

Student Number:

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Teacher: Ms Mandile, Mrs Underwood, Mrs Dunn- please circle



Trial Examination 2021

HSC Year 12 Mathematics Advanced

General Instructions

- Reading time – 10 minutes
- Working time – 3 hours
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided at the back of this paper
- For questions in Section II, show relevant mathematical reasoning and/or calculations

Total Marks: 100

Section I – 10 marks (pages 2–6)

- Attempt Questions 1–10
- Allow about 15 minutes for this section

Section II – 90 marks (pages 7–29)

- Attempt Questions 11–30
- Allow about 2 hours and 45 minutes for this section

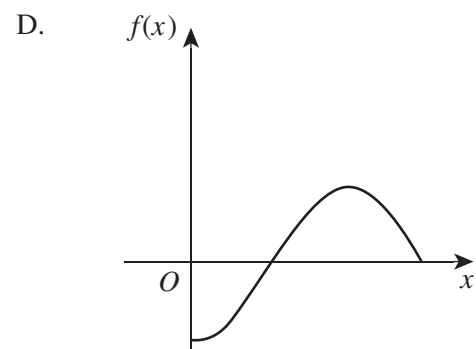
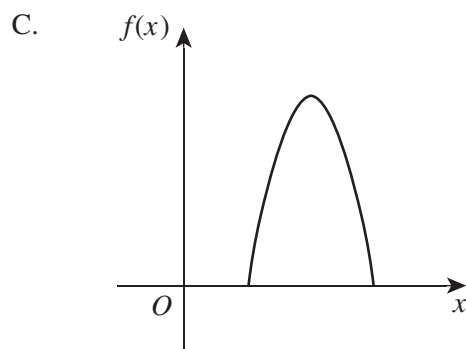
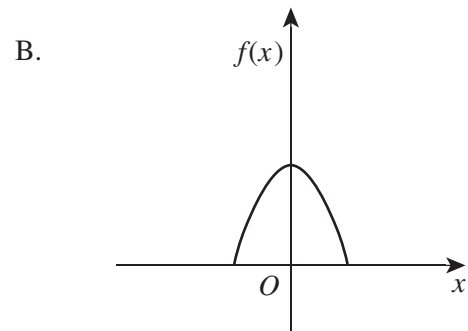
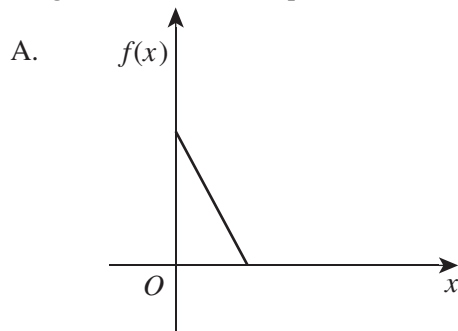
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SECTION I**10 marks****Attempt Questions 1–10****Allow about 15 minutes for this section**

For Questions 1–10, use the multiple choice answer sheet provided at the end of the paper.

- 1** If each of the functions below was integrated over the domain shown, which integral would **not** be equal to the area enclosed by the function and the x-axis?



- 2 The table shows the future value of a \$1 annuity for varying interest rates over different time periods.

<i>Time period</i>	<i>Interest rate</i>				
	<i>1%</i>	<i>2%</i>	<i>3%</i>	<i>4%</i>	<i>5%</i>
1	1.0000	1.0000	1.0000	1.0000	1.0000
2	2.0100	2.0200	2.0300	2.0400	2.0500
3	3.0301	3.0604	3.0909	3.1216	3.1525
4	4.0604	4.1216	4.1836	4.2465	4.3101
5	5.1010	5.2040	5.3091	5.4163	5.5256
6	6.1520	6.3081	6.4684	6.6330	6.8019
7	7.2135	7.4343	7.6625	7.8983	8.1420
8	8.2857	8.5830	8.8923	9.2142	9.5491

What is the present value of an annuity, correct to the nearest dollar, that would provide a future value of \$34 800 after 4 years at 2% per annum, compounded half-yearly?

