



STUDENT NAME: _____

CLASS TEACHER: _____

2024 | HURSLTONE AGRICULTURAL HIGH SCHOOL

Year 12 HSC Assessment Task 1

Mathematics Advanced

Examiners

- Ms. T Tarannum, Ms. S Gutesa, Mr. R Jansen

**General
Instructions**

- Reading time – 3 minutes
- Working time – 50 minutes
- Write using black or blue pen
- NESA-approved calculators may be used
- A Reference sheet is provided
- Your handwritten notes are permitted to be used
- In Questions 7 – 9, show relevant mathematical reasoning and/or calculations

**Total marks:
36****Section I – 6 marks**

- Attempt Questions 1 – 6
- Allow about 6 minutes for this section

Section II – 30 marks

- Attempt Questions 7 – 9
- Allow about 44 minutes for this section

Section I

6 marks

Attempt Questions 1 - 6

Allow about 6 minutes for this section

Use the Multiple-Choice Answer Sheet provided for Questions 1 – 6.

Question 1

What is the gradient of the tangent to the curve $y = 3e^{-x}$ at the point $\left(2, \frac{3}{e^2}\right)$?

- A. $-3e^2$
- B. $\frac{3}{e^2}$
- C. $-\frac{3}{e^2}$
- D. $3e^2$

Question 2

If $a = \log_{10} 7$, what is the value of $\log_{10} \frac{1}{70}$?

- A. $-(1 + a)$
- B. $(1 + a)^{-1}$
- C. $\frac{a}{10}$
- D. $\frac{1}{10a}$

Question 3

A sales team sells 1200 calculators in its first month of operation. They plan to increase their sales by 150 calculators each month.

How many calculators do they plan to sell over an entire two-year period?

- A. 2 550
- B. 7 200
- C. 24 300
- D. 70 200

Question 4

How many terms are in the series $27 + 40 + 53 + \dots + 209$?

- A. 4
- B. 15
- C. 14
- D. 13

Question 5

Which expression is a term of the geometric series $-2y + 6y^2 - 18y^3 + \dots$?

- A. $39366y^9$
- B. $-39366y^9$
- C. $39366y^{10}$
- D. $-39366y^{10}$

Question 6

What is the solution to $\sum_{k=1}^7 (-1)^k 2^k$?

- A. -86
- B. 43
- C. -21
- D. 42

End of Section 1

Mathematics Advanced

Mathematics Extension 1

Mathematics Extension 2

REFERENCE SHEET

Measurement

Length

$$l = \frac{\theta}{360} \times 2\pi r$$

Area

$$A = \frac{\theta}{360} \times \pi r^2$$

$$A = \frac{h}{2}(a + b)$$

Surface area

$$A = 2\pi r^2 + 2\pi rh$$

$$A = 4\pi r^2$$

Volume

$$V = \frac{1}{3}Ah$$

$$V = \frac{4}{3}\pi r^3$$

Functions

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For $ax^3 + bx^2 + cx + d = 0$:

$$\alpha + \beta + \gamma = -\frac{b}{a}$$

$$\alpha\beta + \alpha\gamma + \beta\gamma = \frac{c}{a}$$

$$\text{and } \alpha\beta\gamma = -\frac{d}{a}$$

Relations

$$(x - h)^2 + (y - k)^2 = r^2$$

Financial Mathematics

$$A = P(1 + r)^n$$

Sequences and series

$$T_n = a + (n - 1)d$$

$$S_n = \frac{n}{2}[2a + (n - 1)d] = \frac{n}{2}(a + l)$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(1 - r^n)}{1 - r} = \frac{a(r^n - 1)}{r - 1}, r \neq 1$$

$$S = \frac{a}{1 - r}, |r| < 1$$

Logarithmic and Exponential Functions

$$\log_a a^x = x = a^{\log_a x}$$

$$\log_a x = \frac{\log_b x}{\log_b a}$$

$$a^x = e^{x \ln a}$$

Trigonometric Functions

$$\sin A = \frac{\text{opp}}{\text{hyp}}, \quad \cos A = \frac{\text{adj}}{\text{hyp}}, \quad \tan A = \frac{\text{opp}}{\text{adj}}$$

$$A = \frac{1}{2}ab \sin C$$

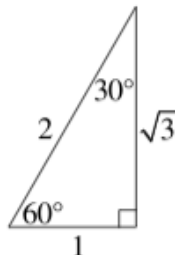
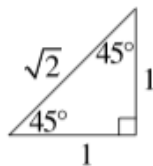
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

