Killara HIGH SCHOOL
MATHEMATICS DEPARTMENT

Student Number

2023 **YEAR 12**

Mathematics Advanced

Assessment Task 2

Date: 6th of March 2023

Q	Marks
MC	**/5**
6	**/2**
7	**/2**
8	**/7**
9	**/2**
10	**/3**
11	**/8**
12	**/2**
13	**/3**
14	**/3**
15	**/7**
Total	**/44**

General Instructions:

- Reading time – 5 minutes
- Working time – 55 minutes
- Write using blue or black pen
- NESA approved calculators may be used
- Show relevant mathematical reasoning and/or calculations
- No white-out may be used

Total Marks: 44

Section I - 5 marks

- Allow about 5 minutes for this section

Section II - 39 marks

- Allow about 50 minutes for this section

This question paper must not be removed from the examination room.

This assessment task constitutes 20% of the course.

BLANK PAGE

Section I

5 marks
Allow about 5 minutes for this section.

Use the multiple-choice sheet for Question 1 – 5

---

1 For the hyperbola $y = \frac{x-2}{x-4}$ find the horizontal and vertical asymptotes.

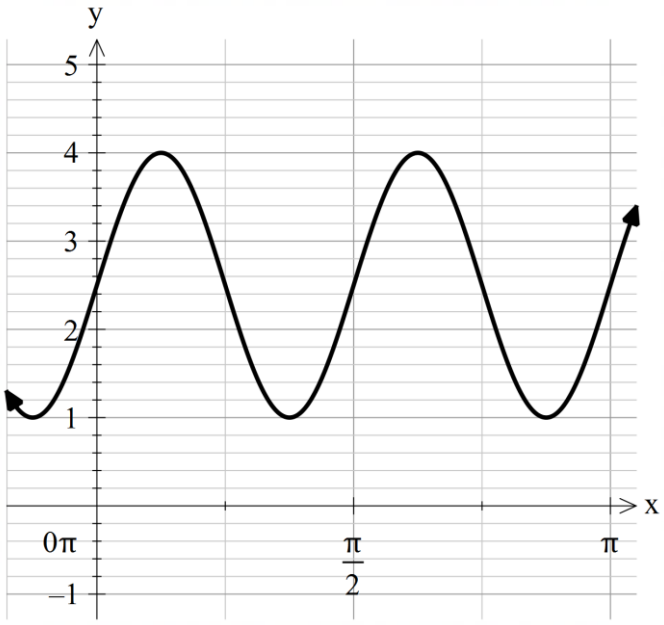
(A) $x = 2 \; and \; y = 2$

(B) $x = 1 \; and \; y = 4$

(C) $x = 4 \; and \; y = 1$

(D) $x = 4 \; and \; y = 4$

2 The diagram below shows the graph of $y = a\sin(bx) + 2.5$



Which of the following could be the values of a and b?

(A) $a = -1.5, \quad b = \frac{1}{4}$

(B) $a = 1.5, \quad b = \frac{1}{4}$

(C) $a = -1.5, \quad b = 4$

(D) $a = 1.5, \quad b = 4$

3 The graph of $y = 2\cos(3x)$ is transformed into $y = \frac{1}{2}\cos(6x)$. Which of the following accurately represents the transformation

(A) A vertical dilation by a factor of $\frac{1}{4}$, a horizontal dilation by a factor of $\frac{1}{2}$

(B) A vertical dilation by a factor of $\frac{1}{4}$, a horizontal dilation by a factor of 2

(C) A vertical dilation by a factor of $\frac{1}{2}$, a horizontal dilation by a factor of $\frac{1}{2}$

(D) A vertical dilation by a factor of $\frac{1}{2}$, a horizontal dilation by a factor of 2

