Mrs Cooper	**Ms Davis**	**Name:** …………………………………………
Mr Feng	**Mrs Greenberg**	
Mrs Kerr	**Mrs Kydd**	**Teacher:**……………………………………..
Ms Lau	**Mrs Millar**	
Ms Mimmo	**Mrs Squires**	

Pymble Ladies' College

MATHEMATICS

HSC ASSESSMENT TASK 2

Term 1
2022

Time Allowed: 60 minutes

General Instructions

- Reading time - 5 minutes
- Working time - 60 minutes
- Write using black or blue pen.
- Erasable pens are not to be used.
- Show all necessary working for questions in Section II.
- Answer in the spaces provided.
- Approved calculators may be used.
- Diagrams are not drawn to scale.
- Marks may be deducted for careless or untidy work.
- A reference sheet is provided.

Mark	**/37**
Rank	**/**
Highest Mark	**/37**

SECTION I

Multiple Choice.
(Each question in this section is worth 1 mark)
Answer the Multiple Choice questions by placing a circle around your answer.

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1 If $f(x) = \tan(x)$ then what is the value of $f'\left(\frac{\pi}{3}\right)$?

(A) $\sqrt{3}$

(B) $2\sqrt{3}$

(C) 4

(D) 8

2 If $f'(x) = 3x^2 - 2x$ and $f(4) = 0$ then which function is $f(x)$?

(A) $f(x) = x^3 - x^2$

(B) $f(x) = x^3 - x^2 + 48$

(C) $f(x) = x^3 - x^2 - 48$

(D) $f(x) = 6x - 2$

3 What is the period and the range of $f(x) = 3\sin\left(\frac{2x}{5}\right) - 2$?

(A) 5π and $[-3,3]$

(B) 5π and $[-5,1]$

(C) $\frac{5\pi}{2}$ and $[-3,3]$

(D) $\frac{5\pi}{2}$ and $[-5,1]$

4 What is the first derivative of the function $y = \dfrac{e^{2x}}{2x}$?

(A) 1

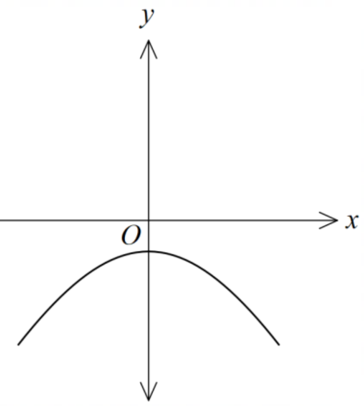
(B) $\dfrac{e^{2x}(1-2x)}{2x^2}$

(C) e^{2x}

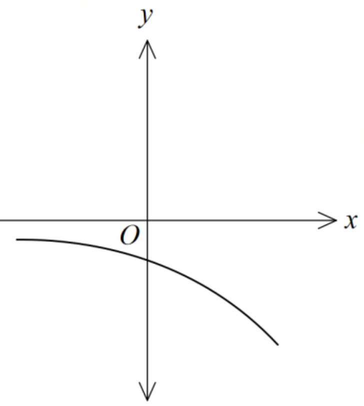
(D) $\dfrac{e^{2x}(2x-1)}{2x^2}$

5 The function $f(x)$ has the property that $f(x)$, $f'(x)$ and $f''(x)$ are negative for all real values of x.
Which of the following could be the graph of $f(x)$?

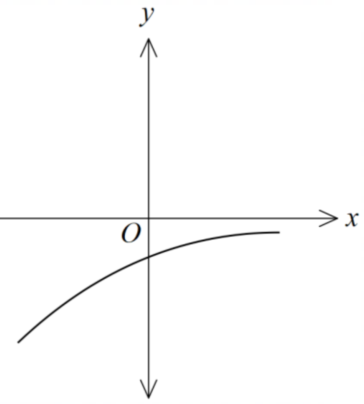
(A)



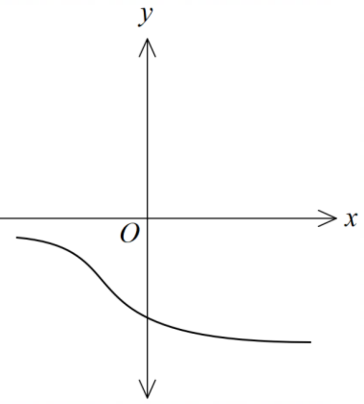
(B)