- A. \$4055
 B. \$4200
 C. \$8195
 D. \$8443
- 3 Which series has a limiting sum of 10?

- A. $1+2+3+4$
 B. $9+1+0.1\dots$
 C. $5+2.5+1.25\dots$
 D. $1.25+2.5+5\dots$

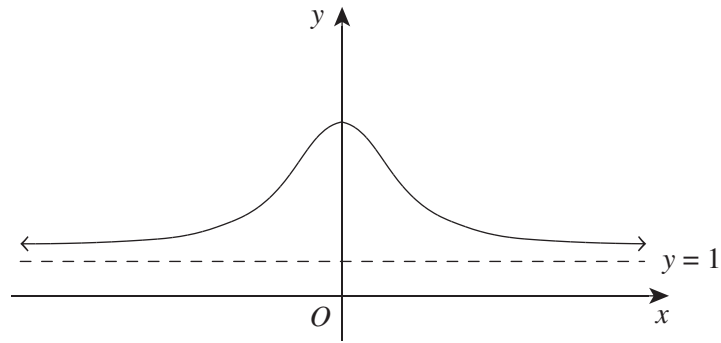
- 4 The table shows the accidents recorded on a motorway.

	Cause		
	Speeding	Alcohol consumption	Driver fatigue
Fatal	35	41	12
Non-fatal	75	149	55

What is the probability that an accident is fatal, given that alcohol consumption was NOT the cause?

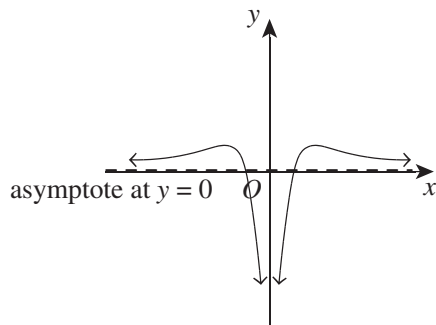
- A. $\frac{47}{367}$
- B. $\frac{47}{177}$
- C. $\frac{47}{88}$
- D. $\frac{177}{367}$
- 5 What is the domain and range of a circle with the equation $x^2 + 4x + y^2 = 5$?
- A. domain = $[-2 - \sqrt{5}, -2 + \sqrt{5}]$; range = $[-\sqrt{5}, \sqrt{5}]$
- B. domain = $[2 - \sqrt{5}, 2 + \sqrt{5}]$; range = $[-\sqrt{5}, \sqrt{5}]$
- C. domain = $[-1, 5]$; range = $[-3, 3]$
- D. domain = $[-5, 1]$; range = $[-3, 3]$
- 6 What is the maximum value of $y = 2\sin\left(\frac{x}{3}\right) + 1$ and the value of x occurring in the domain $[0, 2\pi]$?
- A. The maximum value of $y = 2\sin\left(\frac{x}{3}\right) + 1$ is 2 occurring at $x = \frac{\pi}{2}$.
- B. The maximum value of $y = 2\sin\left(\frac{x}{3}\right) + 1$ is 3 occurring at $x = \frac{\pi}{2}$.
- C. The maximum value of $y = 2\sin\left(\frac{x}{3}\right) + 1$ is 2 occurring at $x = \frac{3\pi}{2}$.
- D. The maximum value of $y = 2\sin\left(\frac{x}{3}\right) + 1$ is 3 occurring at $x = \frac{3\pi}{2}$.

- 7 The graph shows the function $y = f(x)$ with a horizontal asymptote at $y = 1$.

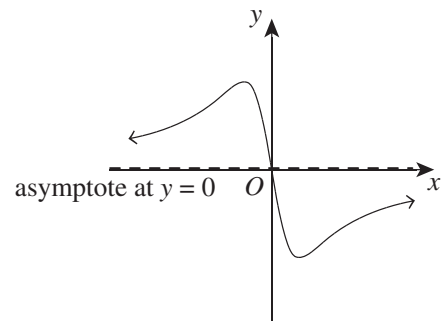


Which of the following could be the graph of $y = f'(x)$?

