TRINITY GRAMMAR SCHOOL

Mathematics Department

(NESA Student Number | Year 12 only)

2020

Year 12

Mathematics

TRIAL EXAMINATION

HSC ASSESSMENT TASK 4

Date of Assessment Task: Friday, 21 August 2020

General Instructions

- Reading time – 10 minutes
- Working time – 3 hours
- Write using black pen
- NESA approved calculators may be used
- A reference sheet is provided
- In Questions 11 – 34, show relevant mathematical reasoning and/or calculations
- Write your NESA Student Number (Year 12 HSC) on the question paper and on any answer sheets used to write your responses to the questions submitted
- Extra writing paper has been provided at the back of the booklet for additional working space. This will only be marked if you have noted your use of the additional space in a given question.

Total marks: 100

Section I – 10 marks (pages 3 – 6)

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section

Section II – 90 marks (pages 7 – 23)

- Attempt Questions 11 – 34
- Allow about 2 hour and 45 minutes for this section

- **HSC Assessment Weighting:** 30%

Section I

10 marks
Attempt Questions 1 – 10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1 – 10.

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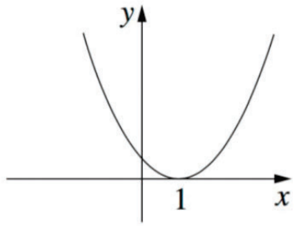
1 Which equation best represents the given graph?



NOT TO SCALE

A. $f(x) = x^2 + 1$

B. $f(x) = x^2 - 1$

C. $f(x) = (x + 1)^2$

D. $f(x) = (x - 1)^2$

2 A machine produces cylindrical pipes. The mean diameter of the pipes is 7.5 cm and the standard deviation is 0.02 cm. Pipes with a diameter less than 7.46 cm or greater than 7.54 cm must be discarded. Assuming a normal distribution, what percentage of pipes are discarded?

A. 0.15%

B. 0.3%

C. 2.5%

D. 5%

3 Which expression is equivalent to $\tan\theta + \cot\theta$?

A. $\operatorname{cosec}\theta + \sec\theta$

B. $\sec\theta\operatorname{cosec}\theta$

C. 1

D. 2

4 Which interval below shows the domain of the function $y = \frac{1}{\sqrt{x+2}}$?

A. $(-\infty, \infty)$

B. $(-2, \infty)$

C. $[-2, \infty]$

D. $(-2, \infty)$

5 What is the common difference d of the arithmetic series $\ln 8 + \ln 16 + \ln 32 + \cdots$?

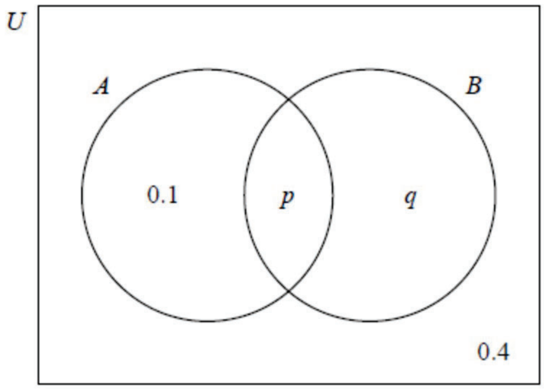
A. 1

B. 2

C. $\ln 2$

D. $\ln 8$

6 The Venn Diagram below shows the probabilities of A and B. If $P(A) = 0.25$. What is the value of $P(B)$?



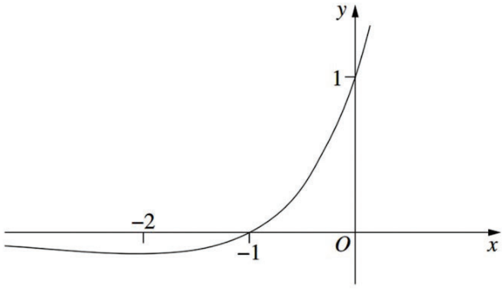
A. 0.15

B. 0.35

C. 0.50

D. 0.60

7 The graph of $y = e^x(1 + x)$ is shown below.



How many solutions are there to the equation $e^x(1 + x) = x^3 + 1$?

A. 0

B. 1

C. 2

D. 3

8 For what values of k does the quadratic $x^2 - (k + 2)x + 4 = 0$ have real roots?

A. $-6 \leq k \leq 2$

B. $-6 < k < 2$

C. $k \leq -6, k \geq 2$

D. $k < -6, k > 2$

9 What is the area bounded by $f(x) = (x + 2)(x - 3)$ and the *x*-axis over the domain $[0,5]$?

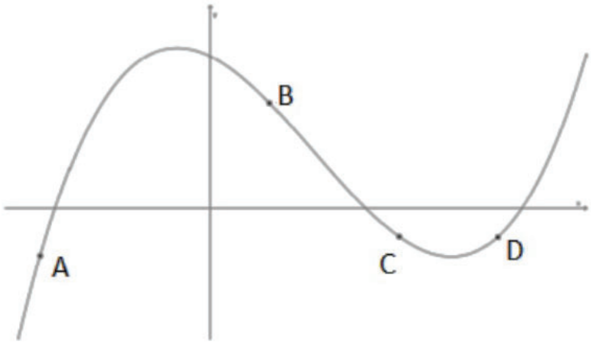
A. $\frac{5}{6}$

B. $\frac{11}{3}$

C. $\frac{157}{6}$

D. $\frac{167}{6}$

10 Which point below satisfies the conditions $y < 0, \frac{dy}{dx} > 0$ and $\frac{d^2y}{dx^2} < 0$?



A. Point *A*

B. Point *B*

C. Point *C*

D. Point *D*

Section II

90 marks
Attempt Questions 11 – 34
Allow about 2 hours and 45 minutes for this section

Answer each question in the space provided. Extra pages are available at the back of the booklet.

