



--	--	--	--	--	--	--	--	--

Student Number

2021 Glenwood High School
Year 12 Trial HSC Examination
Assessment Task 4

Mathematics Advanced

General**Instructions**

- Reading Time – 10 minutes
- Working time – 3 hours
- Write using black pen
- NESA approved calculators may be used
- A reference sheet will be provided
- For questions in Section II, show relevant mathematical reasoning and/or calculations

Total marks:**100****Section I – 10 marks** (pages 2 – 4)

- Attempt Questions 1- 10
- Allow about 15 minutes for this section

Section II – 90 marks (pages 5 – 36)

- Attempt Questions 11 - 36
- Allow about 2 hours and 45 minutes for this section

Section I

10 marks

Attempt Questions 1–10

Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1 – 10

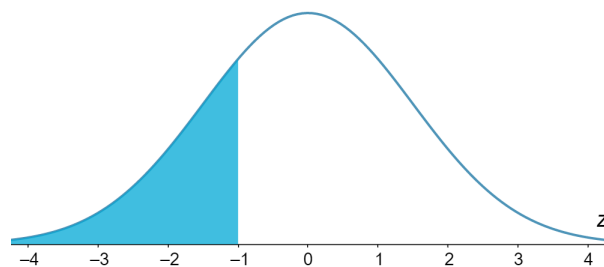
1. The point $P(8, -3)$ lies on the graph of $y = f(x)$.
Find the coordinates of the image of P if the function is transformed to :
 $y = -2f(x + 7) + 5$.

- A. $(1, 11)$
- B. $(1, -1)$
- C. $(15, 11)$
- D. $(15, -1)$

2. Which interval gives the range of the function $y = 6 \sin x - 10$?

- A. $[6, -10]$
- B. $[4, 16]$
- C. $[-16, -4]$
- D. $[-4, 16]$

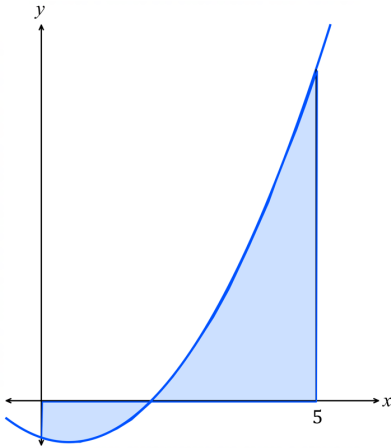
3. The graph shows the area under a normal distribution curve.



Which of the following is equivalent to the shaded area?

- A. $P(z \geq 1)$
- B. $P(z \geq -1)$
- C. $1 - P(-1 \leq z \leq 1)$
- D. $P(z \leq 1)$

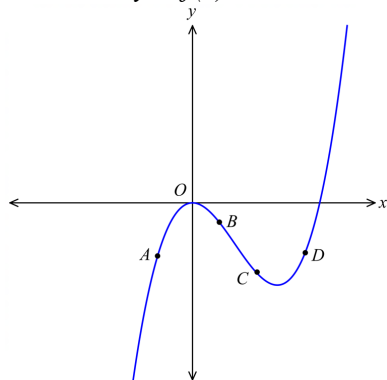
4. Which expression will give the area of the shaded region bounded by the curve $y = x^2 - x - 2$, the x -axis and the lines $x = 0$ and $x = 5$?



- A. $A = \left| \int_0^1 (x^2 - x - 2) dx \right| + \int_1^5 (x^2 - x - 2) dx$
- B. $A = \int_0^1 (x^2 - x - 2) dx + \left| \int_1^5 (x^2 - x - 2) dx \right|$
- C. $A = \left| \int_0^2 (x^2 - x - 2) dx \right| + \int_2^5 (x^2 - x - 2) dx$
- D. $A = \int_0^2 (x^2 - x - 2) dx + \left| \int_2^5 (x^2 - x - 2) dx \right|$
5. What is the equation of the tangent to the curve $y = \sin x$ at the origin?
- A. $y = -x$
- B. $y = \cos x$
- C. $y = \sin x$
- D. $y = x$
6. An amount of \$5000 is invested and compounded annually at 6%. After how many years will the investment first exceed twice its initial value?
- A. 11
- B. 12
- C. 13
- D. 14

7. What is the amplitude and period for the function $f(x) = 4\sin\left(\frac{x+\pi}{3}\right)$?
- A. Amplitude 3 and period $\frac{\pi}{2}$
 - B. Amplitude 3 and period 2π
 - C. Amplitude 4 and period $\frac{2\pi}{3}$
 - D. Amplitude 4 and period 6π
8. Given that $\log_a b = 3.75$ and $\log_a c = 0.25$
Find $\log_a (bc)^2$.
- A. 0.9375
 - B. 3.5
 - C. 8
 - D. 0.879
9. How many solutions does $2\sin^2 x - 1 = 0$ have for $-\frac{\pi}{2} \leq x \leq \pi$.
- A. 1
 - B. 2
 - C. 3
 - D. 4

10. The graph of the relation $y = f(x)$ is shown below:



At which point on the curve is $f'(x) < 0$ and $f''(x) < 0$?