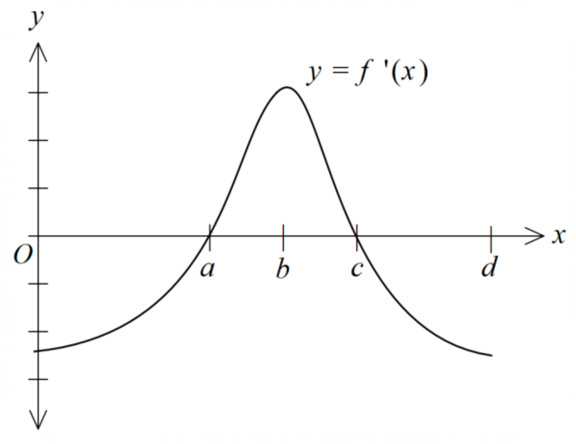
(C)



(D)



6 The graph of $f'(x)$, the derivative of $f(x)$, is shown below.
The domain of $f(x)$ is $x \in [0, d]$.



Which of the following statements is true?

(A) $f(x)$ has a point of inflexion at $x = a$

(B) $f(x)$ has a point of inflexion at $x = b$

(C) $f(x)$ has a local minimum at $x = c$

(D) $f(x)$ has a local maximum at $x = b$

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SECTION II

(31 marks)

For Questions 7-14, your responses should include relevant mathematical reasoning and/or calculations. **Marks**

Question 7 (3 marks)

Differentiate the following:

(a) $y = \ln\left(3x^2 + 1\right)$ **1**

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(b) $y = 5^{3x}$ **1**

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(c) $f\left(x\right) = \cos^2\left(4x\right)$ **1**

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Question 8 (3 marks)

Solve $1-2\cos\left(\dfrac{x}{2}\right)=0$ for $x\in[-\pi,\pi]$ **3**

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Question 9 (3 marks)

Find the equation of the tangent to the curve $y=x\sin x$ at the point $\left(-\dfrac{\pi}{2},\dfrac{\pi}{2}\right)$. **3**

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Question 10 (2 marks)

Evaluate $\displaystyle\int_1^e \frac{1}{2x}\,dx$ **2**

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Question 11 (7 marks)

Let $f\left(x\right)=\frac{1}{2}x^4-2x^3+6$.

(a) Find the coordinates of any stationary points and point(s) of inflection of $y=f\left(x\right)$ and determine their nature. **4**

(b) Sketch the graph of $y=f\left(x\right)$ showing all important features. **3**

You do not need to find the x-intercepts.

Question 12 (5 marks)

Find:

(a) $\int 3x^2 \;\; dx$ **1**

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(b) $\int \left(\frac{9}{x^4} + 8e^{4x} \right) \; dx$ **2**

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(c) $\int \frac{1}{(2x-5)^2} \; dx$ **2**

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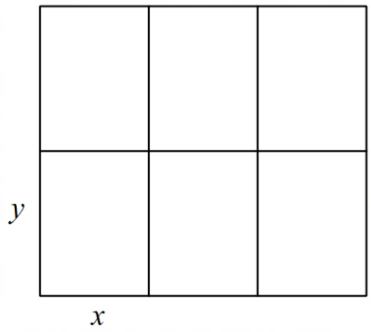
Question 13 (4 marks)

A council is setting aside an area of land to create six fenced plots for a community garden. Each plot will be a rectangle measuring x by y metres as shown in the diagram.

The area of the land being set aside is $108\ m^2$.



(a) Show that the total length of fencing, L metres, is given by $L(x) = 9x + \dfrac{144}{x}$ **2**

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(b) Find the value of x that minimises the length of fencing required. **2**

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Question 14 (4 marks)

The height of the tide at a particular beach today is given by the function

$$h(t) = 0.8\sin\left(\frac{4\pi t}{25} + \frac{\pi}{2}\right),$$

where h is the height of water, in metres, relative to the mean sea level and t is the time in hours after midnight.

(a) At what rate was the height of the tide changing at that beach at 9.00 a.m. today? **2**

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(b) At what time will the first low tide of the day occur? **2**