In Questions 11 – 35, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (1 mark)

Find the exact value of x if $17^x = 2$. **1**

Question 12 (2 marks)

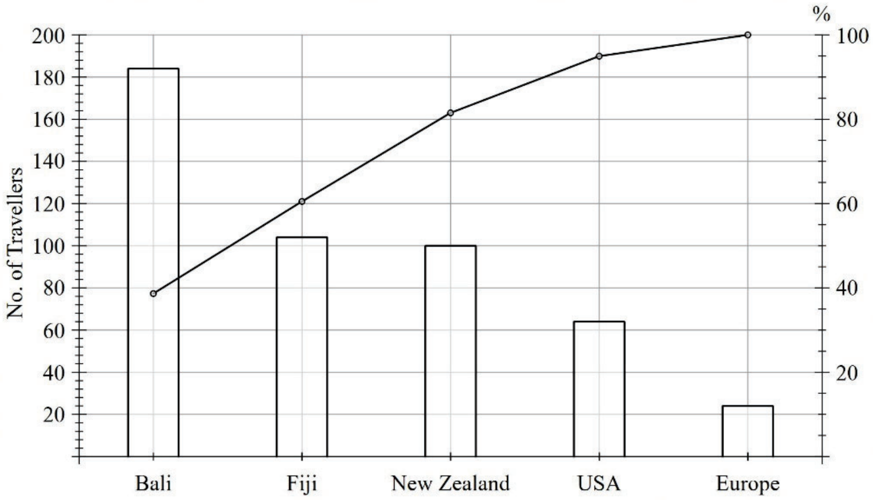
If $m(k) = k^2$, simplify the expression for $m(k+1) - m(k)$. **2**

Question 13 (2 marks)

Given that $f(x) = 3x^2 + 1$ and $g(x) = x - 2$, what is the value of $f(g(4))$? **2**

Question 14 (2 marks)

The pareto chart below summarises the data collected by an airport, identifying popular destinations for travellers.



(a) Using the chart, how many travellers were surveyed? **1**

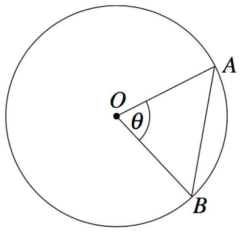
(b) What percentage difference is there between the number of travellers to Fiji compared to America? **1**

Question 15 (3 marks)

Solve the equation $4(4^x) - 5(2^x) + 1 = 0$. **3**

Question 16 (5 marks)

The diagram shows a circle with centre O and radius 4 centimetres. The points A and B lie on the circumference of the circle and $\angle AOB = \theta$.



NOT TO SCALE

(a) There are two possible values of θ for which the area of ΔAOB is $4\sqrt{3}$ cm^2. One value of θ is $\frac{\pi}{3}$. Find the other value. **2**

(b) Suppose that $\theta = \frac{\pi}{3}$.

(i) Find the area of the sector AOB. **1**

(ii) Find the exact length of the perimeter of the minor segment bounded by the chord AB and the arc AB. **2**

Question 17 (4 marks)

In a scientific study, men in various stages of their career were evaluated and their ages followed a normal distribution. The age of a man 3 standard deviations below the mean age was 25. The age of a man 3 standard deviations above the mean age was 55.

(a) What is the mean age of the men in the study and what was the standard deviation? **2**

(b) A random person was selected from the data and it is noted that he was 48. What z-score correlates to the man's age? **1**

(c) What is the probability that a man chosen at random from this group is less than 35 years old? **1**

Question 18 (3 marks)

Solve the equation $\log_2 x + \log_2(x - 3) = 2$. **3**

Question 19 (3 marks)

Find $\frac{dy}{dx}$ given that $y = \log_e\left(\frac{2x+1}{3x-7}\right)$. **3**

Question 20 (6 marks)

Find the derivatives of the following expressions:

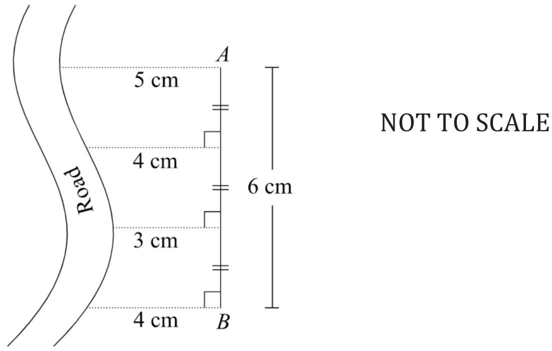
(a) $x^5 - 3x^2 + 9$ **2**

(b) $x^3 e^x$ **2**

(c) $\dfrac{3x}{\sin x}$ **2**

Question 21 (3 marks)

Simon measures the dimensions of his property on a map, which is a block of land bounded on one side by a road. He notes the measurements on the map as shown in the diagram below. Simon knows that the length of his property, corresponding to 6 cm on the map, is 90 metres. **3**



Using the trapezoidal rule, calculate the approximate area of the land, in squared metres.

Question 22 (3 marks)

A curve has a gradient function $\frac{dy}{dx} = e^{\frac{x}{2}} + \sin 2x$ and passes through the point $(0, 0)$. Find the equation of the curve. **3**

Question 23 (5 marks)

A control tower spots Ship A 100 km away at a bearing of 285ºT and Ship B 46 km away at a bearing of 073ºT. Ship B calls for Ship A to send a rescue team.

(a) How far will the rescue team need to travel from Ship A to reach Ship B? Give your answer correct to the nearest kilometre. **2**

(b) At what bearing will the rescue team from Ship A need to travel to reach Ship B? Round your answer to the nearest minute. **3**

Question 24 (6 marks)

The velocity of a particle is given by $v = \frac{5}{2t+1}$ ms^{-1}. Initially, the particle is 3 m to the left of the origin.

(a) Find the expression for the displacement x. **2**

(b) Find the exact time when the particle is at the origin. **2**

(c) Show that the acceleration of the particle is always negative. **2**

Questions 11 to 24 are worth 48 marks in this task.

Question 25 (3 marks)

Evaluate p and q in the discrete probability distribution table below, given that $E(X) = 2.3$. **3**

x	1	2	3	4
$P(X = x)$	p	q	0.3	0.2

Question 26 (3 marks)

Find the stationary points of $f(x) = x^3 - 6x^2 + 9x - 4$ and determine their nature. **3**

Question 27 (6 marks)

Evaluate the following integrals. Give all answers in exact values.

(a) $\displaystyle\int_{-2}^{3} \frac{1}{x^3}\,dx$ **2**

(b) $\displaystyle\int_{\frac{\pi}{2}}^{\pi} \sin 2x\,dx$ **2**

(c) $\displaystyle\int_{1}^{3} \frac{12}{4x-1}\,dx$ **2**

Question 28 (6 marks)

Ten high school students have their height and the length of their left foot measured. The results are recorded in the table below.