$$l = r\theta$$

$$A = \frac{1}{2}r^2\theta$$



Trigonometric identities

$$\sec A = \frac{1}{\cos A}, \quad \cos A \neq 0$$

$$\operatorname{cosec} A = \frac{1}{\sin A}, \quad \sin A \neq 0$$

$$\cot A = \frac{\cos A}{\sin A}, \quad \sin A \neq 0$$

$$\cos^2 x + \sin^2 x = 1$$

Compound angles

$$\sin(A + B) = \sin A \cos B + \cos A \sin B$$

$$\cos(A + B) = \cos A \cos B - \sin A \sin B$$

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\text{If } t = \tan \frac{A}{2} \text{ then } \sin A = \frac{2t}{1 + t^2}$$

$$\cos A = \frac{1 - t^2}{1 + t^2}$$

$$\tan A = \frac{2t}{1 - t^2}$$

$$\cos A \cos B = \frac{1}{2}[\cos(A - B) + \cos(A + B)]$$

$$\sin A \sin B = \frac{1}{2}[\cos(A - B) - \cos(A + B)]$$

$$\sin A \cos B = \frac{1}{2}[\sin(A + B) + \sin(A - B)]$$

$$\cos A \sin B = \frac{1}{2}[\sin(A + B) - \sin(A - B)]$$

$$\sin^2 nx = \frac{1}{2}(1 - \cos 2nx)$$

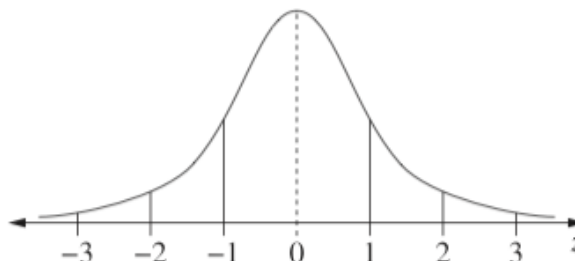
$$\cos^2 nx = \frac{1}{2}(1 + \cos 2nx)$$

Statistical Analysis

$$z = \frac{x - \mu}{\sigma}$$

An outlier is a score
less than $Q_1 - 1.5 \times IQR$
or
more than $Q_3 + 1.5 \times IQR$

Normal distribution



- approximately 68% of scores have z -scores between -1 and 1
- approximately 95% of scores have z -scores between -2 and 2
- approximately 99.7% of scores have z -scores between -3 and 3

$$E(X) = \mu$$

$$\operatorname{Var}(X) = E[(X - \mu)^2] = E(X^2) - \mu^2$$

Probability

$$P(A \cap B) = P(A)P(B)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}, \quad P(B) \neq 0$$

Continuous random variables

$$P(X \leq x) = \int_a^x f(x) dx$$

$$P(a < X < b) = \int_a^b f(x) dx$$

Binomial distribution

$$P(X = r) = {}^nC_r p^r (1 - p)^{n-r}$$

$$X \sim \operatorname{Bin}(n, p)$$

$$\Rightarrow P(X = x)$$

$$= \binom{n}{x} p^x (1 - p)^{n-x}, \quad x = 0, 1, \dots, n$$

$$E(X) = np$$

$$\operatorname{Var}(X) = np(1 - p)$$

Differential Calculus

Function

Derivative

$$y = f(x)^n$$

$$\frac{dy}{dx} = n f'(x) [f(x)]^{n-1}$$

$$y = uv$$

$$\frac{dy}{dx} = u \frac{dv}{dx} + v \frac{du}{dx}$$

$$y = g(u) \text{ where } u = f(x)$$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

