Killara HIGH SCHOOL
MATHEMATICS DEPARTMENT

Centre Number

Student Number

2022 **YEAR 12**

Mathematics Advanced

Task 2

Date: 06/03/2022

Q	Marks
MC	/4
5	/8
6	/2
7	/6
8	/5
9	/3
10	/6
11	/8
12	/4
13	/4
Total	/50

General Instructions:

- Reading time – 5 minutes
- Working time – 55 minutes
- Write using blue or black pen
- NESA approved calculators may be used
- Show relevant mathematical reasoning and/or calculations
- No white-out may be used

Total Marks: 50

Section I - 4 marks

- Allow about 5 minutes for this section

Section II - 46 marks

- Allow about 50 minutes for this section

This question paper must not be removed from the examination room.

This assessment task constitutes 25% of the course.

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Section I

4 marks
Allow about 5 minutes for this section

Use the multiple-choice sheet for Question 1–5

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1 Find the derivative of e^{2x}.

(A) e^2

(B) $\frac{e^{2x}}{2}$

(C) e^{2x}

(D) $2e^{2x}$

2 Given that $g(a) = 3$, and $g'(a) = 6$. Evaluate $\frac{d}{dx}[\big(g(x)\big)^3]$ when $x = a$.

(A) 27

(B) 54

(C) 162

(D) 324

3 If A and B are events such that $P(A|B) = P(B|A)$, then

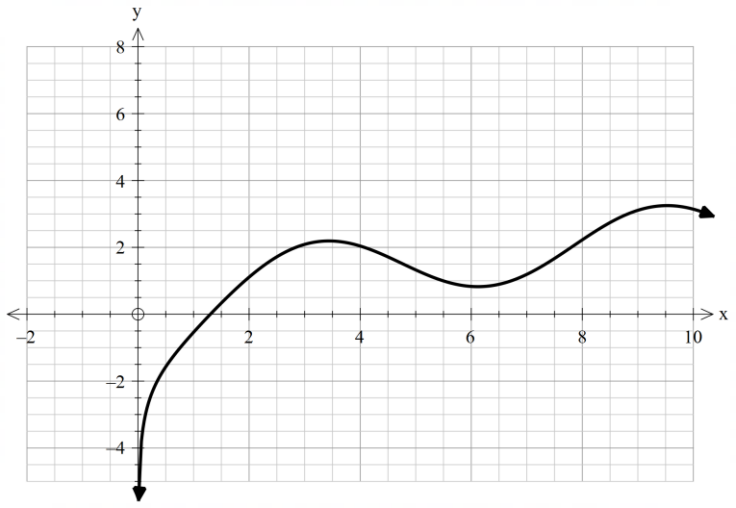
(A) $n(A) < n(B)$

(B) $P(A) = P(B)$

(C) $A \cap B = \phi$

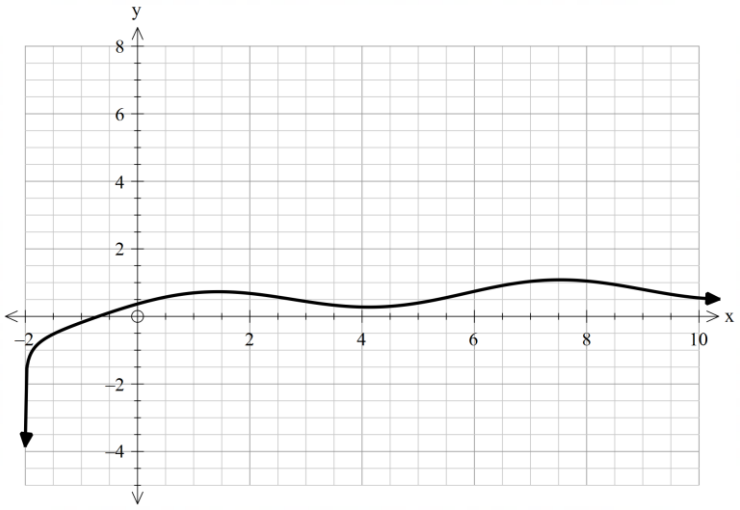
(D) $A = B$

The sketch below shows the function $g(x)$.

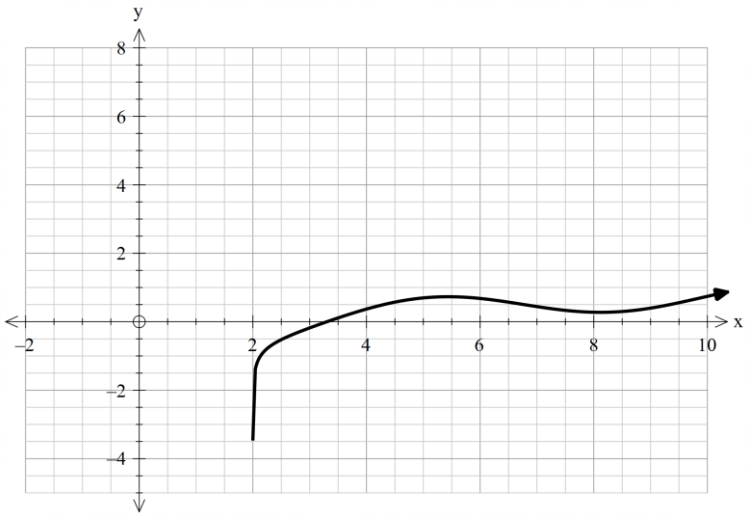


Which of the following diagrams represents $3g(x-2)$?