4 If $f(x) = x^2\cos x$, find $f'(x)$:

(A) $f'(x) = -2sinx + 2xcosx$

(B) $f'(x) = -x^2sinx + 2xcosx$

(C) $f'(x) = x^2sinx + 2xcosx$

(D) $f'(x) = x^2sinx + 2xsinx$

5 If the tangent to the graph of $y = e^{ax}$, $a \neq 0$, at $x = c$ passes through the origin, then c is equal to:

(A) 0

(B) $\frac{1}{a}$

(C) 1

(D) a

End of Section I

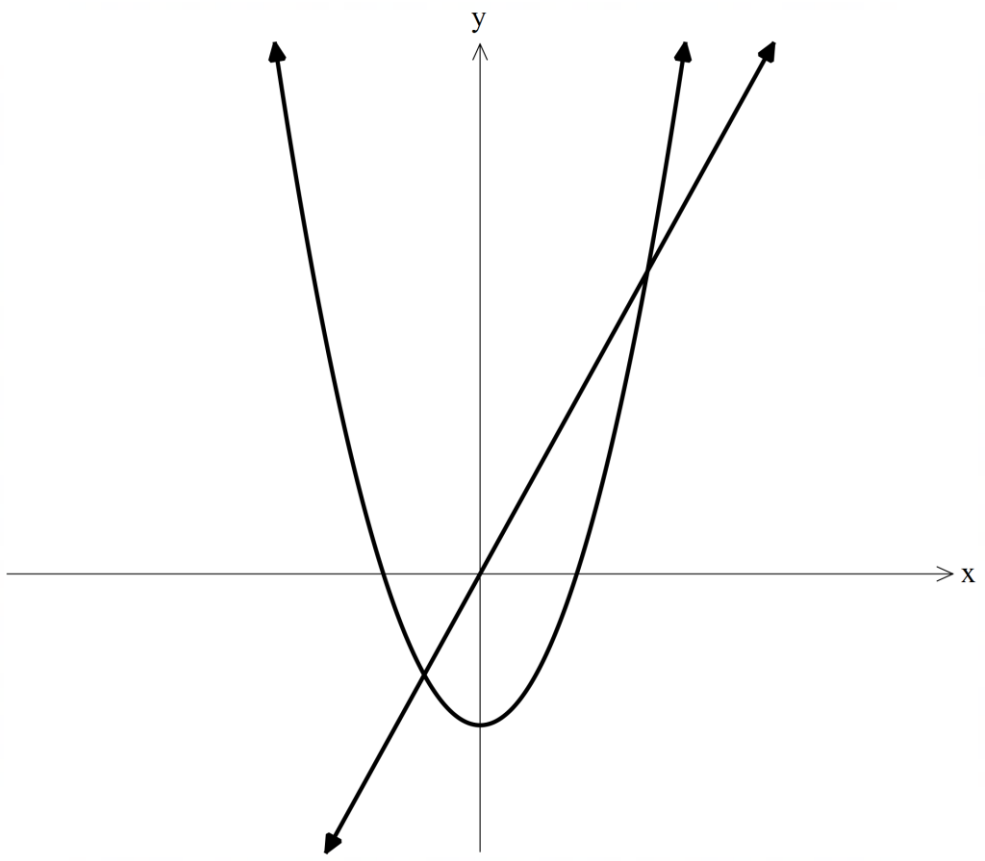
Examination continues next page

Section II

In Questions 6 – 18, your response should include relevant mathematical reasoning and/or calculations.

Question 6 (2 marks)

The functions $y = x^2 - 3$ and $y = 2x$ are sketched below. **2**



Hence, or otherwise, solve $x^2 - 3 \leq 2x$

.....

.....

.....

.....

.....

.....

.....

.....

Question 7 (2 marks)

Solve the following equation for the domain $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$ **2**

$$\sin(2x) = \sqrt{3}\cos(2x)$$

Question 8 (7 marks)

Differentiate the following with respect to x:

(a) $$y = \frac{\log_e x}{e^x}$$ **3**

(b) $y = \tan(\log_e x)$ **2**

(c) $y = x \sin(x°)$ **2**

Examination Continues on Next Page

Question 9 (2 marks)

Solve the following equations for the domain $0 \leq x \leq 2\pi$

$$\sqrt{2}\sin\left(x - \frac{\pi}{4}\right) = -1$$ **2**

Question 10 (3 marks)

The function $f(x) = \ln(x)$ undergoes the following transformations, in the following order: **3**

A horizontal dilation by a factor of 2, a vertical dilation by a factor of 3 and a vertical translation up by $\ln(8)$.

