



SYDNEY BOYS HIGH SCHOOL

NESA Number:

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Name:

Class:

2021

YEAR 11
TERM 4
HSC ASSESSMENT TASK #1

Mathematics Advanced

General Instructions

- Reading time – 5 minutes
- Working time – 60 minutes
- Write using black pen
- NESA approved calculators may be used
- A reference sheet is provided with this paper
- Marks may **NOT** be awarded for messy or badly arranged work
- For questions in Section II, show ALL relevant mathematical reasoning and/or calculations

Total Marks: 39

Section I – 5 marks (pages 2 – 5)

- Attempt Questions 1 – 5
- Allow about 7 minutes for this section

Section II – 34 marks (pages 4 – 17)

- Attempt Questions 6 – 20
- Allow about 53 minutes for this section

Examiner: AYW

Section I

5 marks

Attempt Questions 1 – 5

Allow about 7 minutes for this section

Use the multiple-choice answer sheet for Questions 1 - 5

1. The curve $y = ax^2 + 8x - 4$ has a stationary point at $x = 2$. What is the value of a ?
- A. -3
- B. -2
- C. 2
- D. 3
2. If $y = \frac{1}{x^3}$, then $\frac{dy}{dx}$ equals which of the following?
- A. $3x^{-2}$
- B. $-\frac{x^{-2}}{2}$
- C. $-3x^{-4}$
- D. x^2
3. What is the solution to the inequation $(1-x)(x+2) \leq 0$?
- A. $x \leq -2, x \geq 1$
- B. $x \leq -1, x \geq 2$
- C. $-2 \leq x \leq 1$
- D. $-1 \leq x \leq 2$

4. A and B are events of a sample space.

Given that $P(A|B) = q$, $P(B) = q^2$ and $P(A) = q^{\frac{1}{3}}$, what is $P(B|A)$ equal to?

A. q

B. $q^{\frac{4}{3}}$

C. $q^{\frac{7}{3}}$

D. $q^{\frac{8}{3}}$

5. If $\int_0^{10} h(x)dx = 7$ and $\int_4^{10} h(x)dx = -2$, what is the value of $\int_4^0 h(x)dx$?

A. 9

B. 5

C. -7

D. -9



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Mathematics Advanced

Section II – Part A (12 marks)

Marks:

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These spaces provide guidance for the expected length of response. Your responses should include relevant mathematical reasoning and/or calculation.

Section II
Part A
Attempt Questions 6 - 13

12 marks

Answer each question in the spaces provided. A blank page is provided at the end of each question to allow rewriting of a part.

Your responses should include relevant mathematical reasoning and/or calculations.

Question 6

Differentiate $y = 3x^4 - 8x + 7$

1

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Question 7

Differentiate $y = \frac{2}{x\sqrt{x}}$, leaving your answer in index form.

1

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Question 8

Evaluate $\lim_{x \rightarrow -7} \frac{x^2 + 2x - 35}{x + 7}$

1

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Question 9

The quadratic function, $y = x^2 - 6x + c$, is to be drawn on a number plane.

- (a)

Find the discriminant of the equation above.

1

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- (b)

Hence or otherwise, find the value of c for which the graph, of the equation above, will touch the x -axis.

1

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Question 10

Find a primitive function of $f(x) = 4x^3 - 6x^2 - 11$.

1

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Question 11

The probability that the Dragon Football team will win this weekend is 0.75.
The probability that the Sharks Football team will win this weekend is 0.6.
What is the probability that ONLY one team will win this weekend?

2

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Question 12

The polynomials $P(x)$ and $Q(x)$ have degrees m and n respectively where $m > n$.
Write down the degree of

(a) $P(x) - Q(x)$ **1**

.....
.....

(b) $P(x) \times Q(x)$ **1**

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Question 13

Find the equation of the normal to the curve $y = 2x^2 - 5x + 1$ at the point $(1, -2)$. **2**

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End of Part A

This image shows a full page of a handwriting practice worksheet. It consists of approximately 28 horizontal rows. Each row is defined by two parallel dotted lines, creating a series of uniform gaps for letter height. The entire page is otherwise blank, with no margins, text, or other markings.



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HSC ASSESSMENT TASK #1

Mathematics Advanced

Section II – Part B (11 marks)

Marks:

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These spaces provide guidance for the expected length of response. Your responses should include relevant mathematical reasoning and/or calculation.

