Killara HIGH SCHOOL

MATHEMATICS DEPARTMENT

Student Number

2024 **Year 12 Examination**

Mathematics Advanced

Task 2

Date: 12/03/2024

Q	Marks
MC	/5
6	/6
7	/2
8	/2
9	/3
10	/3
11	/2
12	/4
13	/4
14	/5
15	/5
Total	/41

General Instructions

- Reading time – 5 minutes
- Working time – 55 minutes
- Write using blue or black pen
- Calculators approved by NESA may be used
- A reference sheet is provided
- Show relevant mathematical reasoning and/or calculations
- No white-out may be used

Total Marks: 41

Section I - 5 marks

- Allow about 5 minutes for this section

Section II - 36 marks

- Allow about 50 minutes for this section

This question paper must not be removed from the examination room.

This assessment task constitutes 20% of the course.

Section I

5 marks
Attempt Questions 1–5
Allow about 5 minutes for this section.

Use the multiple-choice sheet for Question 1–5.

---

1 Find the derivative of $f(x) = e^{2x}$ with respect to x.

A. $\frac{d}{dx}f(x) = \frac{1}{2}e^{x}$

B. $\frac{d}{dx}f(x) = \frac{1}{2}e^{2x}$

C. $\frac{d}{dx}f(x) = 2e^{x}$

D. $\frac{d}{dx}f(x) = 2e^{2x}$

2 The vertical and horizontal asymptotes of the following function

$$f(x) = \frac{x - 3}{2 - x}$$

are

A. $x = 2, \quad y = -1$

B. $x = 2, \quad y = 1$

C. $x \neq 2, \quad y = -\frac{3}{2}$

D. $x \neq 2, \quad y = \frac{3}{2}$

3 What are the changes in amplitude and period when the function $f(x) = 2\cos\frac{x}{2}$ is transformed into $g(x) = \cos\frac{x}{4}$?

A. The amplitude is halved and the period is halved

B. The amplitude is halved and the period is doubled

C. The amplitude is doubled and the period is halved

D. The amplitude is doubled and the period is doubled

4 Differentiate $y = \frac{\tan x}{x}$ with respect to x.

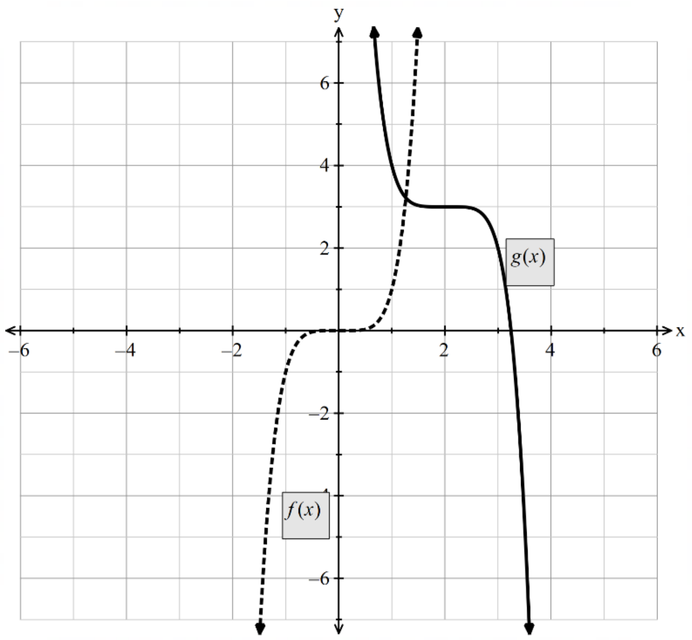
A. $\frac{dy}{dx} = \frac{\sec x^2 - \tan x}{x^2}$

B. $\frac{dy}{dx} = \frac{\tan x - \sec^2 x}{x^2}$

C. $\frac{dy}{dx} = \frac{x\sec^2 x - \tan x}{x^2}$

D. $\frac{dy}{dx} = \frac{\tan x - x\sec^2 x}{x^2}$

5 What are the transformations from $f(x)$ to $g(x)$?



A. Shift to right by 2 units, shift up by 3 units, reflect on x-axis

B. Shift to left by 2 units, shift up by 3 units, reflect on x-axis

C. Shift to right by 2 units, shift up by 3 units, reflect on y-axis

D. Shift to left by 2 units, shift up by 3 units, reflect on y-axis

End of Section I

Section II

36 marks
Attempt Questions 6–15
Allow about 50 minutes for this section.

