# Mathematics Advanced

## HSC marking feedback 2025

### General feedback

Students should:

- show relevant mathematical reasoning and/or calculations
- read the question carefully to ensure that they do not miss important components of the question
- have a clear understanding of key words in the question and recognise the intent, such as show, solve, evaluate, hence, calculate, derive
- use the Reference Sheet where applicable
- ensure the solution is legible and follows a clear sequence
- engage with any stimulus material provided and refer to it in their response when required by the question
- check that their solution answers the question
- round off numerical solutions only at the final step of the solution
- construct graphs neatly and with precision and display all relevant information as required by the question
- interpret information presented in graphs across a range of contexts
- understand when to use relevant calculator functions
- carefully note any information in the question which supplies units of measurement.

### Section II

### Question 11 (a)

In better responses, students were able to:

- find $t$ using the axis of symmetry
- substitute the value of $t$ into the equation of the quadratic to find $h$.

Areas for students to improve include:

- recognising the difference between intercepts and the vertex.

## Question 11 (b)

In better responses, students were able to:

- use symmetry to find the value of $t$
- substitute $h = 12$ and correctly solve for $t$.

Areas for students to improve include:

- recognising the symmetrical shape of a quadratic
- solving a quadratic by factorising.

## Question 12

In better responses, students were able to:

- differentiate correctly, particularly the term with the negative index
- find the equation of the tangent by collecting the like terms of the equation correctly
- substitute the point and gradient into the equation of a straight line to find $y = 19x - 25$ using either form of the equation of a line.

Areas for students to improve include:

- identifying the most appropriate method of differentiation – working with negative indices
- recognising the difference between finding the gradient of a tangent and locating a maximum or minimum point
- knowing the difference between finding the derivative at a point ($f'(a) = m$) and solving to find value of $x$ that produces a particular derivative, $f'(x) = a$
- correctly substituting values into $y - y_1 = m(x - x_1)$.

## Question 13

In better responses, students were able to:

- form three equations connecting the terms of the geometric sequence, or form two equations connecting the terms $p$ and $q$ to solve for the required pronumerals
- use the formula for the $n^{th}$ term of a geometric sequence to establish the common ratio $r$ and the values of $p$ and $q$
- use the formula for geometric sequence.

Areas for students to improve include:

- recognising a sequence as an arithmetic or geometric sequence
- writing a pattern with the information given to try to establish a common ratio
- using the Reference Sheet to determine the $n^{th}$ term of a geometric sequence.

## Question 14 (a)

In better responses, students were able to:

- demonstrate a strong understanding through accurately selecting the correct terminology to describe both form and direction
- identify that the form is linear and the distribution has a negative direction.

Areas for students to improve include:

- using the correct language for form and/or direction
- refining their knowledge of statistical terms and key words used in the syllabus for bivariate data analysis.

## Question 14 (b)

In better responses, students were able to:

- identify the slope and $\boldsymbol{y}$-intercept for the equation of the least-squares regression line
- clearly interpret the meaning of the slope and $\boldsymbol{y}$-intercept within the context of the scenario, rather than simply explaining what slope and $\boldsymbol{y}$-intercepts represent in general terms.

Areas for students to improve include:

- interpreting the slope and $y$-intercept in context of the question with values shown
- interpreting the slope and $y$-intercept correctly, not just identifying the values
- understanding that the slope is not the same as the correlation coefficient.

## Question 14 (c)

In better responses, students were able to:

- correctly substitute $x = 42$ into the equation to calculate the answer.

Areas for students to improve include:

- showing correct substitution into the formula
- understanding what each variable in an equation represents.

### Question 14 (d)

In better responses, students were able to:

- convert 2 hours to minutes and substitute $x = 120$ minutes into the formula
- identify that the graph goes below zero and cannot have a negative number of minutes of exercise
- recognise 2 hours is outside the dataset
- understand the meaning of the key word ‘extrapolate’.

Areas for students to improve include:

- understanding that the values outside the dataset are not reliable
- knowing extending the line of regression gives the negative answer for time
- knowing the meaning of the key word extrapolate.

### Question 15 (a)

In better responses, students were able to:

- find the value of $k$ correctly and relate it to the amplitude
- find the value of $a$ by correctly using knowledge of periodicity
- substitute the values of $k$ and $a$ into the sine function in terms of $t$.

Areas for students to improve include:

- understanding the amplitude is the coefficient of the sine equation
- understanding that the value of constant $a$ can be calculated knowing that period $= \frac{2\pi}{a}$.

