Barker College

Student Number

Year 12, 2022 HSC Mathematics Advanced Assessment Task 1 (40 Marks)

Sequences and Series
Series and Finance
Conditional Probability
Discrete Probability Distributions

Staff Involved: 8:30 AM

PCB AXD GPF ALY*

ESP RAS JZT RJW WEDNESDAY 9 FEBRUARY

155 copies

General Instructions:

- Working time – 55 minutes
- Write using black pen
- NESA approved calculators may be used
- A reference sheet is provided separately
- Show relevant mathematical reasoning and/or calculations
- Marks may not be awarded for careless or poorly arranged working

Section I – 19 marks

Sequences, Series and Finance

- Attempt Questions 1 - 7

Section II – 21 marks

Probability

- Attempt Questions 8 - 13

Section I: Sequences, Series and Finance (19 marks)

1. $7, x, 175$ are the first three terms of an **arithmetic** sequence.
Find x.

Trial and error will not be awarded any marks. We require an algebraic solution. **1 mark**

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2. For what values of x does the series $1 - \frac{x}{2} + \frac{x^2}{4} - \frac{x^3}{8} + \cdots$ have a limiting sum? **2 marks**

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3. What is meant by the phrase "limiting sum" or "infinite sum" of a series? **1 mark**

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4. Can there be a geometric series with a limiting sum of $\frac{5}{8}$ and a first term of 2? Fully justify your answer. **3 marks**

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5. The balance of a reducible interest loan changes each time period (eg month).

Explain why the amount of interest you pay each time period reduces over time, if the interest rate remains the same. **1 mark**

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6. A series is defined by $S_n = 8n + 8n^2$ where S_n is the sum of the first n terms of the series.

i) Explain why $T_1 = 16$, where T_n is the n^{th} term in the corresponding sequence. **1 mark**

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ii) Find S_{11} and S_{12} **1 mark**

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iii) Hence find T_{12} **1 mark**

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iv) Find a simplified expression for T_n in terms of n. **2 marks**

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7. A loan of $10 000 is to be repaid by equal annual instalments with the first repayment made at the end of the first year. The interest rate is 10% p.a., calculated yearly. The annual repayment is $M.

i) Show that the amount (A_2) owing at the end of the second year, after the repayment is made, is:

$A_2 = \$12100 - 2.1M$ **2 marks**

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ii) If the loan (including interest charges) is exactly repaid at the end of n years, show that: **4 marks**

$$M = \frac{1000}{1 - \frac{1}{(1.1)^n}}$$

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End of Section I

Student number

Section II: Probability (21 marks)

8. The table below shows the number of students who play tennis and soccer.

	Tennis	Not Tennis	**Totals**
Soccer	12	23	**35**
Not Soccer	15	8	**23**
Totals	**27**	**31**	**58**

Find the probability that a student chosen at random plays tennis, given that they **do not** play soccer.

1 mark

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9. For events A and B, $P(A) = 0.2, P(B) = 0.4$, and $P(B|A) = 0.1$.

Find $P(A|B)$. **2 marks**

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10. The probability that my soccer team will win if it is raining is 0.7. The probability that we win if it is not raining is 0.4.

The chance of rain for our next game is 0.3. What is the probability that we win? **3 marks**

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11. A complete discrete probability distribution table is shown.

x	-5	0	2	3
$P(X = x)$	0.5	0.3	0.15	x

i) Find x as a decimal. **1 mark**

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ii) Calculate the expected value of the distribution. **1 mark**

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iii) Calculate the standard deviation of the distribution, correct to 3 decimal places. **3 marks**

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iv) This distribution represents the overall financial return in dollars on a game of chance. Positive x values represent an overall financial gain while negative values represent a loss.

Would you expect to win money in the long run? Why/why not? **1 mark**

In your answer **refer to value(s)** from above.

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12. A chance experiment was conducted where 3 coins were tossed and the number of tails was recorded. This was repeated 50 times and the results are shown below.

Number of Tails	**Frequency**
0	4
1	13
2	25
3	8

i) Complete the probability distribution table for this experiment. **2 marks**

x	0	1	2	3
$P(X = x)$				

ii) Calculate $E(X)$ for this distribution. **1 mark**

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iii) Calculate $\mathrm{Var}(X)$. **2 marks**

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iv) All three coins are identical and are biased.

Describe one fact from the data that supports this. **1 mark**

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13. A discrete random variable has the probability distribution as shown in the table where n is a finite positive integer. The ... in the table represent the pattern continuing.

x	0	1	2	...	$n-1$
$P(X = x)$	$\frac{1}{n}$	$\frac{1}{n}$	$\frac{1}{n}$	...	$\frac{1}{n}$

Show that **3 marks**

$$E(X) = \frac{n-1}{2}$$

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End of Paper

SECTION I

1.
$$T_2 - T_1 = T_3 - T_2$$
$$x - 7 = 175 - x$$
$$2x = 182$$
$$x = 91 \checkmark$$

2.
$$\left|-\frac{x}{2}\right| < 1$$
$$\frac{x}{2} < 1 \qquad \frac{x}{2} > -1$$
$$x < 2 \qquad x > -2$$
$$\therefore -2 < x < 2 \checkmark$$

3. A SERIES WHERE THE SUM OF AN INFINITE NO. OF TERMS CONVERGES TO A FINITE SUM.

4. $a = 2$, $r = ?$, $S_\infty = 5/8$

$$\frac{5}{8} = \frac{2}{1-r}$$
$$5(1-r) = 16$$
$$5 - 5r = 16$$
$$-5r = 11$$
$$r = -2.2$$

NO. DOES NOT MEET CRITERIA $|r| < 1$.