- A. A
- B. B
- C. C
- D. D

END OF SECTION I

**2021 TRIAL HIGHER SCHOOL CERTIFICATE
EXAMINATION**

--	--	--	--	--	--	--	--	--

Student Number

Mathematics Advanced**Section II Answer Booklet 1****90 marks****Attempt Questions 11 – 36****Allow about 2 hour and 45 minutes for this section****Booklet 1 — Attempt Questions 11–20 (31 marks)****Booklet 2 — Attempt Questions 21–36 (59 marks)****Instructions**

- Write your Student Number at the top of this page.
- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of responses.
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided on pages 14–16 of Booklet 1. If you use this space, clearly indicate which question you are answering.

Question 11 (2 marks)

A function $f(x)$ has the derivative $f'(x) = 4x^3 - 6x^2 + 5$ and $f(1) = 10$.

2

Find the function.

.....

.....

.....

.....

.....

.....

.....

.....

Question 12 (2 marks)

Solve $2 \sin^2\left(\frac{x}{3}\right) = 1$ for the domain $[-\pi, \pi]$

2

.....

.....

.....

.....

.....

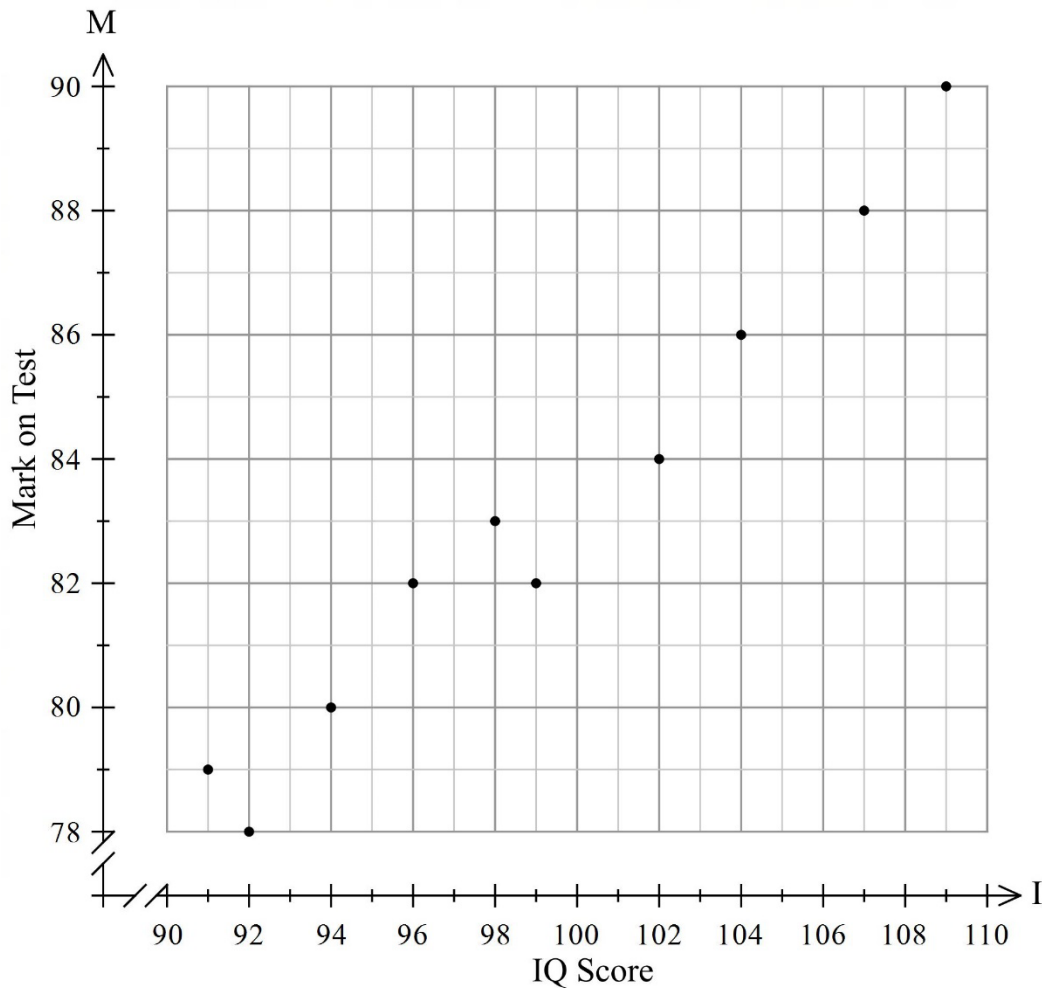
.....