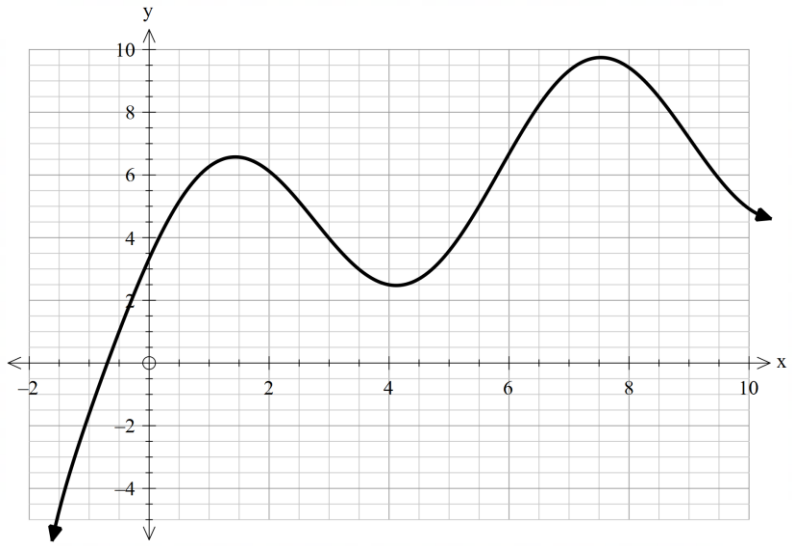
4 (A)



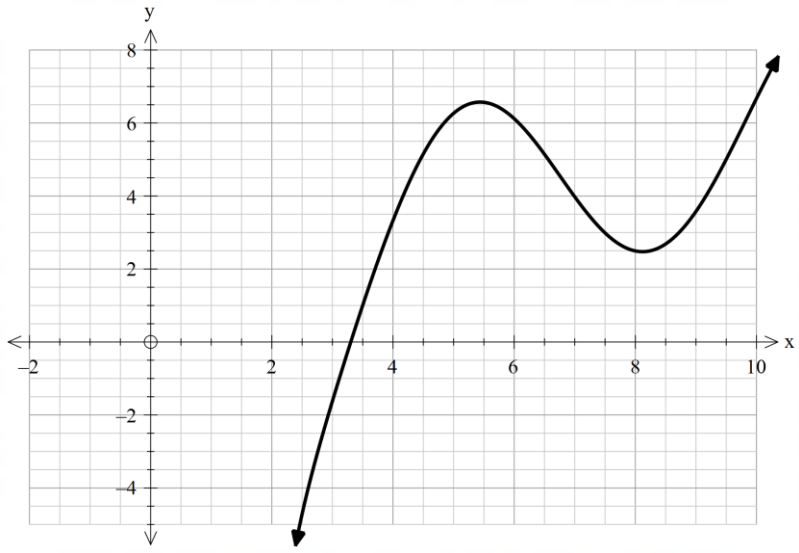
(B)



(C)



(D)



End of Section I

Examination continues on next page

Section II

In Questions 6 – 17, your response should include relevant mathematical reasoning and/or calculations.

Question 5 (8 marks)

(a) Differentiate $y = \cos(4x)$. **1**

(b) Differentiate $f(x) = (\tan x - 1)^{13}$. **2**

(c) Differentiate **2**

$$y = \ln\left(\frac{x^2 + 2x + 1}{x - 3}\right)$$

(d) The point P(π,0) lies on the curve $y = e^x \sin x$. Find the equation to the tangent to the curve at P. **3**

Question 6 (2 marks)

The graph of $f(x) = -x^2 + x$ undergoes the following transformations in the order shown below: **2**

Translate to the left by $\frac{1}{2}$

Downward translation by $\frac{1}{4}$

Reflection in the x-axis

Find the equation of the resulting graph.

Question 7 (6 marks)

At a fundraiser event, Lauren runs a dice game in which a player rolls two standard dice, and the sum of the uppermost faces is calculated.

(a) Show that the probability of the sum being less than five is $\frac{1}{6}$. **1**

(b) Lauren sets the prizes for the game as follows: **1**
If the sum is less than five, the player wins \$20. If the sum is greater than eight, the player wins \$10. Otherwise, the player receives no money.

Complete the probability distribution table shown below.

Amount won (X)	20	10	0
$P(X = x)$			

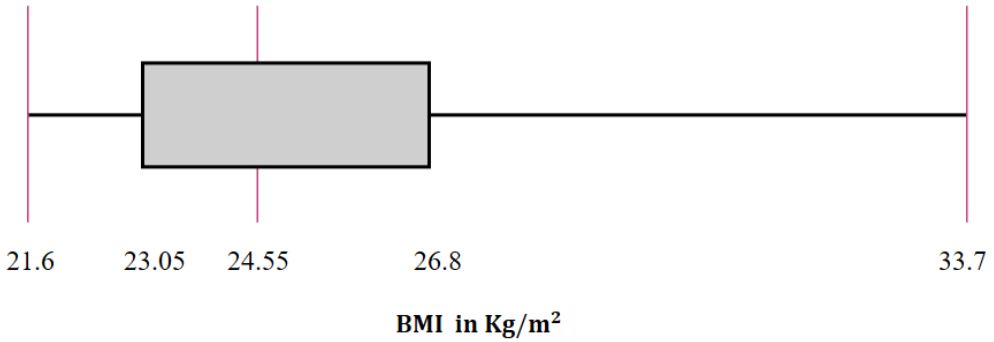
(c) Calculate $E(X)$, the expected amount won by the player. **1**