In Questions 6 – 15, your response should include relevant mathematical reasoning and/or calculations.

Question 6 (6 marks)

Differentiate the following with respect to *x*:

(a) $y = \frac{1}{3}(x+4)^6$ **2**

(b) $y = \log_e(2x^2 + 3)$ **2**

(c) $f(x) = \dfrac{\sin(3x-1)}{\cos x}$ **2**

Question 7 (2 marks)

Solve $\sin(2x) = -\frac{\sqrt{3}}{2}$, for $-\pi \le x \le \pi$. **2**

Question 8 (2 marks)

If $y = xe^{2x}$, prove that $\frac{dy}{dx} - 2y = e^{2x}$. **2**

Do NOT write in this area.

Question 9 (3 marks)

Find the x-values for the points on the graph of $y = \tan x$ at which the normal has slope of $-\frac{1}{2}$, where $0 \leq x \leq \pi$. **3**

Question 10 (3 marks)

Justin invested \$40 000 into an account with an interest rate of 4.5% compounded per annum. After t years, his account's worth is modelled by

$$A(t) = 40000e^{0.045t}$$

Where A is the total investment at the end of t years.

(a) Find the total investment after 10 years. **1**

(b) Find the rate of change in the value of the account at the end of 10 years. Answer to 2 decimal places. **2**

Do NOT write in this area.

Question 11 (2 marks)

The following transformations are applied to the function $y = \ln(x)$: **2**

i) Reflected on x-axis

ii) Vertical dilation of factor $\frac{1}{2}$

iii) Horizontal translation to the left by 2 units

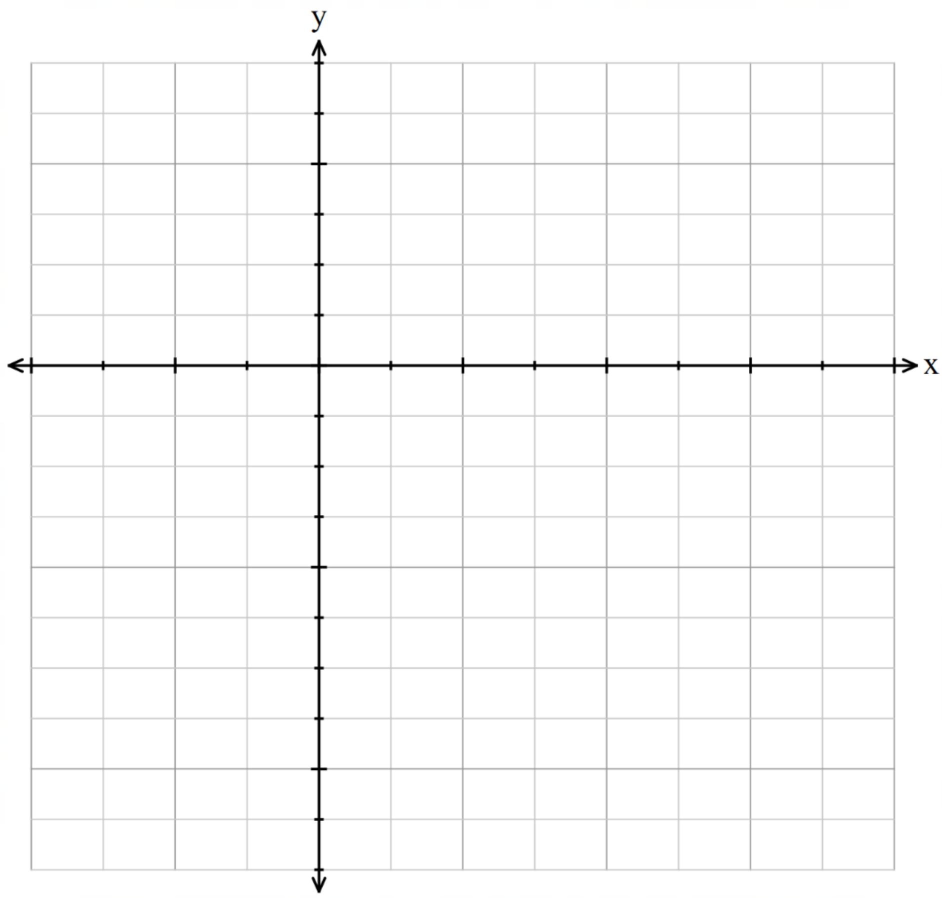
iv) Vertical translation down by 3 units.