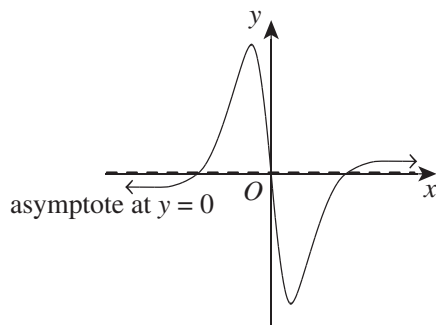
A.



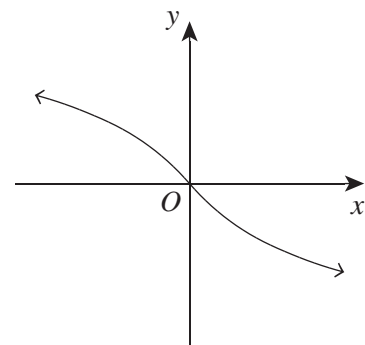
B.



C.



D.



- 8 Given that $\int_2^6 f(x) dx = 3$, what is the value of $\int_4^6 f(2(x-3)) dx$?

- A. $\frac{1}{2}$
 B. $\frac{3}{2}$
 C. 3
 D. 8

- 9 Given that $f(x)$ is an even function and $g(x)$ is an odd function, which of the following statements is true?

- A. $f(x) \times g(x)$ is an even function.
- B. $f(x) + g(x)$ is an odd function.
- C. $f[g(x)]$ is an even function.
- D. $g[f(x)]$ is an odd function.

- 10 The discrete random variable X has the following probability distribution.

x	-1	0	1	2	3
$P(X=x)$	$\frac{1}{10}$	a	b	b	$2b$

What are the possible values of a and b ?

- A. $a = \frac{1}{3}$ $b = \frac{37}{120}$
- B. $a = \frac{1}{3}$ $b = \frac{17}{120}$
- C. $a = \frac{1}{3}$ $b = \frac{27}{40}$
- D. $a = \frac{1}{3}$ $b = \frac{3}{40}$

HSC Year 12 Mathematics Advanced

Section II Answer Booklet

Section II

90 marks

Attempt Questions 11–30

Allow about 2 hours and 45 minutes for this section

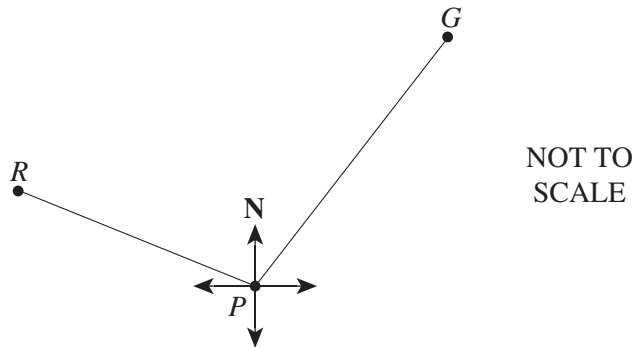
Instructions

- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
 - Your responses should include relevant mathematical reasoning and/or calculations.
 - Extra writing space is provided on pages 30–32 of the Booklet. If you use this space, clearly indicate which question you are answering.
-

Please turn over

Question 11 (3 marks)

Ringo and Greg start at location P and drive in different directions. Ringo drives at a speed of 70 km/h on a bearing of 285° , while Greg drives at a speed of 60 km/h on a bearing of 030° , as shown in the diagram.



- (a) Show that $\angle GPR = 105^\circ$.

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- (b) What will be the distance between Ringo and Greg after travelling for 2 hours?
Give your answer correct to the nearest kilometre.

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Question 12 (3 marks)

In a chess tournament, each player has a ‘chess rating’, which is a value from 1600 to 2000.

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Danny wants to know if there is a correlation between the number of years a player has spent playing chess and their chess rating.