Find the final function

Question 11 (8 marks)

(a) Let $f(x) = 2\sin(3x) - 3$.

(i) State the period of $f(x)$. **1**

.....

.....

.....

(ii) State the amplitude of $f(x)$. **1**

.....

.....

.....

(iii) State the range of $f(x)$. **1**

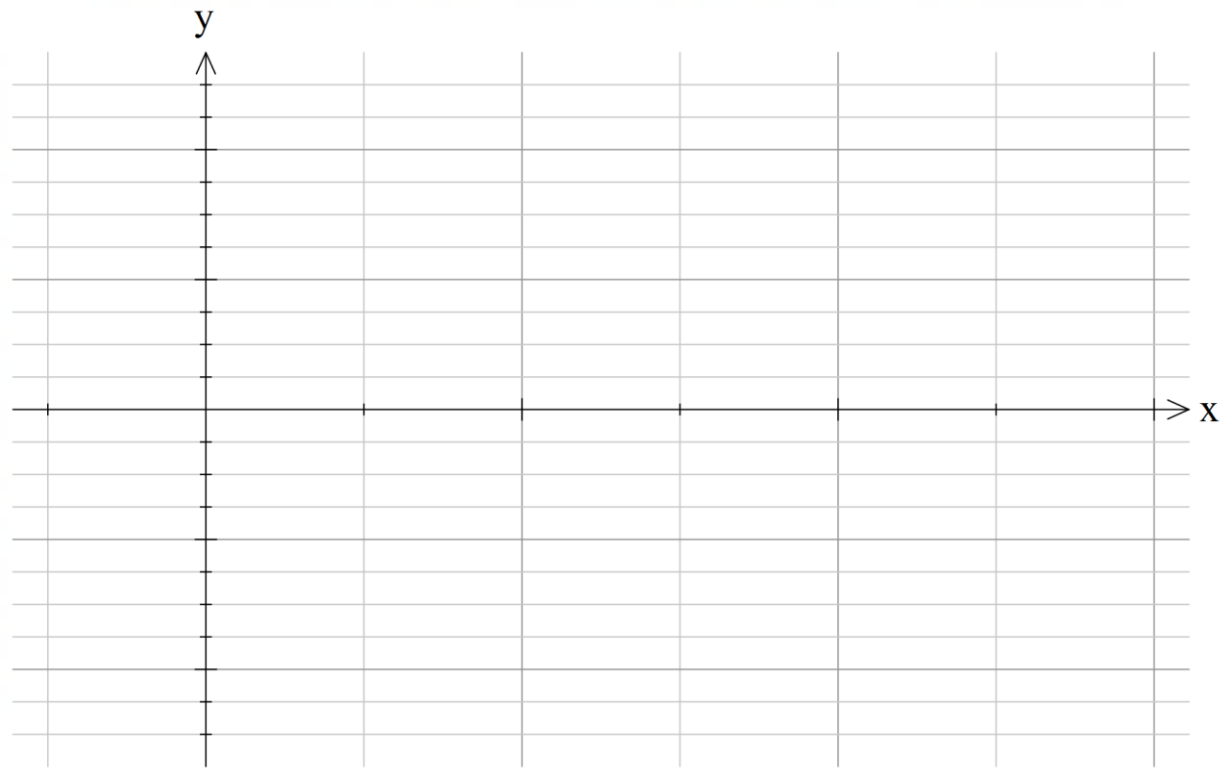
.....

.....

.....

.....

