

Mrs Cooper
Mrs Greenberg
Ms Lau
Ms Smith
Mrs Ward

Mr Feng
Mrs Kerr
Mrs Millar
Mrs Squires

Name:

Teacher:



Pymble Ladies' College

MATHEMATICS
HSC ASSESSMENT TASK 2

Term 1
2023

General Instructions

- Reading time - 5 minutes
- Working time - 55 minutes
- Write using black or blue pen.
- Erasable pens are not to be used.
- Show all necessary working for questions in Section II.
- Approved calculators may be used.
- Diagrams are not drawn to scale.
- Marks may be deducted for careless or untidy work.
- Reference sheet is provided.

Mark	/40
Rank	/
Highest Mark	/40

BLANK PAGE

Mathematics Advanced

Year 12

Name:

Teacher's Name:.....

Section I – Multiple Choice Answer Sheet

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample: $2 + 4 =$ (A) 2 (B) 6 (C) 8 (D) 9
 A ☐ B ☒ C ☐ D ☐

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ☒ B ☒ C ☐ D ☐

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word **correct** and drawing an arrow as follows.

A ☒ B ☒ C ☐ D ☐
 correct

Assessment Task 2

1 A ☐ B ☐ C ☐ D ☐

2 A ☐ B ☐ C ☐ D ☐

3 A ☐ B ☐ C ☐ D ☐

4 A ☐ B ☐ C ☐ D ☐

5 A ☐ B ☐ C ☐ D ☐

6 A ☐ B ☐ C ☐ D ☐

BLANK PAGE

SECTION I

Multiple Choice.

Use the multiple-choice answer sheet for Questions 1-6

(Each question in this section is worth 1 mark)

1 What is the period and amplitude of $y = 3 \cos 2x$?

(A) Amplitude = 2, Period = $\frac{2\pi}{3}$

(B) Amplitude = 3, Period = π

(C) Amplitude = π , Period = 3

(D) Amplitude = $\frac{2\pi}{3}$, Period = 2

2 Which of the following expressions is equal to $\int \frac{2}{2x+1} dx$?

(A) $2 \log_e |(2x+1)| + C$

(B) $\frac{1}{2} \log_e |(2x+1)| + C$

(C) $\log_e |(2x+1)| + C$

(D) $\log_e |(x+1)| + C$

3 What is the derivative of $e^{\frac{1}{x^2}}$?

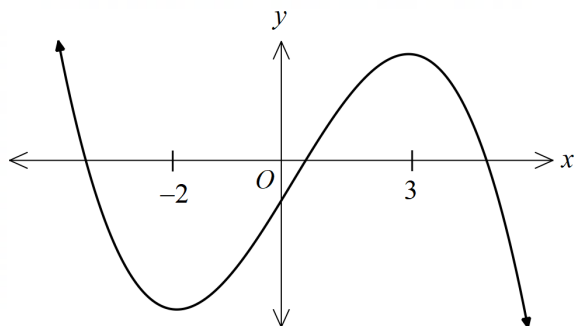
(A) $-2e^{\frac{1}{x^2}}$

(B) $\frac{1}{x^2} e^{\frac{1}{x^2}}$

(C) $-2x^{-3} e^{\frac{1}{x^2}}$

(D) $-2xe^{x^{-3}}$

- 4 The diagram shows part of $y = f(x)$ which has a local minimum at $x = -2$ and a local maximum at $x = 3$.



Which of the following shows the correct relationship between $f''(-2)$, $f(0)$ and $f'(3)$?

- (A) $f(0) < f'(3) < f''(-2)$
(B) $f(0) < f''(-2) < f'(3)$
(C) $f''(-2) < f'(3) < f(0)$
(D) $f''(-2) < f(0) < f'(3)$
- 5 What is the value of $\int_0^1 2xe^{x^2} dx$?
- (A) $e - 1$
(B) e
(C) $\frac{e}{2}$
(D) $2e$

- 6 Which of the following satisfies the equation $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 3y = 0$?
- (A) $y = \ln(3x)$
(B) $y = \ln(-3x)$
(C) $y = e^{3x}$
(D) $y = e^{-3x}$

SECTION II

(34 marks)

Answer in the spaces provided.