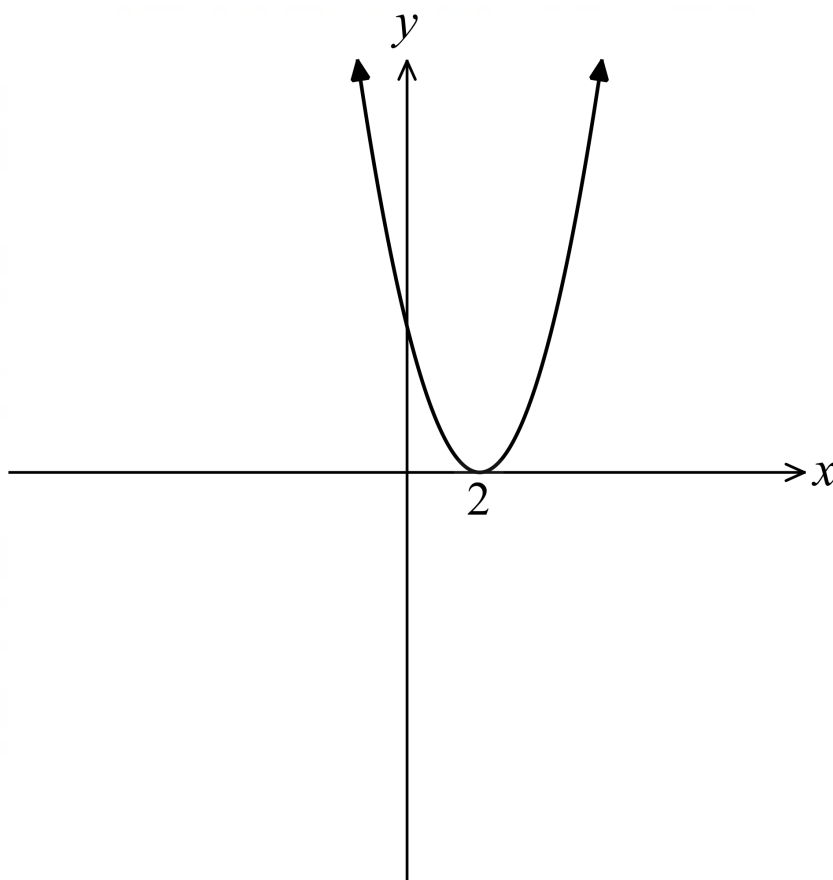
Section II**Part B****11 marks****Attempt Questions 14 - 17**

Answer each question in the spaces provided. A blank page is provided at the end of each question to allow rewriting of a part.

Your responses should include relevant mathematical reasoning and/or calculations.

Question 14

Consider the function $y = f(x)$ whose gradient function, $y = f'(x)$, is given in the diagram below.



- (a) Comment on the sign of the gradient of $f(x)$ for all real x .

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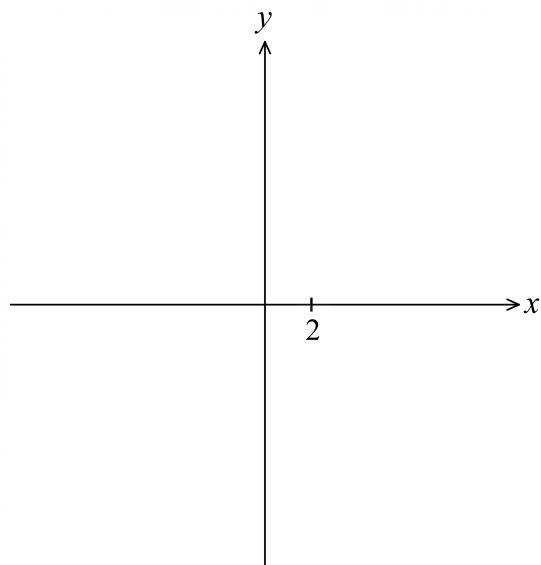
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Question 14 continues on page 11

Question 14 (continued)

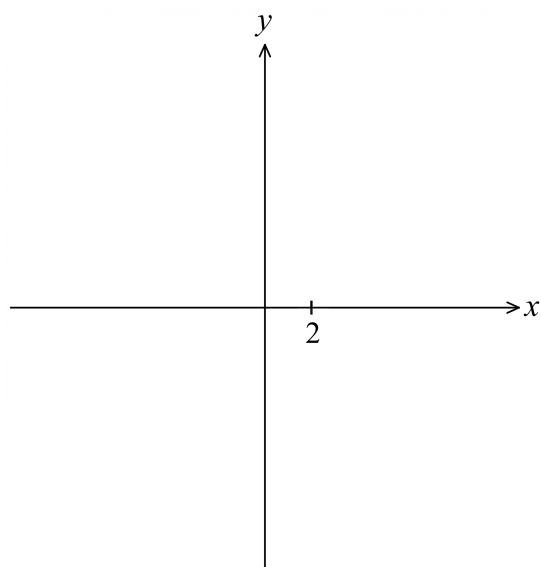
(b) Sketch a possible graph of $y = f(x)$.