(b) Hence, sketch one period of $y = 2\sin(3x) - 3$ **3**



(c) On the same axis sketch $y = -x - 2$. Hence, state how many solutions there are for $2\sin(3x) - 3 = -x - 2$. **2**

.....

.....

.....

.....

Question 12 (2 marks)

Differentiate: **2**

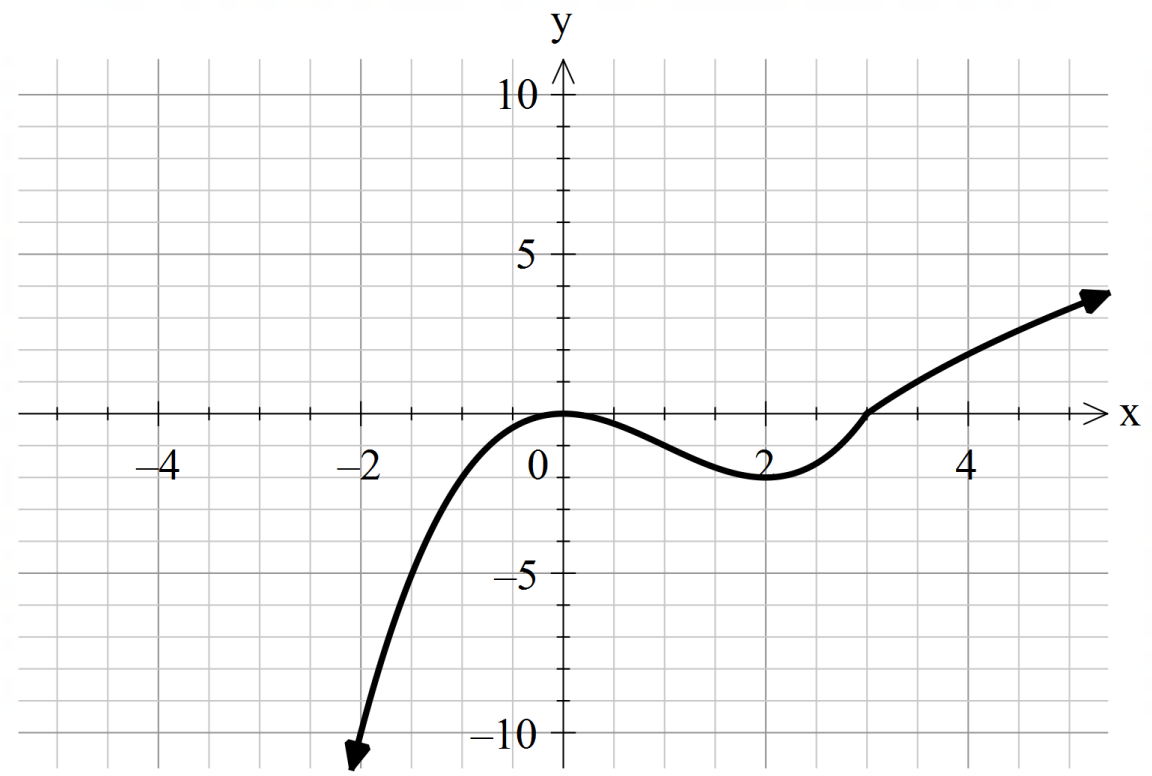
$$y = \ln\left(\frac{x}{\sqrt{x^2 - 1}}\right)$$

Question 13 (3 marks)

Find the equation of the tangent to the function $y = 5^x$ at the point $(2, 25)$. **3**

Question 14 (3 marks)

The graph below shows the function $f(x)$. On the same axis sketch $2f(2(x+1))$. **3**



Question 15 (7 marks)

A cart is attached to a spring that is connected to a wall. The cart is pulled 8 metres away from the wall and then released, so that it rolls back and forth towards and away from the wall as the spring stretches and contracts. The distance from the cart to the wall varies with time according to the formula:

$$C = k\sin(at + b) + c$$

Suppose the distance between the cart and the wall varies between 4 metres and 8 metres, and that it takes 6 seconds for the cart to go from the point farthest to the wall to the point that is closest from the wall.