Danny randomly surveyed 10 players at the tournament. The results are shown in the table.

	<i>Years spent playing chess (x)</i>	<i>Chess rating (y)</i>
<i>Player 1</i>	1	1650
<i>Player 2</i>	2	1550
<i>Player 3</i>	2	1600
<i>Player 4</i>	3	1500
<i>Player 5</i>	4	1900
<i>Player 6</i>	4	1650
<i>Player 7</i>	6	1700
<i>Player 8</i>	8	1950
<i>Player 9</i>	10	1950
<i>Player 10</i>	15	2000

Beth, a competitor in the tournament, has spent 5 years playing chess.

Use the equation of the least-squares regression line for this data set to determine Beth’s chess rating. Give your answer correct to the nearest whole number.

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Question 13 (continued)

The revenue generated from an event, R , is calculated using $R = N \times P$.

- (b) What is the maximum revenue generated from this event?

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- (c) The manager of this event wants the tennis arena to be limited to 15 000 spectators.

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Show that this will result in a revenue loss of \$390 625.

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End of Question 13

Question 14 (3 marks)

Find the equation of the tangent to the curve $y = \frac{1}{4x+1}$ at the point (0, 1).

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Question 15 (4 marks)

(a) Differentiate $y = \frac{1}{2} \ln(x^2)$ with respect to x .

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(b) Differentiate $y = \frac{e^x}{\sin x}$ with respect to x .

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Question 16 (2 marks)

Evaluate $\int_0^{\frac{\pi}{4}} x + \sin x \, dx$.

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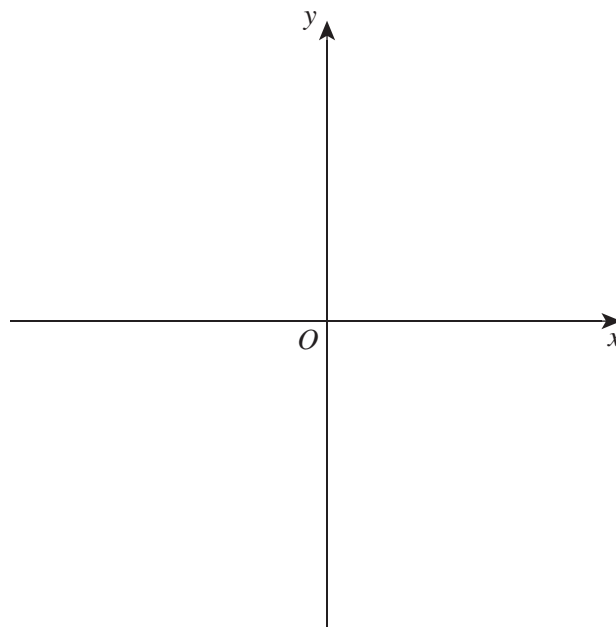
Question 17 (3 marks)

The function $y = f(x)$ is continuous for all values of x . The following is known of the function's properties.

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- When $x < -1$, $f'(x) > 0$ and $f''(x) < 0$.
- When $x > -1$, $f'(x) > 0$ and $f''(x) > 0$.
- $f(-1) = f'(-1) = f''(-1) = 0$

On the axes below, sketch the graph of $y = f(x)$, labelling any x -intercepts.



Question 18 (3 marks)Solve the equation $\log_{10}(x - 21) = 2 - \log_{10}x$.**3**

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Question 19 (3 marks)

An arithmetic sequence has the first term 5 and a common difference of d . The sum of the first 20 terms is 4 times the sum of the first 10 terms.

3Find the value of d .

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Question 20 (4 marks)

Shirley uses her car every day. Sometimes, she needs a few attempts to start her car. Let the discrete random variable X represent the number of attempts needed.

The probability function for the random variable X is given by

$$P(X = x) = \begin{cases} a & x = 1 \\ \frac{1}{2}P(X = x - 1) & x = 2, 3, 4, 5 \\ 0 & \text{for all other values of } x \end{cases}$$

- (a) Show that $a = \frac{16}{31}$.