State the transformed function.

Question 12 (4 marks)

Given two functions of $f(x) = \ln(x+1)$ and $g(x) = (x-1)^2 - 4$.

(a) Graph $f(x)$ and $g(x)$ on the same graphing template given below, label any key features which may include any asymptotes, $x -$ and $y -$intercepts and vertex. **3**



(b) Hence find the number of solutions for the equation $\ln(x+1) = (x-1)^2 - 4$. **1**

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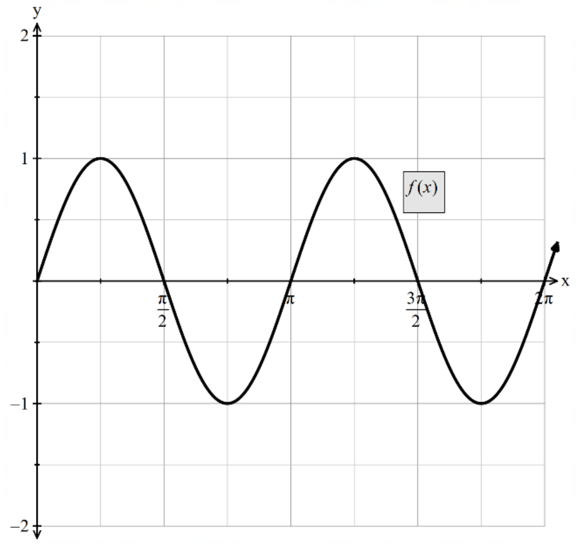
Question 13 (4 marks)

Given the graph $f(x) = sin(ax)$ as shown below:



(a) Determine the value of a, the coefficient of x. **1**

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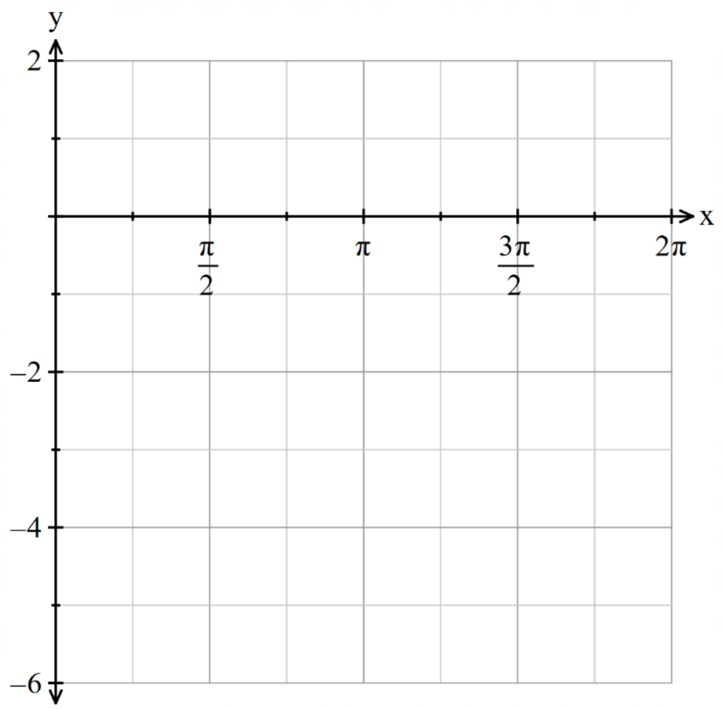
(b) The function $f(x) = sin(ax)$ is then transformed to the function **3**

$$g(x) = 2\,sin\left(a\left(x + \frac{\pi}{4}\right)\right) - 3.$$

Sketch $g(x)$ on the graph template below. labelling minimum and maximum points.



Question 14 (5 marks)

Tom is jumping on a trampoline. He starts at a minimum height of $30cm$ below the trampoline frame. Tom can reach a maximum height of 50cm above the trampoline frame.

The model function is

$$H(t) = \ a \sin\big(2\pi(t - c)\big) + 10,$$

where H is the height in cm above the trampoline frame and t is the time in seconds.

(a) Explain why the amplitude of the function, $a = 40$. **1**

(b) Show that $c = \frac{1}{4}$, if $0 < c < 1$. **2**

Do NOT write in this area.