1



(c) Sketch $y = f''(x)$.

1



Question 15

Find the values of a , b and c for which $5x^2 + 7x - 8 = ax(x+1) + bx^2 + c(x-2)$ for all real x .

2

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Question 16

A school decide to do a fundraiser for a charity, 70 tickets were sold and 2 prizes were offered. Two tickets were drawn without replacement to determine the prize winners.

Joe bought some of the tickets. The probability that he won both prizes was $\frac{1}{161}$.

(a) Find the number of tickets bought by Joe. **2**

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(b) What is the probability of Joe winning at least one prize? **2**

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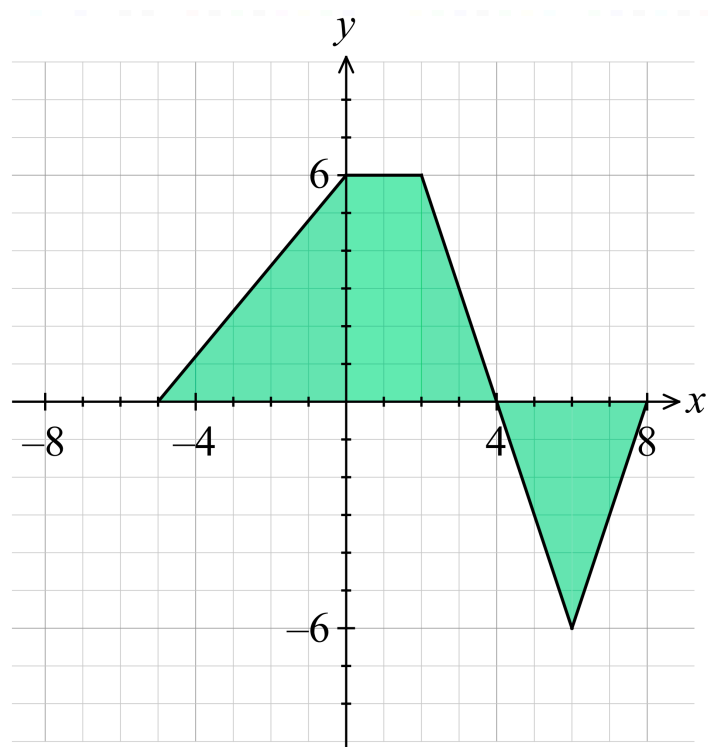
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Question 17

The diagram below shows $y = f(x)$.



Find the value of $\int_{-5}^8 f(x) dx$.

2

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End of Part B

[illegible]



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Mathematics Advanced

Section II – Part C (11 marks)

Marks:

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These spaces provide guidance for the expected length of response. Your responses should include relevant mathematical reasoning and/or calculation.

Section II
Part C
Attempt Questions 18 - 20

11 marks

Answer each question in the spaces provided. A blank page is provided at the end of each question to allow rewriting of a part.

Your responses should include relevant mathematical reasoning and/or calculations.

Question 18

Estimate $\int_2^6 \frac{2}{x(x-1)} dx$ by using the trapezoidal rule with 4 sub-intervals.

3

Leave your answer in exact form.

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Question 19

Find, from first principles, the derivative of $y = \frac{2}{(x-3)^2}$.

3

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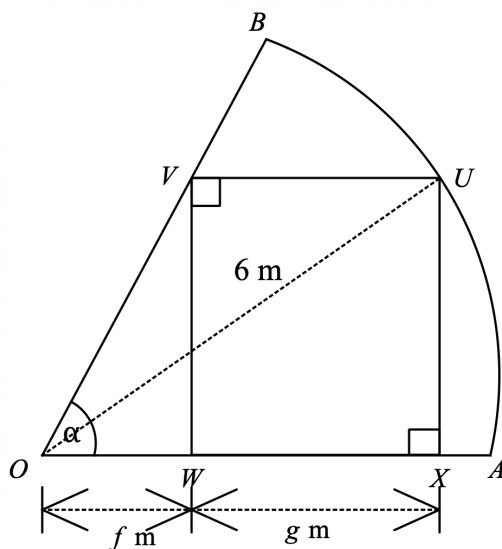
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Question 20

The diagram below shows a rectangle $UVWX$ drawn in a sector OAB of a circle, radius 6 m .
Let Angle $AOB = \alpha$, $\tan \alpha = 3$, $OW = f\text{ m}$ and $WX = g\text{ m}$.



- (a) Show that $UX = 3f$.

1

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- (b) Hence show $(f + g)^2 = 36 - 9f^2$.

1

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Question 20 continues on page 18

Question 20 (continued)

- (c) Find the maximum perimeter of the rectangle in exact form.