(a) By sketching a graph of the motion of the cart, or otherwise, explain why the position of C can be modelled by **2**

$$C = 2\sin\left(\frac{\pi}{6}t + \frac{\pi}{2}\right) + 6$$

(b) How many seconds before the cart is 5m from the wall for the first time? **1**

(c) Find an expression for the velocity and acceleration of the cart and describe its motion at t=3. **4**

END OF THE EXAMINATION

Extra writing space

If you use this space, clearly indicate which question you are answering.

Extra writing space

If you use this space, clearly indicate which question you are answering.

Killara HIGH SCHOOL
MATHEMATICS DEPARTMENT

Centre Number								
Student Number								

Mathematics Advanced Task 2

Section 1 - Multiple Choice Answer Sheet
Use this multiple-choice sheet for questions 1 – 5. Detach this sheet.

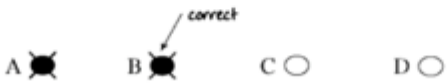
Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample: 2 + 4 = (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

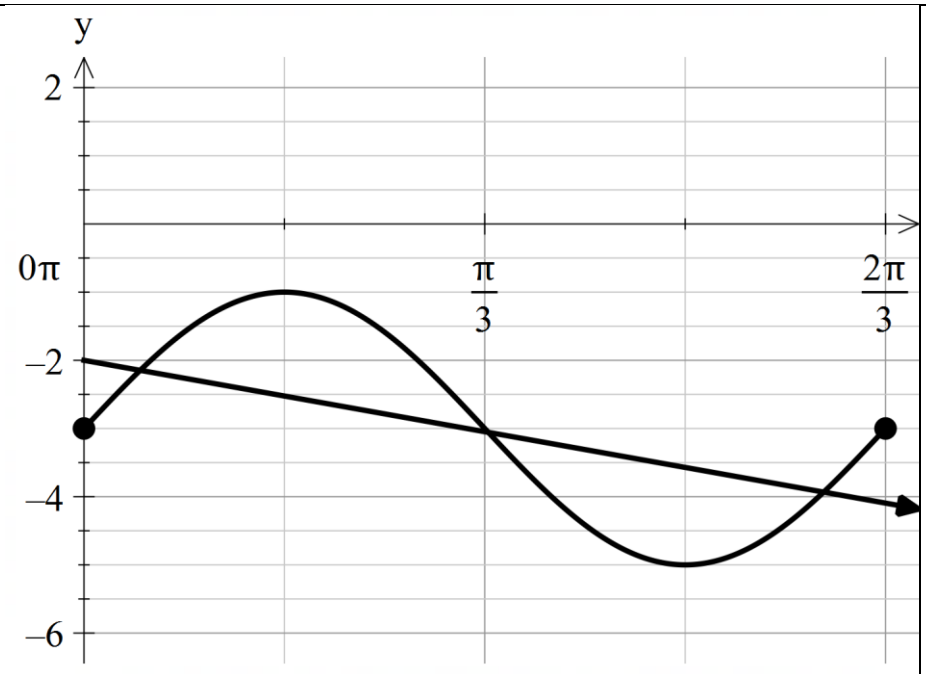
If you change your A ● B ✕ C ○ D ○ mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word *correct* and drawing an arrow as follows.