(d) Lauren charges \$5 for each roll of the two dice. In the long run, what is Lauren's expected profit/loss from this game **1**

(e) If Lauren wants to make a long-term profit per game of 20% of what she charges. By forming an equation, determine what Lauren should charge a player for each roll of the two dice? **2**

Question 8 (5 marks)

Body mass index (BMI), in kilograms per square metre, was recorded for a sample of 32 men and displayed as a box-and whisker plot as shown below:



(a) Describe the shape of the distribution. **1**

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(b) Determine whether the score of 33.7 is an outlier. **2**

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(c) People with a BMI of 25 or over are considered overweight. **2**
Approximately, how many of these men would be considered as overweight?
Justify your answer with appropriate working.

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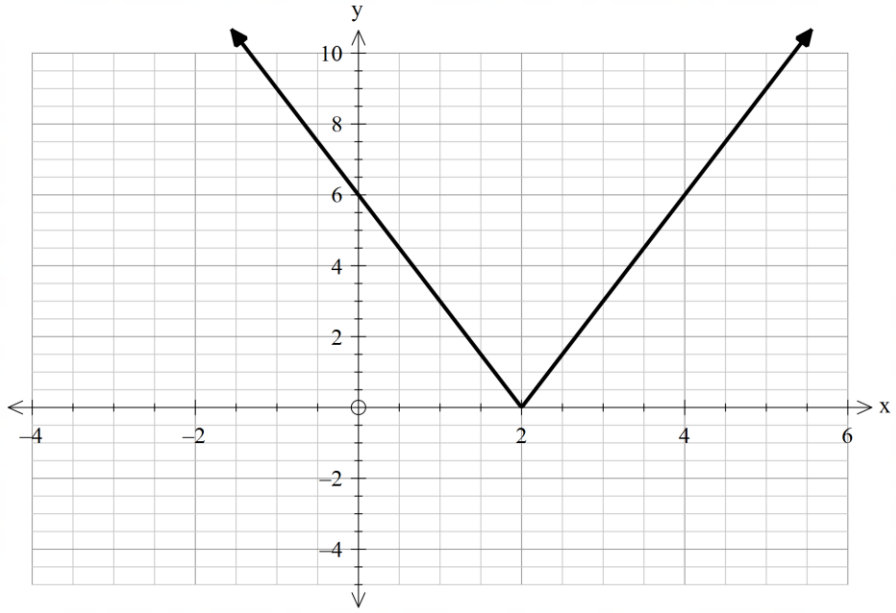
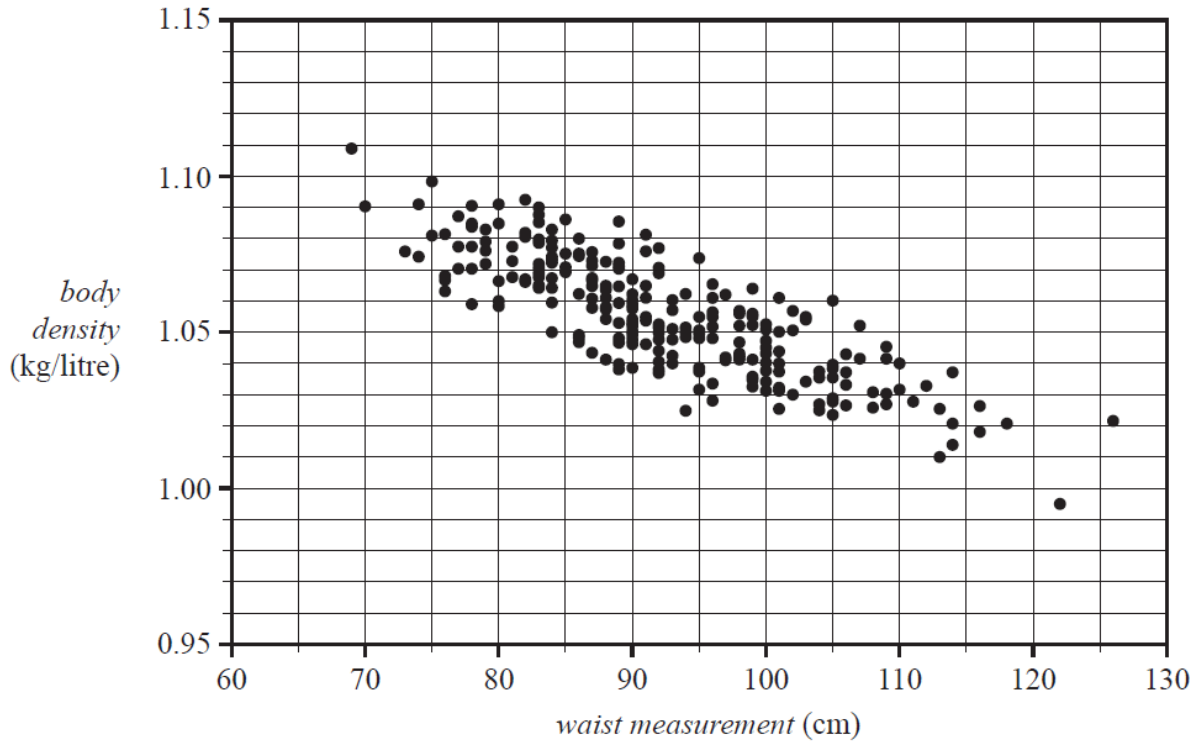
Question 9 (3 marks)

Below is a sketch of $y = |3(x - 2)|$. Using the graph, or otherwise, find the values of m for which $|3(x - 2)| = mx + 2$ has exactly one solution. **3**



Question 10 (6 marks)

A few simple body measurements that may explain the BMI were studied. The scatterplot below shows Body density, in kilograms per litre, plotted against waist measurement, in centimetres, for 250 men.