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End of Assessment

NSW Education Standards Authority

HIGHER SCHOOL CERTIFICATE

Mathematics Advanced
Mathematics Extension 1
Mathematics Extension 2

REFERENCE SHEET

Measurement

Length

$$l = \frac{\theta}{360} \times 2\pi r$$

Area

$$A = \frac{\theta}{360} \times \pi r^2$$

$$A = \frac{h}{2}(a + b)$$

Surface area

$$A = 2\pi r^2 + 2\pi rh$$

$$A = 4\pi r^2$$

Volume

$$V = \frac{1}{3}Ah$$

$$V = \frac{4}{3}\pi r^3$$

Functions

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For $ax^3 + bx^2 + cx + d = 0$:

$$\alpha + \beta + \gamma = -\frac{b}{a}$$

$$\alpha\beta + \alpha\gamma + \beta\gamma = \frac{c}{a}$$

and $\alpha\beta\gamma = -\frac{d}{a}$

Relations

$$(x - h)^2 + (y - k)^2 = r^2$$

Financial Mathematics

$$A = P(1 + r)^n$$

Sequences and series

$$T_n = a + (n - 1)d$$

$$S_n = \frac{n}{2}[2a + (n - 1)d] = \frac{n}{2}(a + l)$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(1 - r^n)}{1 - r} = \frac{a(r^n - 1)}{r - 1}, \ r \neq 1$$

$$S = \frac{a}{1 - r}, \ |r| < 1$$

Logarithmic and Exponential Functions

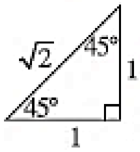
$$\log_a a^x = x = a^{\log_a x}$$

$$\log_a x = \frac{\log_b x}{\log_b a}$$

$$a^x = e^{x \ln a}$$

Trigonometric Functions

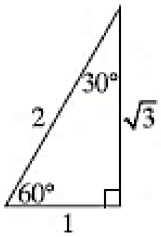
$\sin A = \frac{\text{opp}}{\text{hyp}}, \quad \cos A = \frac{\text{adj}}{\text{hyp}}, \quad \tan A = \frac{\text{opp}}{\text{adj}}$



$A = \frac{1}{2}ab\sin C$

$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$c^2 = a^2 + b^2 - 2ab\cos C$



$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$

$l = r\theta$

$A = \frac{1}{2}r^2\theta$

Trigonometric identities

$\sec A = \frac{1}{\cos A}, \ \cos A \neq 0$

$\operatorname{cosec} A = \frac{1}{\sin A}, \ \sin A \neq 0$

$\cot A = \frac{\cos A}{\sin A}, \ \sin A \neq 0$

$\cos^2 x + \sin^2 x = 1$

Compound angles

$\sin(A + B) = \sin A\cos B + \cos A\sin B$

$\cos(A + B) = \cos A\cos B - \sin A\sin B$

$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$

If $t = \tan\frac{A}{2}$ then $\sin A = \frac{2t}{1 + t^2}$

$\cos A = \frac{1 - t^2}{1 + t^2}$

$\tan A = \frac{2t}{1 - t^2}$

$\cos A\cos B = \frac{1}{2}\left[\cos(A - B) + \cos(A + B)\right]$

$\sin A\sin B = \frac{1}{2}\left[\cos(A - B) - \cos(A + B)\right]$

$\sin A\cos B = \frac{1}{2}\left[\sin(A + B) + \sin(A - B)\right]$

$\cos A\sin B = \frac{1}{2}\left[\sin(A + B) - \sin(A - B)\right]$

$\sin^2 nx = \frac{1}{2}(1 - \cos 2nx)$

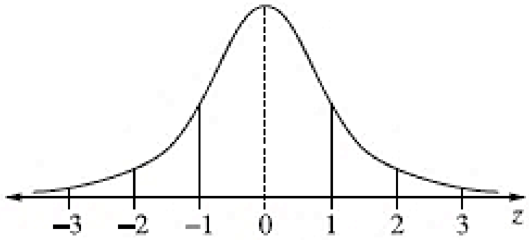
$\cos^2 nx = \frac{1}{2}(1 + \cos 2nx)$

Statistical Analysis

$z = \frac{x - \mu}{\sigma}$

An outlier is a score
less than $Q_1 - 1.5 \times IQR$
or
more than $Q_3 + 1.5 \times IQR$

Normal distribution



- approximately 68% of scores have z-scores between –1 and 1
- approximately 95% of scores have z-scores between –2 and 2
- approximately 99.7% of scores have z-scores between –3 and 3

$E(X) = \mu$

$\operatorname{Var}(X) = E\left[(X - \mu)^2\right] = E\left(X^2\right) - \mu^2$

Probability

$P(A \cap B) = P(A)P(B)$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A|B) = \frac{P(A \cap B)}{P(B)}, \ P(B) \neq 0$