.....

.....

Question 13 (2 marks)

The bivariate data in the scatterplot below compares the recorded IQ score of students with their mark on a class test out of 100.



- (a) Describe the direction and strength of the correlation between IQ score and test marks.

1

.....

.....

- (b) The equation of the least-squares regression line is $M = 0.615I + 22.176$.

1

Using the equation, predict the test mark of a student with an IQ of 114. Answer to the nearest whole number.

.....

.....

.....

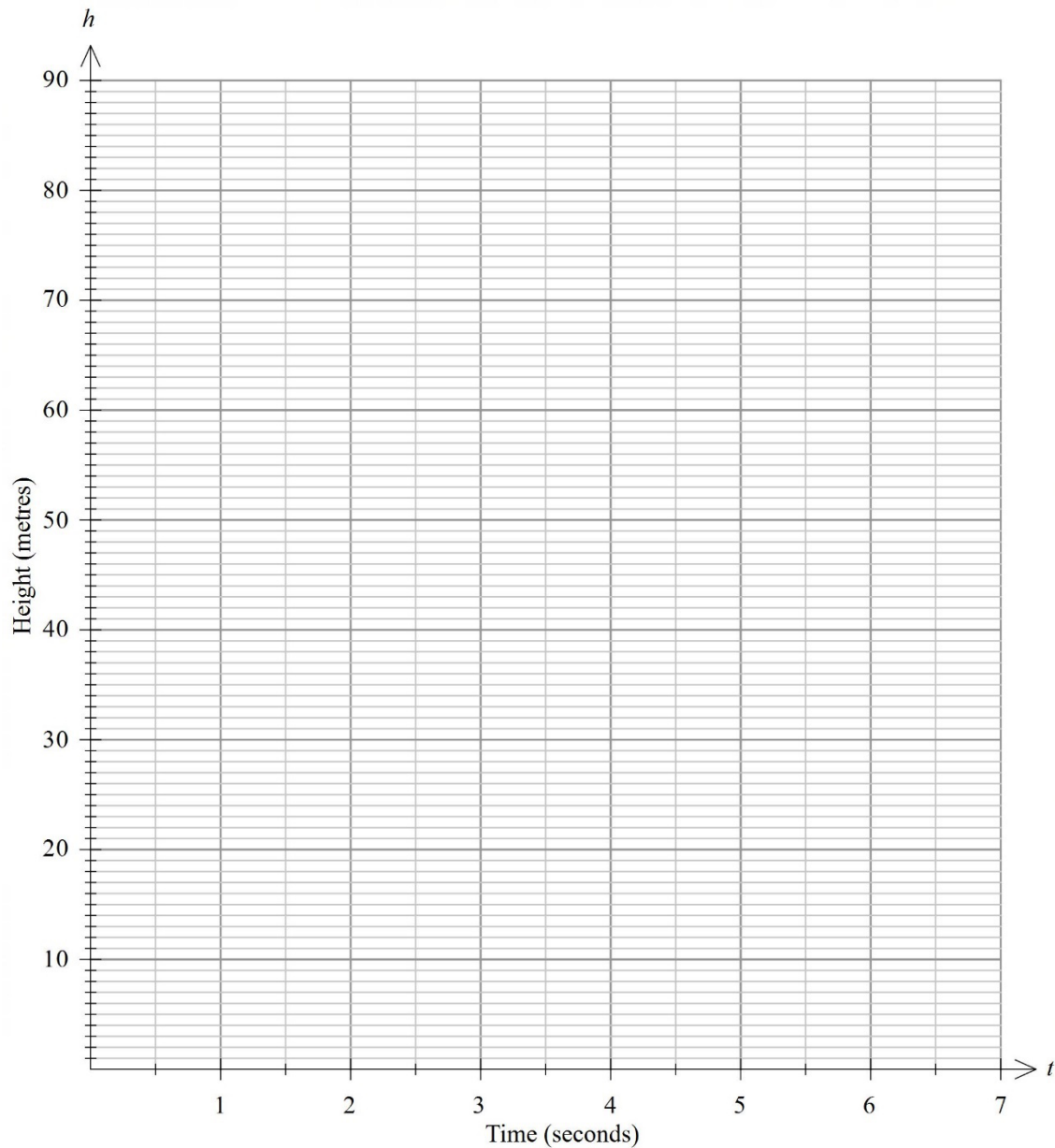
Question 14 (5 marks)

From the top of a tower, which is 60 metres high, Isaac throws a stone upward and allows it to fall under gravity.

