SYDNEY BOYS HIGH SCHOOL

NESA Number:

Name:

Maths Class: Please circle

A	B	1	2
S	U	M	
Z2		Z4	

2024

YEAR 11
TERM 4
HSC TASK 1

Mathematics Advanced

General Instructions

- Reading time – 5 minutes
- Working time – 90 minutes
- Write using black pen
- Only NESA approved calculators may be used
- A reference sheet is provided with this paper
- Marks may **NOT** be awarded for messy or badly arranged work
- All answers, unless otherwise indicated, should be left in simplifies exact form.
- **In Questions 11– 26, show ALL relevant mathematical reasoning and/or calculations**

Total Marks: 64

Section I – 10 marks (pages 2 – 5)
- Attempt Questions 1 – 10
- Allow about 15 minutes for this section

Section II – 54 marks (pages 6 – 23)
- Attempt Questions 11 – 26
- Allow about 75 minutes for this section

Examiner: *External Examiner*

Total mark:

Section I

10 marks

Attempt Questions 1–10

Allow about 15 minutes for this section.

Use the multiple-choice answer sheet for Questions 1–10.

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1 Which of the following is approximately equal to 1°?

A. 0.00175 radians

B. 0.0175 radians

C. 0.175 radians

D. 1.75 radians

2 What is the domain of the function

$$f(x)=\sqrt{x}+\frac{1}{\sqrt{2-x}}$$

A. (0, 2)

B. [0, 2)

C. (0, 2]

D. [0, 2]

3 Which of the following functions has a horizontal tangent at (0, 0)?

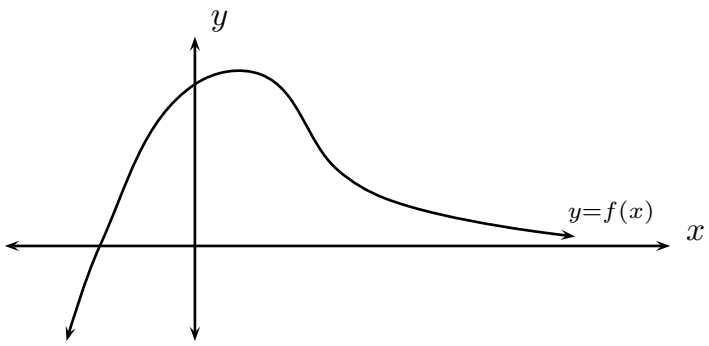
A. $y=x^{-\frac{1}{3}}$

B. $y=x^{\frac{1}{3}}$

C. $y=x^{\frac{3}{4}}$

D. $y=x^{\frac{4}{3}}$

4 A function $f(x)$ is graphed below.
The x-axis is an asymptote for the function.



Which of the following statements is true?

A. $f(x)$ has a global minimum at $y = 0$

B. $f(x)$ has an asymptote at $x = 0$

C. $f(x)$ has a single point of inflection

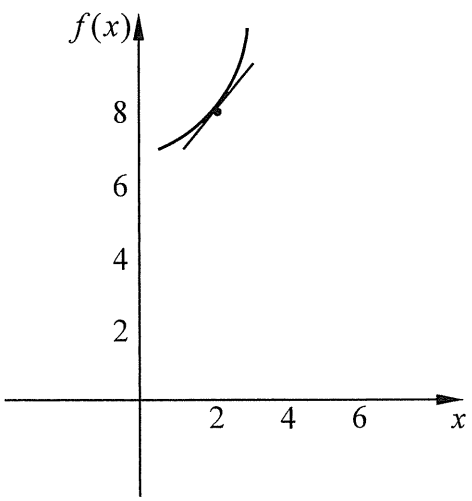
D. $f(x)$ has two stationary points

5 A function f has the following properties:

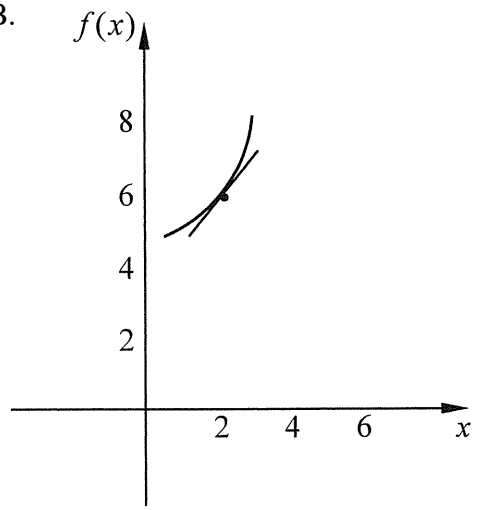
$$f(2)=8 \qquad f'(2)=6 \qquad f''(2)=2$$

Which sketch below matches the graph of the function near $x = 2$?

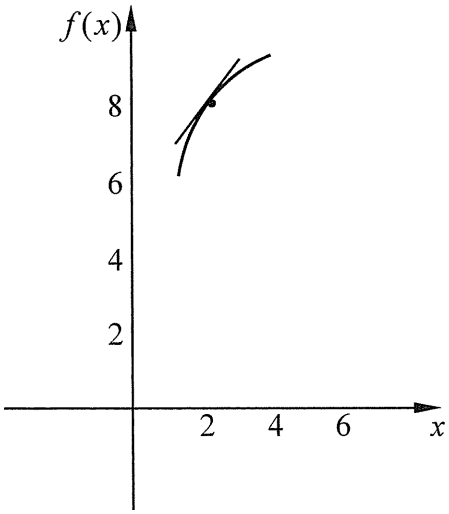
A.



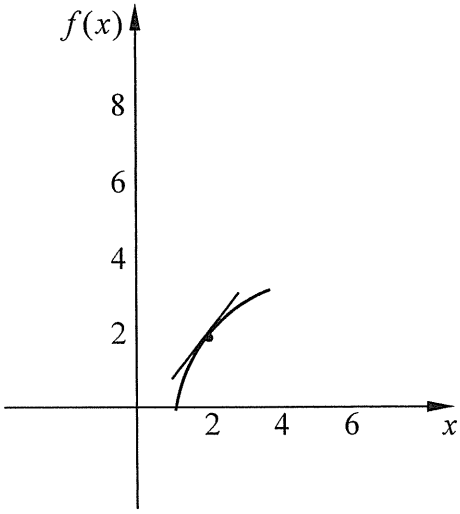
B.



C.



D.



6 Two events, A and B are independent, and $P(A)$ and $P(B)$ are both non-zero.

Which of the following expressions gives the probability that event A occurs and then event B also occurs?

A. $P(B \mid A)$

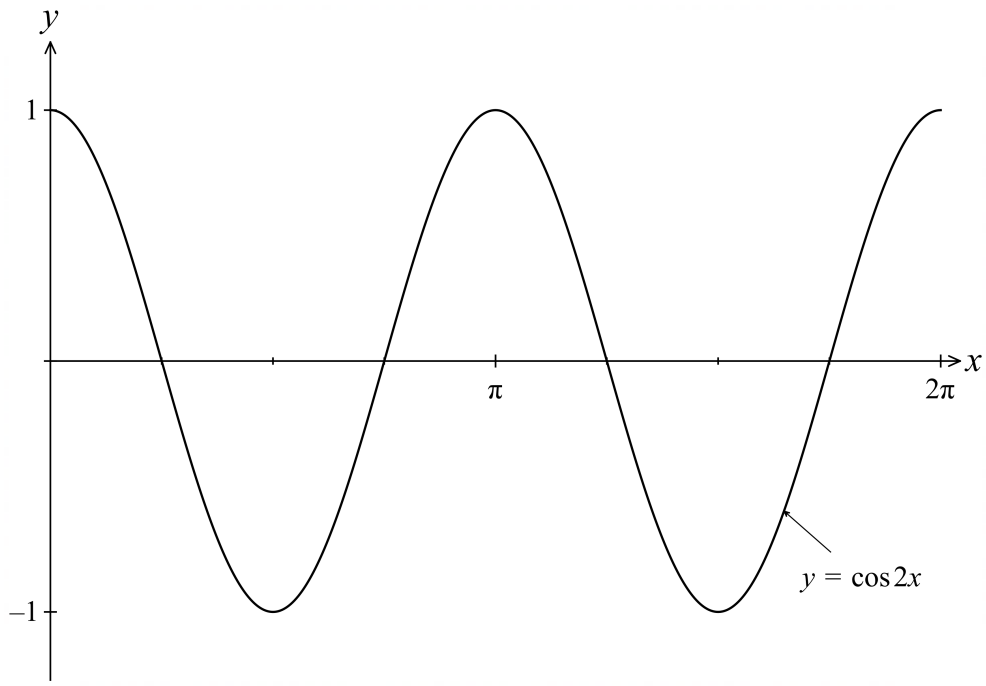
B. $P(A \mid B)$

C. $P(A \cup B)$

D. $P(A)P(B)$

7 How many solutions does $6\cos 2x = x$ have for $0 \le x \le 2\pi$?

The graph below is that of $y = \cos 2x$.



A. 1

B. 2

C. 3

D. 4

8 The values for two continuous functions, f and g, and their derivatives are given in the tables below.

	$x = 0$	$x = 2$
$f(x)$	2	–1
$f'(x)$	–1	2

	$x = 0$	$x = 2$
$g(x)$	1	–1
$g'(x)$	0	3

If $h(x) = f(g(x))$ what is the value of $h'(0)$?