Trendline: Body Density $= 1.195 - 0.001512 \times$ Waist measurement

$R^2 = 0.6783$

When a least-squares line is fitted to the scatterplot using a software, the equation of the line and R^2 are as shown in the diagram. However, the trendline has been removed from the scatterplot.

(a) Draw the graph of this least-squares line on the scatterplot above. **2**

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(b) Calculate Pearson's correlation coefficient and then evaluate the appropriateness of using this coefficient to determine whether waist measurement can be an alternate reliable measure to explain body density. **4**

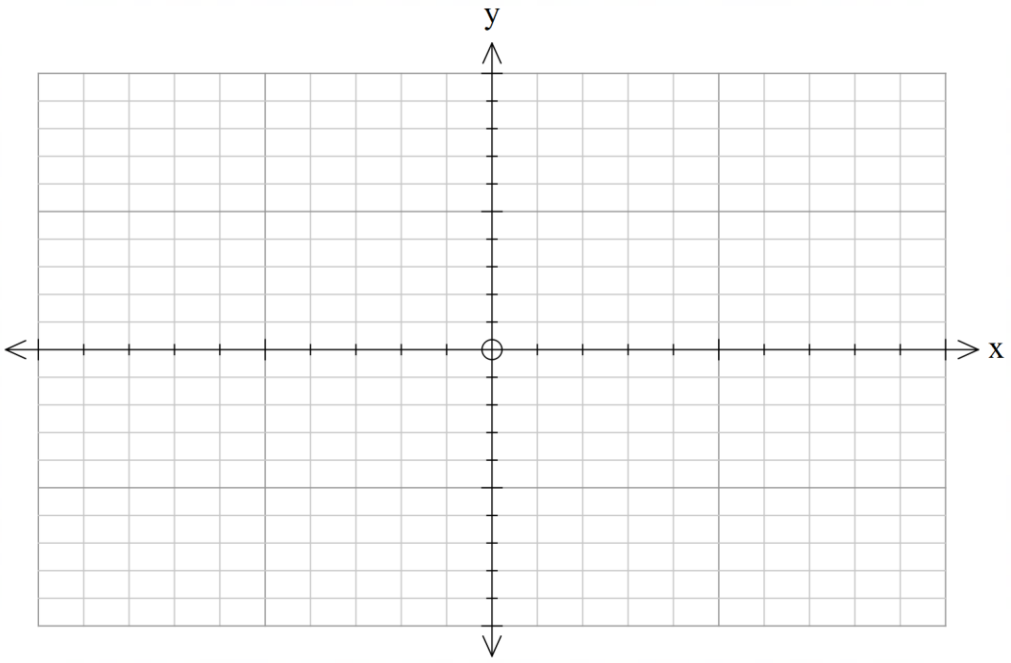
Question 11 (10 marks)

(a) Differentiate the function $y = xe^x$. **1**

(b) Discuss the nature of the function for extreme values of x, i.e. $x \to \pm\infty$. **2**

(c) Find the stationary point on $y = xe^x$, and explain why this point will be a minimum turning point. **1**

(d) Hence, sketch the graph of $y = xe^x$. **3**



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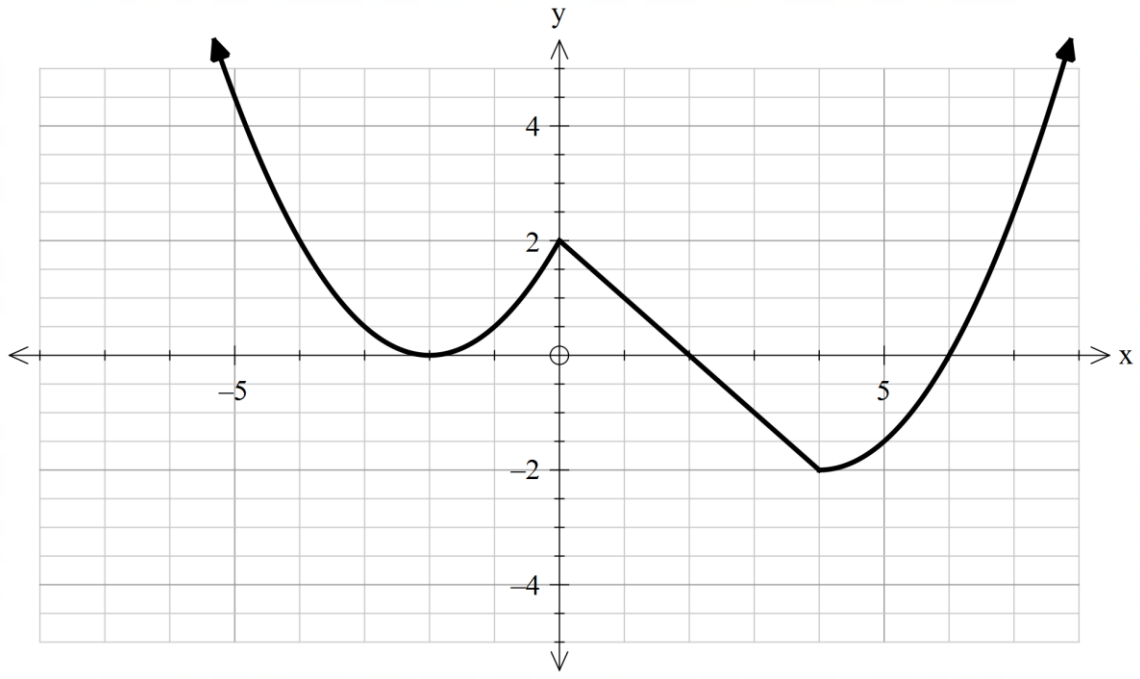
Question 12 (4 marks)