(c) At what time does Tom first reach a height of 30cm above the trampoline? Answer to 2 decimal places. **2**

Question 15 (5 marks)

Given two functions $f(x) = \dfrac{-x+2}{x^2-4}$ and $g(x) = \dfrac{-x+3}{(x-1)^2-4} - 1$.

(a) Describe the transformation from $f(x)$ to $g(x)$. **2**

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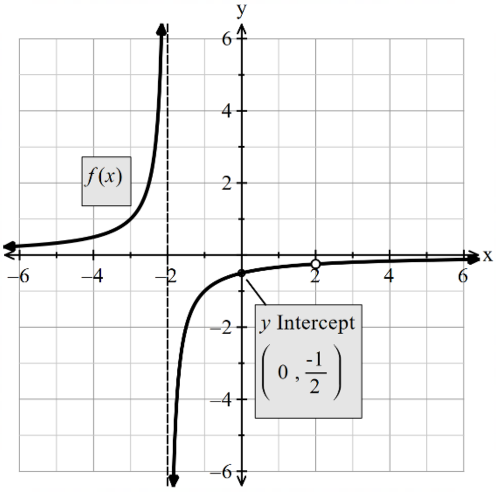
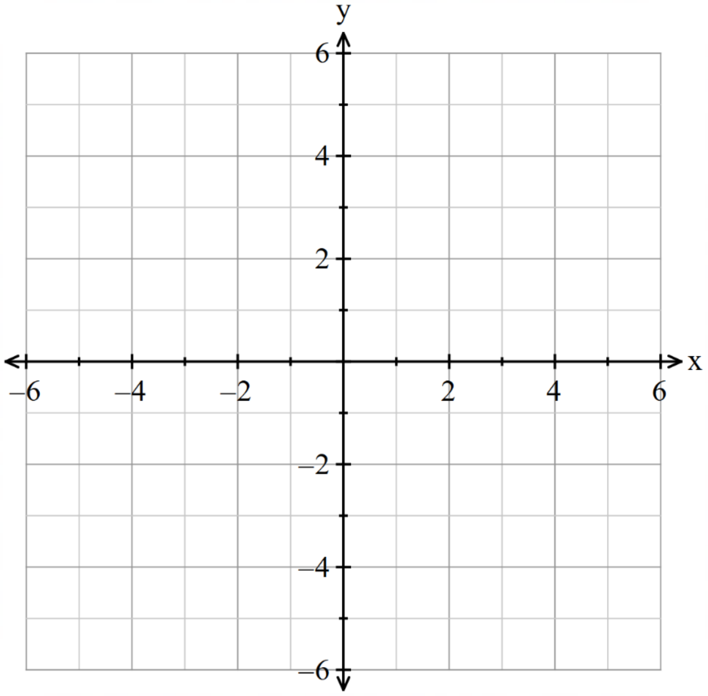
(b) The graph of $f(x) = \dfrac{-x+2}{x^2-4}$ as shown below. **3**



Sketch $g(x) = \dfrac{-x+3}{(x-1)^2-4} - 1$. Label key features.



Do NOT write in this area.

End of paper

Section II extra writing space

If you use this space, clearly indicate which question you are answering.

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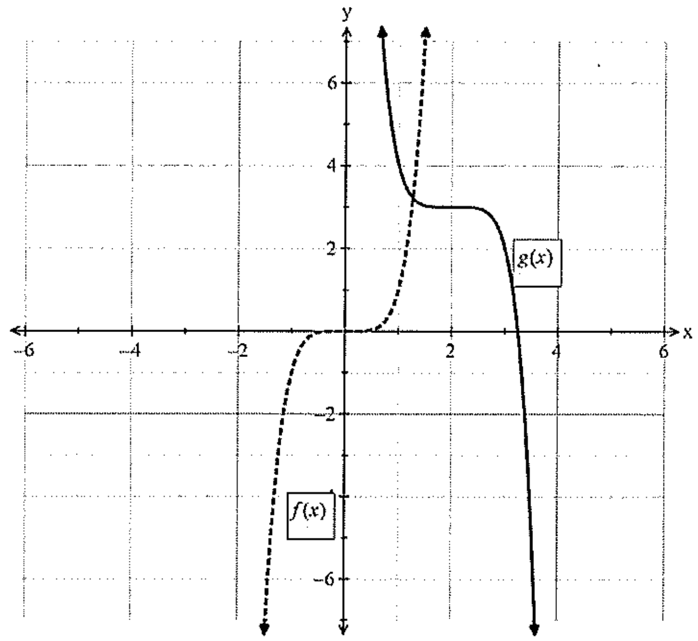


A. Shift to right by 2 units, shift up by 3 units, reflect on x-axis

B. Shift to left by 2 units, shift up by 3 units, reflect on x-axis

C. Shift to right by 2 units, shift up by 3 units, reflect on y-axis

D. Shift to left by 2 units, shift up by 3 units, reflect on y-axis

End of Section I

Do NOT write in this area.