A. –3

B. –1

C. 0.

D 3

9 How many real solutions x are there to the equation $x|x| + 1 = 3|x|$?

A. 1 B. 2

C. 3 D. 4

[Note that $|x|$ is equal to x if $x \geq 0$, and equal to $-x$ otherwise]

10 For $f(x)$, a differentiable function, $f'(x) > 0$ and $f''(x) < 0$.

If a and b are in the domain of $f(x)$ and $a < b$ which statement is NOT true?

A. $f(a) < f(b)$ B. $0 < f'(a) < f'(b)$

C. $f'(b) < \dfrac{f(b) - f(a)}{b - a}$ D. $f'(a) > \dfrac{f(b) - f(a)}{b - a}$

NESA Number:

Mark:

Year 11 Mathematics Advanced

TERM 4 HSC TASK #1

Part A

All relevant mathematical reasoning and/or calculations need to be shown

Solutions that rely wholly on calculator technology may not necessarily receive full marks.

Work that is written in the non-lined spaces will not be marked.

Section II

Part A – 20 marks

Attempt all questions.

Answer each question in the space provided.

A blank page is provided at the end of each part to allow rewriting of a question.

Your response should include relevant mathematical reasoning and/or calculations.

Question 11 (2 marks)

What is the equation of the tangent to $y = x^2 - 3$ at $x = -1$? **2**

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Question 12 (2 marks)

Solve $2x^2 - 5x + 2 \geq 0$ **2**

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Question 13 (2 marks)

On the graph of $y = f(x)$ there is a point (2, –3). **2**

What is the corresponding point on $y = -f\left(\frac{x}{2}\right) - 5$?

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Question 14 (2 marks)

If $f'(x) = 2x^3 + 5$, find $f(x)$ if $f(0) = 2$. **2**

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Question 15 (4 marks)

For events A and B from a sample space, $P(A \mid B) = \frac{3}{4}$ and $P(B) = \frac{1}{3}$.

(a) Calculate $P(A \cap B)$. **1**

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(b) Calculate $P(\overline{A} \cap B)$, where $\overline{A}$ denotes the complement of A. **1**

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(c) If A and B are independent, calculate $P(A \cup B)$. **2**

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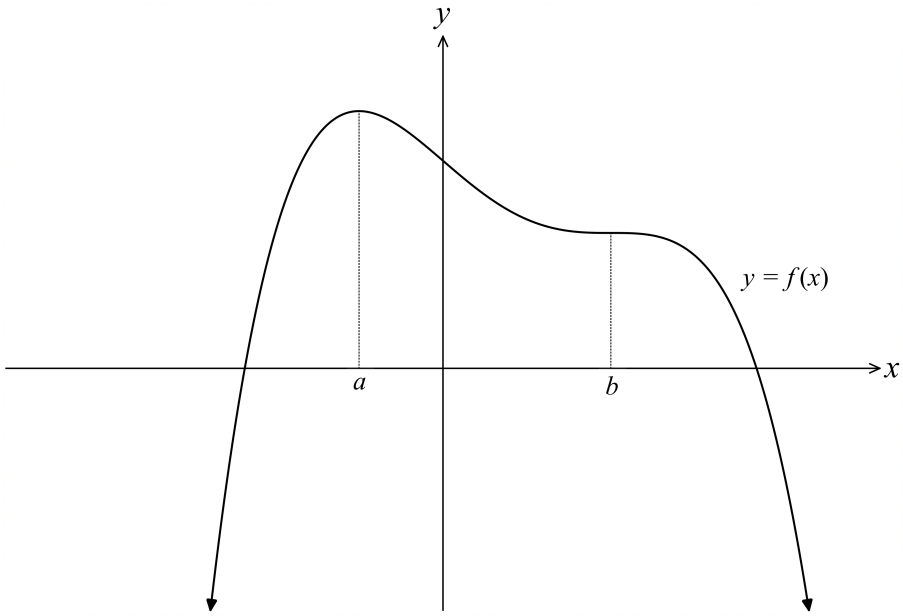
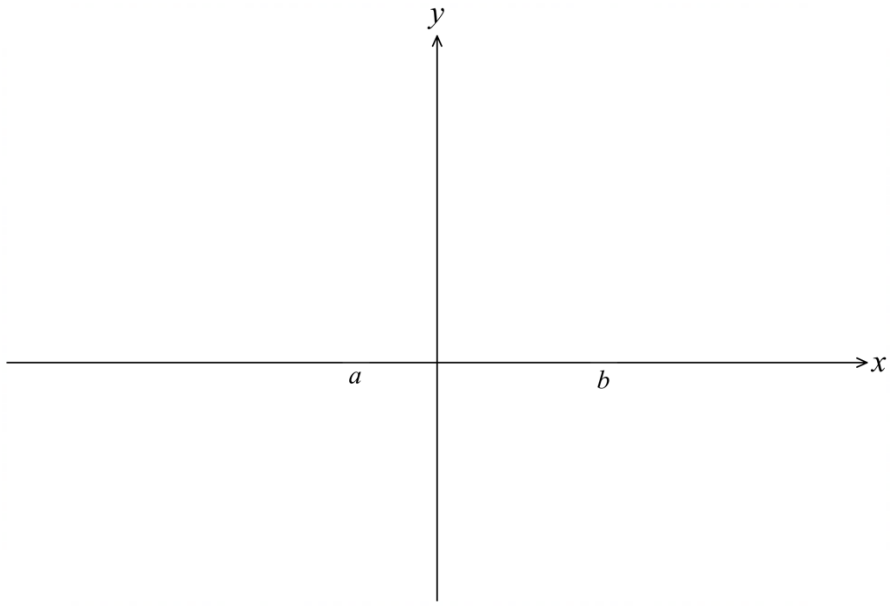
Question 16 (2 marks)

The following diagram shows the graph of $y = f(x)$. **2**

A maximum occurs when $x = a$ and a horizontal point of inflexion when $x = b$.

Sketch, on the number plane below, a possible graph of $y = f'(x)$.





Question 17 (3 marks)

Let $f(x) = \sqrt{2-x}$ and $g(x) = \sqrt{x}$.

(a) Find the rule for the composite function $f(g(x))$. **1**

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(b) Find the domain of $f(g(x))$. **2**

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Question 18 (3 marks)

Consider the function $f(x) = \dfrac{1}{(x+4)^2}$.

(a) Find $f''(x)$. **2**

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(b) Hence, explain why $y = f(x)$ is concave up for all real except $x = -4$. **1**

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End of Part A

Section II Part A – ONLY extra writing space

If you use this space, clearly indicate which question you are answering.

NESA Number:

Mark:

Year 11 Mathematics Advanced

TERM 4 HSC TASK #1

Part B

ALL relevant mathematical reasoning and/or calculations need to be shown

Solutions that rely wholly on calculator technology may not necessarily receive full marks.

Work that is written in the non-lined spaces will not be marked.

Section II

Part B – 17 marks

Attempt all questions.

Answer each question in the space provided.

A blank page is provided at the end of each part to allow rewriting of a question.

Your response should include relevant mathematical reasoning and/or calculations.

Question 19 (4 marks)

In a bag there are two fair coins, for which P(H) = 0.5, and a trick coin with a head on both sides, hence for the trick coin P(H) = 1.

(a) You take a coin without looking and flip it. What is the probability of getting a heads? **2**

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(b) You take a coin out without looking and flip it, and it comes up heads. What is the probability that you chose the trick coin? **2**

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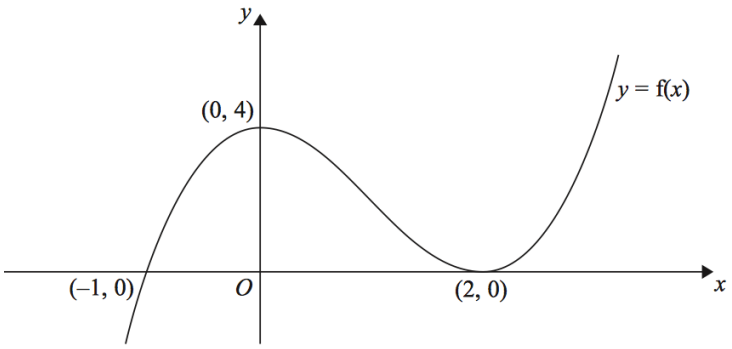
Any work on this page will not be marked

Question 20 (4 marks)

The figure shows a sketch of the curve C with equation $y = f(x)$.



The curve C passes through the point $(-1, 0)$ and touches the x-axis at the point $(2, 0)$.

The curve C has a maximum at the point $(0, 4)$.

(a) The equation of the curve C can be written in the form $y = x^3 + ax^2 + bx + c$, where a, b, and c are integers. **2**

Calculate the values of a, b, and c.