Extra writing space is provided at the end of this section.

Your responses should include relevant mathematical reasoning and/or calculations.

Marks

Question 7 (10 marks)

(a) Differentiate:

(i) $y = \ln(4x - 2)$

1

.....
.....

(ii) $y = e^{3x} \cos 2x$

2

.....
.....
.....
.....
.....
.....

(iii) $f(x) = \ln\left(\frac{5-2x}{5x-2}\right)$

2

.....
.....
.....
.....
.....

(iv) $f(x) = \cos(\sqrt{x})$

2

.....
.....
.....
.....

Question 7 continues on next page

- (b) Show that $f'(x) = \sec x + x \tan x \sec x$ when $f(x) = \frac{x}{\cos x}$.

3

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

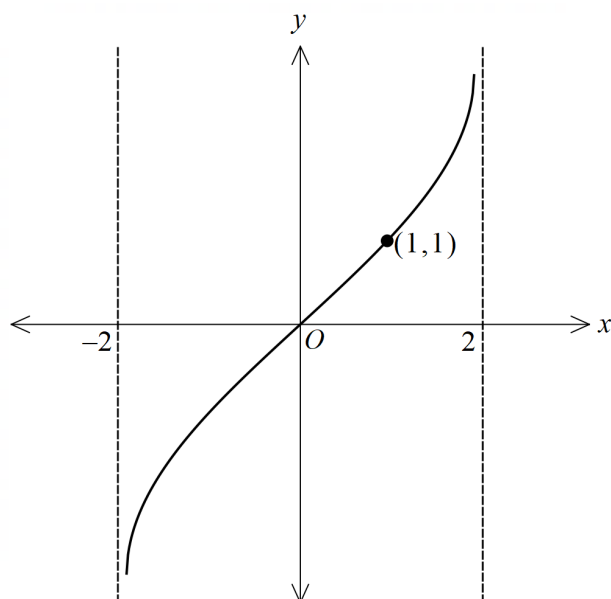
.....

Question 8 (2 marks)

Marks

The graph of $y = a \tan bx$ is shown below for $-2 < x < 2$.

2



Find the values of a and b .

.....

.....

.....

.....

.....

.....

.....

.....

.....

Question 9 (8 marks)

Marks

A function $f(x)$ is defined by $f(x) = x^3 - x^2 - x + 1$.

- (a) Find the coordinates of the stationary points and determine their nature.

4

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

- (b) Find the point of inflection.

2

.....

.....

.....

.....

.....

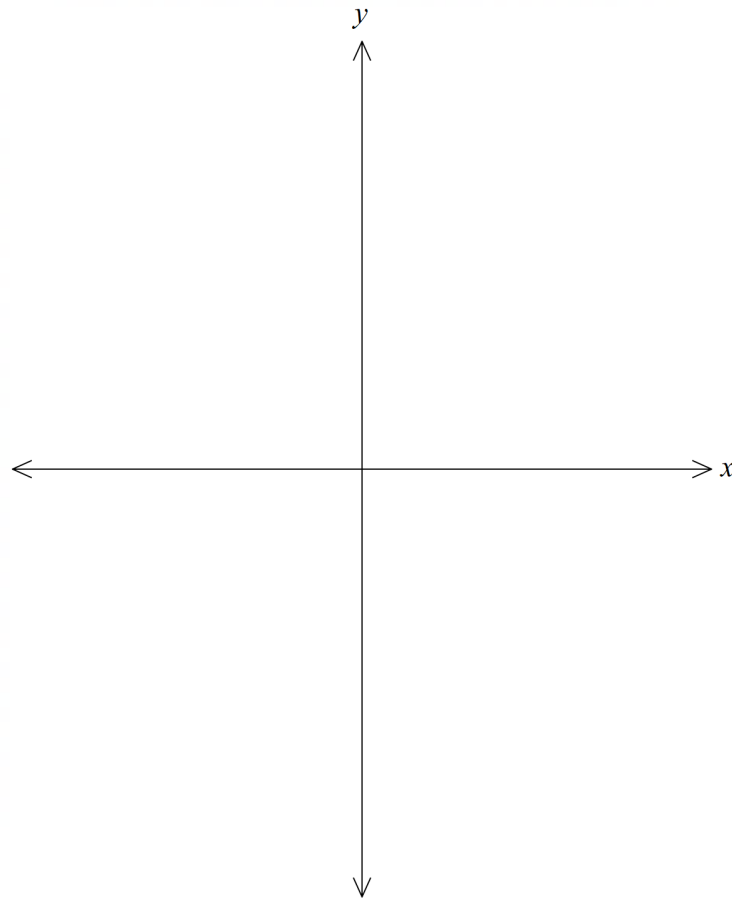
.....