The height of the stone is modelled by the equation, $h = 20t - 5t^2 + 60$, where h is the height (in metres) of the stone above ground level and t is the time in seconds since the stone was thrown.

- (a) On the grid below, draw the graph of this model and label it as *Stone*.

2



.....

.....

.....

Question 14 continues next page

Question 14 (continued)

- (b) At the same time as the stone is thrown, Jay releases a toy glider from the top of the tower. 2
The glider drifts down, losing height at a constant rate of 5 metres per second.
Draw the graph modelling the height of the glider on the grid on the previous page and determine the equation which describes the model. Label the graph *Glider*.

.....

.....

.....

- (c) Determine the height of the glider when the stone reaches the ground. 1

.....

.....

Question 15 (3 marks)

- Show that $\frac{1}{y} \left(\frac{d^2 y}{dx^2} \right) \div \frac{dy}{dx} = 2$ when $y = \tan x$ 3

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

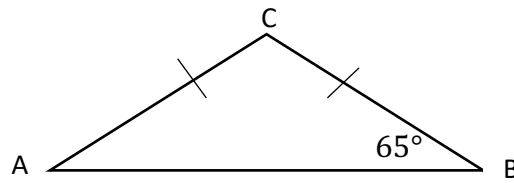
.....

.....

.....

Question 16 (4 marks)

- (a) In the diagram, ABC is a triangle where $AC = CB$, $\angle ABC = 65^\circ$ and the area of the triangle is 70 cm^2 .



NOT TO SCALE

Show that $AC = \sqrt{\frac{140}{\sin 50^\circ}}$

2

.....

.....

.....

.....

.....

.....

.....

.....

- (b) Find the perimeter of triangle ABC , correct to 1 decimal place

2

.....

.....

.....

.....

.....

.....

.....

.....

Question 17 (4 marks)

The concentration of a pesticide in the soil is modelled by the equation given by

$$P = P_0 e^{-kt}$$

where P_0 and k are constants, and t is the number of days after the application of the pesticide.

- (a) Once in the soil, the half-life of the pesticide is the time taken for its concentration to be reduced to one half of its initial value. If the half-life of the pesticide is 25 days, find the value of k . Answer correct to 2 significant figures

2

.....

.....

.....

.....

.....

.....

.....

.....

- (b) What is the percentage decrease in concentration of the pesticide after 80 days? Answer to the nearest whole number

2

.....

.....

.....

.....

.....

.....

.....

.....

.....

Question 18 (3 marks)

Solve $2 \log_2(x) - \log_2(x + 3) = 2$.

3

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

Question 19 (2 marks)

Find $\int \frac{4 \sin\left(\frac{5x}{3}\right)}{7} dx$

2

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

Question 20 (4 marks)

Six red balls and four green balls are placed in a hat. One ball is selected and is not replaced back into the hat. A second ball is then selected.

- (a) Draw a tree diagram to show the possible outcomes. Include the probability on each branch. **2**

- (b) Find the probability that a red and green ball is selected. **1**

.....

.....

.....

- (c) Find the probability that at least one red ball is selected. **1**

.....

.....

.....

Section II Extra writing space

If you use this space, clearly indicate which question you are answering.

[illegible]

Section II Extra writing space

If you use this space, clearly indicate which question you are answering.

[illegible]

Section II Extra writing space

If you use this space, clearly indicate which question you are answering.

[illegible]

**2021 TRIAL HIGHER SCHOOL CERTIFICATE
EXAMINATION**

--	--	--	--	--	--	--	--	--

Student Number

Mathematics Advanced**Section II Answer Booklet 2****Booklet 2 — Attempt Questions 21–36 (59 marks)****Instructions**

- Write your Student Number at the top of this page.
- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of responses.
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided on pages 37–40 of Booklet 2. If you use this space, clearly indicate which question you are answering.

Question 21 (5 marks)

A particle moves such that its velocity at a given time is given by:

$$v(t) = 8 - 16 \sin t$$

- (a) What is the initial acceleration of the particle? 1

.....