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- (b) The probability distribution table for the discrete random variable X is shown.

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x	1	2	3	4	5
$P(X = x)$	$\frac{16}{31}$	$\frac{8}{31}$	$\frac{4}{31}$	$\frac{2}{31}$	$\frac{1}{31}$

Shirley claims that after an entire year, she needs an average of two attempts to successfully start her car.

Calculate the expected value and comment on the accuracy of Shirley's claim.

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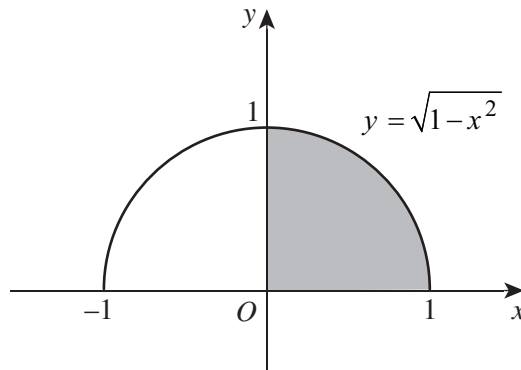
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Question 21 (4 marks)

The graph of $y = \sqrt{1-x^2}$ is shown.



The table gives the values of $\sqrt{1-x^2}$ for x .

x	0	0.25	0.5	0.75	1
$\sqrt{1-x^2}$	1	0.968		0.661	0

- (a) Complete the table above. Answer to 3 decimal places. 1
- (b) Hence, use the trapezoidal rule to find an approximation of the shaded area, to 3d.p. 2

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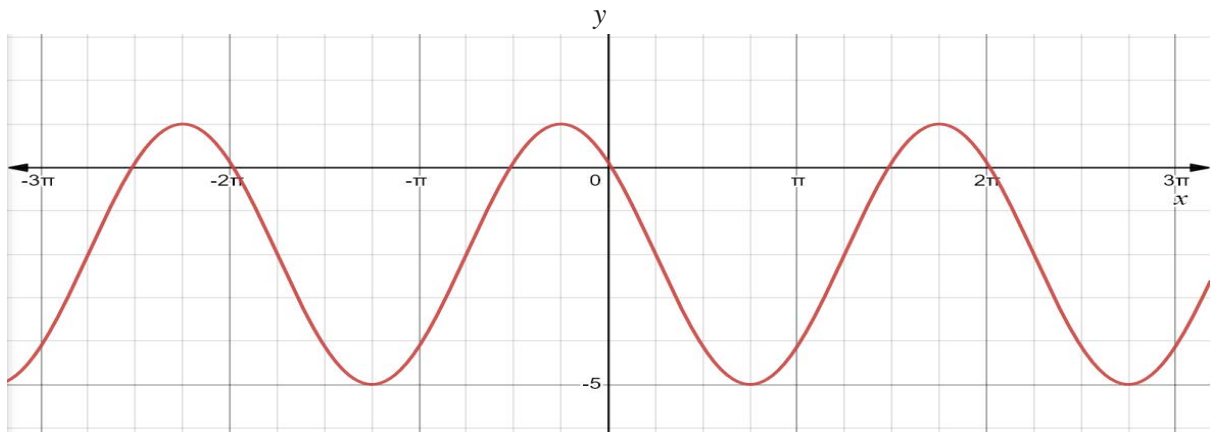
- (c) How could a better approximation of the shaded area be obtained using the trapezoidal rule? 1

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Question 22 (3 marks)

The graph is a function of the form $y = k \cos(x + a) + b$



Find the equation of the function.