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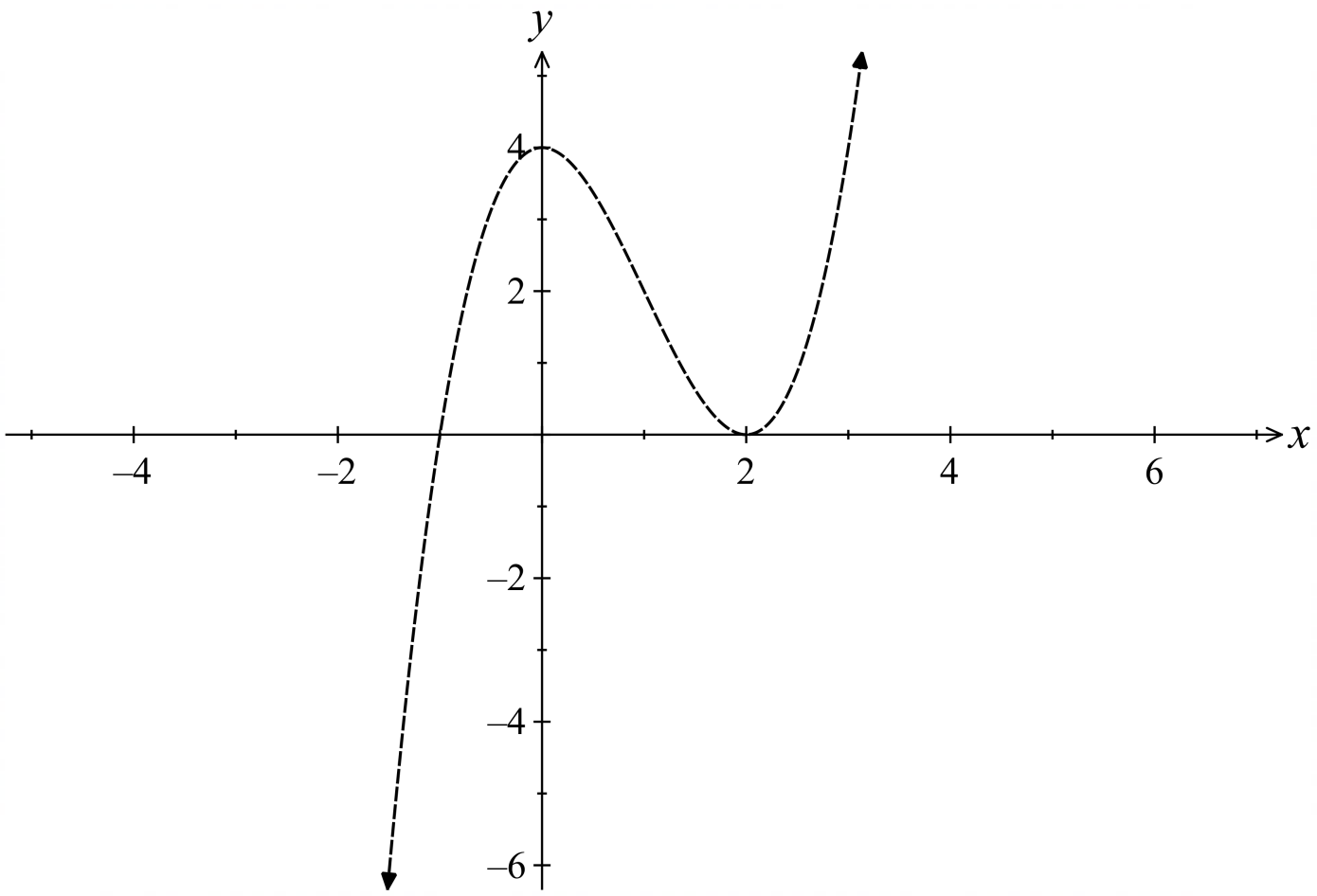
Question 20 continues on page 15

(b) Sketch the curve with equation $y = f\left(\frac{1}{2}x\right)$ on the number plane provided below. **2**

Show clearly the coordinates of all the points where the curve crosses or meets the coordinate axes.



End of Question 20

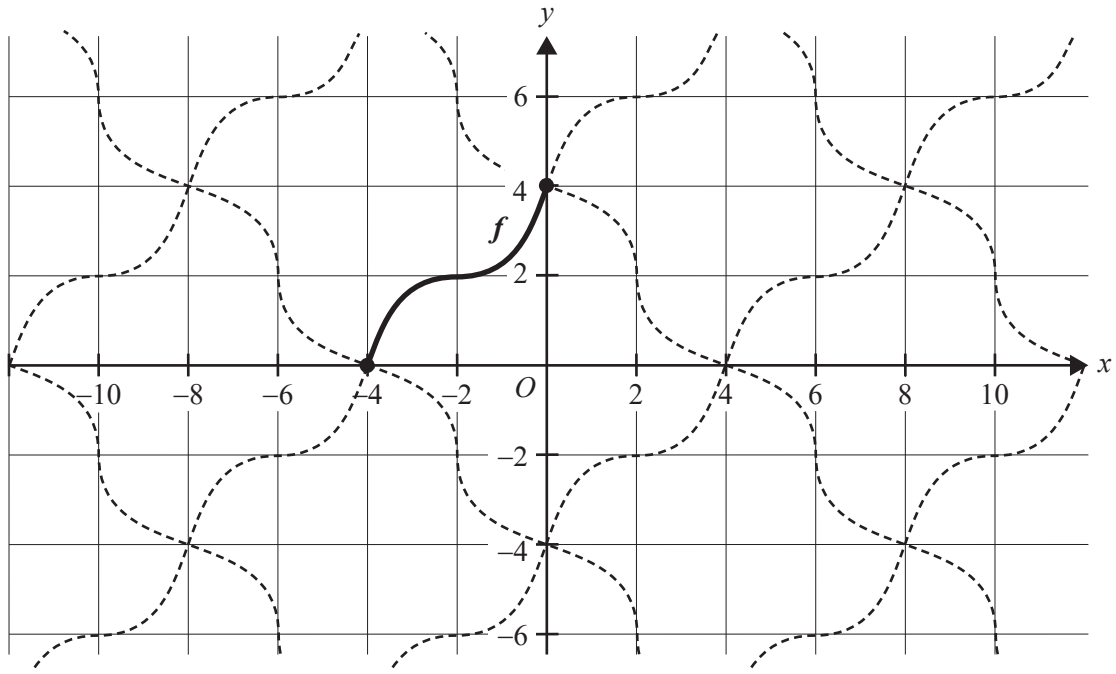
Question 21 (3 marks)

Let f be a cubic function defined on [–4, 0], with

$$f(x) = a(x-b)^3 + c,$$

where a, b, and c are real numbers.

Jorge has created a tiling pattern using transformations of the cubic function f.
On the graph below, the function f is shown in **bold**. The rest of Jorge's tiling pattern is shown with dashed lines, which are all the result of transformations of f.



(a) The transformation of f given by $f_2(x) = f(x-4) - 4$ describes a different edge of one of the tiles. **1**

Trace over the edge described by f_2 on the graph above, and label this edge f_2.

(b) Given that f has a stationary point of inflexion at (–2, 2) and an intercept at (0, 4), Find the values of a, b, and c. **2**

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Question 22 (6 marks)

Consider the curve $y = 3x^3 - 9x + 3$.

(a) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$. **2**

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(b) Find all the stationary points of the curve **2**
Determine the nature of each stationary point.

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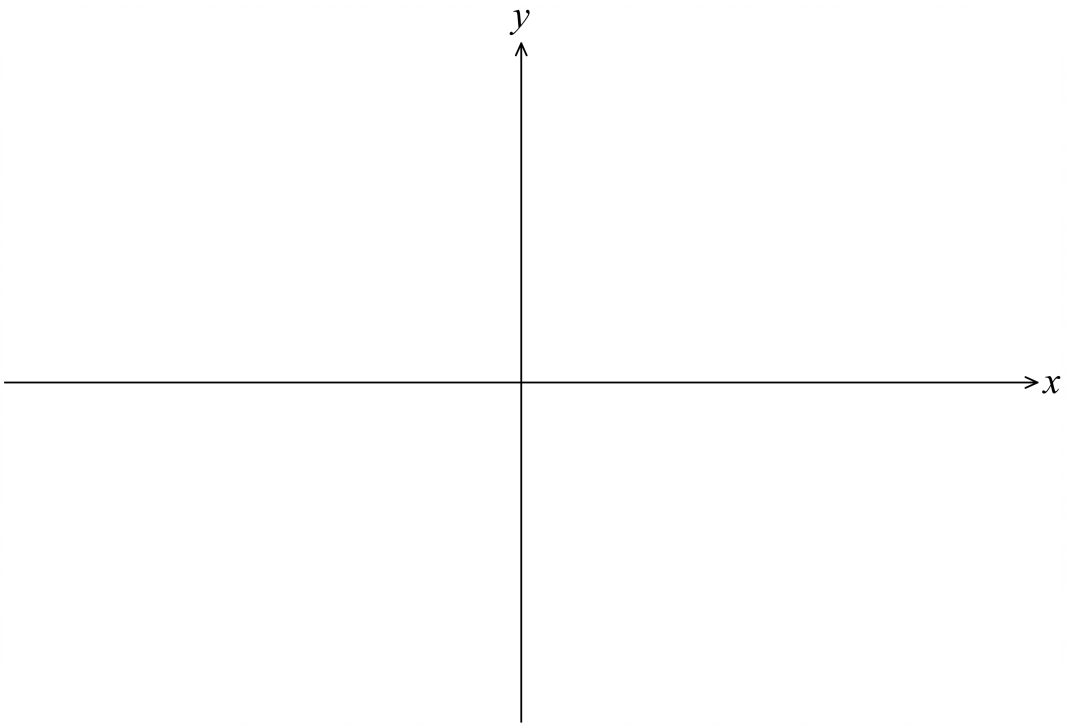
(c) Sketch the curve, clearly indicating all the stationary points and the y-intercept. **2**
Note: You are not required to find the x-intercepts or any points of inflexion.



End of Part B

Section II Part B – ONLY extra writing space

If you use this space, clearly indicate which question you are answering.

NESA Number:

Mark:

Year 11 Mathematics Advanced

TERM 4 HSC TASK #1

Part C

ALL relevant mathematical reasoning and/or calculations need to be shown

Solutions that rely wholly on calculator technology may not necessarily receive full marks.

Work that is written in the non-lined spaces will not be marked.

Section II

Part C – 17 marks

Attempt all questions.

Answer each question in the space provided.

A blank page is provided at the end of each part to allow rewriting of a question.

Your response should include relevant mathematical reasoning and/or calculations.

