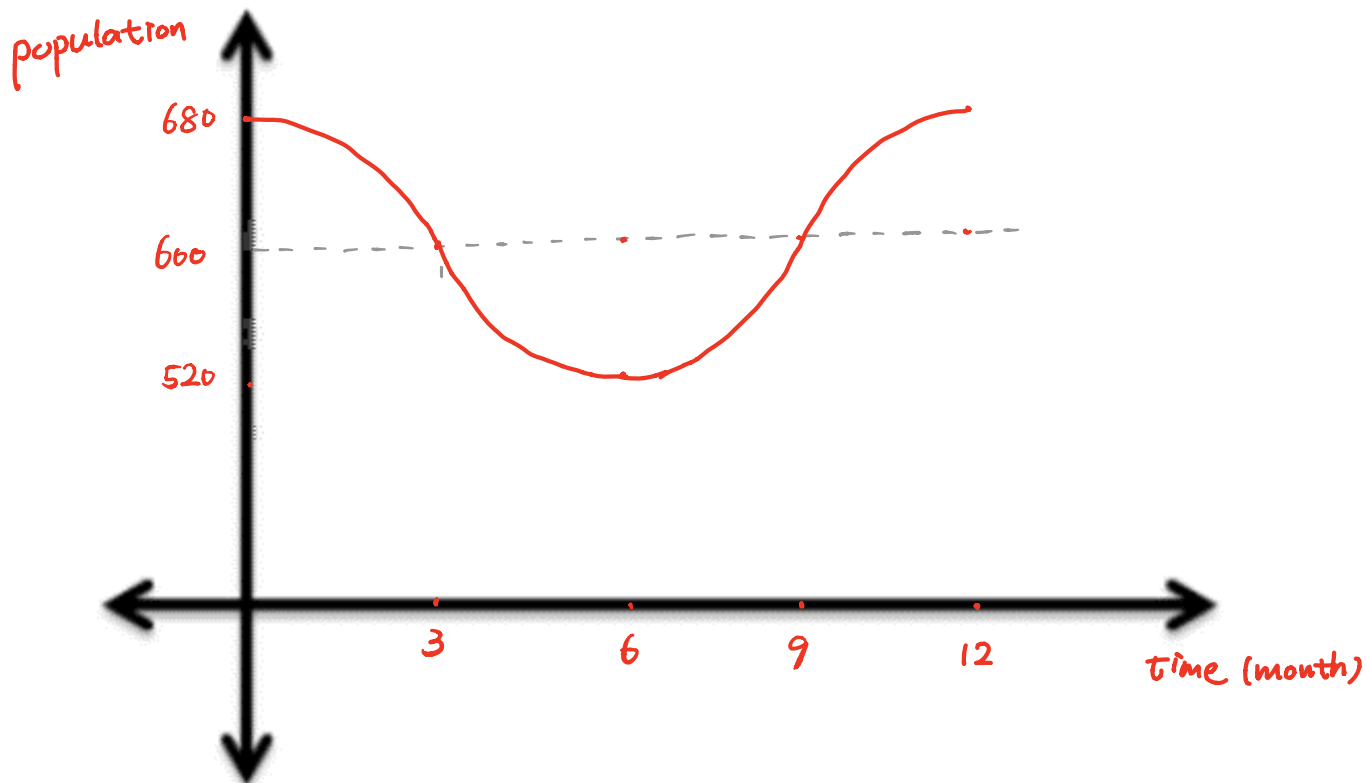
$$\cos\left(\frac{\pi}{6}t\right) = -\frac{1}{2} \quad (1 \text{ mark})$$

$$\frac{\pi}{6}t = \frac{2\pi}{3} \text{ or } \frac{4\pi}{3}$$

$$t = 4 \text{ or } 8 \quad (1 \text{ mark})$$

- (ii) Sketch the graph of $P(t)$ for $0 \leq t \leq 12$. 2

- period
- amplitude correct
- correct cos graph



14. (a) Find the horizontal and vertical asymptote of the graph $y = \frac{x}{x^2-4}$

2

vertical asymptote: $x^2-4=0$

$x=2$ and $x=-2$ (1 mark)