### Question 15 (b)

In better responses, students were able to:

- draw a correct sine graph with period 20 and amplitude 4
- draw half the sine graph in the domain $0 \leq t \leq 10$.

Areas for students to improve include:

- calculating the period of the sine graph
- understanding that only part of the sine graph is drawn as the period is double the given domain
- drawing trigonometric graphs where the horizontal axis is in terms of integers, instead of radians or degrees
- understanding the coefficient of the sine equation is the amplitude of the graph.

## Question 15 (c)

In better responses, students were able to:

- recognise that solutions can be obtained by inspection rather than calculations
- indicate the solution on their graph and transfer this to mathematical notation
- read the scale on the number plane correctly to identify the correct end points of the domain
- identify the correct domain on the graph by excluding the stationary points.

Areas for students to improve include:

- calculating the value of each subdivision on the grid
- understanding a stationary point is not part of the domain where the graph is decreasing
- using the correct words, such as ‘between’, to express that the end points are not included in the domain
- using the correct notation, including inequality symbols and interval notation, to express the correct domain where both graphs are decreasing
- ensuring the correct domain is stated where both graphs are decreasing.

## Question 16 (a)

In better responses, students were able to:

- differentiate the function correctly
- find both the $x$ and $f(x)$ values
- determine the nature of the stationary points using either the first derivative and a table of values, or via the second derivative.

Areas for students to improve include:

- using the product or quotient rule correctly
- using a table to determine the nature of the stationary points or substituting the $x$-values into the second derivative and finding the answer to determine the nature
- factorising out the common factor of $e^x$ in the numerator and dividing through by $e^x$ in the denominator correctly, to obtain the correct derivative.

### Question 16 (b)

In better responses, students were able to:

- draw a smooth curve
- show the stationary points at (0,0) and (2, 0.54)
- apply the work done in Q16a into Q16b, in order to complete the correct graph.

Areas for students to improve include:

- understanding the given scale, drawing a smooth curve, labelling their stationary points
- finding the value of $y$ for each stationary point, following on from finding the value of $x$.

### Question 17 (a)

In better responses, students were able to:

- write the correct numerical expression for $A_1$ in terms of the borrowed amount, interest applied at 1.005 and the repayment amount
- write $A_2$ correctly in terms of $A_1$ before expanding the factor 1.005.

Areas for students to improve include:

- understanding the relationship between $A_1$ and $A_2$
- checking that large numbers have been repeatedly written with the correct number of zeros
- understanding how the amount remaining at the end of each month is calculated
- understanding the interest is charged on the principal for the whole month before the monthly repayment is subtracted.

### Question 17 (b)

In better responses, students were able to:

- express $A_3$ or $A_n$ as a series correctly
- apply the sum of the geometric sequence formula correctly to sum the geometric series in $A_n$
- use applicable algebraic techniques to expand the bracket and collect like terms.

Areas for students to improve include:

- expressing $A_n$ as a series showing the correct first term and correct last term
- using the Reference Sheet to ensure the correct formula is applied to sum the geometric sequence
- using appropriate algebraic techniques to expand brackets and divide decimals.

## Question 17 (c)

In better responses, students were able to:

- rearrange the formula correctly to make $1.005^n$ the subject
- use logarithms correctly to solve an index equation
- establish the connection between parts (b) and (c) of the question, and apply the formula given in the previous part to answer the question.

Areas for students to improve include:

- solving an index equation by making $1.005^n$ the subject
- applying logarithms laws correctly to solve an index equation
- recognising they need to use the previous solution (from part (b))
- understanding how to use their calculator for logarithms
- rounding numerical solutions only at the final step of the solution.

## Question 18

In better responses, students were able to:

- substitute correctly into the given expressions to find the composite function
- sketch accurately the hyperbola $\frac{3}{x-1}+5$ with labelled asymptotes $x=1$ and $y=5$
- recognise the vertical translation (5 units up) of the hyperbola $\frac{3}{x-1}+5$
- use accurate notation $(-\infty,5)\cup(5,\infty)$ or all real *y*, $y\neq 5$ to state the range.

Areas for students to improve include:

- sketching the hyperbola $\frac{3}{x-1}+5$
- considering the translations $g(f(x))=\frac{3}{x-1}+5$ and identifying and applying translations correctly)
- using accurate mathematical notation to correctly express the range.

## Question 19

In better responses, students were able to:

- convert the ratio 1: 2: 3 into accurate probability values for $P(A), P(B)$ and $P(C)$
- construct a two-stage tree diagram with all correct values
- multiply along the correct branches of the tree diagram to calculate $P(A \cap W) = \frac{1}{6} \times 0.5$
- calculate $P(Win) = \frac{1}{6} \times 0.5 + \frac{1}{3} \times 0.4 + \frac{1}{2} \times 0.2 = \frac{19}{60}$
- interpret that the question requires the application of conditional probability.