Given the graph of $f(x)$ as shown below sketch $f\big(2(x-1)\big)-1$



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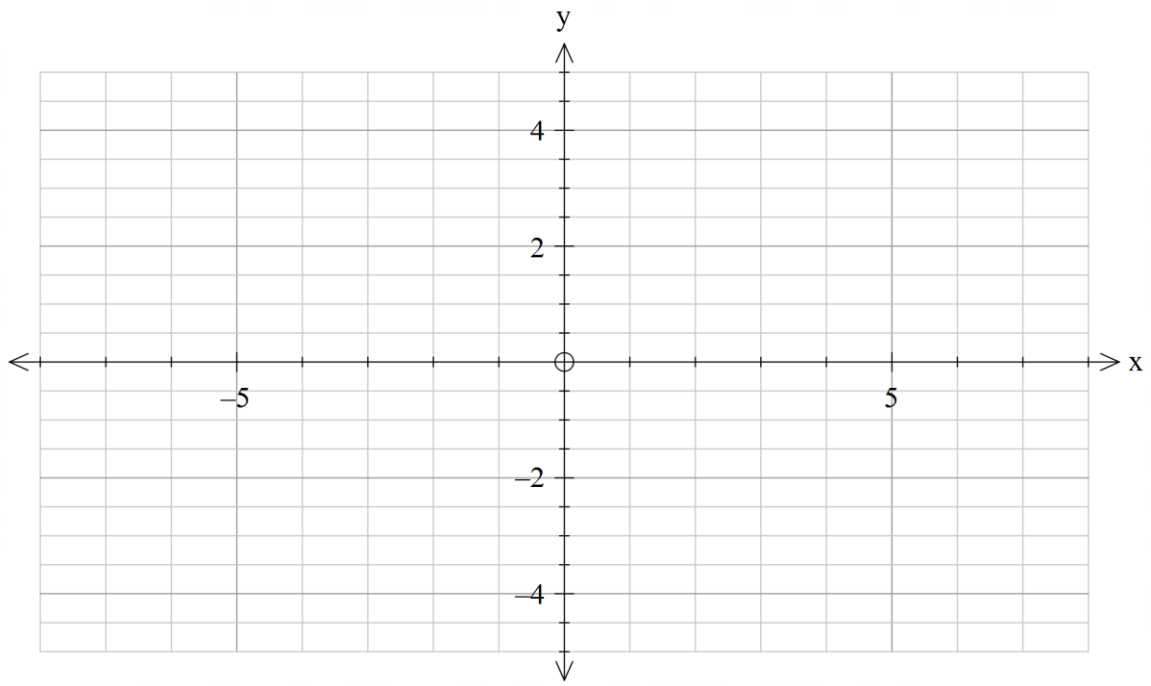

Question 13 (4 marks)

A laboratory blood test is 99% effective in detecting a certain disease when it is in fact, present. However, the test also returns a false positive result for 0.5% of the healthy people tested (That is, if a healthy person is tested, then the probability that the test will imply the person has disease is 0.005). **4**

If 0.1% of the population actually has the disease, what is the probability that a person has the disease given that their test result is positive?

End of Examination

Extra writing space

If you use this space, clearly indicate which question you are answering.

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Section I – Multiple Choice Answer Sheet

Use this multiple-choice answer sheet for questions 1 – 5. Detach this sheet.

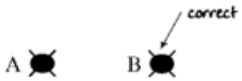
Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample: 2 + 4 = (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

If you change your A ● B ✕ C ○ D ○ mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word *correct* and drawing an arrow as follows.