Section II

42 marks
Attempt Questions 6–15
Allow about 50 minutes for this section.

In Questions 6 – 15, your response should include relevant mathematical reasoning and/or calculations.

Question 6 (6 marks)

Differentiate the following with respect to x:

(a) $y = \frac{1}{3}(x+4)^6$ **2**

$\frac{dy}{dx} = \frac{6}{3}(x+4)^5$

$= 2(x+4)^5$

(b) $y = \log_e(2x^2+3)$ **2**

$y = \ln(2x^2+3)$

$\frac{dy}{dx} = \frac{4}{2x^2+3}$

(c) $f(x) = \frac{\sin(3x-1)}{\cos x}$ **2**

$f'(x) = \frac{3\cos(3x-1)\cos x - (\sin 3x-1)\sin x}{\cos^2 x}$

$= \frac{3\cos(3x-1)\cos x + (\sin 3x-1)\sin x}{\cos^2 x}$

Question 7 (2 marks)

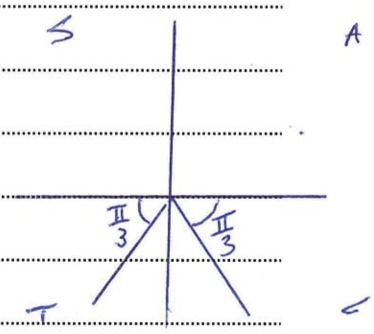
Solve $\sin(2x) = -\frac{\sqrt{3}}{2}$, for $-\pi \leq x \leq \pi$. **2**

$2x = \pi + \frac{\pi}{3}, \; 2\pi - \frac{\pi}{3}$

$2x = \frac{4\pi}{3}, \; \frac{5\pi}{3}$

$x = \frac{4\pi}{6}, \; \frac{5\pi}{6}$

$x = \frac{2\pi}{3}, \; \frac{5\pi}{6}, \; -\frac{2\pi}{3}, \; -\frac{5\pi}{6}$



Question 8 (2 marks)

If $y = xe^{2x}$, prove that $\frac{dy}{dx} - 2y = e^{2x}$. **2**

$\frac{dy}{dx} = e^{2x} + 2xe^{2x}$ Since $y = xe^{2x}$

$\therefore \frac{dy}{dx} = e^{2x} + 2y$

$\frac{dy}{dx} - 2y = e^{2x}$

Question 9 (3 marks)

Find the x-value for the points on the graph of $y = \tan x$ at which the normal has slope of $-\frac{1}{2}$, where $0 \leq x \leq \pi$. 3

slope of normal $= -\frac{1}{2}$ $\therefore y' = 2$

$y = \tan x$

$y' = \sec^2 x$

$2 = \frac{1}{\cos^2 x}$

$\cos^2 x = \frac{1}{2}$

$\cos x = \pm \frac{\sqrt{2}}{2}$

$\therefore x = \frac{\pi}{4}$, or $\pi - \frac{\pi}{4}$

$x = \frac{\pi}{4}$ or $\frac{3\pi}{4}$.

Question 10 (3 marks)

Justin invested \$40 000 into an account with an interest rate of 4.5% compounded per annum. After t years, his account's worth is modelled by

$$A(t) = 40000e^{0.045t}$$

Where A is the total investment at the end of t years.

(a) Find the total investment after 10 years. 1

$A(t) = 40000e^{0.045t}$

$A(10) = 40000e^{0.045(10)}$

$A(10) = 62732.4874$

$\doteq \$62732.50$

(b) Find the rate of change in the value of the account at the end of 10 years. Answer to 2 decimal places. 2

$A'(t) = 40000[e^{0.045t} \cdot (0.045)]$

$A'(t) = 1800e^{0.045t}$

$A'(10) = 1800e^{0.045(10)}$

$\doteq \$2822.96/\text{year}$.