Height (cm)	140	164	182	173	154	158	149	181	174	177
Foot (cm)	20	24	28	26	22	23	24	30	27	27

(a) Using technology, calculate Pearson's correlation coefficient for the data. Give your answer to 3 decimal places. **1**

(b) Describe the strength of the association between height and the length of the left foot for these students. **1**

(c) Using technology, determine the equation of the least-squares regression line that allows left foot length to be predicted from height. Give all values correct to one decimal place. **2**

(d) Using the least-squares regression equation, estimate the height of a student with a left foot length of 25 cm. **2**

Question 29 (2 marks)

There is a daily flight from Sydney to Auckland. In June, the probability of the flight departing on time given that there is fine weather is 0.7, and the probability of the flight departing on time given that the weather is not fine is 0.5. Also in June, the probability of a day being fine in 0.3. **2**

Find the probability that on a particular day in June that the weather is fine, given that the flight departs on time.

Question 30 (4 marks)

A cup of coffee contains 100 mg of caffeine, which leaves the body at a continuous rate of 17% per hour.

(a) Write an equation for the amount of caffeine, A, in the body t hours after drinking a cup of coffee. **2**

(b) How long does it take for the body to only have a quarter of the caffeine left? Give your answer correct to 2 decimal places. **2**

Question 31 (3 marks)

Find the exact values of x that satisfy the equation $2\cos 2x - \sqrt{3} = 0$ over the domain $[0, 3\pi]$. **3**

Question 32 (4 marks)

Consider the geometric series $1 + \left(2\sqrt{3} - 3\right) + \left(2\sqrt{3} - 3\right)^2 + \cdots$

(a) Show that this series has a limiting sum. **2**

(b) Find the exact value of the limiting sum. Express your answer with a rational denominator. **2**

Question 33 (5 marks)

The Bay of Fundy in Canada has the largest tides in the world. The difference between low and high water levels is 15 metres. At a particular point, the depth of water, y, in metres is given as a function of time, t, in hours since midnight by $y = k\cos\big(a(t-b)\big) + c$, where k, a, b and c are constants.

(a) Low tide at the Bay of Fundy results in a depth of 1.5 metres of water. High tide was first noted at 4 am, and the time difference between subsequent high tides is 12 hours. With the aid of a diagram, write the equation of y to model the Bay of Fundy tides. **4**

(b) What is the depth of the water 2 hours after high tide? **1**

Question 34 (6 marks)

A prize fund of \$300 000 is established by the council of a school to provide scholarships. The fund is in an account that earns interest at the rate of 6% p.a. compounded monthly. At the end of each year, a prize of \$25 000 is withdrawn from the fund. Let A_n denote the amount remaining in the fund after the n^{th} year, just after the prize has been awarded for the year.

(a) Show that $A_3 = 300\,000(1.005)^{36} - 25000(1.005)^{24} - 25000(1.005)^{12} - 25000$. **2**

(b) Show that $A_n = 300000(1.005)^{12n} - 405332.15(1.005^{12n} - 1)$. **2**

(c) For how many years can the council provide the full amount of the scholarship? **2**

END OF TASK

Extra Writing Space

If you require extra writing space, please use the paper included below. Indicate clearly which question you are attempting. Note in the original question that you have used this space.

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Mathematics Department

(NESA Student Number | Year 12 only)

2020

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Mathematics

TRIAL EXAMINATION

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- **HSC Assessment Weighting:** 30%

Section I

10 marks
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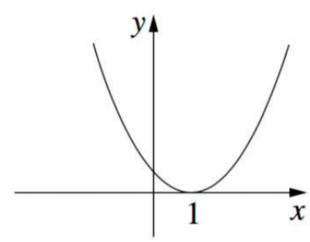
1 Which equation best represents the given graph?



NOT TO SCALE

A. $f(x) = x^2 + 1$

B. $f(x) = x^2 - 1$

C. $f(x) = (x + 1)^2$

D. $f(x) = (x - 1)^2$

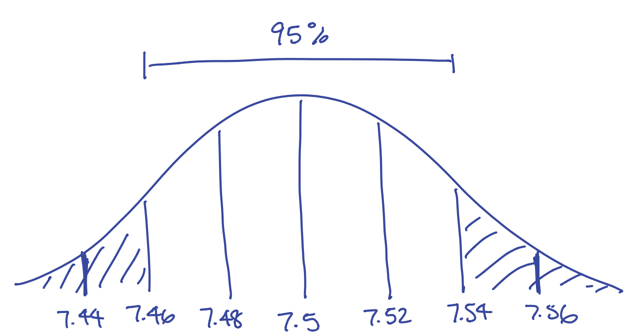
2 A machine produces cylindrical pipes. The mean diameter of the pipes is 7.5 cm and the standard deviation is 0.02 cm. Pipes with a diameter less than 7.46 cm or greater than 7.54 cm must be discarded. Assuming a normal distribution, what percentage of pipes are discarded?