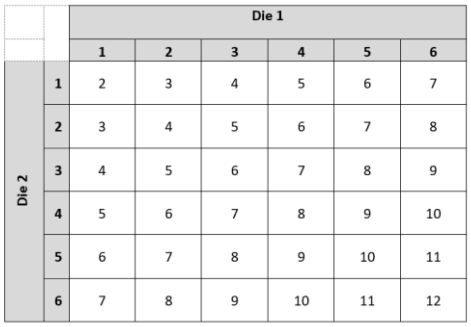
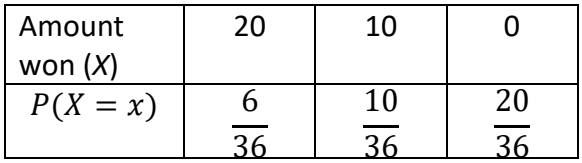
1.	A ○	B ○	C ○	D ○
2.	A ○	B ○	C ○	D ○
3.	A ○	B ○	C ○	D ○
4.	A ○	B ○	C ○	D ○

2022 Advanced Mathematics Task 2 Marking Scheme

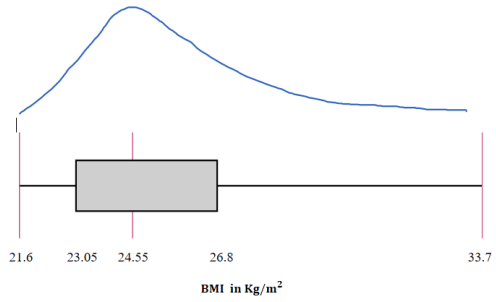
1.	D		
2.	$\frac{d}{dx}\left(g(x)\right)^3) = 3g'(x)\left(g(x)\right)^2$ $= 3 * 6 * 3^2$ $= 162$ C		
3.	$P(A\|B) = P(B\|A)$ $\frac{P\,(A \cap B)}{P(B)} = \frac{P(B \cap A)}{P(A)}$ Hence, P(A)=P(B) **B**		
4.	D		

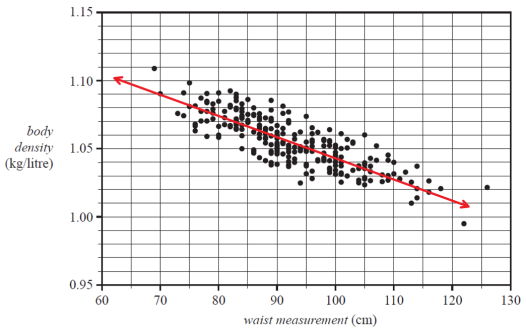
5.(a)	$y = \cos(4x)$ $\frac{dy}{dy} = -4\sin\,(4x)$	1 mark correct solution	
(b)	$f(x) = (\tan x - 1)^{13}$ $f'(x) = 13\sec^2 x\,(\tan x - 1)^{12}$	1 mark: attempts the chain rule, but makes 1 error 2 marks: correct solution	
(c)	$y = \ln\left(\frac{x^2 + 2x + 1}{x - 3}\right)$ $= \ln\left(\frac{(x+1)^2}{x-3}\right)$ $= 2\ln(x+1) - \ln(x-3)$ $\frac{dy}{dx} = \frac{2}{x+1} - \frac{1}{x-3}$	1 mark: Uses log laws/attempts to use the quotient rule 2marks: Correct derivative. Does not require simplification/bringing into a single fraction	
(d)	$y = e^x \sin x$ $\frac{dy}{dx} = e^x \sin x + e^x \cos x$ Sub point $P(\pi, 0)$ $\frac{dy}{dx} = 0 + e^\pi$ $= e^\pi$ Use the point gradient forgery $y - 0 = e^\pi(x - \pi)$ $y = e^\pi - \pi e^\pi$	1 mark: correct derivation 1 marks: finding the gradient from the derivative 1 mark: Using the point gradient form (or equivalent) to find the equation of the tangent	
6.	$f(x) = -x^2 + x$ The transformations on $f(x)$ are: $f(x) \to f\left(x + \frac{1}{2}\right) \to f\left(x + \frac{1}{2}\right) - \frac{1}{4}$ $\to -f\left(x + \frac{1}{2}\right) + \frac{1}{4}$ Thus, the transformed graph will be, $-f\left(x + \frac{1}{2}\right) + \frac{1}{4}$ $-f\left(x + \frac{1}{2}\right) + \frac{1}{4}$	2 marks: Correct solution with all working 1 mark: At least one of the transformation is applied correctly/	

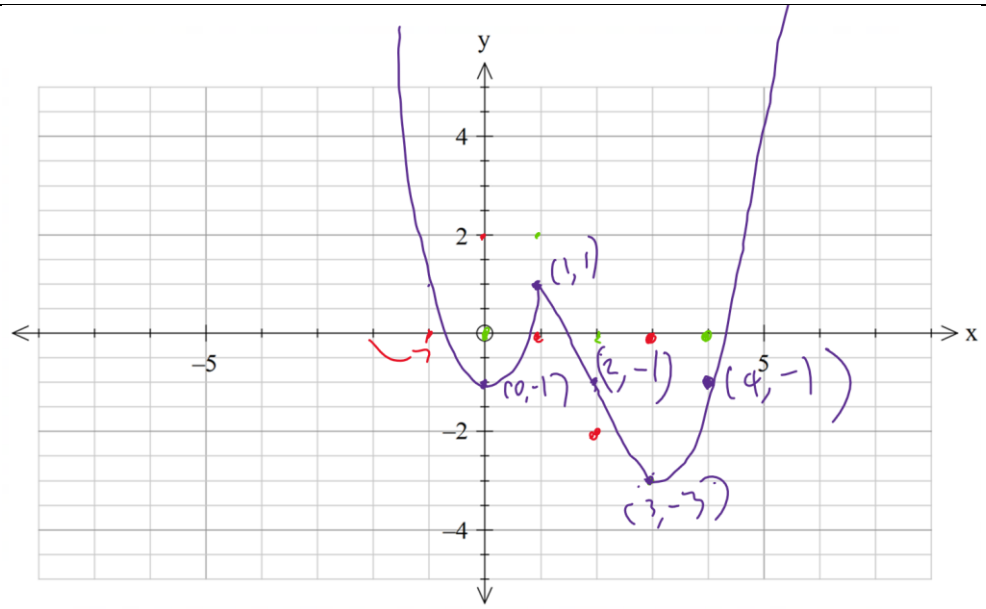
	$= -\left[-\left(x+\frac{1}{2}\right)^2+\left(x+\frac{1}{2}\right)+\frac{1}{4}\right]$ $=\left(x+\frac{1}{2}\right)^2-\left(x+\frac{1}{2}\right)+\frac{1}{4}$ $=x^2$		