Question 23 (6 marks)

A curve with equation $y = f(x)$ passes through the point (4, 9).

It is known that $f'(x) = \frac{3\sqrt{x}}{2} - \frac{9}{4\sqrt{x}} + 2,\ x > 0$.

(a) Find $f(x)$. **3**

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(b) Point P lies on the curve. The normal to the curve at P is parallel to the line $2y + x = 0$. **3**
Find the x-coordinate of P.

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Question 24 (3 marks)

The decibel level of a sound may be calculated using the formula

$$L = 10\log_{10}(10^{12} I)$$

where L is the loudness of the sound in decibels (dB) and I is the intensity of the sound.

(a) Find the equation of I in terms of L. **2**

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(b) The loudness of a jet engine is 150 dB. **1**
What is the magnitude of the sound intensity?

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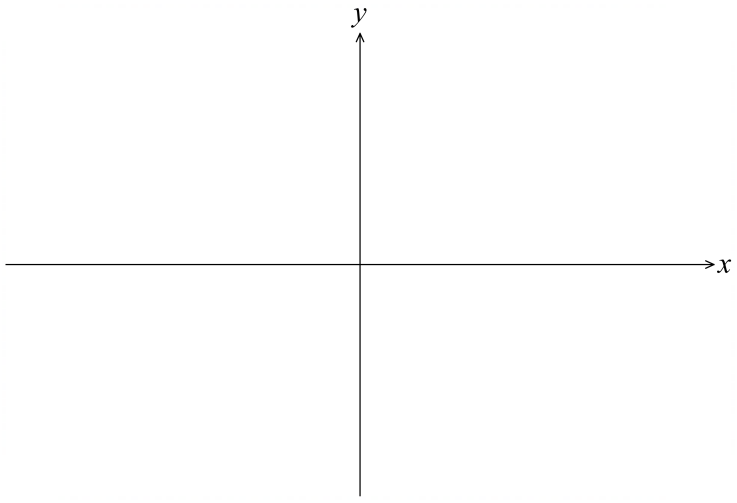
Question 25 (3 marks)

(a) Sketch $y = |x| - 1$ and $y = 2x + 2$ neatly on the same number plane. **2**



(b) Hence, solve the inequality $|x| - 2x > 3$. **1**

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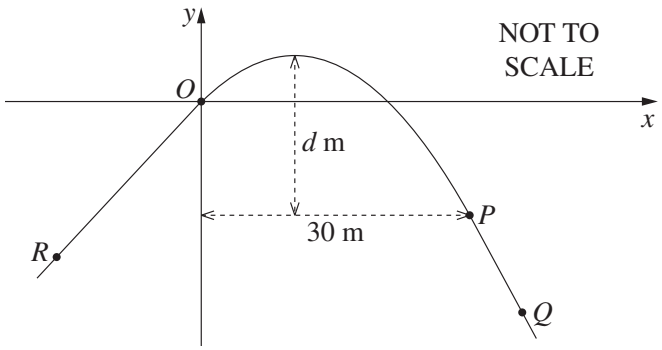
Question 26 (5 marks)

The diagram illustrates the design for part of a roller-coaster track.

The section RO is a straight line with slope 1.2, and the section PQ is a straight line with slope –1.8.

The section OP is a parabola $y = ax^2 + bx$, where a and b are constants.

The horizontal distance from the y-axis to P is 30 m.



In order that the ride is smooth, the straight line sections must be tangent to the parabola at O and at P.

(a) Find the values of a and b so that the ride is smooth. **3**

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(b) Find the distance d, from the vertex of the parabola to the horizontal line through P, as shown on the diagram. **2**

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End of paper

Section II Part C – ONLY extra writing space

If you use this space, clearly indicate which question you are answering.

SYDNEY BOYS HIGH SCHOOL

2024

YEAR 11

HSC TASK 1

Mathematics Advanced .

Sample Solutions

NOTE: This process of checking your mark is about reading the solutions and the comments.

Before putting in an appeal re marking, first consider that the mark is not linked to the amount of writing you have done.

Just because you have shown some 'working' does not justify that your solution is worth any marks.

Students who used pencil, an erasable pen and/or whiteout, may NOT be able to appeal.

MC Answers

1	2	3	4	5	6	7	8	9	10
B	B	D	C	A	D	C	C	C	B

It is very evident that students are not reading carefully the questions!

Read the question! Don't rush or assume that one keyword gives it away.
READ!

Do you want to improve your result in the next task? READ carefully

Section I Multiple Choice Solutions

1 Which of the following is approximately equal to 1°?

A. 0.00175 radians

B. 0.0175 radians

C. 0.175 radians

D. 1.75 radians

$$180^\circ = \pi^c \Rightarrow 1^\circ = \frac{\pi^c}{180}$$

A	2
B	124
C	2
D	3

2 What is the domain of the function

$$f(x) = \sqrt{x} + \frac{1}{\sqrt{2-x}}$$

A. (0, 2)

B. [0, 2)

C. (0, 2]

D. [0, 2]

$$\left.\begin{array}{ll} \sqrt{x} & : x \geq 0 \\ \dfrac{1}{\sqrt{2-x}} & : x < 2 \end{array}\right\} \Rightarrow 0 \leq x < 2$$

A	13
B	98
C	9
D	11

3 Which of the following functions has a horizontal tangent at (0, 0)?

A. $y = x^{-\frac{1}{3}}$

B. $y = x^{\frac{1}{3}}$

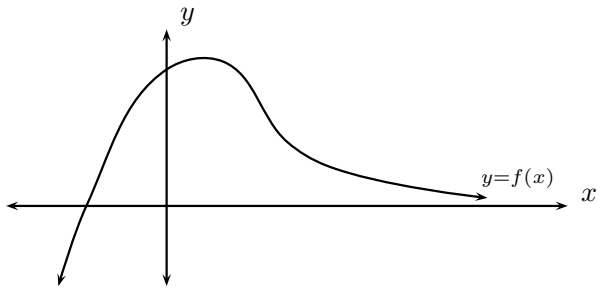
C. $y = x^{\frac{3}{4}}$

D. $y = x^{\frac{4}{3}}$

D is the only one with a power greater than 1.

A	7
B	13
C	17
D	94

4 A function $f(x)$ is graphed below.
The x-axis is an asymptote for the function.



Which of the following statements is true?

A. $f(x)$ has a global minimum at $y = 0$

B. $f(x)$ has an asymptote at $x = 0$

C. $f(x)$ has a single point of inflection

D. $f(x)$ has two stationary points

Know your definitions!

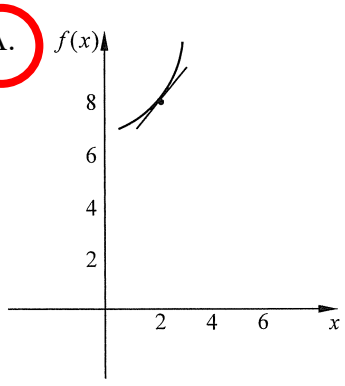
A	2
B	10
C	106
D	13

5 A function f has the following properties:

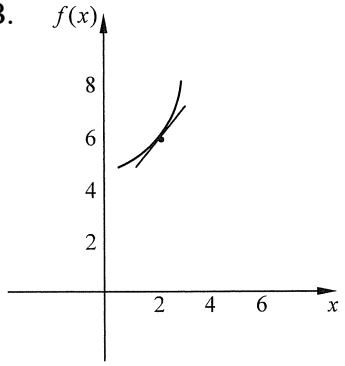
$f(2)=8$ **(2, 8)** $f'(2)=6$ **increasing** $f''(2)=2$ **concave up**

Which sketch below matches the graph of the function near $x = 2$?

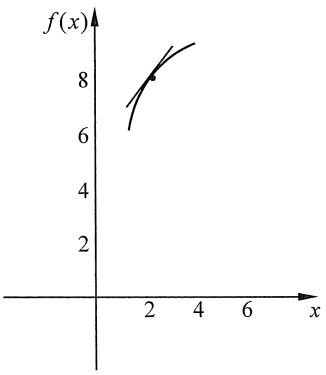
A.



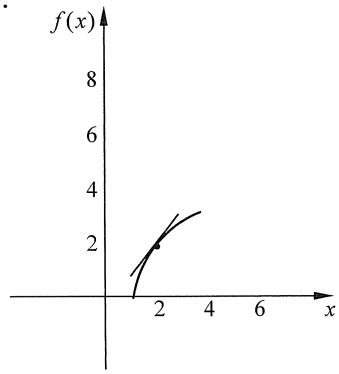
B.



C.



D.



A	119
B	4
C	8
D	0

6 Two events, A and B are **independent**, and $P(A)$ and $P(B)$ are both non-zero.

Which of the following expressions gives the probability that event A occurs **and** then event B also occurs?