A. 0.15%

B. 0.3%

C. 2.5%

D. 5%



3 Which expression is equivalent to $\tan\theta + \cot\theta$?

A. $\operatorname{cosec}\theta + \sec\theta$

B. $\sec\theta \operatorname{cosec}\theta$

C. 1

D. 2

$$= \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}$$

$$= \frac{\sin^2\theta + \cos^2\theta}{\cos\theta\sin\theta}$$

$$= \frac{1}{\cos\theta\sin\theta}$$

$$= \sec\theta \operatorname{cosec}\theta$$

4 Which interval below shows the domain of the function $y = \frac{1}{\sqrt{x+2}}$?

A. $(-\infty, \infty)$

B. $(-2, \infty)$

C. $[-2, \infty]$

D. $(-2, \infty)$

$x + 2 > 0$
$x > -2$

5 What is the common difference d of the arithmetic series $\ln 8 + \ln 16 + \ln 32 + \cdots$?

A. 1

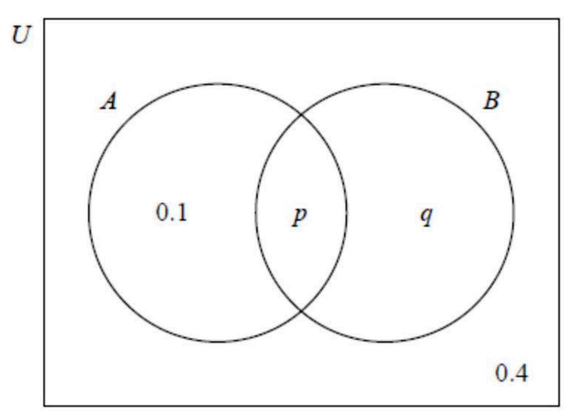
B. 2

C. $\ln 2$

D. $\ln 8$

$\ln 2^3 + \ln 2^4 + \ln 2^5$
$3\ln 2 + 4\ln 2 + 5\ln 2$
$+\ln 2$

6 The Venn Diagram below shows the probabilities of A and B. If $P(A) = 0.25$. What is the value of $P(B)$?



A. 0.15

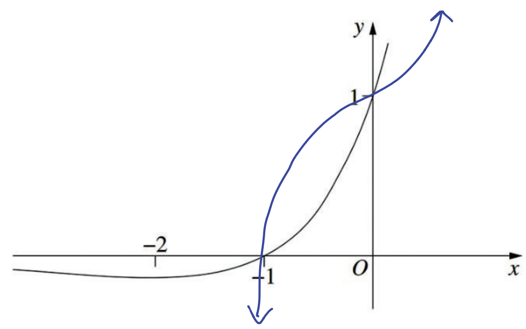
B. 0.35

C. 0.50

D. 0.60

$P(B) = 1 - 0.4 - 0.1$
$= 0.5$

7 The graph of $y = e^x(1 + x)$ is shown below.



NOT TO SCALE

How many solutions are there to the equation $e^x(1 + x) = x^3 + 1$?

A. 0

B. 1

C. 2

D. 3

8 For what values of k does the quadratic $x^2 - (k + 2)x + 4 = 0$ have real roots?

A. $-6 \le k \le 2$

B. $-6 < k < 2$

C. $k \le -6, k \ge 2$

D. $k < -6, k > 2$

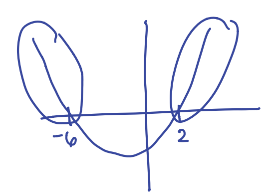
$b^2 - 4ac \ge 0$

$(-(k+2))^2 - 4(1)(4) \ge 0$

$k^2 + 4k + 4 - 16 \ge 0$

$k^2 + 4k - 12 \ge 0$

$(k+6)(k-2) \ge 0$



9 What is the area bounded by $f(x) = (x + 2)(x - 3)$ and the x-axis over the domain $[0,5]$?

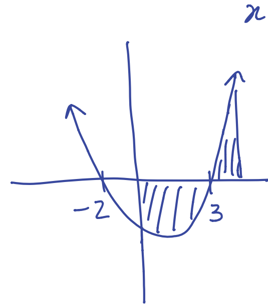
$x^2 - x - 6$

A. $\frac{5}{6}$

B. $\frac{11}{3}$

C. $\frac{157}{6}$

D. $\frac{167}{6}$



$-\int_0^3 (x^2 - x - 6)\,dx + \int_3^5 (x^2 - x - 6)\,dx$

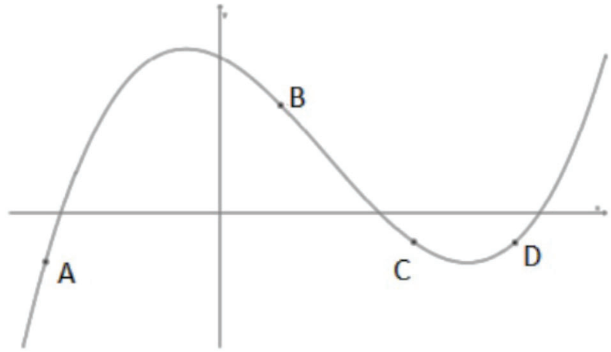
$= -\left[\frac{x^3}{3} - \frac{x^2}{2} - 6x\right]_0^3 + \left[\frac{x^3}{3} - \frac{x^2}{2} - 6x\right]_3^5$

$= -\left(-\frac{27}{2}\right) + \left(-\frac{5}{6} + \frac{27}{2}\right)$

$= -\frac{5}{6} + \frac{54}{2}$

$= \frac{157}{6}$

10 Which point below satisfies the conditions $y < 0$, $\frac{dy}{dx} > 0$ and $\frac{d^2y}{dx^2} < 0$?



A. Point A

B. Point B

C. Point C

D. Point D

• below x-axis (A/C/D)
• increasing (A/D)
• concave down (A)

Section II

90 marks
Attempt Questions 11 – 34
Allow about 2 hours and 45 minutes for this section

Answer each question in the space provided. Extra pages are available at the back of the booklet.

In Questions 11 – 35, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (1 mark)

Find the exact value of x if $17^x = 2$. **1**

$x = \log_{17} 2$

$x = \frac{\log 2}{\log 17}$ or $\frac{\ln 2}{\ln 17}$ ✓

Question 12 (2 marks)

If $m(k) = k^2$, simplify the expression for $m(k+1) - m(k)$. **2**

$m(k+1) - m(k) = (k+1)^2 - k^2$ ✓

$= k^2 + 2k + 1 - k^2$

$= 2k + 1$ ✓

Question 13 (2 marks)

Given that $f(x) = 3x^2 + 1$ and $g(x) = x - 2$, what is the value of $f(g(4))$? **2**

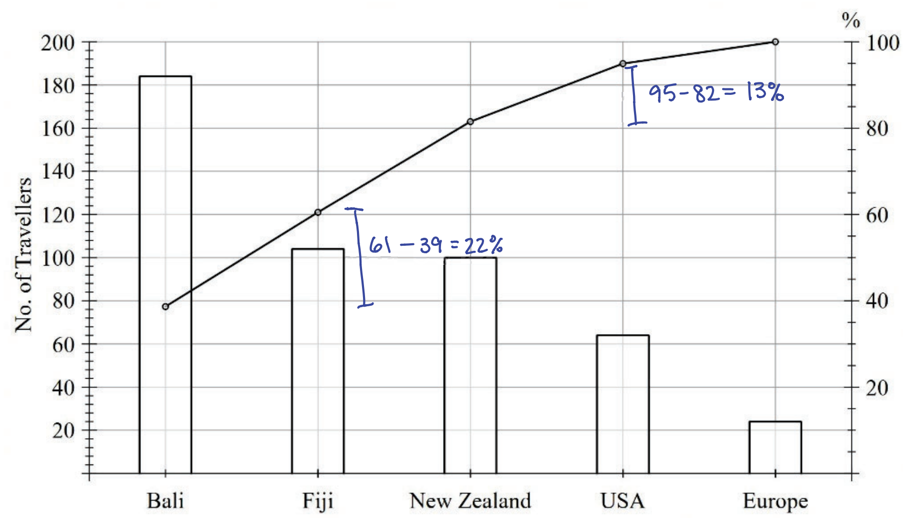
$g(4) = 4 - 2 = 2$ ✓

$f(2) = 3(2)^2 + 1 = 13$

$\therefore f(g(4)) = 13$ ✓

Question 14 (2 marks)