3

[illegible]

End of paper

[illegible]



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HSC ASSESSMENT TASK #1**

Mathematics Advanced

Sample Solutions

Quick MC Answers

- | | |
|---|---|
| 1 | B |
| 2 | C |
| 3 | A |
| 4 | D |
| 5 | D |

NOTE: Before putting in an appeal re-marking, first consider that the mark is not linked to the amount of ink you have used.

Just because you have shown ‘working’ does not justify that your solution is worth any marks.

If you have used whiteout or pencil, you can not appeal.

-
1. The curve $y = ax^2 + 8x - 4$ has a stationary point at $x = 2$. What is the value of a ?

A. -3

Axis of symmetry is the location of the stationary point.

☒ B. -2

$$\therefore x = -\frac{b}{2a} = -\frac{8}{2a} = 2 \Rightarrow a = -2$$

C. 2

D. 3

2. If $y = \frac{1}{x^3}$, then $\frac{dy}{dx}$ equals which of the following?

A. $3x^{-2}$

$$y = \frac{1}{x^3} = x^{-3}$$

B. $-\frac{x^{-2}}{2}$

☒ C. $-3x^{-4}$

D. x^2

3. What is the solution to the inequation $(1-x)(x+2) \leq 0$?

☒ A. $x \leq -2, x \geq 1$

A or C looking at the x -intercepts

B. $x \leq -1, x \geq 2$

$$(1-x)(x+2) \leq 0 \Leftrightarrow (x-1)(x+2) \geq 0$$

C. $-2 \leq x \leq 1$

D. $-1 \leq x \leq 2$

4. A and B are events of a sample space.

Given that $P(A|B) = q$, $P(B) = q^2$ and $P(A) = q^{\frac{1}{3}}$, what is $P(B|A)$ equal to?

A. q

$$P(A|B) = \frac{P(A \cap B)}{P(B)} \Rightarrow P(A \cap B) = P(A|B) \cdot P(B) = q^3$$

B. $q^{\frac{4}{3}}$

$$P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{q^3}{q^{\frac{1}{3}}}$$

C. $q^{\frac{7}{3}}$

☒ D. $q^{\frac{8}{3}}$

5. If $\int_0^{10} h(x)dx = 7$ and $\int_4^{10} h(x)dx = -2$, what is the value of $\int_4^0 h(x)dx$?

A. 9

$$\int_0^{10} h(x)dx = \int_0^4 h(x)dx + \int_4^{10} h(x)dx = 7$$

B. 5

C. -7

$$\therefore \int_0^4 h(x)dx = 9 \Rightarrow \int_4^0 h(x)dx = -9$$

☒ D. -9

Section II
Part A
Attempt Questions 6 - 13

12 marks

Answer each question in the spaces provided. A blank page is provided at the end of each question to allow rewriting of a part.

Your responses should include relevant mathematical reasoning and/or calculations.

Question 6

Differentiate $y = 3x^4 - 8x + 7$

1

$$\frac{dy}{dx} = 12x^3 - 8$$

Extremely well done

Question 7

Differentiate $y = \frac{2}{x\sqrt{x}}$, leaving your answer in index form.

1

$$\frac{dy}{dx} = 2x^{-3/2}$$

$$= -3x^{-5/2}$$

$-\frac{1}{2}$ NOT SIMPLIFY

$-\frac{1}{2}$ NOT IN INDEX FORM

Poorly done most tried to use the quotient rule or did not realise $x\sqrt{x} = x^{3/2}$

Question 8

Evaluate $\lim_{x \rightarrow -7} \frac{x^2 + 2x - 35}{x + 7}$

1

$$= \lim_{x \rightarrow -7} \frac{(x-5)(x+7)}{x+7}$$

$$= -7 - 5$$

$$= -12$$

Very poorly done. Most did not understand what was required. Notation was poor.

Question 9

The quadratic function, $y = x^2 - 6x + c$, is to be drawn on a number plane.

- (a) Find the discriminant of the equation above.

1

$$\begin{aligned}\Delta &= b^2 - 4ac \\ &= (-6)^2 - 4(1)c \\ &= 36 - 4c\end{aligned}$$

$-\frac{1}{2}$ for $\Delta = \sqrt{b^2 - 4ac}$
Reasonably well done

- (b) Hence or otherwise, find the value of c for which the graph, of the equation above, will touch the x -axis.

1

Touch when $\Delta = 0$

$$\begin{aligned}0 &= 36 - 4c \\ c &= 9\end{aligned}$$

No half marks
"Touch" does not include
crossing the x -axis hence
 $c \leq 9$ was incorrect

Question 10

Find a primitive function of $f(x) = 4x^3 - 6x^2 - 11$.