A. $P(B|A)$ B. $P(A|B)$

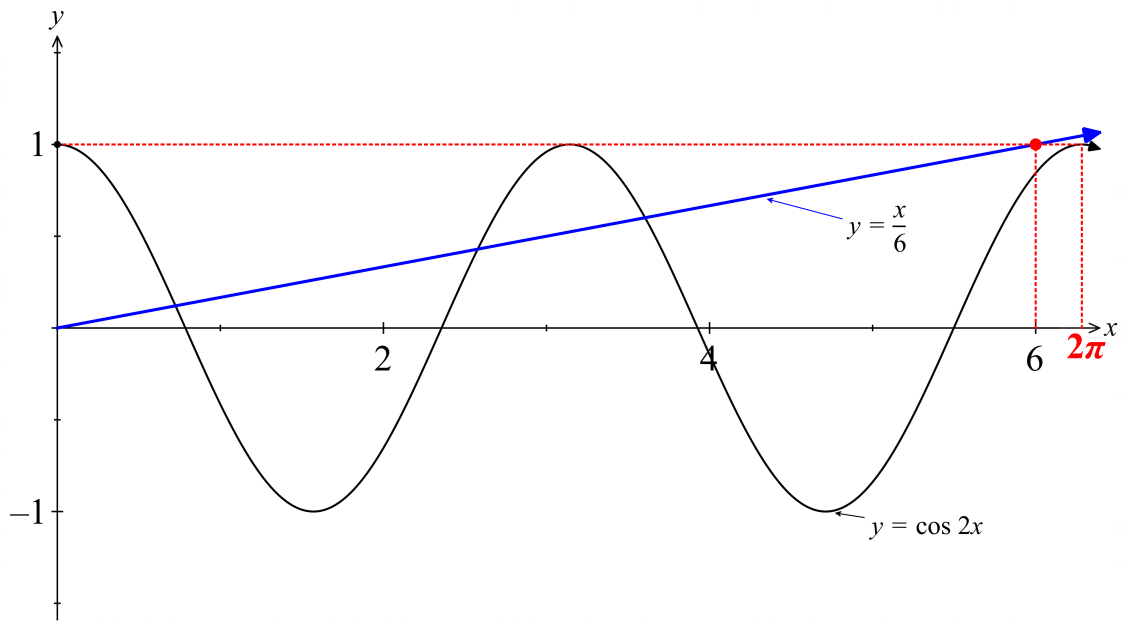
C. $P(A \cup B)$ D. $P(A)P(B)$

Know your definitions and your Reference sheet.

A	23
B	7
C	15
D	86

7 How many solutions does $6\cos 2x = x$ have for $0 \le x \le 2\pi$?

The graph below is that of $y = \cos 2x$.



$6\cos 2x = x \Rightarrow \cos 2x = \dfrac{x}{6}$

A. 1 B. 2

C. 3 D. 4

A	11
B	16
C	39
D	64

8 The values for two continuous functions, f and g, and their derivatives are given in the tables below.

	$x=0$	$x=2$
$f(x)$	2	-1
$f'(x)$	-1	2

	$x=0$	$x=2$
$g(x)$	1	-1
$g'(x)$	0	3

If $h(x)=f(g(x))$ what is the value of $h'(0)$?

A. -3 B. -1

C. 0. D 3

$h'(x) = f'(g(x)) \times g'(x)$

$h'(0) = f'(g(0)) \times g'(0)$

$= f'(1) \times 0$

$= 0$

A	13
B	33
C	79
D	3

9 How many real solutions x are there to the equation $x|x| + 1 = 3|x|$?

A. 1 B. 2

C. 3 D. 4

[Note that $|x|$ is equal to x if $x \geq 0$, and equal to $-x$ otherwise]

If $x \geq 0$ then $x|x| + 1 = 3|x| \Rightarrow x^2 + 1 = 3x$

$\therefore x^2 - 3x + 1 = 0 \quad -(1)$

$\Delta = 5$ and $3 > \sqrt{5}$

If $x < 0$ then $x|x| + 1 = 3|x| \Rightarrow -x^2 + 1 = -3x$

$\therefore x^2 - 3x - 1 = 0 \quad -(2)$

$\Delta = 13$ and $3 \not> \sqrt{13}$

$\therefore$ equation (1) has two positive roots, but equation (2) has one positive root which is invalid.

A	12
B	36
C	24
D	59

10 For $f(x)$, a differentiable function, $f'(x) > 0$ **(increasing)** and $f''(x) < 0$ **(concave down)**.

If a and b are in the domain of $f(x)$ and $a < b$ which statement is NOT true?

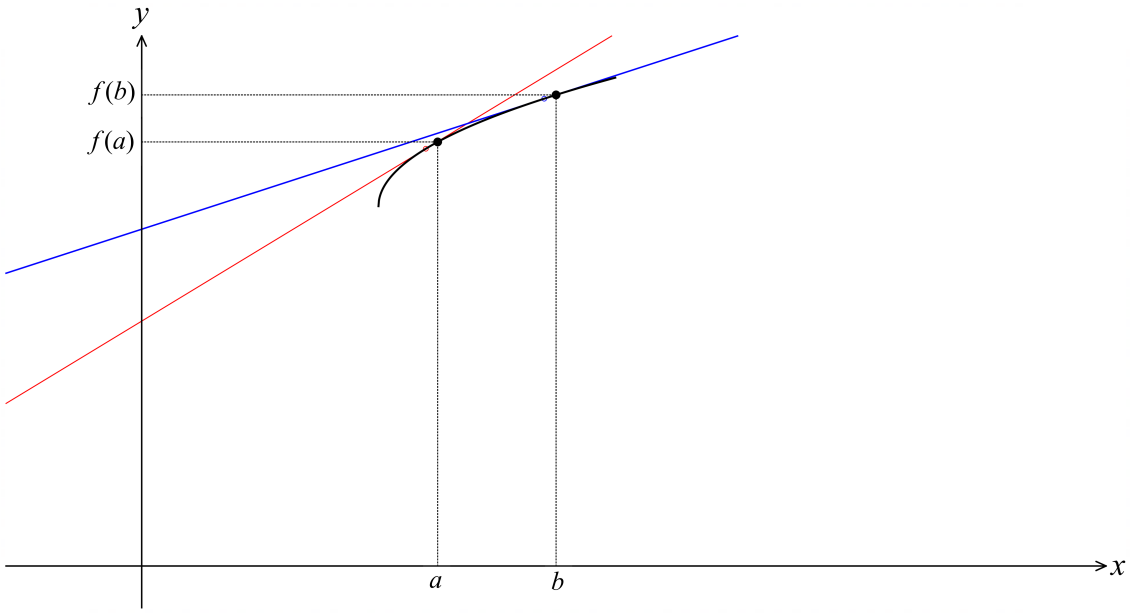
A. $f(a) < f(b)$

B. $0 < f'(a) < f'(b)$

C. $f'(b) < \dfrac{f(b) - f(a)}{b - a}$

D. $f'(a) > \dfrac{f(b) - f(a)}{b - a}$

The graph is decreasing and concave down.



A	7
B	67
C	31
D	25

End of Section I solutions

PART A Solutions & Comments 20 Marks

Question 11 (2 marks)

What is the equation of the tangent to $y = x^2 - 3$ at $x = -1$? 2

$y' = 2x$

At $x = -1$; $m = y' = -2$, $y = (-1)^2 - 3 \therefore y = -2$

Eqn of tangent at $(-1, -2)$ $m = -2$

$y - -2 = -2(x - -1)$

$y + 2 = -2x - 2$

$2x + y + 4 = 0$ or $y = -2x - 4$

Criteria	Marks
Correct solution	2
Correct gradient andcorresponding *y*-intercept	1
Correct gradient or corresponding *y*-intercept	0.5

Marker's comment:

Generally, well done.

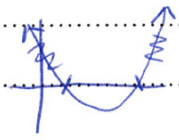
There were some silly mistakes in manipulating algebraic equations.

Question 12 (2 marks)

Solve $2x^2 - 5x + 2 \geq 0$ 2

$(2x - 1)(x - 2) = 0$

$x = 1/2$, $x = 2$



$x \leq 1/2$, $x \geq 2$

Criteria	Marks
Correct solution	2
Correct solution for the quadratic	1

Marker's comment:

Generally, well done.

Question 13 (2 marks)

On the graph of $y = f(x)$ there is a point $(2, -3)$. 2

What is the corresponding point on $y = -f\left(\frac{x}{2}\right) - 5$?

Horizontal dilation by a factor of 2.

$(2, -3) \Rightarrow (2 \times 2, (-3 \times -1) - 5)$

$= (4, -2)$

Criteria	Marks
Correct solutions	2
One correct coordinate	1

Marker's comment:

The horizontal dilation factor is 2, not $\frac{1}{2}$.

Many students did not link the concept of reflecting about the *x*-axis to multiplying the *y*-value by –1

Students needed to read the question carefully as the question asked for a point (*x*, *y*).

Question 14 (2 marks)

If $f'(x) = 2x^3 + 5$, find $f(x)$ if $f(0) = 2$. 2

$f(x) = \frac{2x^4}{4} + 5x + C$

$f(0) = 2 = 0 + 0 + C \quad \therefore C = 2$

$\therefore f(x) = \frac{x^4}{2} + 5x + 2$

Criteria	Marks
Correct solution	2
Correctly completed the process of anti-differentiation without the constant	1

Marker's comment:

Generally, well done.

No half marks

Question 15 (4 marks)

For events A and B from a sample space, $P(A \mid B) = \frac{3}{4}$ and $P(B) = \frac{1}{3}$.

(a) Calculate $P(A \cap B)$. 1

$P(A|B) = \frac{P(A \cap B)}{P(B)}$

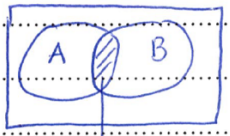
$3/4 \times 1/3 = P(A \cap B)$

$P(A \cap B) = 1/4$

Criteria	Mark
Correct solution	1

Marker's comment:
Generally, well done.
No half marks

(b) Calculate $P(\overline{A} \cap B)$, where $\overline{A}$ denotes the complement of A. 1



$A \cap B = 1/4$

$P(\overline{A} \cap B) = 1/3 - 1/4$

$\therefore P(\overline{A} \cap B) = 1/12$

Criteria	Mark
Correct solution	1

Marker's comment:
Some students were confused with the symbols.
No half marks

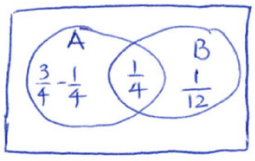
(c) If A and B are independent, calculate $P(A \cup B)$. 2

Since $P(A|B) = 3/4 \therefore P(A) = 3/4$ for independent event.