The pareto chart below summarises the data collected by an airport, identifying popular destinations for travellers.



(a) Using the chart, how many travellers were surveyed? **1**

184 + 104 + 100 + 64 + 24 = 476 travellers ✓

(b) What percentage difference is there between the number of travellers to Fiji compared to America? **1**

$\frac{104}{476} - \frac{64}{476} = 0.084$

$\therefore 8.4\%$ ✓

Question 15 (3 marks)

Solve the equation $4(4^x) - 5(2^x) + 1 = 0$. **3**

Let $u = 2^x \rightarrow 4^x = 2^{2x} = u^2$

$4u^2 - 5u + 1 = 0$ ✓

$4u^2 - 4u - u + 1 = 0$

$4u(u-1) - 1(u-1) = 0$

$(4u-1)(u-1) = 0$

$u = \frac{1}{4} \quad u = 1$ ✓

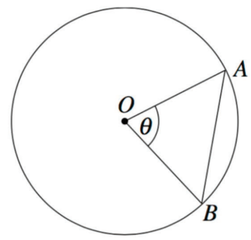
$2^x = \frac{1}{4} \quad 2^x = 1$

$x = -2 \quad x = 0$ ✓

$\therefore x = -2$ or 0

Question 16 (5 marks)

The diagram shows a circle with centre O and radius 4 centimetres. The points A and B lie on the circumference of the circle and $\angle AOB = \theta$.



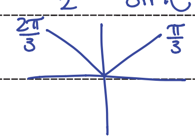
NOT TO SCALE

(a) There are two possible values of θ for which the area of ΔAOB is $4\sqrt{3}$ cm^2. One value of θ is $\frac{\pi}{3}$. Find the other value. **2**

$A = \frac{1}{2}ab\sin C$

$4\sqrt{3} = \frac{1}{2}(4)^2 \sin C$

$\frac{\sqrt{3}}{2} = \sin C$ ✓



$\therefore \frac{2\pi}{3}$ ✓

(b) Suppose that $\theta = \frac{\pi}{3}$.

(i) Find the area of the sector AOB. **1**

$A = \frac{1}{2}r^2\theta$

$= \frac{1}{2}(4)^2\left(\frac{\pi}{3}\right)$

$= \frac{8\pi}{3}$ cm^2 ✓

(ii) Find the exact length of the perimeter of the minor segment bounded by the chord AB and the arc AB. **2**

$l = r\theta$

$= 4\left(\frac{\pi}{3}\right)$ ✓

$c^2 = a^2 + b^2 - 2ab\cos C$

$c^2 = (4)^2 + (4)^2 - 2(4)(4)\cos\frac{\pi}{3}$

$c^2 = 32 - 32\left(\frac{1}{2}\right)$

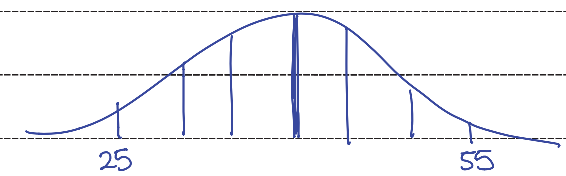
$c^2 = 16$

$c = 4$ $\therefore$ perimeter $= 4 + \frac{4\pi}{3}$ or $4\left(1 + \frac{\pi}{3}\right)$ cm ✓

Question 17 (4 marks)

In a scientific study, men in various stages of their career were evaluated and their ages followed a normal distribution. The age of a man 3 standard deviations below the mean age was 25. The age of a man 3 standard deviations above the mean age was 55.

(a) What is the mean age of the men in the study and what was the standard deviation? **2**



$\frac{25+55}{2} = 40$

$\frac{55-40}{3} = \frac{15}{3} = 5$

$\therefore \mu = 40, \ \sigma = 5$ ✓ ✓

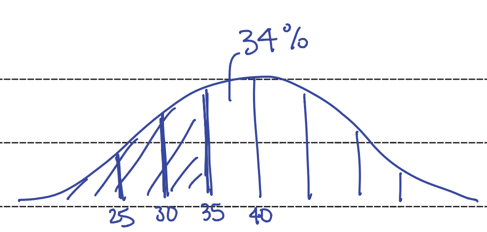
(b) A random person was selected from the data and it is noted that he was 48. What *z*-score correlates to the man's age? **1**

$Z = \frac{x-\mu}{\sigma}$

$= \frac{48-40}{5}$

$= 1.6$ ✓

(c) What is the probability that a man chosen at random from this group is less than 35 years old? **1**



$50\% - 34\% = 16\%$ ✓

Question 18 (3 marks)

Solve the equation $\log_2 x + \log_2(x-3) = 2$. **3**

$\log_2(x(x-3)) = 2$

$x^2 - 3x = 2^2$

$x^2 - 3x - 4 = 0$

$(x-4)(x+1) = 0$

$x = 4, -1$

Domain: $x > 3$

$\therefore x = 4$

Question 19 (3 marks)

Find $\frac{dy}{dx}$ given that $y = \log_e\left(\frac{2x+1}{3x-7}\right)$. **3**

$y = \ln(2x+1) - \ln(3x-7)$ ✓

$\frac{dy}{dx} = \frac{2}{2x+1} - \frac{3}{3x-7}$ ✓

$= \frac{6x-14 - 6x-3}{(2x+1)(3x-7)}$

$= \frac{-17}{(2x+1)(3x-7)}$ ✓

Question 20 (6 marks)