Account is increasing at a rate of \$2822.96/year at the end of 10 years.

Question 11 (2 marks)

The following transformations are applied to the function $y = \ln(x)$: **2**

i) Reflected on x-axis

ii) Vertical dilation of factor $\frac{1}{2}$

iii) Horizontal translation to the left by 2 units

iv) Vertical translation down by 3 units.

State the transformed function.

(i) $y = -\ln x$

(ii) $2y = -\ln x$

$y = -\frac{1}{2}\ln x$

(iii) $2y = -\ln(x+2)$

$y = -\frac{1}{2}\ln(x+2)$

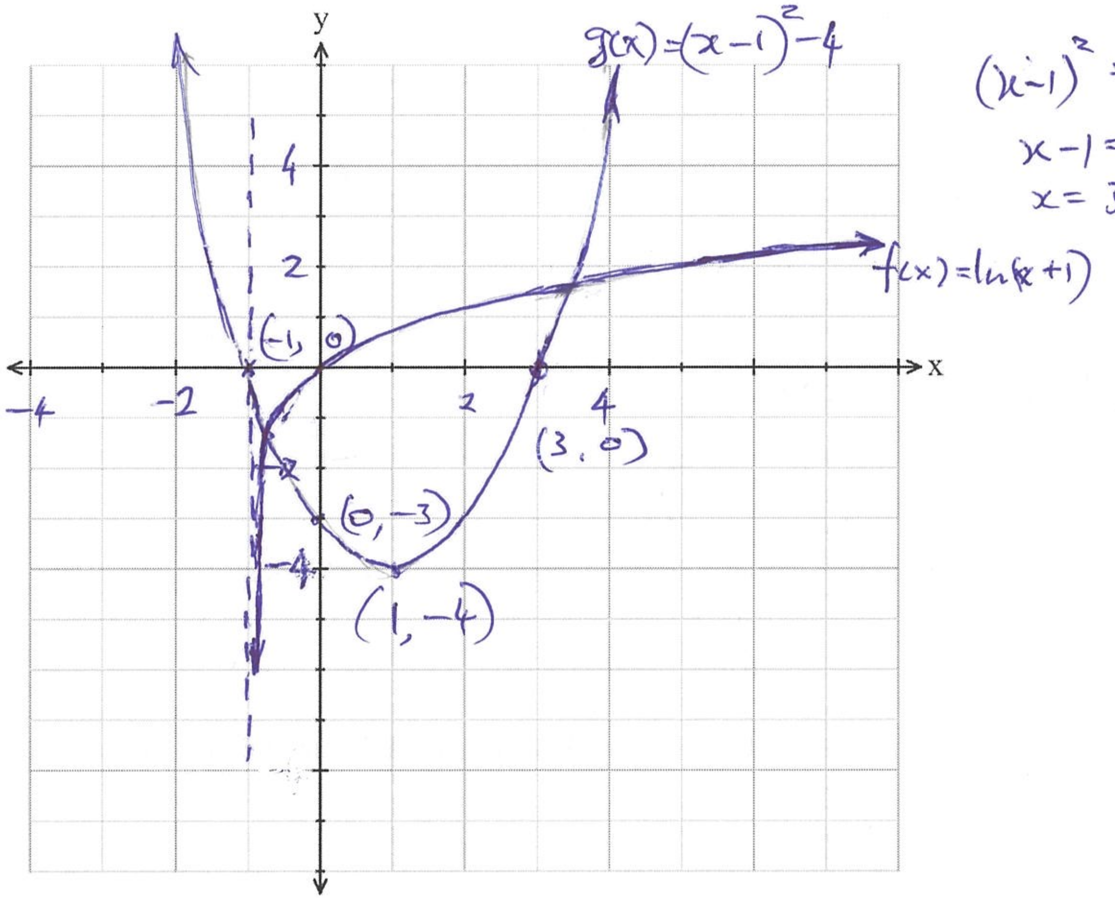
(iv) $y = -\frac{1}{2}\ln(x+2) - 3$

Do NOT write in this area.

Question 12 (4 marks)

Given two functions of $f(x) = \ln(x+1)$ and $g(x) = (x-1)^2 - 4$.