7. (a)	Sum table  From the table, $P(sum<5)=\frac{6}{36}=\frac{1}{6}$ Or The possible pairs are (1, 1). (1,2), (2,1), (1,3), (2,2), (3,1)	1 mark: Using the two way table or lists, demonstrates the number of possible outcomes. (The solution must contain $\frac{6}{36}$ and then simplifies to finish the proof.	
(b)		2 marks: Gives the correct probabilities and completes the P.D. table.	
(c)	$E(X)=\Sigma x\,P(x)$ $=20\times\frac{6}{36}+10\times\frac{10}{36}+0\times\frac{20}{36}$ $=\frac{220}{36}$ $=\$6.11$ The expected amount won per game is $\$6.11$	1 mark: correctly demonstrates $\Sigma xP(x)$ from their table and gives the expected value	
(d)	Expected payout = \$6.11- \$5 = \$1.11 Lauren would incur an average loss of \$1.11 per game.	1 mark: Gives the correct option with justification based on their working. (Must calculate the difference between	

		expected value and the cost of entry.	
(e)	Let the price be P. E(X) = $-0.2P$ $$-0.2P = 20 \times \frac{6}{36} + 10 \times \frac{10}{36} - P$$ $$-0.2P = 6.11 - P$$ $$0.8P = 6.11$$ $$P = 6.11 \div 0.8 = \$7.64$$	1 mark: Equates E(X) to $-0.2P$ 1 mark: Solves to give P	

8. (a)	Positively skewed  	1 mark: Gives the correct shape	
(b)	Upper bound for data = $Q_3 - 1.5\ IQR$ = $26.8 + 1.5 \times (26.8 - 23.05)$ = 32.425 < 33.7 That is, 33.7 is outside the upper bound. Hence, 33.7 is an outlier.	1 mark: Calculates upper bound (working should be shown) 1 mark: Clearly states 32.425<33.7 and hence an outlier	
(c)	24.55 < 25 < 26.8. That is 25 is between the median and Q_3 score. Thus, the percentage of overweight people need to between 25 to 50%. $32 \times 0.5 = 16$ and $32 \times \frac{1}{4} = 8$ We expect between 9 to 15 over-weight people	2 marks: Any answer 9 to 15 with working (Reasoning based on Median and Q_3 percentage must be shown) 1 mark: gives a number 8 to 15. (Do not accept 16)	
9	$m \leq -3, m \geq 3, m = -1$	1 mark: correctly identified 3 and -3 as the boundaries, but had incorrect inequalities or found $m = -1$ 2 marks: correctly identified the inequalities OR found correct boundaries and incorrect inequalities 3 marks: all 3 correct	

10. (a)	The trendline is known. when w= 70, Body density = $1.195 - 0.001512 \times 70 = 1.089$ when w= 120, Body density = $1.195 - 0.001512 \times 120 = 1.014$  	2 marks: Correct line as per the trendline given 1 mark: A line is drawn	
(b)	It is given that $R^2 = 0.6783$ Hence, r = $-0{,}8236$ 1 mark (Band 4 and 5) The correlation coefficient shows a strong negative relationship between the variables (Band 4) From the scatter plot, the model is linear. (Band 3) The Pearson's correlation coefficient is a measure for linear models, and in this case the r-value is strong. Hence, it is appropriate to use the correlation coefficient to measure the association and Waist measurement can be used as a reliable simpler measure to explain the body density. (Band 6)	1 mark: correctly determines the correlation coefficient 1 mark: Interprets value of the correlation coefficient 1 mark: correctly identifies the scatter plot is linear 1 mark: shows logical organisation to correctly identify that the correlation coefficient should be used, and the strength shows waist measurement can be used as a simpler, yet reliable measure.	