1

$$F(x) = x^4 - 2x^3 - 11x + C$$

Generally well done
"+C" not necessary as only
"a" primitive was required

Question 11

The probability that the Dragon Football team will win this weekend is 0.75.

2

The probability that the Sharks Football team will win this weekend is 0.6.

What is the probability that ONLY one team will win this weekend?

$$\begin{aligned}P(D \sim S) + P(\sim D S) \\ &= (0.75)(0.4) + (0.25)(0.6) \quad \textcircled{1} \\ &= 0.45 \quad \textcircled{1}\end{aligned}$$

Very well done, Working
showed a lot of
understanding

OR

$$\begin{aligned}1 - P(\text{Both win}) - P(\text{Both lose}) \\ &= 1 - (0.75)(0.6) - P(0.25)(0.4) \\ &= 0.45\end{aligned}$$

Question 12

The polynomials $P(x)$ and $Q(x)$ have degrees m and n respectively where $m > n$.

Write down the degree of

(a) $P(x) - Q(x)$
 m

Very poorly done. $m-n$ was a common response.
 No half marks

1

(b) $P(x) \times Q(x)$
 $m+n$

Very poorly done. most answered mn
 No $\frac{1}{2}$ marks

1

Question 13

Find the equation of the normal to the curve $y = 2x^2 - 5x + 1$ at the point $(1, -2)$.

2

$\frac{dy}{dx} = 4x - 5$

$\left(\frac{1}{2}\right)$

Reasonably well done. Some put the derivative equal to zero
 Final arithmetic was poor

@ $x=1$ $m_T = \frac{dy}{dx} = -1$

$m_N = \frac{-1}{m_T} = +1$

$\left(\frac{1}{2}\right)$

$y - y_1 = m_N(x - x_1)$

$y + 2 = 1(x - 1)$

$y = x - 3$

$x - y - 3 = 0$

$\left(\frac{1}{2}\right)$

End of Part A

Section II
Part B
Attempt Questions 14 - 17

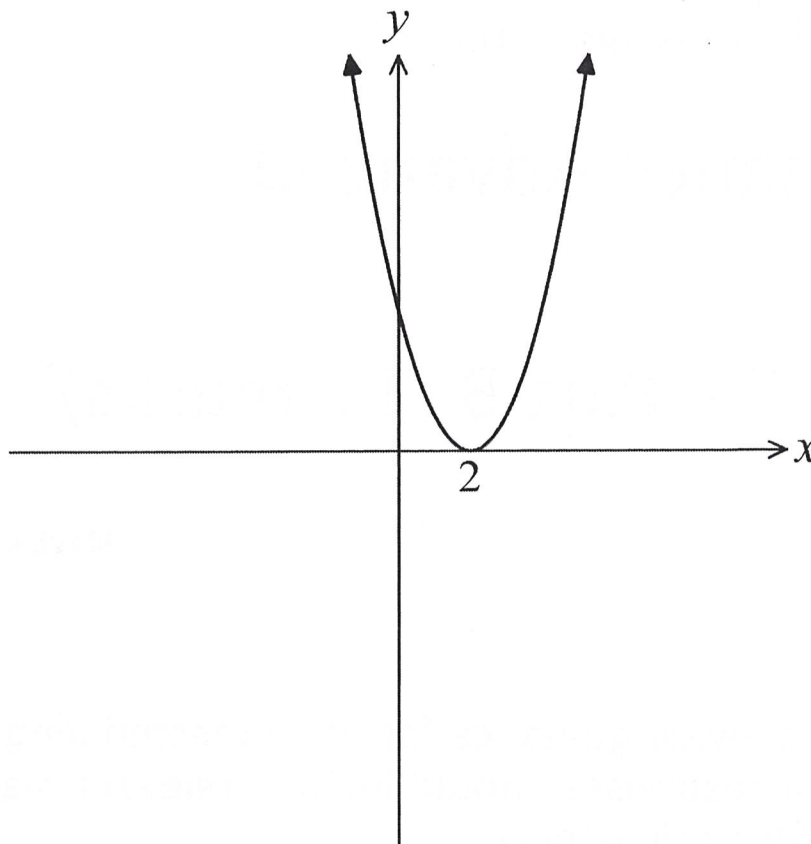
11 marks

Answer each question in the spaces provided. A blank page is provided at the end of each question to allow rewriting of a part.

Your responses should include relevant mathematical reasoning and/or calculations.

Question 14

Consider the function $y = f(x)$ whose gradient function, $y = f'(x)$, is given in the diagram below.



- (a) Comment on the sign of the gradient of $f(x)$ for all real x .