Areas for students to improve include:

- converting ratios into accurate probability values
- using a visual solution like a tree diagram to demonstrate their understanding
- recognising key words in the question, such as ‘given’, which implies conditional probability
- understanding that the word ‘respectively’ means ‘in the same order’.

## Question 20

In better responses, students were able to:

- calculate $n = 84$ , $r = \frac{6}{12}\% = 0.005$ and identify from the future value table, the interest factor of 104.07393
- use the future value formula correctly: $A = 21000\left(1 + \frac{6}{12}\%\right)^{84}$
- calculate the monthly deposit correctly: $A \div 104.07393 = \$306.78$.

Areas for students to improve include:

- converting the interest rate $(r)$ and number of periods $(n)$ to the compounding period
- recognising when to use the future value formula and when to use the provided future value interest factors tables
- understanding when to divide by the interest factor and when to multiply.

### Question 21 (a)

In better responses, students were able to:

- use alternative methods to find the mode after the maximum turning point of the probability density function couldn't be found using $f'(x)=\dfrac{-1}{x^2}$
- provide a sketch of $f(x)=\dfrac{1}{x}$ for $1 \le x \le e$
- explain that $f(x)=\dfrac{1}{x}$ for $1 \le x \le e$ is decreasing for the given domain
- evaluate the end points $f(1)=\dfrac{1}{1}=1$ and $f(e)=\dfrac{1}{e} \approx 0.37$ to determine the maximum value.

Areas for students to improve include:

- demonstrating understanding of how the shape of a probability density function relates to the mode
- using precise mathematical language, such as 'decreasing', 'endpoint', and 'maximum value', in justifications.

### Question 21 (b)

In better responses, students were able to:

- apply the concept of 'area under the probability density function' to determine the 25th percentile
- integrate accurately and substitute limits appropriately
- rewrite logarithmic expressions in exponential form to solve for $x$.

Areas for students to improve include:

- understanding that definite integration determines percentiles of a probability density function
- selecting and justifying appropriate bounds of integration
- integrating accurately, showing clear use of limits.

## Question 22

In better responses, students were able to:

- use the LHS to work through the proof
- use a common denominator to connect the LHS
- recognise $(sin^2\theta + cos^2\theta)^2 = sin^4\theta + cos^4\theta + 2sin^2\theta cos^2\theta$
- use $sin^2\theta + cos^2\theta = 1$
- accurately apply trigonometric Pythagorean identities
- provide a solution with a legible and clear sequence.

Areas for students to improve include:

- working with just one side of the proof
- recognising that $sin\,^4\,\theta + cos\,^4\,\theta \neq (sin\,^2\,\theta \cdot cos\,^2\,\theta)^2$
- manipulating fractions and common denominators
- showing each step when rearranging trigonometric functions.

## Question 23 (a)

In better responses, students were able to:

- find the number of male sheep correctly using the given information: $\frac{12600}{21} = 600$
- express 15 male sheep as a percentage of 600 male sheep: $\frac{15}{600} = 0.025 = 2.5\%$
- use the percentage to equate to a $z$-score of 2
- calculate $x$ kg using the normal distribution curve or the $z$-score formula.

Areas for students to improve include:

- calculating the number of male sheep correctly according to the given ratio
- identifying the difference between a $z$-score and a percentage
- using and labelling a diagram to help to understand where the values are placed within the normal distribution
- using the Reference Sheet to match the percentage break-up to a $z$-score.

### Question 23 (b)

In better responses, students were able to:

- express 300 female sheep as a percentage of the 12 000 female sheep correctly
- recognise the female percentage (or $z$-score) is the same as the male percentage from part (a) of the question
- recognise and state that with a smaller mean and standard deviation, the percentage of female sheep greater than $x$ would be less than 2.5%, or that the $z$-score of $x$ would be greater than 2.

Areas for students to improve include:

- calculating the correct number and percentage of female sheep
- understanding the concepts, $z$-score, mean, standard deviation in more detail, and how these concepts affect each other
- providing an accurate explanation, justified with mathematical reasoning and calculations.

### Question 24

In better responses, students were able to:

- recognise that the tangent will have the same gradient as each function at the point of intersection
- differentiate both functions and equate them to 1
- use the point of intersection and the derivative of quadratic to solve for $a$
- use the point of intersection, $a$ and the quadratic to solve for $c$.