(a) Graph $f(x)$ and $g(x)$ on the same graphing template given below, labelling key features of both graphs. **3**



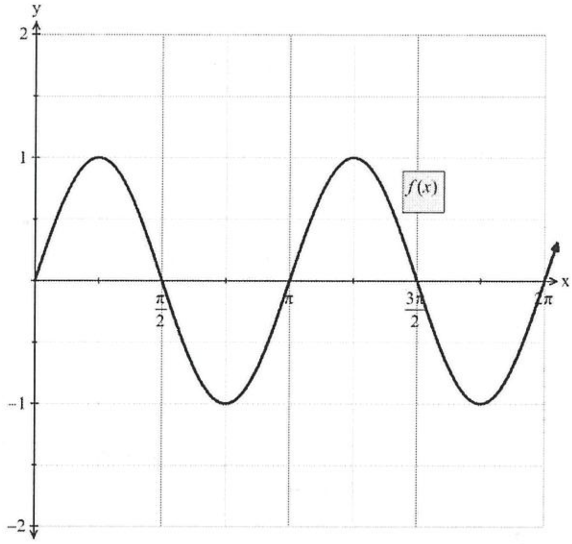
$(x-1)^2 = 4$
$x - 1 = \pm 2$
$x = 3, -1$

(b) Hence find the number of solutions for the equation $\ln(x+1) = (x-1)^2 - 4$. **1**

2 solutions.

Question 13 (4 marks)

Given the graph $f(x) = sin(ax)$ as shown below:



(a) Determine a **1**

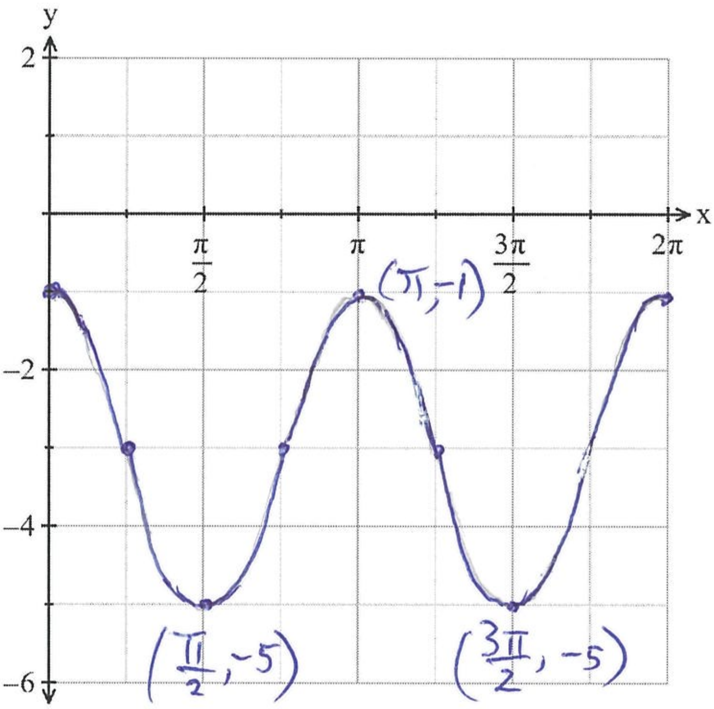
$\frac{2\pi}{2} = \pi$

$a = 2$

(b) The function $f(x) = sin(ax)$ is then transformed to the function **3**

$$g(x) = 2\,sin\left(a\left(x + \frac{\pi}{4}\right)\right) - 3.$$

Sketch $g(x)$ on the graph template below. labelling minimum and maximum points.



Question 14 (5 marks)

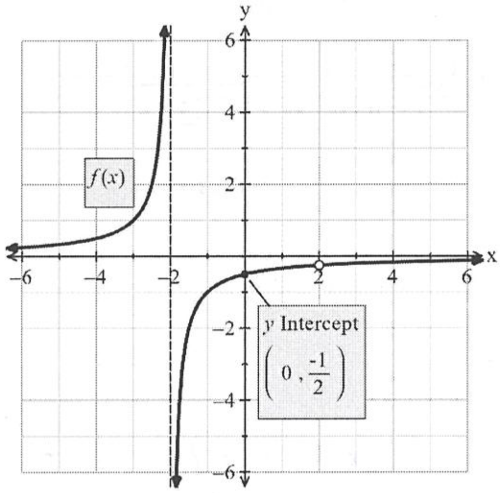
Given two functions $f(x) = \dfrac{-x+2}{x^2-4}$ and $g(x) = \dfrac{-x+3}{(x-1)^2-4} - 1$.