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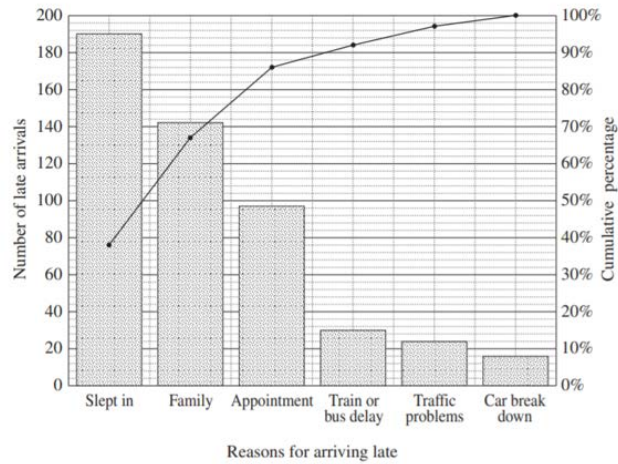
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Question 23 (2 marks)

A school collected data related to the reasons given by students for arriving late. The Pareto chart shows the data collected.



- (a) How many of the main causes above does it take to account for 80% of the problems?

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- (b) Approximately what percentage of students gave the reason 'appointment'?

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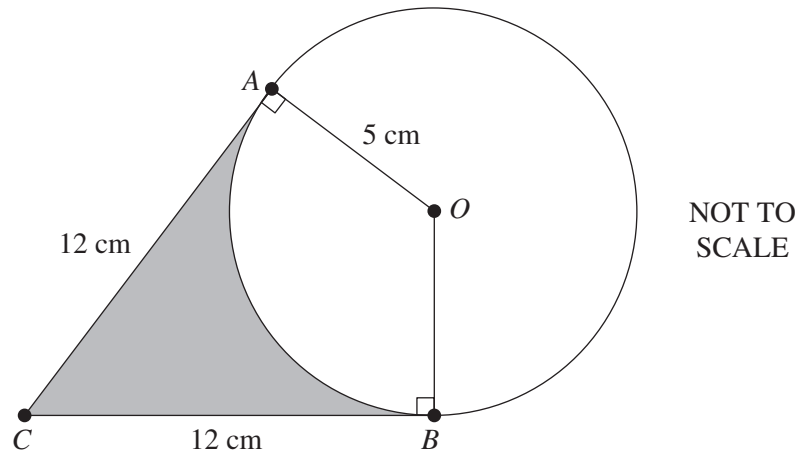
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Question 24 (4 marks)

The diagram shows a circle with centre O and radius 5 cm. Tangents are drawn from the points A and B , which meet at point C .



- (a) Show that $\angle AOB = 2.4$ radians correct to two significant figures.

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- (b) Hence, find the area of the shaded region.

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Question 25 (6 marks)

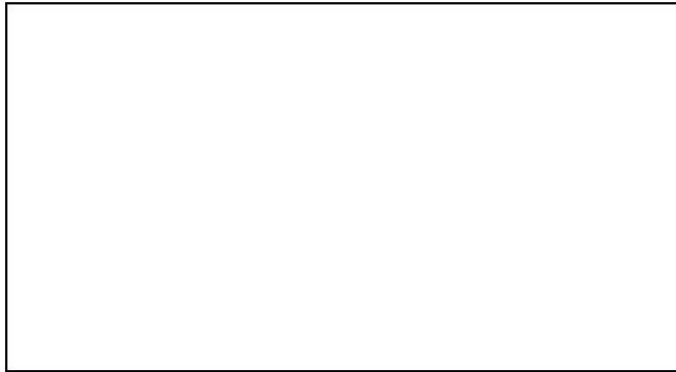
A teacher surveyed his class of 24 students about the subjects that they have chosen to study in the following year.

The results indicate that:

- 8 students chose to study neither Physics nor Chemistry
- 8 students chose to study Chemistry
- 12 students chose to study Physics.

(a) Draw a Venn diagram showing this information.

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(b) A student choosing to study Chemistry is event 1. A student choosing to study Physics is event 2.

2

If a student is chosen at random, determine whether these events are independent of each other.

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Question 25 continues on page 21

Question 25 (continued)

(c) The entire year group is given the same survey. The results indicate that:

2

- the probability that a student chose to study Chemistry is $\frac{1}{3}$
- the probability that a student chose to study Physics is $\frac{2}{5}$
- given that a student chose to study Chemistry, the probability that they chose to study Physics is $\frac{3}{7}$.