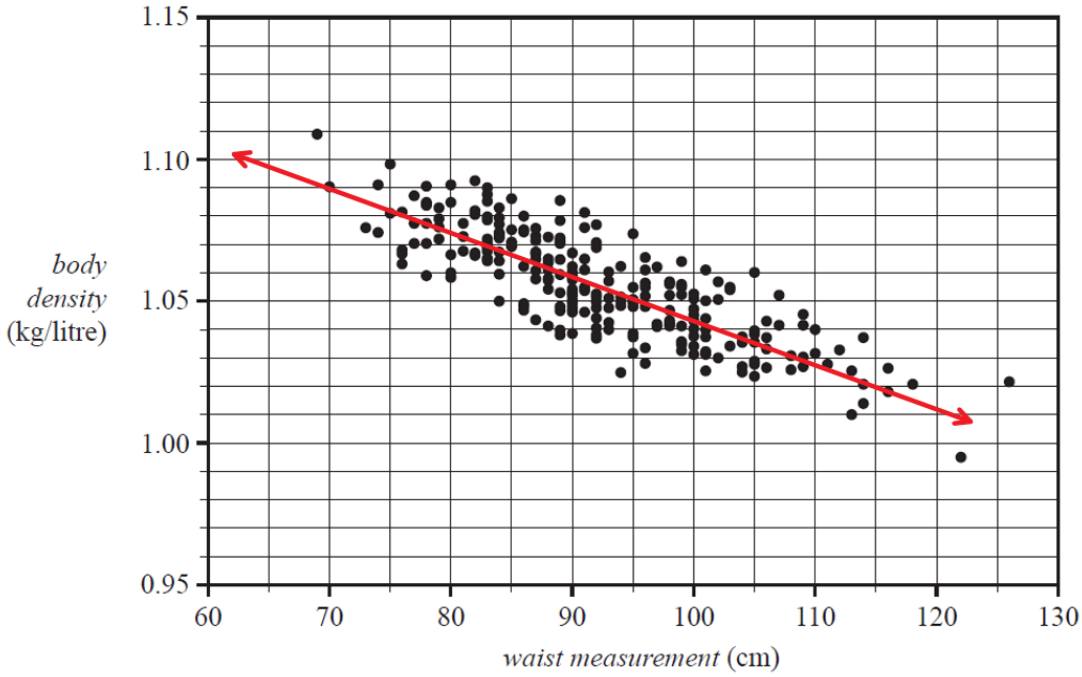


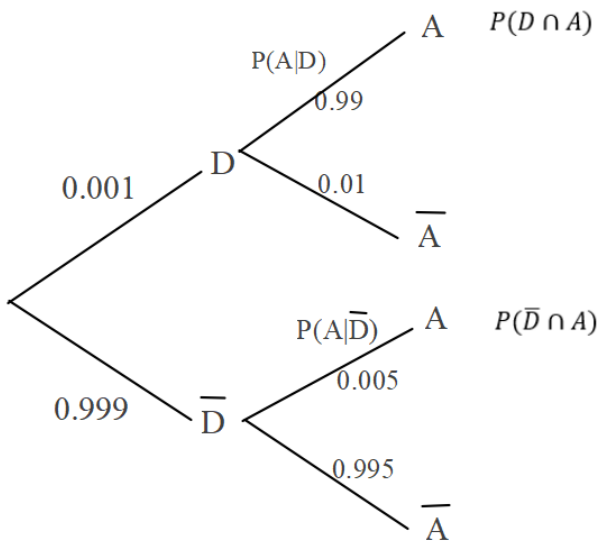
1 mark:
Evidence of at least 1 correct transformation

2 marks:
evidence of at least two correct transformations or all correct but misses two of the 5 points

3 marks:
correct but misses one of the 4 points

4 marks:
All five points correct



13.	Let D and $\overline{D}$ are the events that a person has disease and the person does not have disease. **Method 1: Tree Diagram**  $P(D) = \frac{0.1}{100} = 0.001,\ \ P(\overline{D}) = 0.999$ Let A be the event that the test result is positive. $\therefore P(A\|D) = 0.99$ and $P(A\|\overline{D}) = 0.005$ Probability that a person has the disease given that his test result is positive = $= \frac{\text{P(positive and have disease)}}{P(\text{Positive})}$ $\frac{P(AD)}{P(AD) + P(A\overline{D})}$ $= \frac{P(D) \times P(A\|D)}{P(D) \times P(A\|D) + P(\overline{D}) \times P(A\|\overline{D})}$ $= \frac{0.001 \times 0.99}{0.001 \times 0.99 + 0.999 \times 0.005}$ $= \frac{22}{133}$ **Method 2: Two-way table** Calculations using a two-way table is shown below	1 mark for each process • Tree diagram with $P(D)$ and $P(\overline{D})$ shown • $P(A\|D)$ and $P(A\|\overline{D})$ Shown on tree diagram • Writes the correct expression for Probability that a person has the disease given that his test result is positive $\frac{P(AD)}{P(AD) + P(A\overline{D})}$ • Substitutes the correct probabilities from the tree diagram, and calculates the probability correctly	

		D	$\bar{D}$		Marking: 2 marks: Calculates all four probabilities: $A \cap B, A \cap \bar{B},$ $\bar{A} \cap B, \bar{A} \cap \bar{B}$ 1 mark: calculates P(A) 1 mark: Substitutes the correct probabilities from the two-way table, and calculates the probability correctly	
	A	$P(\text{A} \cap \text{D})$ 0.001×0.99	$P(\text{A} \cap \bar{\text{D}})$ 0.999×0.005	P(A) 5.985×10^{-3}		
	$\bar{A}$	$P(\bar{\text{A}} \cap \text{D})$ 0.001×0.01	$P(\bar{\text{A}} \cap \bar{\text{D}})$ 0.999×0.995	$\text{P}(\bar{\text{A}})$ 0.994015		
		$P(D)$	$P(\bar{D})$			

		Die 1					
		1	**2**	**3**	**4**	**5**	**6**
Die 2	**1**	2	3	4	5	6	7
	2	3	4	5	6	7	8
	3	4	5	6	7	8	9
	4	5	6	7	8	9	10
	5	6	7	8	9	10	11
	6	7	8	9	10	11	12