Areas for students to improve include:

- differentiating logarithmic functions
- recognising that they cannot equate a derivative to a function
- solving equations correctly with fractions
- recognising that $e$ is a number, not a variable.

### Question 25 (a)

In better responses, students were able to:

- apply correctly the product rule $\frac{d}{dx}(x\cos x)=(-x\sin x+\cos x)$ or $\frac{d}{dx}(-x\cos x)=(x\sin x-\cos x)$
- simplify expressions involving positive and negative terms correctly.

Areas for students to improve include:

- differentiating trigonometric functions correctly.

### Question 25 (b)

In better responses, students were able to:

- recognise and use the result from part (a) of the question $\int_0^{2025\pi} x\sin x\,dx = [\sin x - x\cos x]_0^{2025\pi}$
- substitute correctly into the given function $x = 2025\pi$ and $x = 0$ into $[\sin x - x\cos x]_0^{2025\pi}$

Areas for students to improve include:

- recognising that 'hence' implies the use of a previous result
- recording of the function $= [\sin x - x\cos x]_0^{2025\pi}$ and appropriate substitution accurately
- checking calculator mode (radians) when performing calculus operations.

### Question 25 (c)

In better responses, students were able to:

- use $|A_n| = (2n-1)\pi$ to determine the terms $\pi, 3\pi, 5\pi, \ldots (2\times 2025 - 1)\pi$
- recognise that the total area is given by the sum of an arithmetic sequence.
- correctly apply the sum of an arithmetic sequence formula $S_n = \frac{2025}{2}(\pi + 4049\pi)$ or $S_n = \frac{2025}{2}[2\times\pi + (2025-1)\times 2\pi]$

Areas for students to improve include:

- engaging with the formula provided as a guide in the question $|A_n| = (2n-1)\pi$
- considering absolute value notation negates concerns about areas below the $x$-axis $|A_n| = (2n-1)\pi$
- identifying the values of $a, d, n$ and/or $l$ clearly before substituting into the arithmetic sequence formula.

## Question 26 (a)

In better responses, students were able to:

- manipulate the equation for circumference and perimeter to show $r = \frac{100-3x}{2\pi}$
- formulate the area of the equilateral triangle in terms of $x$
- combine the area of the circle and triangle
- manipulate surds and algebraic fractions.

Areas for students to improve include:

- using a diagram
- manipulating algebra to change the subject of an equation
- using the sine rule or Pythagoras to find the area of the equilateral triangle
- ensuring the solution follows a clear sequence
- making sure that every step is neatly written in the solution to a question using the key word 'show'.

## Question 26 (b)

In better responses, students were able to:

- recognise that calculus is not required
- state the concavity of the quadratic referencing the coefficient of $x^2$
- find the area at the extremities of the domain
- correctly find and determine the nature of the stationary point
- recognise the maximum area will occur at the extremities of the domain
- justify that $x$ can only exist in the domain $0 \leq x \leq \frac{100}{3}$
- find the value of both endpoints $x = 0$ and $x = \frac{100}{3}$ to show that the maximum area occurs when $x = 0$.

Areas for students to improve include:

- showing when a quadratic has a minimum or maximum turning point, from the value of the coefficient of $x^2$
- differentiating functions with constant denominators
- checking stationary points
- recognising minimum stationary points mean domain extremities will be possible maximum values of the function
- recognising that calculus is not always required when the term maximum or minimum is used.

## Question 27 (a)

In better responses, students were able to:

- apply the formula for the trapezoidal rule correctly with two applications from the Reference Sheet
- find the value of $b - a$, and the function values
- write correct numerical expressions to represent the required area.

Areas for students to improve include:

- understanding the concept of 2 applications
- understanding how to find $b - a$ for a trapezoidal rule problem
- using a table of values to help them correctly apply the process
- understanding the difference between function values and applications.

## Question 27 (b)

In better responses, students were able to:

- integrate the function correctly
- demonstrate an understanding of and the process to show that $\ln \frac{1}{2} = -\ln 2$
- substitute correctly into the integrand
- show all lines of working in putting together their solution.

Areas for students to improve include:

- showing their steps to submit a full solution to the problem
- applying correct integration steps in getting to their solution
- using the Reference Sheet to help them integrate exponential functions
- changing between equivalent logarithms.

## Question 27 (c)

In better responses, students were able to:

- write the inequality the correct way, demonstrating an understanding of what the trapezoidal rule does in 'overestimating' the area (on this occasion)
- understand inequalities in this particular context
- use the earlier parts of Question 27 to manipulate the inequality.