$\therefore P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \frac{3}{4} + \frac{1}{3} - \frac{1}{4}$

$\therefore P(A \cup B) = \frac{5}{6}$



Criteria	Marks
Correct solution	2
Correctly identified P(A) for independent events	1

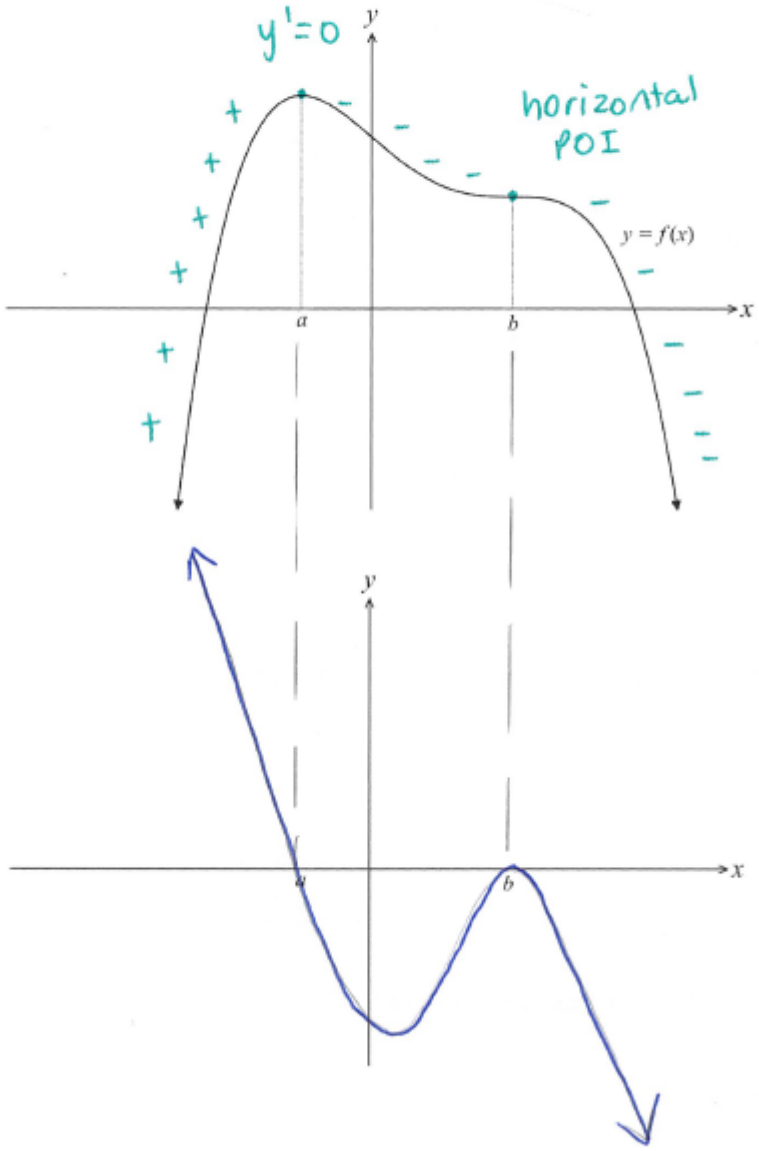
Marker's comment:
Students were confused by independent and mutually exclusive events for the correct solution.

Question 16 (2 marks)

The following diagram shows the graph of $y = f(x)$. **2**

A maximum occurs when $x = a$ and a horizontal point of inflexion when $x = b$.

Sketch, on the number plane below, a possible graph of $y = f'(x)$.



Criteria	Marks
Correct solution	2
Correct indicated the x-intercepts for the derivative graph	1

Marker's comment:

Students can draw their graph with a pencil, then trace it with a pen, and rub the pencil marks off. The graph should be smooth and continuous.

Question 17 (3 marks)

Let $f(x) = \sqrt{2-x}$ and $g(x) = \sqrt{x}$.

(a) Find the rule for the composite function $f(g(x))$. 1

$f(g(x)) = \sqrt{2-\sqrt{x}}$

Criteria	Mark
Correct solution	1

Marker's comment:

Generally, well done.

Some students need to read the question carefully as the question is not asking for domain or range.

No half marks

(b) Find the domain of $f(g(x))$. 2

$g(x) = \sqrt{x}$, $x \geq 0$

For $f(g(x))$, $2 - \sqrt{x} \geq 0$

$\therefore \quad 2 \geq \sqrt{x} \geq 0$

$4 \geq x \geq 0$

$\therefore \quad 0 \leq x \leq 4$

Criteria	Marks
Correct solution	2
For writing $x \leq 4$ only	1.5
Recognising the domain for g(x)	0.5

Marker's comment:

Students need to factor in the domain of g(x).

Some students need to revise their knowledge of algebraic inequalities.

Question 18 (3 marks)

Consider the function $f(x) = \dfrac{1}{(x+4)^2}$.

(a) Find $f''(x)$. 2

$f(x) = (x+4)^{-2}$

$f'(x) = -2(x+4)^{-3}$

$f''(x) = 6(x+4)^{-4}$

$f''(x) = \dfrac{6}{(x+4)^4}$

Criteria	Marks
Correct solution	2
Find the first derivative correctly	1

Marker's comment:

Generally, well done.

(b) Hence, explain why $y = f(x)$ is concave up for all real except $x = -4$. 1

$(x+4)^4 > 0$ and $6 > 0$

$\therefore \dfrac{6}{(x+4)^4} > 0$

$f'' > 0$ is concave up except $x = -4$ where it is the vert asy.

Criteria	Mark
Correct solution	1

Marker's comment:

This is like a "Show that" question and as such students may need to clearly show something that they think is obvious.

As a result, students needed to ***explain why*** the second derivative for this function is positive.
Students who didn't lost a half mark.

Also, students needed to give a reason for the exclusion of $x = -4$ (it is undefined, not a point of inflexion).

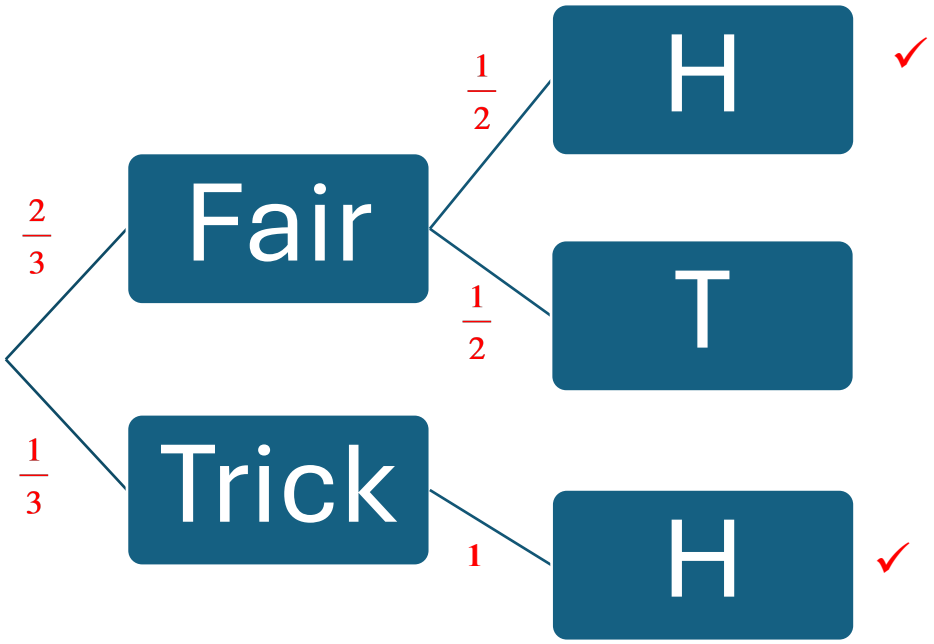
End of Part A solutions

Part B Solutions & Comments — 17 marks

Question 19 (4 marks)

In a bag there are two fair coins, for which P(H) = 0.5, and a trick coin with a head on both sides, hence for the trick coin P(H) = 1.

(a) You take a coin without looking and flip it. What is the probability of getting a heads? **2**



$$P(H) = \frac{2}{3} \times \frac{1}{2} + \frac{1}{3} \times 1 = \frac{2}{3}$$

(b) You take a coin out without looking and flip it, and it comes up heads. **2**
What is the probability that you chose the trick coin?

The question is asking for P(Trick Coin | Heads)

The sample space is the result from Part (a) i.e. $\frac{2}{3}$

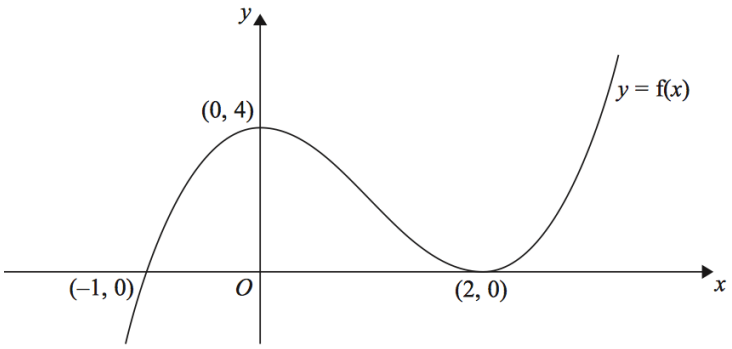
$$\therefore P(\text{Trick Coin} \mid \text{Heads}) = \frac{\frac{1}{3}}{\frac{2}{3}} = \frac{1}{2}$$

Comments:
For a relatively straightforward question, it was generally not done well.
Why? Students did not read the question

In part (a), if students wrote down a bald answer the maximum they could get was 1 mark.
Students who tried to use the formula from the reference generally got it wrong.
Using a diagram is the best way to solve this question, but students didn't rise to it.