1.	A○	B○	C○	D○
2.	A○	B○	C○	D○
3.	A○	B○	C○	D○
4.	A○	B○	C○	D○
5.	A○	B○	C○	D○

1.	C	
2.	D	
3.	A	
4.	B	
5.	B	
6.	$x^2 - 3 = 2x$ most students got to this point. solved for $x = 3$ or -1... some students left it as $x \geq 3$ or $x \leq -1$ which was incorrect. $x^2 - 2x - 3 = 0$ $(x-3)(x+1) = 0$ $x = 3\ or\ -1$ $-1 \leq x \leq 3$	1 mark solve for x 1 mark correct inequality
7.	New bounds $-\pi \leq 2x \leq \pi$ $\tan(2x) = \sqrt{3}$ most students got tan. many got $\frac{\pi}{6}$. $2x = \frac{\pi}{3}, -\frac{\pi}{3}$ $x = \frac{\pi}{6}, -\frac{\pi}{6}$ however many students did not achieve the negative.	1 mark for finding the base angle with working 1 mark for finding all correct solutions
8		
a)	$y = (\ln x) \div e^x$ $\frac{dy}{dx} = \frac{\frac{e^x}{x} - e^x \ln(x)}{e^{2x}}$ $= \frac{\frac{1}{x} - \ln(x)}{e^x}$ → If you left it here you got 3 marks. also $= \frac{1 - x\ln(x)}{x e^x}$ This was popular.	1 for correct derivations of e^x and $\ln(x)$ 1 for correct use of the quotient rule 1 mark for cancelling out the e^x terms
b)	$y = \tan(\ln(x))$ $\frac{dy}{dx} = \frac{\sec^2(\ln(x))}{x}$ most students left the answer as $\frac{1}{x}\sec^2 \ln x$.	1 mark for differentiating the trigonometric function correctly. Including keeping the log function consistent. 1 mark for applying the chain rule correctly and multiplying by $\frac{1}{x}$
c)	$y = x\sin(x°)\ \ \left|(x^o = \frac{\pi x}{180})\right.$ most students forgot deg to rad conversion $\frac{dy}{dx} = \sin\left(\frac{\pi x}{180}\right) + \frac{\pi x}{180}\cos\left(\frac{\pi x}{180}\right)$ The rest was Algebra $\frac{dy}{dx} = \sin(x°) + \frac{\pi x}{180}\cos(x°)$ or $\frac{\pi x}{180}\ for\ x^o$	1 mark for correctly applying the product rule (without applying any correction or

		converting to radians) 2 marks correctly applied product rule and corrected converted to radians.
9.	New bounds $\frac{-\pi}{4} \leq x - \frac{\pi}{4} \leq \frac{7\pi}{4}$ $\sqrt{2}\sin\left(x - \frac{\pi}{4}\right) = -1$ $\sin\left(x - \frac{\pi}{4}\right) = -\frac{1}{\sqrt{2}}$ $x - \frac{\pi}{4} = -\frac{\pi}{4}, \pi + \frac{\pi}{4}, \frac{7\pi}{4}$ $x = 0, \frac{6\pi}{4}, 2\pi$ $x = 0, \frac{3\pi}{2}, 2\pi$	1 mark for a correct solution 2 marks for all 3 solutions
10.	$f(x) = \ln(x)$ Horizontal dialation $f\left(\frac{x}{2}\right) = \ln\left(\frac{x}{2}\right)$ Vertical dilation $3f\left(\frac{x}{2}\right) = 3\ln\left(\frac{x}{2}\right)$ $3f\left(\frac{x}{2}\right) = 3\ln(x) - 3\ln(2)$ Vertical translation $3f\left(\frac{x}{2}\right) + \ln(8) = 3\ln(x) - \ln(8) + \ln(8)$ $= 3\ln(x)$	1 for 2 correct transformations and 2 for all 3 transformations correct 3 for all 3 transformations correct and final function simplified.
11.		
a)	Period $Period = \frac{2\pi}{3}$	
b)	$a = 2$	
c)	$-5 \leq y \leq -1$	

d)		1 mark correct scale x and y 1 mark shape (with correct scale) 1 mark only sketched one period
e)		1 mark correctly sketch $y = -x - 2$ with the same scale as the sine function 1 mark correctly interprets the two sketches
12	$y = \ln\left(\frac{x}{\sqrt{x^2-1}}\right)$ $\frac{dy}{dx} = \ln x - \frac{1}{2}\ln(x^2-1)$ $\frac{dy}{dx} = \frac{1}{x} - \frac{2x}{2(x^2-1)}$ $\frac{dy}{dx} = \frac{1}{x} - \frac{x}{x^2-1}$ $= \frac{x^2-1-x^2}{x(x^2-1)}$ $= -\frac{1}{x^3-x}$	1 mark for applying log laws/using quotient law to find the numerator of the derivative 2 marks for deriving (did not need to FULLY simplify, if using quotient rule had to get to a rational denominator with no negative powers in the numerator.)
13.	$y = 5^x$	1 mark for deriving