(a) Describe the transformation from $f(x)$ to $g(x)$. **2**

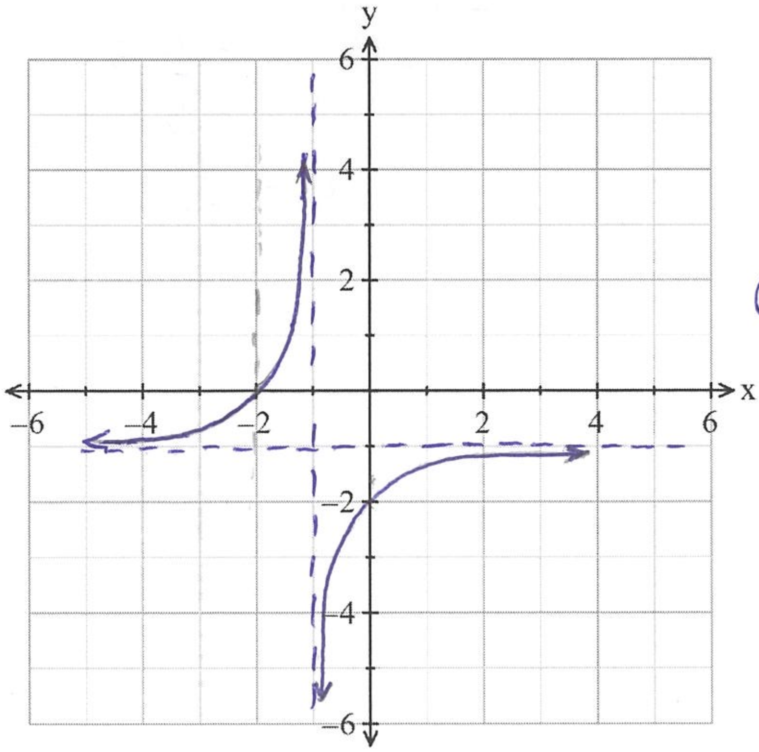
$g(x) = \dfrac{-(x-1)+2}{(x-1)^2-4} - 1$

Horizontal translate to right by 1 unit then vertical translate down by 1 unit.

(b) The graph of $f(x) = \dfrac{-x+2}{x^2-4}$ as shown below. **3**



Sketch $g(x) = \dfrac{-x+3}{(x-1)^2-4} - 1$. Label key features.



x-intercept is $x=-2$

y-intercept is $y=-2$

Question 15 (5 marks)

Tom is jumping on a trampoline. He starts at a minimum height of $30cm$ below the trampoline frame. Tom can reach a maximum height of 50cm above the trampoline frame.

The model function is

$$H(t) = a\sin(2\pi(t-c)) + 10,$$

where H is the height in cm above the trampoline frame and t is the time in seconds.

(a) Explain why the amplitude of the function, $a = 40$. **1**

Min height $= -30$
max height $= 50$
Amplitude $= \frac{50-(-30)}{2}$
$= \underline{40\,cm}$

(b) Show that $c = \frac{1}{4}$, if $0 < c < 1$. **2**

when $t = 0$, $40\sin(2\pi \times (-c)) + 10 = -30$
$\sin(2\pi \times (-c)) = \frac{-40}{40}$
$2\pi \times (-c) = \sin^{-1}(-1)$
$-2\cancel{\pi}c = -\frac{1}{2}\cancel{\pi}$
$c = -\frac{1}{2} \times -\frac{1}{2}$
$c = \underline{\frac{1}{4}}$

(d) At what time does Tom first reach a height of 30cm above the trampoline? Answer to 2 decimal places. **2**

$30cm = 40cm \sin(2\pi(t-\frac{1}{4}))+10$

$\frac{30-10}{40} = \sin(2\pi(t-\frac{1}{4}))$

$\frac{1}{2} = \sin(2\pi t - \frac{2\pi}{4})$

$\frac{1}{2} = \sin(2\pi t - \frac{\pi}{2})$

$2\pi t - \frac{\pi}{2} = \frac{\pi}{6}$

$2\pi t = \frac{\pi}{6} + \frac{\pi}{2}$

$\cancel{2}\pi t = \frac{2\pi}{3}$

$t = \frac{1}{3}$

End of paper

Do NOT write in this area.