Question 20 (4 marks)

The figure shows a sketch of the curve C with equation $y = f(x)$.



The curve C passes through the point $(-1, 0)$ and touches the x-axis at the point $(2, 0)$.

The curve C has a maximum at the point $(0, 4)$.

(a) The equation of the curve C can be written in the form $y = x^3 + ax^2 + bx + c$, where a, b, and c are integers. **2**

Calculate the values of a, b, and c.

$y = (x+1)(x-2)^2$

$= (x+1)(x^2 - 4x + 4)$

$= x^3 - 4x^2 + 4x$

$\quad + x^2 - 4x + 4$

$= x^3 - 3x^2 + 4$

$\therefore a = -3, b = 0, c = 4$

Comments:

There was no need to do any calculus, just know the definitions for a polynomial graph.

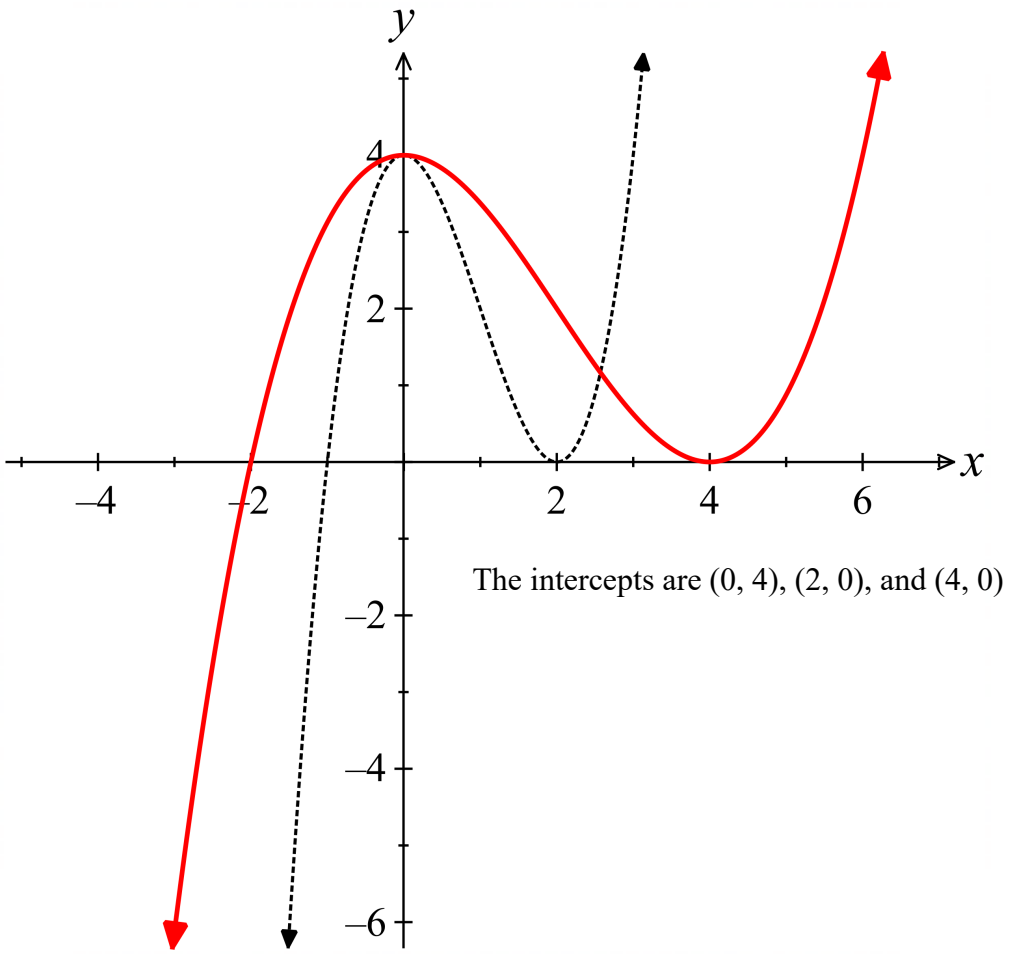
If students got all 3 values right	2 marks
If students only got 2 values right	1 mark
If students got only 1 value right	$\frac{1}{2}$ mark

The solution to Question 20 (b) is on B3

(b) Sketch the curve with equation $y = f\left(\frac{1}{2}x\right)$ on the number plane provided below. **2**

Show clearly the coordinates of all the points where the curve crosses or meets the coordinate axes.

$y = f\left(\frac{1}{2}x\right)$ is a horizontal dilation by a factor of 2.



The intercepts are (0, 4), (2, 0), and (4, 0)

Comments:
Students could not get full marks if they didn't 'show clearly the coordinates'.

Students who dilated vertically could not score any marks.

Students who dilated by a factor of $\frac{1}{2}$ also did not fare well here.

End of Question 20

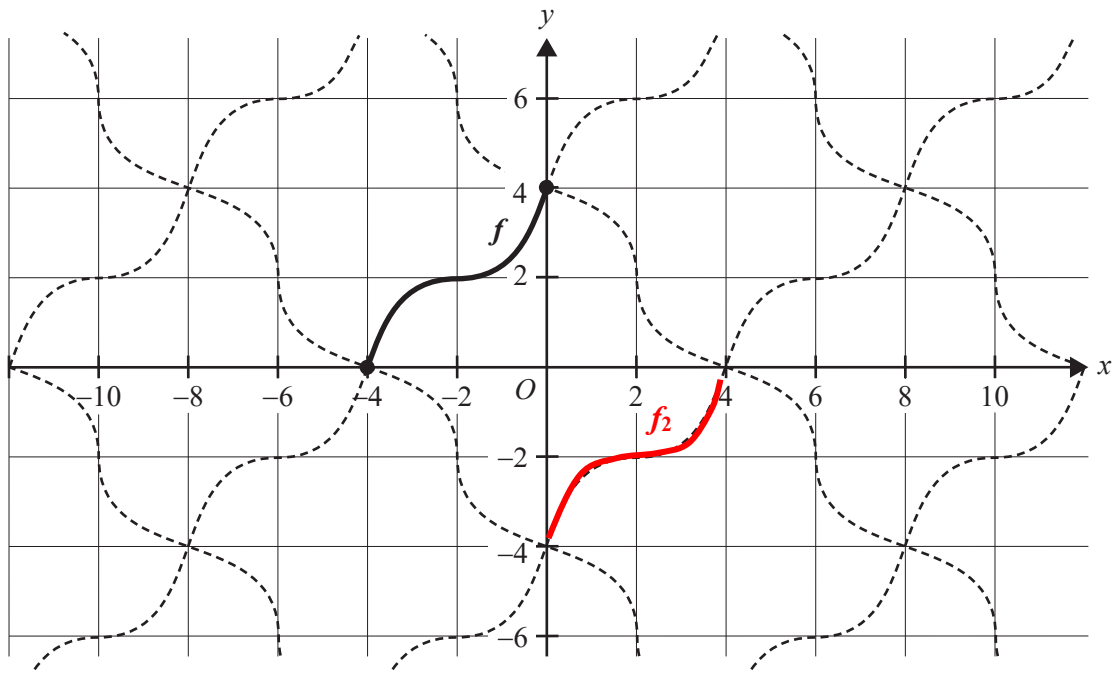
Question 21 (3 marks)

Let f be a cubic function defined on [–4, 0], with

$$f(x) = a(x-b)^3 + c,$$

where a, b, and c are real numbers.

Jorge has created a tiling pattern using transformations of the cubic function f.
On the graph below, the function f is shown in **bold**. The rest of Jorge's tiling pattern is shown with dashed lines, which are all the result of transformations of f.



(a) The transformation of f given by $f_2(x) = f(x-4) - 4$ describes a different edge of one of the tiles. Trace over the edge described by f_2 on the graph above, and label this edge f_2. **1**

$f_2(x)$ is translated 4 to the right and 4 down.

(b) Given that f has a stationary point of inflexion at (–2, 2) and an intercept at (0, 4), Find the values of a, b, and c. **2**

The graph $y = a(x - b)^3 + c$ i.e. a cubic shifted b horizontally and c vertically.

The graph $y = x^3$ has a stationary point of inflexion at (0, 0) and so this is shifted to (b, c) i.e. (–2, 2)

To get a, substitute $x = 0$: $f(0) = 4 = -ab^3 + c$

$\therefore 4 = 8a + 2 \Rightarrow a = ¼$

$\therefore a = ¼, b = -2, c = 2$

Comments

Not done well!

The method outlined above was not used, rather the pages were filled with simultaneous equations

Question 22 (6 marks)

Consider the curve $y = 3x^3 - 9x + 3$.

(a) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$. **2**

$\frac{dy}{dx} = 9x^2 - 9 = 9(x-1)(x+1)$ $\qquad$ $\frac{d^2y}{dx^2} = 18x$

Comments
Extremely well done.

(b) Find all the stationary points of the curve **2**
Determine the nature of each stationary point.