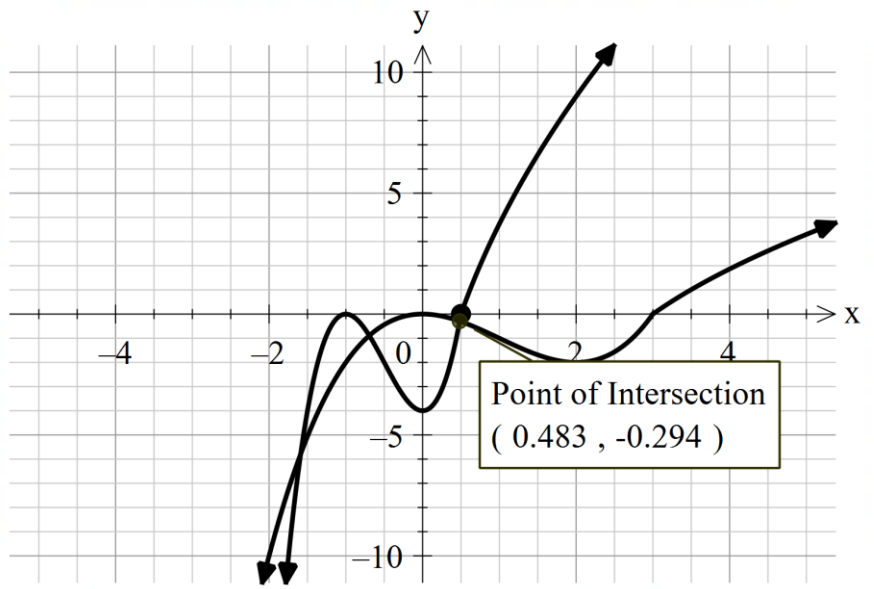
	$$\frac{dy}{dx} = \ln(5) * 5^x$$ $$\frac{dy(2)}{dx} = 25\ln 5$$ Using point gradient formula $$y - 25 = 25\ln(x)\,(x-2)$$ $$y = 25x\ln(x) - 50\ln(x) + 25$$ or $$y = 25\ln(x)(x-2) + 25$$	correctly 1 mark for find the value of the tangent at $(2,25)$ 1 mark for finding the gradient function

Markers comments:
Question 9: Many students didn't consider either 0 or 2π, and only received one mark for a solid understanding of the concept

Question 10: Many students did not fully simplify their answer, or incorrectly used log laws that didn't account for the coefficient of 3 in the question.
A startling number of students applied $\frac{1}{3}$ as a vertical dilation!

Question 11: Generally well done for parts a and b. In part c many students sketched the linear function without consideration of the scale they have constructed for the trig function

Question 12: Students who used the log laws generally received two marks, students who attempted to use the quotient rule often made a small, avoidable error or struggled to express their answer simply.