1

Since $f'(x)$ is all positive except at $x=2$.
 $\therefore$ The gradient of $f(x)$ is positive for all real x and the gradient is zero at $x=2$.

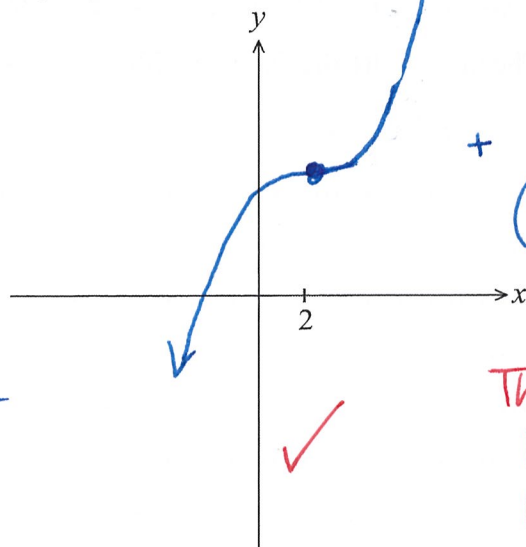
Question 14 continues on page 11

* This question is NOT asking students to describe the y' graph.
 $\therefore$ no asking increasing/decreasing function

Question 14 (continued)

(b) Sketch a possible graph of $y = f(x)$.

Since $x=2$ is a double root for $y = f'(x)$
 $\therefore x=2$ is a horizontal (stationary) point of inflexion.



(1/2)

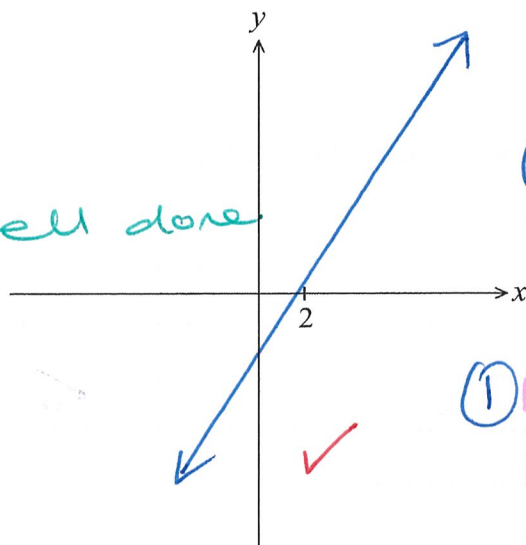
cubic w/ $x=2$ as pt. of inf.

(1/2)

clearly show a hort. pt. of inf

The y-intercept can be anywhere as long as $x=2$ is the hort. pt. of inf.

(c) Sketch $y = f''(x)$.



(1/2)

linear with x-inter = 2

(1)

linear with x-inter = 2 w/ positive gradient

* Generally well done

Question 15

Find the values of a , b and c for which $5x^2 + 7x - 8 = ax(x+1) + bx^2 + c(x-2)$ for all real x .

2

$$ax^2 + ax + bx^2 + cx - 2c$$

$$(a+b)x^2 + (a+c)x - 2c$$

$$\equiv 5x^2 + 7x - 8$$

(1/2)

$$-2c = -8 \quad \therefore c = 4$$

$$a + 4 = 7 \quad \therefore a = 3$$

$$a + b = 5 \quad \therefore b = 2$$

/2

* Students must expand the expression and equate the coefficients carefully.

4

Question 16

A school decide to do a fundraiser for a charity, 70 tickets were sold and 2 prizes were offered. Two tickets were drawn without replacement to determine the prize winners.

Joe bought some of the tickets. The probability that he won both prizes was $\frac{1}{161}$.

- (a) Find the number of tickets bought by Joe.

2

let the no. of tickets bought by Joe be x .
winning both prizes = $\frac{x}{70} \times \frac{(x-1)}{69} = \frac{1}{161}$ ✓

Only ① if ignored

$$x(x-1) = \frac{1}{161} \times 70 \times 69$$

$$x^2 - x - 30 = 0$$

$$(x-6)(x+5) = 0 \quad \frac{1}{2}$$

but the rest of working are correct

$$\therefore x = 6, x = -5$$

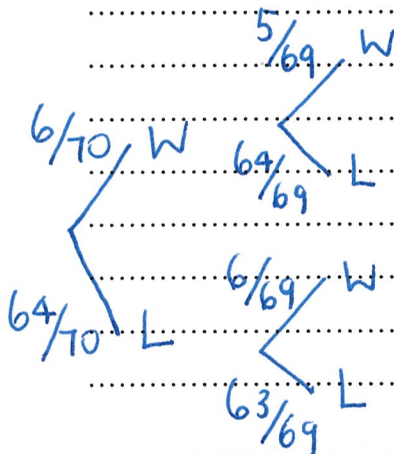
$$x > 0 \therefore x = 6 \quad \frac{1}{2}$$

Joe bought 6 tickets

* Some students had difficulties to form and solve the eqn.