$$y = \frac{u}{v}$$

$$\frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

$$y = \sin f(x)$$

$$\frac{dy}{dx} = f'(x) \cos f(x)$$

$$y = \cos f(x)$$

$$\frac{dy}{dx} = -f'(x) \sin f(x)$$

$$y = \tan f(x)$$

$$\frac{dy}{dx} = f'(x) \sec^2 f(x)$$

$$y = e^{f(x)}$$

$$\frac{dy}{dx} = f'(x) e^{f(x)}$$

$$y = \ln f(x)$$

$$\frac{dy}{dx} = \frac{f'(x)}{f(x)}$$

$$y = a^{f(x)}$$

$$\frac{dy}{dx} = (\ln a) f'(x) a^{f(x)}$$

$$y = \log_a f(x)$$

$$\frac{dy}{dx} = \frac{f'(x)}{(\ln a) f(x)}$$

$$y = \sin^{-1} f(x)$$

$$\frac{dy}{dx} = \frac{f'(x)}{\sqrt{1 - [f(x)]^2}}$$

$$y = \cos^{-1} f(x)$$

$$\frac{dy}{dx} = -\frac{f'(x)}{\sqrt{1 - [f(x)]^2}}$$

$$y = \tan^{-1} f(x)$$

$$\frac{dy}{dx} = \frac{f'(x)}{1 + [f(x)]^2}$$

Integral Calculus

$$\int f'(x) [f(x)]^n dx = \frac{1}{n+1} [f(x)]^{n+1} + c$$

where $n \neq -1$

$$\int f'(x) \sin f(x) dx = -\cos f(x) + c$$

$$\int f'(x) \cos f(x) dx = \sin f(x) + c$$

$$\int f'(x) \sec^2 f(x) dx = \tan f(x) + c$$

$$\int f'(x) e^{f(x)} dx = e^{f(x)} + c$$

$$\int \frac{f'(x)}{f(x)} dx = \ln |f(x)| + c$$

$$\int f'(x) a^{f(x)} dx = \frac{a^{f(x)}}{\ln a} + c$$

$$\int \frac{f'(x)}{\sqrt{a^2 - [f(x)]^2}} dx = \sin^{-1} \frac{f(x)}{a} + c$$

$$\int \frac{f'(x)}{a^2 + [f(x)]^2} dx = \frac{1}{a} \tan^{-1} \frac{f(x)}{a} + c$$

$$\int u \frac{dv}{dx} dx = uv - \int v \frac{du}{dx} dx$$

$$\int_a^b f(x) dx$$

$$\approx \frac{b-a}{2n} \left\{ f(a) + f(b) + 2[f(x_1) + \dots + f(x_{n-1})] \right\}$$

where $a = x_0$ and $b = x_n$

Combinatorics

$${}^nP_r = \frac{n!}{(n-r)!}$$

$$\binom{n}{r} = {}^nC_r = \frac{n!}{r!(n-r)!}$$

$$(x+a)^n = x^n + \binom{n}{1}x^{n-1}a + \cdots + \binom{n}{r}x^{n-r}a^r + \cdots + a^n$$

Vectors

$$|\underline{u}| = |x\underline{i} + y\underline{j}| = \sqrt{x^2 + y^2}$$

$$\underline{u} \cdot \underline{v} = |\underline{u}| |\underline{v}| \cos \theta = x_1x_2 + y_1y_2,$$

$$\text{where } \underline{u} = x_1\underline{i} + y_1\underline{j}$$

$$\text{and } \underline{v} = x_2\underline{i} + y_2\underline{j}$$

$$\underline{r} = \underline{a} + \lambda \underline{b}$$

Complex Numbers

$$\begin{aligned} z = a + ib &= r(\cos \theta + i \sin \theta) \\ &= re^{i\theta} \end{aligned}$$

$$\begin{aligned} [r(\cos \theta + i \sin \theta)]^n &= r^n(\cos n\theta + i \sin n\theta) \\ &= r^n e^{in\theta} \end{aligned}$$

Mechanics

$$\frac{d^2x}{dt^2} = \frac{dv}{dt} = v \frac{dv}{dx} = \frac{d}{dx} \left(\frac{1}{2} v^2 \right)$$

$$x = a \cos(nt + \alpha) + c$$

$$x = a \sin(nt + \alpha) + c$$

$$\ddot{x} = -n^2(x - c)$$

SECTION II

Student Name: _____

Class Teacher : _____

30 marks

Attempt Questions 7 - 9

Allow about 44 minutes for this section.

Answer each question in the spaces provided. Extra working space is available after each question. If you need to use this extra space, you must clearly indicate this in the main solution space, and then clearly indicate the question number and part that you are answering in the extra space.

For questions in Section II, your responses should include relevant mathematical reasoning and/or calculations.

2024 Mathematics Advanced Task 1 Section II

Question 7 (10 marks)	Marks
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(a) Solve $\log_3(2x - 1) + \log_3(x - 4) = 2$.	3
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Question 7 continued on next page

(b)

i) Sketch the graph of $y = 2^x + 1$.