14.		1 mark for evidence of each translation.
15.		
a)	$C = 2\sin\left(\frac{\pi}{6}t + \frac{\pi}{2}\right) + 6$ The distance between the cart and the wall varies between 4 metres and 8 metres. Therefore, the mean position is $\frac{8+4}{2} = 6\ cm.$ Therefore, the amplitude is $8 - 6\ or\ 6 - 4\ or\ \frac{8-4}{2} = 2$ cm. Period = 12 seconds. The coefficient of $t = \frac{2\pi}{12} = \frac{\pi}{6}$. $C = 2\sin\left(\frac{\pi}{6}t + \theta\right) + 6$ $t = 0 \quad C = 8$ $8 = 2\sin(\theta) + 6$ $\sin(\theta) = 1$ $\therefore \theta = \frac{\pi}{2} + 2k\pi \ , \forall\ k \in Z$ For convenience let us make $k = 0$	1 mark for demonstrating the rational for 2 of the constants 2 marks for demonstrating the rational for all 5 constants
b)	$2\sin\left(\frac{\pi}{6}t + \frac{\pi}{2}\right) + 6 = 5$ $2\sin\left(\frac{\pi}{6}t + \frac{\pi}{2}\right) = -1$ $\sin\left(\frac{\pi}{6}t + \frac{\pi}{2}\right) = -\frac{1}{2}$ $\frac{\pi}{6}t + \frac{\pi}{2} = \frac{7\pi}{6}, \frac{11\pi}{6}$ $\frac{\pi}{6}t = \frac{4\pi}{6}, \frac{8\pi}{6}$ $t = 4, 8$ $t = 4$ is the first solution	1 mark for the correct answer

c)	$C = 2\sin\left(\frac{\pi}{6}t + \frac{\pi}{2}\right) + 6$ $C' = \frac{\pi}{3}\cos\left(\frac{\pi}{6}t + \frac{\pi}{2}\right)$ $C'' = -\frac{\pi^2}{18}\sin\left(\frac{\pi}{6}t + \frac{\pi}{2}\right)$ at $t = 3$ $C'(3) = \frac{\pi}{3}\cos\left(\frac{\pi}{2} + \frac{\pi}{2}\right)$ $= -\frac{\pi}{3}$ $C''(0) = \frac{\pi}{3}\sin\left(\frac{\pi}{2} - \frac{\pi}{2}\right)$ $= 0$ The cart is moving towards the wall at $\frac{\pi}{3} m/s$ and is not experiencing any acceleration (changing from speeding to slowing down)	1 mark an expression for the velocity 1 mark an expression for acceleration 1 mark finding values at $t = 3$ 1 mark a sentence discussing the motion in context.

Markers comments:

Question 13: Most students can find the derivative of $y = 5^x$. Some students cannot find the gradient properly by substituting $x = 2$ into the gradient function. Most students can apply the gradient point form to find the tangent. Please use exact values instead of decimals when possible.

Question 14: This question has been poorly done. A lot of students struggle with the overall combination of different transformations. It is important to review the order in which different transformations are applied. Students need to label the key features in addition to the general shape of the final function after transformations. A good way to keep track of transformations is list a few points before transformations and find the new points after transformations

Question 15: This question has been poorly done.

a) Students need to explain the derivation of the amplitude 2, phase $\frac{\pi}{2}$, mean position 6 and $\frac{\pi}{6}$ clearly. The proper derivation of any 2 parameters is worth 1 mark. A random sketch without explanation cannot earn you a mark. Note, please do not use the phrase or sentence such as 'it is obvious that the amplitude is 2.' You have do the calculation to show that it is 2. Most students cannot make a connection between this context with the basic concepts of trigonometric functions.

b) Most students have done it really well. However, from $\sin(\frac{\pi}{6}t + \frac{\pi}{2}) = -\frac{1}{2}$, instead of $\frac{7\pi}{6}$ some students wrote $\frac{\pi}{6}t + \frac{\pi}{2} = -\frac{\pi}{6}$, which results in $t = -4$ seconds. As time cannot be negative, no mark will be given. In addition, we cannot simply say the time is 4 seconds when t =-4 seconds.

c) Most students struggle with this one. Please revise the application of differentiation in motion. In addition, a negative velocity does not mean that the cart is slowing down. A minus sign simply refers to the direction. The cart is moving towards the wall. To determine whether a cart is speeding up or slowing down , we have to calculate its acceleration. Regardless, momentarily, the cart has an acceleration of $0\ m/s^2$. It is important for students to gain conceptual understanding first by doing more practice.