- (b) What is the probability of Joe winning at least one prize?

2



At least one prize

$$= 1 - LL$$

$$= 1 - \frac{64}{70} \times \frac{63}{69} \quad \checkmark$$

$$= \frac{19}{115} \quad \checkmark$$

Aw 1 for any correct combination of WW, WL, LW

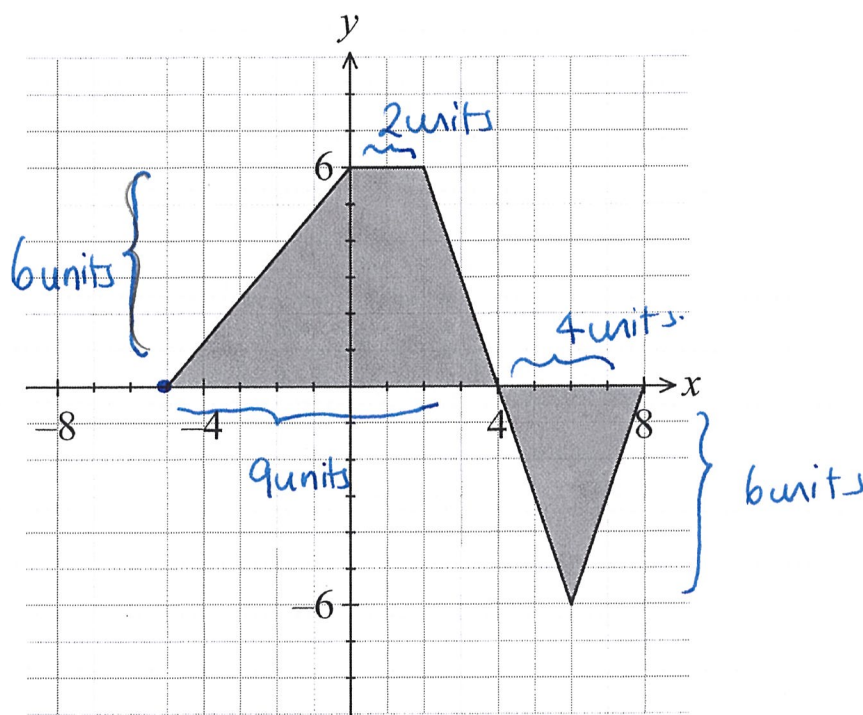
ISE $\rightarrow$ Ignore Subsequent Error

* A probability tree may help using the answer from (a).

4

Question 17

The diagram below shows $y = f(x)$.



Find the value of $\int_{-5}^8 f(x) dx$. $\Rightarrow$ Did not ask for area.

2

$$\begin{aligned}
 &= \frac{1}{2} \times 6 \times (9+2) - \left(\frac{1}{2} \times 6 \times 4 \right) \\
 &= 33 - 12 \\
 &= 21
 \end{aligned}$$

- * The limits are 8 to -5, some students got confused with the label of -4.
- * $\frac{1}{2}$ for 45 as answer.

2

End of Part B

Section II
Part C
Attempt Questions 18 - 20

11 marks

Answer each question in the spaces provided. A blank page is provided at the end of each question to allow rewriting of a part.

Your responses should include relevant mathematical reasoning and/or calculations.

Your

Question 18

Estimate $\int_2^6 \frac{2}{x(x-1)} dx$ by using the trapezoidal rule with 4 sub-intervals.

3

Leave your answer in exact form.

$$A = \frac{w}{2} [E + 2M]$$

where $w = \frac{b-a}{n} = \frac{6-2}{4} = 1$

$$= \frac{1}{2} \left[1 + \frac{1}{15} + 2 \left(\frac{1}{3} + \frac{1}{6} + \frac{1}{10} \right) \right]$$

x	2	3	4	5	6
y	1	$\frac{1}{3}$	$\frac{1}{6}$	$\frac{1}{10}$	$\frac{1}{15}$

$$= \frac{17}{15}$$

Only about a third of the cohort knew the correct formula for the Trap Rule. Even then, many did not find the width of the intervals correctly.