Calculate the probability that a student picked at random has chosen to study Chemistry or Physics.

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End of Question 25

Question 26 (6 marks)

A particle is moving in a straight line with a velocity given by

$$\frac{dx}{dt} = 8 \cos\left(2t - \frac{\pi}{2}\right),$$

where x is the displacement from the origin in metres and t is time measured in seconds.

The particle is initially at rest 4 m to the right of the origin.

- (a) Show that the displacement of the particle is given by $x = 4 \sin\left(2t - \frac{\pi}{2}\right) + 8$. **3**

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- (b) Show that the particle comes to rest again when it is 12 m to the right of the origin **2**
and $t = \frac{\pi}{2}$ s.

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- (c) Find the function for acceleration of the particle **1**

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Question 29 (11 marks)

Jonathan's parents decide to set up an investment account to help pay for his university degree. Jonathan's parents deposit \$1000 into the account every year, which earns interest at 4% per annum, compounded yearly. They deposited the first \$1000 at the beginning of the year when Jonathan is born. Jonathan will receive the full amount at the end of the year he turns 18 years old.

Let A_n be the value of the account after n years.

- (a) Show that $A_3 = 1000(1.04)(1 + 1.04 + 1.04^2)$. **3**

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- (b) Show that $A_{18} = 26\,000(1.04^{18} - 1)$. **3**

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Question 29 continues on page 26

Question 29 (continued)

- (c) The amount in the account continues to earn interest at 4% per annum. Jonathan enrolls in a four-year university course, which costs $\$M$ each year, to be paid at the end of the year. As a result:

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$$A_{19} = 26000(1.04^{18} - 1)(1.04) - M.$$

How much can Jonathan withdraw from the account each year if the entire amount is to be withdrawn over the four years?

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End of Question 29

Question 30 (9 marks)

- (a) (i) Show that the equation $-\frac{1}{2}\sin 4x + 4\cos 4x = 0$ can be written as $\tan 4x = 8$. **1**

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- (ii) Show that the exact value to the first two solutions to the equation $\tan 4x = 8$ in the domain $[0, \pi]$ are **2**

$$x_1 = \frac{1}{4} \tan^{-1} 8$$

$$\text{and } x_2 = \frac{1}{4}(\pi + \tan^{-1} 8).$$

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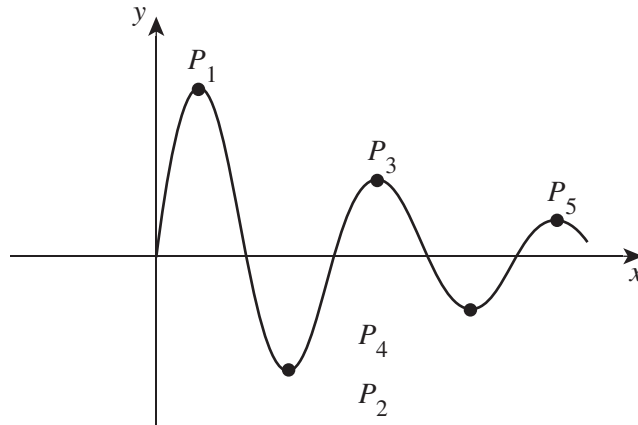
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Question 30 continues on page 28

Question 30 (continued)

- (b) The graph of $y = 10e^{-\frac{1}{2}x} \sin 4x$ in the domain $[0, \infty)$ is shown.



The stationary points are labelled $P_1, P_2, P_3, \dots, P_n$. Let X_n and Y_n be the x - and y -coordinates of P_n respectively. This means the stationary points are $P_1(X_1, Y_1), P_2(X_2, Y_2), \dots, P_n(X_n, Y_n)$.

- (i) Using part (a) write down the x -coordinates of P_1 and P_2 . (Hint: Differentiate the function) **3**

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Question 30 continues on page 29

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Section II extra writing space

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