.....

.....

Question 9 continues on next page

- (c) Sketch the graph of the function, labelling the stationary points, point of inflection and the y -intercept.

2

Question 10 (4 marks)

- (a) Show that the derivative of $y = x \ln x$ is $\frac{dy}{dx} = \ln x + 1$.

2

.....

.....

.....

.....

.....

.....

.....

.....

- (b) Hence, evaluate $\int_1^2 \ln x \, dx$.

2

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

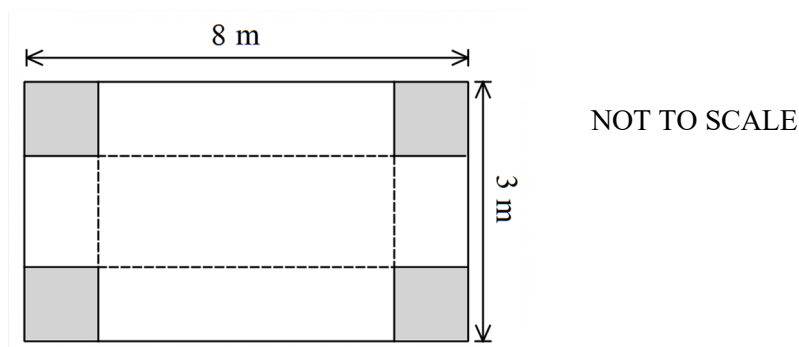
.....

.....

.....

Question 11 (4 marks).

A rectangular piece of metal is 8 m by 3 m. Four equal squares, side x m, are removed from each corner. The edges are then turned up to form a box, open at the top.



- (a) Show that the volume of the box is given by

1

$$V = 4x^3 - 22x^2 + 24x.$$

.....

.....

.....

.....

.....

- (b) Find the value of x which makes this volume a maximum.

3

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

Question 12 (6 marks)

Between 5 am and 5 pm on March 3 2009, the height, h , of the tide in a harbour was given by

$$h = 1 + 0.7 \sin \frac{\pi}{6} t \quad \text{for } 0 \leq t \leq 12$$

where h is in metres and t is in hours, with $t = 0$ at 5 am.

- (a) What is the period of the function h ? **1**

.....
.....
.....

- (b) What was the value of h at low tide, and at what time did low tide occur? **2**

.....
.....
.....
.....
.....
.....
.....
.....

- (c) A ship is able to enter the harbour only if the height of the tide is at least 1.35 m. **3**

Find all times between 5 am and 5 pm on 3 March 2009 during which the ship was able to enter the harbour.

.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....
.....

Section II extra writing space

If you use this space, clearly indicate which question you are answering.

This image shows a full page of a handwriting practice worksheet. It consists of approximately 28 horizontal rows. Each row is defined by two parallel dotted lines, creating a series of uniform gaps for letter height. The entire page is otherwise blank, with no margins, text, or other markings.

BLANK PAGE