Question 19

Find, from first principles,

3

$$f'(x) = \lim_{h \rightarrow 0} \frac{\frac{2}{(x+h-3)^2} - \frac{2}{(x-3)^2}}{h}$$

$$= \lim_{h \rightarrow 0} \frac{2(x-3)^2 - 2(x+h-3)^2}{h(x+h-3)^2(x-3)^2}$$

$$= \lim_{h \rightarrow 0} \frac{2x^2 - 12x + 18 - 2x^2 - 4xh + 12x + 12h - 18 - 2h^2}{h(x-3)^2(x+h-3)^2}$$

$$= \lim_{h \rightarrow 0} \frac{-4x + 12 - 2h}{(x-3)^2(x+h-3)^2} = \frac{-4x + 12}{(x-3)^4}$$

$$f'(x) = \frac{-4(x-3)}{(x-3)^4}$$

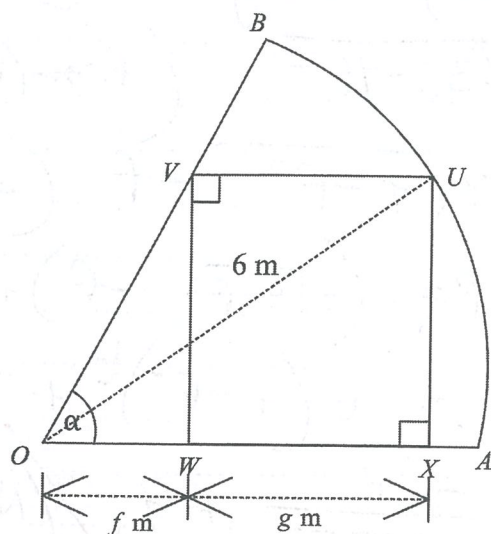
$$f'(x) = \frac{-4}{(x-3)^3}$$

16

No-one got more than half marks for this question. Students did not know to subtract the fractions in order to simplify the numerators.

Question 20

The diagram below shows a rectangle $UVWX$ drawn in a sector OAB of a circle, radius 6 m . Let Angle $AOB = \alpha$, $\tan \alpha = 3$, $OW = f\text{ m}$ and $WX = g\text{ m}$.



- (a) Show that $UX = 3f$.

1

$$\tan \alpha = \frac{VW}{OW}$$

$$\Rightarrow 3 = \frac{VW}{f}$$

$$VW = 3f$$

But $VW = UX$ (opp sides of rect)

$$\therefore UX = 3f$$

Most students got the mark for this question.

- (b) Hence show $(f + g)^2 = 36 - 9f^2$.

1

$$UX^2 = OU^2 - OX^2 \text{ (Pyth. Th)}$$

$$= 6^2 - (f + g)^2$$

$$\text{But } UX = 3f$$

$$\Rightarrow 9f^2 = 6^2 - (f + g)^2$$

$$\Rightarrow (f + g)^2 = 36 - 9f^2$$

In order to get this mark students needed to quote Pythagoras' Theorem or else use the name(s) of the side(s) in the triangle.

Question 20 continues on page 18

For this question $\checkmark = \frac{1}{2}$

- (c) Find the maximum perimeter of the rectangle in exact form.

Perimeter of rect, $P = 2g + 6f$ (1)

Now, $f + g = \sqrt{36 - 9f^2}$ (from (b), also $f, g > 0$)

$\Rightarrow g = 3\sqrt{4 - f^2} - f$ (2) $\checkmark$

Sub (2) in (1) $\Rightarrow P = 2(3\sqrt{4 - f^2} - f) + 6f$

$P = 6(4 - f^2)^{\frac{1}{2}} + 4f$ $\checkmark$

Then $P' = \frac{-6f}{\sqrt{4 - f^2}} + 4$ $\checkmark$ (Note $f \neq \pm 2$)

For st pts $P' = 0$

$\Rightarrow \frac{-6f + 4\sqrt{4 - f^2}}{\sqrt{4 - f^2}} = 0$

$4\sqrt{4 - f^2} = 6f$

$\sqrt{4 - f^2} = \frac{3}{2}f$

$4 - f^2 = \frac{9}{4}f^2$

$\frac{13}{4}f^2 = 4$ $\checkmark$

$f^2 = \frac{16}{13} \Rightarrow f = \frac{4}{\sqrt{13}}$ ($f > 0$)
(≈ 1.109) $\checkmark$

Type	f	1	$\frac{4}{\sqrt{13}}$	1.5
	P'	.535	0	-2.8

$\Rightarrow$ Max at $f = \frac{4}{\sqrt{13}}$ $\checkmark$

Then, Max $P = 6\sqrt{4 - \frac{16}{13}} + \frac{16}{\sqrt{13}}$

$= 6\sqrt{\frac{36}{13}} + \frac{16}{\sqrt{13}} = \frac{52}{\sqrt{13}} = \frac{\sqrt{13} \times 52}{13}$

max $P = 4\sqrt{13}$ m $\checkmark$

Students did not get any marks for this question as they were unable to express g in terms of f

3