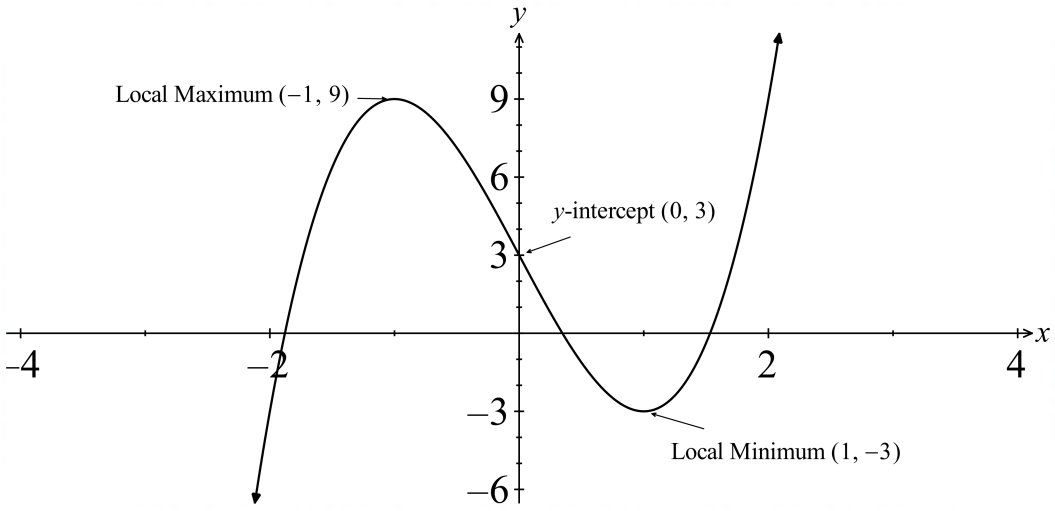
Stationary points are at (1, –3) and (–1, 9).

At (1, –3), $\frac{d^2y}{dx^2} = 18$ so it is a minimum turning point.

At (–1, 9), $\frac{d^2y}{dx^2} = -18$ so it is a maximum turning point.

Comments
It beggars belief that after finding $\frac{d^2y}{dx^2}$ in part (a), that some students would then use the $\frac{dy}{dx}$ method to determine the nature of the stationary points. Possibly these students did not want to finish the paper. That said, students were not penalised for just writing a '+' or '–' or equivalent.

(c) Sketch the curve, clearly indicating all the stationary points and the y-intercept. **2**
Note: You are not required to find the x-intercepts or any points of inflexion.



Comments
Well done.
The curves were generally good.

End of Part B solutions

Section II Part C Solutions and Comments (17 marks)

Attempt all questions.

Answer each question in the space provided.

A blank page is provided at the end of each part to allow rewriting of a question.

Your response should include relevant mathematical reasoning and/or calculations.

Question 23 (6 marks)

A curve with equation $y = f(x)$ passes through the point (4, 9).

It is known that $f'(x) = \dfrac{3\sqrt{x}}{2} - \dfrac{9}{4\sqrt{x}} + 2,\ x > 0.$

(a) Find $f(x)$. **3**

$f(x) = \int \left(\frac{3}{2}x^{\frac{1}{2}} - \frac{9}{4}x^{-\frac{1}{2}} + 2\right) dx$

$= \frac{3}{2}\cdot\frac{2}{3}x^{\frac{3}{2}} - \frac{9}{4}\cdot 2x^{\frac{1}{2}} + 2x + C$

$= x^{\frac{3}{2}} - \frac{9}{2}x^{\frac{1}{2}} + 2x + C$

$f(4) = 4^{\frac{3}{2}} - \frac{9}{2}(4)^{\frac{1}{2}} + 2(4) + C = 9$

$C + 7 = 9$

$\therefore C = 2$

$\therefore f(x) = x\sqrt{x} - \frac{9}{2}\sqrt{x} + 2x + 2$

- Generally well done.
- Most recognised to integrate.
- Some students forgot '+C', so they could not receive full marks.
- Many calculation / arithmetic errors.

(b) Point P lies on the curve. The normal to the curve at P is parallel to the line $2y + x = 0$. **3**
Find the x-coordinate of P.

$y = -\frac{1}{2}x$

$m_N = -\frac{1}{2}$

$\therefore m_T = 2$

$f'(x) = \frac{3}{2}\sqrt{x} - \frac{9}{4\sqrt{x}} + 2 = 2$

$\frac{3}{2}\sqrt{x} - \frac{9}{4\sqrt{x}} = 0$

$\frac{3}{2}\sqrt{x} = \frac{9}{4\sqrt{x}}$

$6x = 9$

$\therefore x = \frac{3}{2}$

- Some students wrote $f'(x) = -\frac{1}{2}$ and made the question a lot harder for themselves.
- Some students had difficulty solving the equation.
- Some students wasted their time trying to find y_P when the question clearly asked for x_P only.

Question 24 (3 marks)

The decibel level of a sound may be calculated using the formula

$$L = 10\log_{10}(10^{12} I)$$

where L is the loudness of the sound in decibels (dB) and I is the intensity of the sound.

(a) Find the equation of I in terms of L. **2**

$\frac{L}{10} = \log_{10}(10^{12} I)$

$10^{12} I = 10^{\frac{L}{10}}$

$I = \frac{10^{\frac{L}{10}}}{10^{12}}$

$= 10^{\frac{L}{10} - 12}$

surprisingly poorly done. students need to revise their log laws.

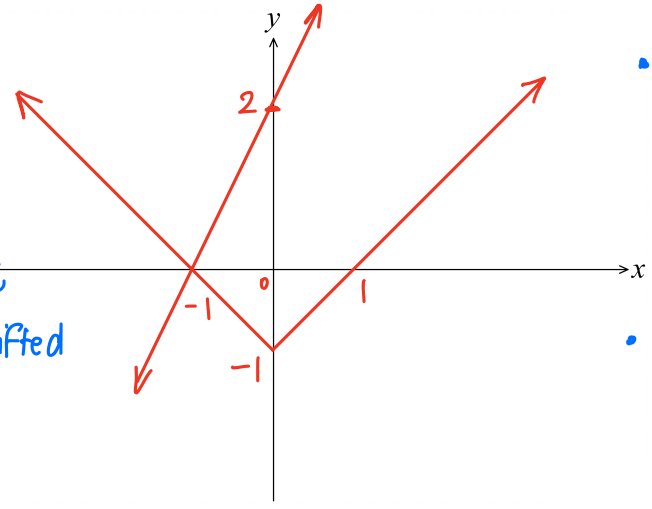
(b) The loudness of a jet engine is 150 dB. **1**
What is the magnitude of the sound intensity?

Generally well done.

$I = 10^{\frac{150}{10} - 12} = 10^{3} = 1000$

Question 25 (3 marks)

(a) Sketch $y = |x| - 1$ and $y = 2x + 2$ neatly on the same number plane. **2**



- Students need to take more care in graph sketching.
- Few errors in translations, $y = |x| - 1$ is $y = |x|$ shifted down by 1 unit.
- Few errors with x and y-intercepts for $y = 2x + 2$.
- students must have POI at $x = -1$ to receive full marks.

(b) Hence, solve the inequality $|x| - 2x > 3$. **1**

$|x| - 1 > 2x + 2$

$\therefore x < -1$

- No half marks.
- Students who solved the inequality algebraically were mostly unsuccessful, regardless, students who used this approach were unable to receive any marks as it was a 'hence' question.

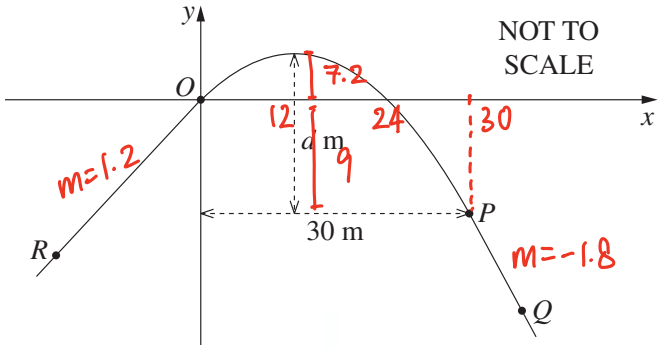
Question 26 (5 marks)

The diagram illustrates the design for part of a roller-coaster track.

The section RO is a straight line with slope 1.2, and the section PQ is a straight line with slope –1.8.

The section OP is a parabola $y = ax^2 + bx$, where a and b are constants.

The horizontal distance from the y-axis to P is 30 m.



In order that the ride is smooth, the straight line sections must be tangent to the parabola at O and at P.

(a) Find the values of a and b so that the ride is smooth. **3**

$\frac{dy}{dx} = 2ax + b$

At $x = 0$, $y' = 2a(0) + b = 1.2$

$\therefore b = 1.2$

At $x = 30$, $y' = 2a(30) + 1.2 = -1.8$

$60a = -3$

$a = -\frac{1}{20}$

$\therefore a = -\frac{1}{20}$, $b = 1.2$

- Few calculation errors, e.g. $a = -20$

(b) Find the distance d, from the vertex of the parabola to the horizontal line through P, as shown on the diagram. **2**

$y = -\frac{1}{20}x^2 + 1.2x$

$Vx = \frac{-b}{2a} = \frac{-1.2}{2(-\frac{1}{20})} = 12$

$Vy = -\frac{1}{20}(12)^2 + 1.2(12) = 7.2$

At P, $x = 30$, $y = -\frac{1}{20}(30)^2 + 1.2(30)$

$= -9$

$\therefore$ Total distance $= 7.2 + 9$

$= 16.2$ m

- Some students misinterpreted 'd'.
- Misconceptions of $Vx = \frac{-b}{a}$ or $\frac{-b}{4a}$.

End of paper