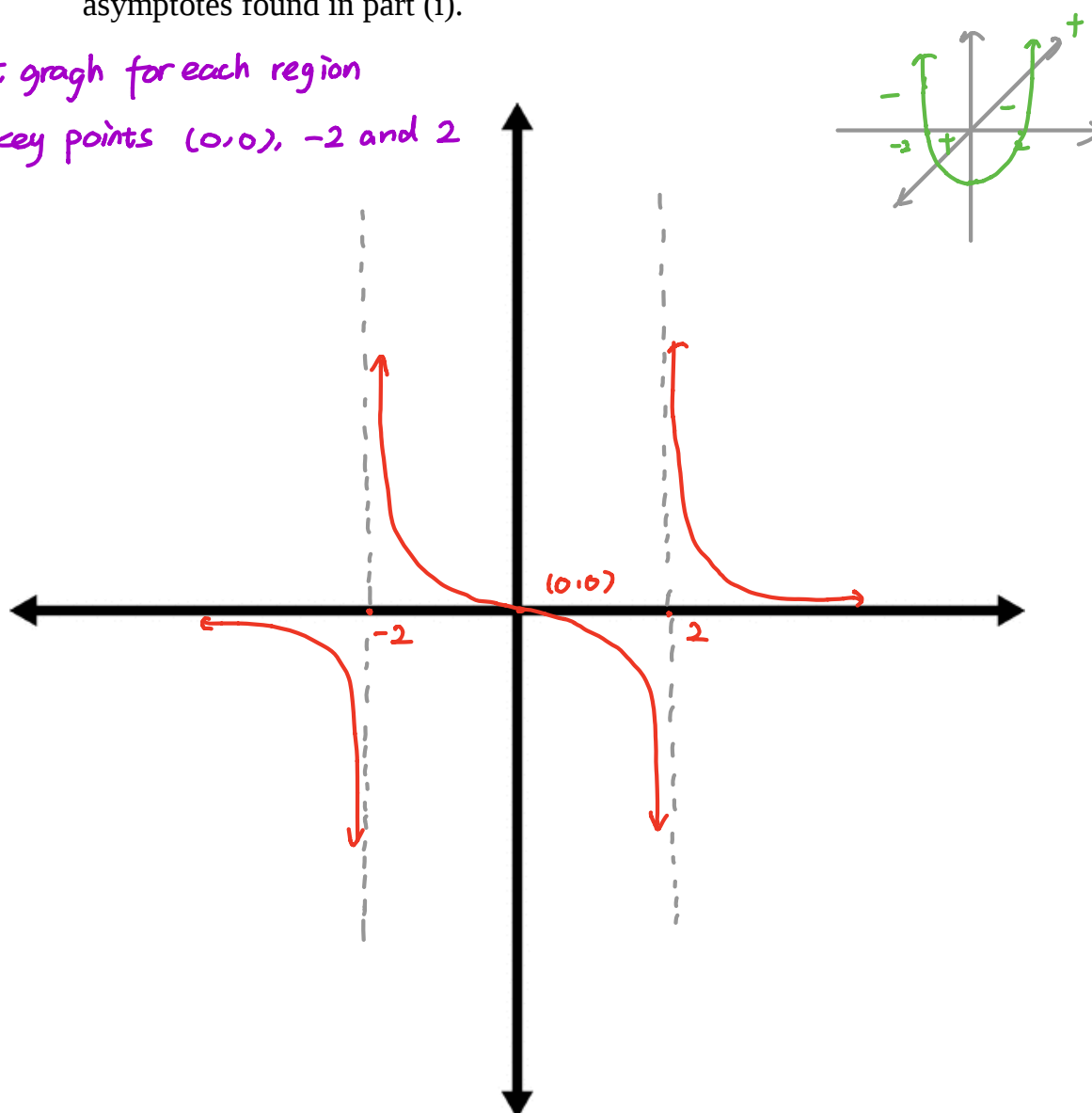
horizontal asymptote: $y = \frac{\frac{1}{x}}{\frac{1}{x^2}} \quad x \rightarrow \infty \quad y \rightarrow 0$

$y=0$ or x -axis (1 mark)

(b) Without the use of calculus, sketch the graph $y = \frac{x}{x^2-4}$, showing all asymptotes found in part (i).

3

- Correct graph for each region
- Label key points $(0,0)$, -2 and 2



Extra writing space

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A ☒ B ☒ C ☐ D ☐
correct

Start Here → A ☐ B ☐ C ☐ D ☐

2. A ☐ B ☐ C ☐ D ☐

3. A ☐ B ☐ C ☐ D ☐