5. INTEREST IS CHARGED AS A % OF AMOUNT OWING. AS THE AMOUNT OWING DECREASES, SO WILL THE AMOUNT OF INTEREST.

6i. $S_1 = T_1$ (THE SUM OF THE FIRST 1 TERMS!)
$$S_1 = (8\times 1) + (8\times 1^2)$$
$$T_1 = 16 \checkmark$$

ii.
$$S_{11} = (8\times 11) + (8\times 11^2)$$
$$= 1056$$
$$S_{12} = (8\times 12) + (8\times 12^2)$$
$$= 1248 \checkmark$$

iii.
$$T_{12} = S_{12} - S_{11}$$
$$= 1248 - 1056$$
$$= 192 \checkmark$$

iv.
$$T_n = S_n - S_{n-1}$$
$$= (8n + 8n^2) - [8(n-1) + 8(n-1)^2]$$
$$= 8n + 8n^2 - (8n - 8 + 8n^2 - 16n + 8)$$
$$T_n = 16n \checkmark$$

7i.
$$A_1 = 10\,000 \times 1.1 - m$$
$$A_2 = A_1 \times 1.1 - m$$
$$= (10\,000 \times 1.1 - m) \times 1.1 - m$$
$$= 10\,000 \times 1.1^2 - m(1.1 + 1)$$
$$= \$12\,100 - 2.1m.$$

ii.
$$A_3 = A_2 \times 1.1 - m$$
$$= [10\,000 \times 1.1^2 - m(1.1+1)] \times 1.1 - m$$
$$= 10\,000 \times 1.1^3 - m(1.1^2 + 1.1 + 1)$$

CONTINUED . . . .

$$A_n = 0 = 10\,000 \times 1.1^n - m(1.1^{n-1} + \ldots + 1.1 + 1)$$

$$m(1.1^{n-1} + \ldots + 1) = 10\,000 \times 1.1^n$$
$$m = \frac{10\,000 \times 1.1^n}{(1.1^{n-1} + \ldots + 1)}$$

$a = 1$, $r = 1.1$, $n = n$
$$S_n = \frac{1(1.1^n - 1)}{1.1 - 1} = \frac{1.1^n - 1}{0.1}$$

$$\therefore m = \frac{10\,000 \times 1.1^n}{\left(\frac{1.1^n - 1}{0.1}\right)}$$
$$= \frac{1000 \times 1.1^n}{1.1^n - 1} \quad \div \frac{1.1^n}{1.1^n}$$
$$\therefore m = \frac{1000}{1 - \frac{1}{1.1^n}}$$

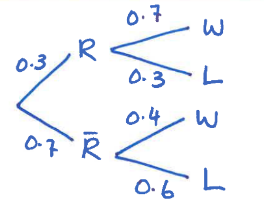
SECTION II

8. $\frac{15}{23} \checkmark$

9.
$$P(B|A) = \frac{P(A\cap B)}{P(A)}$$
$$0.1 = \frac{P(A\cap B)}{0.2}$$
$$\therefore 0.02 = P(A\cap B)$$

$$P(A|B) = \frac{P(A\cap B)}{P(B)}$$
$$= \frac{0.02}{0.4}$$
$$= 0.05 \checkmark$$

10.



$$P(RW) = 0.3 \times 0.7 = 0.21$$
$$P(\bar{R}W) = 0.7 \times 0.4 = 0.28$$
$$\therefore P(W) = 0.21 + 0.28$$
$$= 0.49 \checkmark$$

11.

x	-5	0	2	3	SUM
$p(x)$	0.5	0.3	0.15	0.05	
$xp(x)$	-2.5	0	0.3	0.15	= -2.05
$x^2p(x)$	12.5	0	0.6	0.45	= 13.55

i. $1 - (0.5 + 0.3 + 0.15) = 0.05 \checkmark$

ii. $E(x) = -2.05 \checkmark$ SEE ABOVE

iii.
$$Var(x) = E(x^2) - \mu^2$$
$$= 13.55 - (-2.05)^2$$
$$= 9.3475$$
$$\sigma_n = \sqrt{Var(x)}$$
$$= 3.057 \checkmark$$

iv. NO. EXPECTED VALUE IS NEGATIVE $\therefore$ LOSS IN THE LONG RUN.

12i.

x	0	1	2	3	sum
$P(x)$	0.08	0.26	0.5	0.16	
$xP(x)$	0	0.26	1	0.48	= 1.74
$x^2P(x)$	0	0.26	2	1.44	= 3.7

ii. $E(x) = 1.74$ ✓ SEE ABOVE

iii. $Var(x) = 3.7 - (1.74)^2$

$= 0.6724$ ✓

iv. · $E(x)$ IS GREATER THAN 1.5

· THE CHANCE OF 2 OR 3 TAILS IS $\frac{33}{50}$.

· 2 TAILS OCCUR 50% OF TIME

13.

x	0	1	2	...	$n-1$
$P(x)$	$\frac{1}{n}$	$\frac{1}{n}$	$\frac{1}{n}$	...	$\frac{1}{n}$
$xP(x)$	0	$\frac{1}{n}$	$\frac{2}{n}$	...	$\frac{n-1}{n}$

$E(x) = 0 + \frac{1}{n} + \frac{2}{n} + \dots \frac{n-1}{n}$

$= \frac{1+2+3+\dots n-1}{n}$

· NUMERATOR IS AN AP.

$a = 1$
$d = 1$
$n = (n-1)$

$S_{n-1} = \frac{n-1}{2}\left[2\times1 + ((n-1)-1)\times 1\right]$

$= \frac{n-1}{2}(2 + n - 2)$

$= \frac{n(n-1)}{2}$

CONTINUED...

$\therefore E(x) = \frac{\frac{n(n-1)}{2}}{n}$

$= \frac{n-1}{2}$