Continuous random variables

$P(X \leq r) = \int_a^r f(x)\,dx$

$P(a < X < b) = \int_a^b f(x)\,dx$

Binomial distribution

$P(X = r) = {}^nC_r p^r(1 - p)^{n-r}$

$X \sim \operatorname{Bin}(n, p)$

$\Rightarrow \ P(X = x)$

$= \binom{n}{x}p^x(1 - p)^{n-x}, \ x = 0, 1, \ldots, n$

$E(X) = np$

$\operatorname{Var}(X) = np(1 - p)$

Differential Calculus

Function	Derivative
$y = f(x)^n$	$\frac{dy}{dx} = nf'(x)[f(x)]^{n-1}$
$y = uv$	$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$
$y = g(u)$ where $u = f(x)$	$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$
$y = \frac{u}{v}$	$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$
$y = \sin f(x)$	$\frac{dy}{dx} = f'(x)\cos f(x)$
$y = \cos f(x)$	$\frac{dy}{dx} = -f'(x)\sin f(x)$
$y = \tan f(x)$	$\frac{dy}{dx} = f'(x)\sec^2 f(x)$
$y = e^{f(x)}$	$\frac{dy}{dx} = f'(x)e^{f(x)}$
$y = \ln f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{f(x)}$
$y = a^{f(x)}$	$\frac{dy}{dx} = (\ln a)f'(x)a^{f(x)}$
$y = \log_a f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{(\ln a)f(x)}$
$y = \sin^{-1} f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{\sqrt{1-[f(x)]^2}}$
$y = \cos^{-1} f(x)$	$\frac{dy}{dx} = -\frac{f'(x)}{\sqrt{1-[f(x)]^2}}$
$y = \tan^{-1} f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{1+[f(x)]^2}$

Integral Calculus

$$\int f'(x)[f(x)]^n dx = \frac{1}{n+1}[f(x)]^{n+1} + c$$

where $n \neq -1$

$$\int f'(x)\sin f(x)dx = -\cos f(x) + c$$

$$\int f'(x)\cos f(x)dx = \sin f(x) + c$$

$$\int f'(x)\sec^2 f(x)dx = \tan f(x) + c$$

$$\int f'(x)e^{f(x)}dx = e^{f(x)} + c$$

$$\int \frac{f'(x)}{f(x)}dx = \ln|f(x)| + c$$

$$\int f'(x)a^{f(x)}dx = \frac{a^{f(x)}}{\ln a} + c$$

$$\int \frac{f'(x)}{\sqrt{a^2-[f(x)]^2}}dx = \sin^{-1}\frac{f(x)}{a} + c$$

$$\int \frac{f'(x)}{a^2+[f(x)]^2}dx = \frac{1}{a}\tan^{-1}\frac{f(x)}{a} + c$$

$$\int u\frac{dv}{dx}dx = uv - \int v\frac{du}{dx}dx$$

$$\int_a^b f(x)dx$$

$$\approx \frac{b-a}{2n}\left\{f(a) + f(b) + 2\left[f(x_1) + \cdots + f(x_{n-1})\right]\right\}$$

where $a = x_0$ and $b = x_n$

Combinatorics

$${}^{n}P_{r} = \frac{n!}{(n-r)!}$$

$$\binom{n}{r} = {}^{n}C_{r} = \frac{n!}{r!\,(n-r)!}$$

$$(x+a)^{n} = x^{n} + \binom{n}{1}x^{n-1}a + \cdots + \binom{n}{r}x^{n-r}a^{r} + \cdots + a^{n}$$

Vectors

$$\left|\underset{\sim}{u}\right| = \left|x\underset{\sim}{i} + y\underset{\sim}{j}\right| = \sqrt{x^{2}+y^{2}}$$

$$\underset{\sim}{u} \cdot \underset{\sim}{v} = \left|\underset{\sim}{u}\right|\left|\underset{\sim}{v}\right|\cos\theta = x_{1}x_{2} + y_{1}y_{2},$$

where $\underset{\sim}{u} = x_{1}\underset{\sim}{i} + y_{1}\underset{\sim}{j}$

and $\underset{\sim}{v} = x_{2}\underset{\sim}{i} + y_{2}\underset{\sim}{j}$

$$\underset{\sim}{r} = \underset{\sim}{a} + \lambda\underset{\sim}{b}$$

Complex Numbers

$$\begin{aligned} z = a + ib &= r(\cos\theta + i\sin\theta) \\ &= re^{i\theta} \end{aligned}$$

$$\begin{aligned} \left[r(\cos\theta + i\sin\theta)\right]^{n} &= r^{n}(\cos n\theta + i\sin n\theta) \\ &= r^{n}e^{in\theta} \end{aligned}$$

Mechanics

$$\frac{d^{2}x}{dt^{2}} = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^{2}\right)$$

$$x = a\cos(nt+\alpha) + c$$

$$x = a\sin(nt+\alpha) + c$$

$$\ddot{x} = -n^{2}(x-c)$$