1

ii) State the domain and range for $y = 2^x + 1$

2

Question 7 continued on next page

(c) After a social media application launched its new website, the number of its members could be modelled by the exponential growth equation $N = N_0 e^{kt}$ where N is the estimate for the number of members (in millions), and t is the time in years after the launch of the new website.

- i) It is estimated that when $t = 2$, the number of members will be 4 million and when $t = 4$, the number of members will be 10 million. Find N_0 and k .

2

- ii) According to the model, how many years after the launch of the website will the number of members reach 40 million?

2

End of Question 7

[illegible][illegible]

Student Name: _____

Class Teacher : _____

Question 8 (10 marks)

Marks

- (a) The sum of the first n terms of a certain arithmetic series is given by

$$S_n = \frac{n(3n + 1)}{2}$$

- i) Calculate S_1 and S_2 .

2

- ii) Find the first three terms of this series.

2

Question 8 continued on next page

iii) Show that the expression for the n th term is $T_n = 3n - 1$.

1

(b) The first three terms of an arithmetic sequence are given below.

$$\log_2 x, \log_2(x + 9) \text{ and } \log_2(x + 45)$$

i) Find the value of x .

2

Question 8 continued on next page

- ii) Using the value of x found in part (b)i), find the value of the 5th term as a logarithm in simplest form.

3

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End of Question 8

[illegible][illegible]

Student Name: _____

Class Teacher : _____

Question 9 (10 marks)

Marks

(a) A geometric sequence is formed by $f(x) + f'(x) + f''(x) + \dots$ where $f(x) = e^{\frac{1}{2}x}$.

i) Find the common ratio r .

2

ii) Find the 15th term of the sequence given above.

1

Question 9 continued on next page

(b) The fourth term of a geometric sequence is 96 and the seventh term is 12.

i) Find the first term and common ratio.

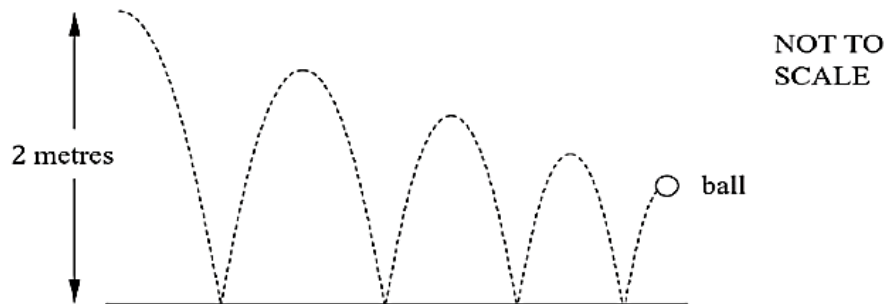
2

ii) Which is the first term smaller than 0.0001?

2

Question 9 continued on next page

- (c) The moon has a lower gravity than Earth, and there is no atmosphere to cause air resistance, so a ball would bounce higher and for much longer on the Moon than on Earth.
- When a ball is dropped on the Moon each bounce is 85% as high as the previous bounce.
- When an identical ball is dropped on Earth each bounce is 40% as high as the previous bounce.



3

Arjit dropped two identical balls on the Moon and on Earth, each from a height of two metres.

Calculate the difference in the total vertical distance travelled by these balls.

[illegible]

End of Question 9

End of Section II

[illegible][illegible]



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2024

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Year 12 HSC Assessment Task 1

Mathematics Advanced

Section I – Multiple Choice Answer Sheet

Allow about 6 minutes for this section

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample: $2 + 4 =$ (A) 2 (B) 6 (C) 8 (D) 9

A ☐ B ☒ C ☐ D ☐

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ☒ B ☒ C ☐ D ☐

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word correct and drawing an arrow as follows.

A ☒ B ☒ ^{correct} C ☐ D ☐

Question:

- | | | | | |
|----|-------------------------|-------------------------|-------------------------|-------------------------|
| 1. | A ☐ | B ☐ | C ☐ | D ☐ |
| 2. | A ☐ | B ☐ | C ☐ | D ☐ |
| 3. | A ☐ | B ☐ | C ☐ | D ☐ |
| 4. | A ☐ | B ☐ | C ☐ | D ☐ |
| 5. | A ☐ | B ☐ | C ☐ | D ☐ |
| 6. | A ☐ | B ☐ | C ☐ | D ☐ |