Areas for students to improve include:

- understanding that the area calculated using the trapezoidal rule is greater than the exact area
- understanding how the inequality changes
- showing their working sequentially and carefully, including each step of the process.

## Question 28 (a)

In better responses, students were able to:

- find the equation correctly, linking the different areas involved in finding the shaded area
- obtain the 3 relevant areas using correct formulae from the Reference Sheet (using radian measure)
- understand and calculate the area of ¼ of a circle to complete their solution.

Areas for students to improve include:

- substituting the radius into the formulae for each area
- knowing the formulae of the 3 areas and how to connect them
- manipulating the equation to find the desired solution.

## Question 28 (b)

In better responses, students were able to:

- draw the line $y = \theta$
- find $\theta = 2.3$
- substitute $\theta = 2.3$ into the formula $l = r\theta$ to find the arc length.

Areas for students to improve include:

- using part (a) of the question for part (b) and seeing the connection
- understanding they are looking for $\theta$, not $y$
- drawing a line onto the graph.

## Question 29 (a)

In better responses, students were able to:

- recognise that $\Delta TOY$ is right angled and choose the appropriate trigonometric ratio to show the required statement
- know and use exact values
- establish the correct ratio for the tangent of the given angle
- state the tangent ratio $\tan 60 = h / OY$ with the angle and sides specifically mentioned.

Areas for students to improve include:

- showing all working out in answering a question with the key word 'show'
- using right angled trigonometric ratios instead of the sine rule to find the required result.

## Question 29 (b)

In better responses, students were able to:

- use the cosine rule correctly using $h$ and the angle $45°$ to find $h$
- recognise that $\Delta TOF$ is an isosceles triangle and that $\angle OYF$ was $45°$, from the given bearing
- apply the cosine rule correctly in $\Delta FOY$ to find the value of $h$
- select the correct formula from the Reference Sheet and obtain and solve the resulting quadratic equation, accepting the positive value and rejecting the negative value.

Areas for students to improve include:

- using the quadratic formula correctly to evaluate the required height
- moving between surds with irrational and rational denominators without error
- manipulating algebraic expressions and equations, which include fractions and surds
- unless it is stated in the question or can be proved, not assuming the existence of right angles.

### Question 29 (c)

In better responses, students were able to:

- find angle *OFY* using the sine rule
- recognise that angle *OYF* is $45°$
- recognise the angle required to provide the most efficient method to find a bearing
- correctly apply the sine rule to find the required angle
- work confidently with bearings, both when comprehending the question and when answering in the context of the question.

Areas for students to improve include:

- recognising that an angle may not be $90°$
- applying simple trigonometric ratios accurately
- creating well-annotated triangles or labelling existing diagrams to support calculations
- demonstrating how angles and bearings relate to each other
- copying the formula to be used accurately from the Reference Sheet and ensuring values are substituted correctly.

### Question 30

In better responses, students were able to:

- establish an algebraical expression with a horizontal translation of $k$ and vertical translation of $k$
- substitute the coordinates (6, 11) into their established equation and write the equation as a quadratic equation in terms of $k$
- solve a quadratic equation correctly by factorisation or using the quadratic formula
- recognise that $k$ is positive and conclude $k$ can only take positive value, and reject any negative value of $k$.

Areas for students to improve include:

- reading the question carefully and translating the function both horizontally and vertically by $k$ units
- understanding that translated horizontally by $k$ units means substituting every $x$ term by $x - k$
- understanding that translated vertically by $k$ units means substituting $y$ by $y - k$ or adding $k$ to the function
- applying appropriate algebraic techniques to correctly expand brackets and collect like terms
- substituting in the coordinates early to minimise errors in an algebraic expansion
- understanding that translating a coordinate instead of a curve is not a thorough solution.

## Question 31

In better responses, students were able to:

- use exact values for cosine to find the related angle
- recognise that $\frac{5\pi}{3}$ and $\frac{7\pi}{3}$ are the second and third solutions to $\cos x = \frac{1}{2}$ respectively
- calculate exact values in radians for the basic equation $\cos x = \frac{1}{2}$
- express answers in radians rather than degrees
- demonstrate the knowledge of varying periods for trigonometric graphs
- state two solutions within the standard domain of [0,2pi].

Areas for students to improve include:

- finding solutions to simple trigonometric equations with specified domains
- calculating the correct angles for $\cos x = \frac{1}{2}$ in the correct quadrants
- converting the required angles from degrees to radians
- understanding the relationship between $p$, the scale factor for horizontal dilation and the period of a trigonometric function.