Find the derivatives of the following expressions:

(a) $x^5 - 3x^2 + 9$ **2**

$5x^4 - 6x$ ✓ ✓

(b) $x^3 e^x$ **2**

$x^3e^x + 3x^2e^x$ or $x^2e^x(x+3)$

✓ ✓

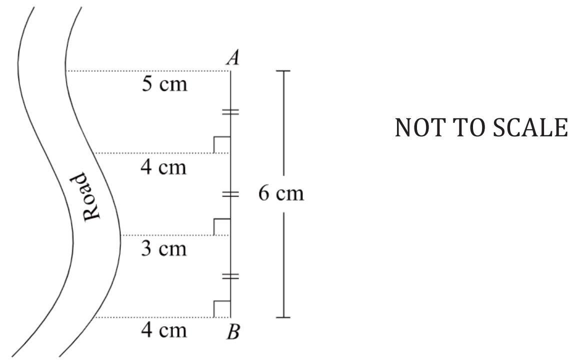
(c) $\dfrac{3x}{\sin x}$ **2**

$$\frac{\sin x \times 3 - 3x\cos x}{\sin^2 x}$$

$$= \frac{3\sin x - 3x\cos x}{\sin^2 x}$$ ✓ ✓

Question 21 (3 marks)

Simon measures the dimensions of his property on a map, which is a block of land bounded on one side by a road. He notes the measurements on the map as shown in the diagram below. Simon knows that the length of his property, corresponding to 6 cm on the map, is 90 metres. **3**



Using the trapezoidal rule, calculate the approximate area of the land, in squared metres.

$\frac{90}{6} = 15$

a			b
60	45	60	75

(90 from a to b)

$$\text{Area} \approx \frac{b-a}{2n}\left[f(a)+f(b)+2\left(f(x_1)+f(x_2)\right)\right]$$

$$\approx \frac{90}{2(3)}\left(60+75+2(45+60)\right)$$

$$\approx 15\,(345)$$

$$\approx 5175 \text{ metres}^2$$

Question 22 (3 marks)

A curve has a gradient function $\frac{dy}{dx} = e^{\frac{x}{2}} + \sin 2x$ and passes through the point $(0, 0)$. Find the equation of the curve. **3**

$$\int\left(e^{\frac{x}{2}}+\sin 2x\right)dx = 2e^{\frac{x}{2}} - \frac{1}{2}\cos 2x + C$$

$$0 = 2e^{\frac{0}{2}} - \frac{1}{2}\cos(2(0)) + C$$

$$0 = 2 - \frac{1}{2} + C$$

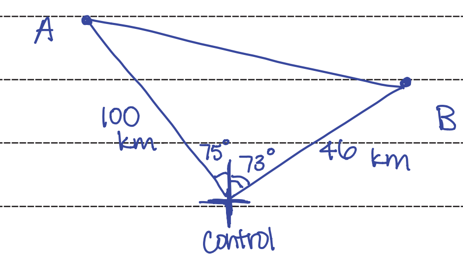
$$-\frac{3}{2} = C$$

$$\therefore\ y = 2e^{\frac{x}{2}} - \frac{1}{2}\cos 2x - \frac{3}{2}$$

Question 23 (5 marks)

A control tower spots Ship A 100 km away at a bearing of 285ºT and Ship B 46 km away at a bearing of 073ºT. Ship B calls for Ship A to send a rescue team.

(a) How far will the rescue team need to travel from Ship A to reach Ship B? **2**
Give your answer correct to the nearest kilometre.



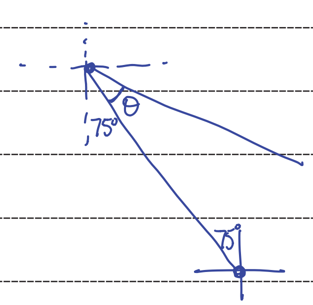
$c^2 = (100)^2 + (46)^2 - 2(100)(46)\cos 148°$ ✓

$c^2 = 19\,918.04$

$c = 141.13$ km

$\therefore$ 141 km ✓

(b) At what bearing will the rescue team from Ship A need to travel to reach Ship B? **3**
Round your answer to the nearest minute.



Bearing $= 180° - 75° - \theta$

$\frac{\sin\theta}{46} = \frac{\sin 148}{141.13}$ ✓

$\sin\theta = \frac{46\sin 148}{141.13}$

$\theta = \sin^{-1}\left(\frac{46\sin 148}{141.13}\right)$

$\theta = 9°56'46''$ ✓

Bearing $= 180° - 75° - 9°56'46''$

$= 95°03'$ ✓

Question 24 (6 marks)

The velocity of a particle is given by $v = \frac{5}{2t+1}$ ms^{-1}. Initially, the particle is 3 m to the left of the origin.

(a) Find the expression for the displacement x. **2**

$\int \frac{5}{2t+1}\,dt = 5 \times \frac{1}{2} \ln|2t+1| + C$

$-3 = \frac{5}{2} \ln|2(0)+1| + C$ ✓

$-3 = \frac{5}{2}(0) + C$

$-3 = C$

$\therefore\ x = \frac{5}{2} \ln|2t+1| - 3$ ✓

(b) Find the exact time when the particle is at the origin. **2**

$0 = \frac{5}{2} \ln|2t+1| - 3$ ✓

$3 = \frac{5}{2} \ln|2t+1|$

$\frac{6}{5} = \ln|2t+1|$

$e^{\frac{6}{5}} = 2t+1$

$e^{\frac{6}{5}} - 1 = 2t$

$\frac{e^{\frac{6}{5}}-1}{2} = t$

$\therefore\ t = \frac{e^{\frac{6}{5}}-1}{2}$ seconds ✓

$\left[\frac{e^{1.2}-1}{2}\right]$

(c) Show that the acceleration of the particle is always negative. **2**

$v = 5(2t+1)^{-1}$

$\frac{dv}{dt} = 5 \times 2 \times -1\,(2t+1)^{-2}$ ✓

$= \frac{-10}{(2t+1)^2}$

The denominator is always positive as it is a squared value. Therefore the numerator of -10 shows that the acceleration is always negative. ✓

Question 11 to 24 are worth 48 marks in this task.

Question 25 (3 marks)

Evaluate p and q in the discrete probability distribution table below, given that $E(X) = 2.3$. **3**

x	1	2	3	4
$P(X = x)$	p	q	0.3	0.2

$p+q+0.3+0.2=1 \rightarrow p+q=0.5$ ①

$p+2q+0.9+0.8=2.3 \rightarrow p+2q=0.6$ ②

② − ① $\quad p+2q=0.6$

$(-)\ p+q=0.5$

$q=0.1$

$p=0.4$

Question 26 (3 marks)

Find the stationary points of $f(x) = x^3 - 6x^2 + 9x - 4$ and determine their nature. **3**

$f'(x) = 3x^2 - 12x + 9$

$0 = 3(x^2 - 4x + 3)$

$0 = x^2 - 4x + 3$

$0 = (x-1)(x-3)$

$\therefore x = 1, 3$

$f''(x) = 6x - 12$

$f''(1) = 6(1) - 12 = -6 \quad \therefore$ concave down (local max)

$f''(3) = 6(3) - 12 = 6 \quad \therefore$ concave up (local min)

Or shown by using a table

$f(1) = 1^3 - 6(1)^2 + 9(1) - 4 = 0$

$f(3) = 3^3 - 6(3)^2 + 9(3) - 4 = -4$

$\therefore$ Local max at $(1, 0)$

Local min at $(3, -4)$

Question 27 (6 marks)

Evaluate the following integrals. Give all answers in exact values.

(a) $\displaystyle\int_{-2}^{3} \frac{1}{x^3}\,dx = \int_{-2}^{3} x^{-3}\,dx$ **2**

$\left[-\frac{1}{2}x^{-2}\right]_{-2}^{3} = \left(-\frac{1}{2}(3)^{-2}\right) - \left(-\frac{1}{2}(-2)^{-2}\right)$ ✓

$= -\frac{1}{18} + \frac{1}{8}$

$= \frac{5}{72}$ ✓

(b) $\displaystyle\int_{\frac{\pi}{2}}^{\pi} \sin 2x\,dx$ **2**

$= \left[-\frac{1}{2}\cos 2x\right]_{\frac{\pi}{2}}^{\pi}$ ✓

$= -\frac{1}{2}\cos 2\pi - \left(-\frac{1}{2}\cos \pi\right)$

$= -\frac{1}{2} - \frac{1}{2}$

$= -1$ ✓

(c) $\displaystyle\int_{1}^{3} \frac{12}{4x-1}\,dx = 3\int_{1}^{3} \frac{4}{4x-1}\,dx$ **2**

$= 3\left[\ln|4x-1|\right]_{1}^{3}$ ✓

$= 3\left(\ln|12-1| - \ln|4-1|\right)$

$= 3\ln\left|\frac{11}{3}\right|$ ✓

Question 28 (6 marks)

Ten high school students have their height and the length of their left foot measured. The results are recorded in the table below.

Height (cm)	140	164	182	173	154	158	149	181	174	177
Foot (cm)	20	24	28	26	22	23	24	30	27	27

(a) Using technology, calculate Pearson's correlation coefficient for the data. Give your answer to 3 decimal places. **1**

$r = 0.935$ ✓

(b) Describe the strength of the association between height and the length of the left foot for these students. **1**

There is a strong and positive correlation between height and the length of the left foot. ✓

(c) Using technology, determine the equation of the least-squares regression line that allows left foot length to be predicted from height. Give all values correct to one decimal place. **2**

$A = -7.298 = -7.30$

$B = 0.196 = 0.20$

✓ ✓

$y = 0.2x - 7.3$

(d) Using the least-squares regression equation, estimate the height of a student with a left foot length of 25 cm. **2**

$25 = 0.20x - 7.30$ ✓

$32.3 = 0.20x$

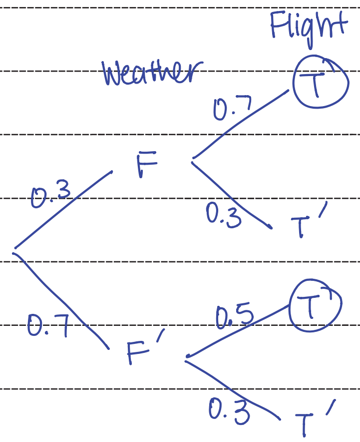
$161.5 = x$

$\therefore$ 161.50 cm tall ✓

Question 29 (2 marks)

There is a daily flight from Sydney to Auckland in June. The probability of the flight departing on time given that there is fine weather is 0.7, and the probability of the flight departing on time given that the weather is not fine is 0.5. Also in June, the probability of a day being fine in 0.3. **2**

Find the probability that on a particular day in June that the weather is fine, given that the flight departs on time.



$$P(F|T) = \frac{P(F \cap T)}{P(T)}$$

$$= \frac{0.3 \times 0.7}{0.3 \times 0.7 + 0.7 \times 0.5}$$ ✓

$$= \frac{0.21}{0.56}$$

$$= \frac{3}{8}$$

$\therefore P(F|T) = \frac{3}{8}$ or 0.375 ✓

Question 30 (4 marks)

A cup of coffee contains 100 mg of caffeine, which leaves the body at a continuous rate of 17% per hour.

(a) Write an equation for the amount of caffeine, A, in the body t hours after drinking a cup of coffee. **2**

$A = 100\,(1 - 0.17)^t$

$A = 100\,(0.83)^t$

✓ ✓

(b) How long does it take for the body to only have a quarter of the caffeine left? Give your answer correct to 2 decimal places. **2**

$25 = 100\,(0.83)^t$ ✓

$\frac{1}{4} = 0.83^t$

$t = \log_{0.83} 0.25$

$t = \frac{\log 0.25}{\log 0.83}$

$t = 7.44$ hours ✓

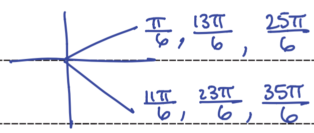
Question 31 (3 marks)

Find the exact values of x that satisfy the equation $2\cos 2x - \sqrt{3} = 0$ over the domain $[0, 3\pi]$. **3**

$2\cos 2x = \sqrt{3}$

$\cos 2x = \frac{\sqrt{3}}{2}$ ✓

Let $u = 2x \rightarrow \cos u = \frac{\sqrt{3}}{2}$ $[0, 6\pi]$



✓

$u = \frac{\pi}{6}, \frac{11\pi}{6}, \frac{13\pi}{6}, \frac{23\pi}{6}, \frac{25\pi}{6}, \frac{35\pi}{6}$

$x = \frac{\pi}{12}, \frac{11\pi}{12}, \frac{13\pi}{12}, \frac{23\pi}{12}, \frac{25\pi}{12}, \frac{35\pi}{12}$ ✓

Question 32 (4 marks)

Consider the geometric series $1 + (2\sqrt{3} - 3) + (2\sqrt{3} - 3)^2 + \cdots$

(a) Show that this series has a limiting sum. **2**

$r = \frac{2\sqrt{3} - 3}{1} = 2\sqrt{3} - 3 \approx 0.464$ ✓

$|r| < 1$ $\therefore$ The series has a limiting sum. ✓

(b) Find the exact value of the limiting sum. Express your answer with a rational denominator. **2**

$S = \frac{a}{1-r}$

$= \frac{1}{1 - (2\sqrt{3} - 3)}$ ✓

$= \frac{1}{4 - 2\sqrt{3}}$

$= \frac{4 + 2\sqrt{3}}{16 - 12}$

$= \frac{2 + \sqrt{3}}{2}$ ✓

Question 33 (5 marks)

The Bay of Fundy in Canada has the largest tides in the world. The difference between low and high water levels is 15 metres. At a particular point, the depth of water, y, in metres is given as a function of time, t, in hours since midnight by $y = k\cos\big(a(t-b)\big) + c$, where k, a, b and c are constants.

(a) Low tide at the Bay of Fundy results in a depth of 1.5 metres of water. High tide was first noted at 4 am, and the time difference between subsequent high tides is 12 hours. With the aid of a diagram, write the equation of y to model the Bay of Fundy tides. **4**

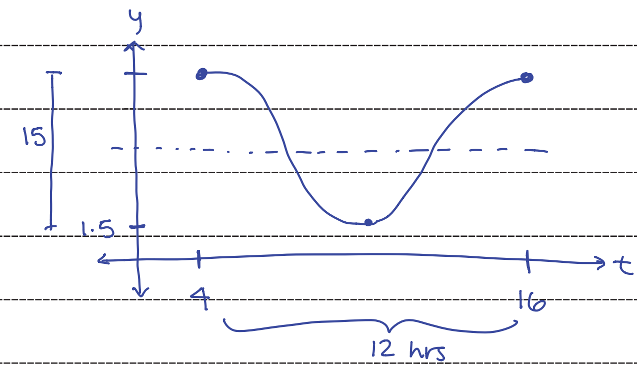
min = 1.5 metres

range = 15 metres

$A = \frac{15}{2} = 7.5$

$D = 1.5 + 7.5 = 9$

$C = 4$ (4 am)



$\frac{2\pi}{B} = 12$

$\frac{2\pi}{12} = B$

$\frac{\pi}{6} = B$

$\therefore\ y = 7.5\cos\left(\frac{\pi}{6}(t-4)\right) + 9$

(b) What is the depth of the water 2 hours after high tide? **1**

2 hrs after 4 am = 6 am

$y = 7.5\left(\cos\left(\frac{\pi}{6}(6-4)\right)\right) + 9$

$= 7.5(0.5) + 9$

$= 12.75$

$\therefore$ 12.75 metres deep

Question 34 (6 marks)

A prize fund of \$300 000 is established by the council of a school to provide scholarships. The fund is in an account that earns interest at the rate of 6% p.a. compounded monthly. At the end of each year, a prize of \$25 000 is withdrawn from the fund. Let A_n denote the amount remaining in the fund after the n^{th} year, just after the prize has been awarded for the year.

(a) Show that $A_3 = 300\,000(1.005)^{36} - 25000(1.005)^{24} - 25000(1.005)^{12} - 25000$. **2**

$r = 1 + \frac{0.06}{12} = 1.005$ 12 times compounded for 1 withdrawal

$A_1 = 300\,000(1.005)^{12} - 25\,000$ ✓

$A_2 = A_1(1.005)^{12} - 25000$

$= (300\,000(1.005)^{12} - 25\,000)(1.005)^{12} - 25000$

$= 300\,000(1.005)^{24} - 25000(1.005)^{12} - 25000$

$A_3 = A_2(1.005)^{12} - 25000$

$= (300\,000(1.005)^{24} - 25000(1.005)^{12} - 25000)(1.005)^{12} - 25000$

$= 300\,000(1.005)^{36} - 25000(1.005)^{24} - 25\,000(1.005)^{12} - 25000$ ✓

as required

(b) Show that $A_n = 300000(1.005)^{12n} - 405332.15(1.005^{12n} - 1)$. **2**

$A_n = 300000(1.005)^{12n} - 25000(1.005)^{12(n-1)} - 25000(1.005)^{12(n-2)} - \ldots - 25000(1.005)^{12} - 25000$

$= 300000(1.005^{12n}) - 25000\,(\underbrace{1.005^{12(n-1)} + 1.005^{12(n-2)} + \ldots + 1.005^{12} + 1}_{\text{This is a GP with } a=1,\ r=1.005^{12},\ n=n})$ ✓

$= 300\,000(1.005^{12n}) - 25000\left(\frac{1(1.005^{12n} - 1)}{1.005^{12} - 1}\right)$

$= 300\,000(1.005^{12n}) - 405332.15(1.005^{12n} - 1)$ as required ✓

(c) For how many years can the council provide the full amount of the scholarship? **2**

$$0 = 300\,000(1.005^{12n}) - 405332.15(1.005^{12n} - 1)$$

$$405332.15(1.005^{12n} - 1) = 300\,000(1.005^{12n})$$

$$1.3511(1.005^{12n} - 1) = 1.005^{12n}$$

$$1.3511(1.005^{12n}) - 1.3511 = 1.005^{12n}$$

$$1.3511(1.005^{12n}) - 1.005^{12n} = 1.3511$$

$$1.005^{12n}(1.3511 - 1) = 1.3511$$

$$1.005^{12n} = \frac{1.3511}{1.3511 - 1}$$

$$1.005^{12n} = 3.8481$$

$$12n = \frac{\log 3.8481}{\log 1.005}$$

$$n = \frac{\log 3.8481}{12 \log 1.005}$$

$$n = 22.52$$

$\therefore$ 22 years

END OF TASK