.....

.....

.....

.....

- (b) Find when the particle first comes to rest. 1

.....

.....

.....

.....

.....

.....

- (c) Given that the particle is initially at the origin, find an equation for the displacement, $x(t)$. 1

.....

.....

.....

.....

.....

.....

Question 21 continues next page

Question 21 (continued)

- (d) Find the distance travelled between the first two times when the particle is at rest.

2

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

Question 22 (2 marks)

Find $\int \frac{x-3}{x^2-6x+5} dx$.

2

.....

.....

.....

.....

.....

.....

Question 23 (3 marks)

The table below shows the future value of a \$1 annuity.

<i>Future value of \$1</i>						
<i>End of year</i>	1%	2%	3%	4%	5%	6%
1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	2.0100	2.0200	2.0300	2.0400	2.0500	2.0600
3	3.0301	3.0604	3.0909	3.1216	3.1525	3.1836
4	4.0604	4.1216	4.1836	4.2465	4.3101	4.3746

- (a) An annuity of \$31 800 is invested at 2% per annum, compounded annually for 4 years. What is the future value of the annuity? 1

.....

.....

.....

- (b) Sophia aims to have a deposit for an apartment of at least \$53 000 in 2 years' time by investing in an annuity. The annuity has an interest rate of 6% p.a. compounding every six months. 2

Calculate Sophia's half-yearly contribution to achieve the deposit.

.....

.....

.....

.....

.....

.....

.....

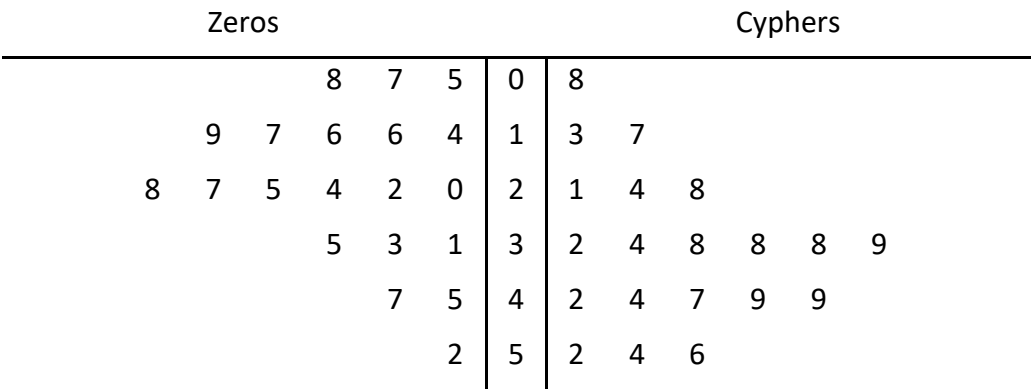
.....

.....

.....

Question 24 (4 marks)

The back-to-back stem-and-leaf plots below compare the points Perry scored in playing two games called *Zeros* and *Cyphers* 20 times each.



(a) Describe the shape of distribution for the *Zeros* game in terms of skewness. 1

.....

.....

(b) Which game had the greater interquartile range? Justify your answer with calculations. 2

.....

.....

.....

.....

.....

.....

(c) Perry scores 56 in his next game of *Zeros*. Is this score and outlier? Justify your answer with calculations. 1

.....

.....

.....

.....

.....

Question 25 (3 marks)

1

- (a) Complete the table of values for $y = \sqrt{1 - x^2}$
Answer to 3 significant figures where required.

x	0	0.125	0.25	0.375	0.5
y			0.968		0.866

2

- (b) By using the Trapezoidal rule with 4 sub-intervals, estimate the integral:

$$\int_0^{\frac{1}{2}} \sqrt{1 - x^2} \, dx$$

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

Questions 11 – 25 are worth 48 marks in total

Question 26 (3 marks)

Two websites give reviews of books. Both sites give each book a rating out of a possible 100. A statistician analysed the ratings awarded by the two websites to 100 books and obtained the statistical results below.

Website	Mean Rating	Standard Deviation
<i>Bookophile</i>	60	12
<i>Great Reads</i>	76	8

- (a) What percentage of books would have a rating between 52 and 76 on the *Great Reads* website? 1

.....

.....

.....

.....

.....

- (b) The book “Moonlighting” got a rating of 78 on *Bookophile* and a rating of 84 on *Great Reads*. Convert both these ratings to z-scores and determine which website rated the book higher in comparison to other books. 2

.....

.....

.....

.....

.....

.....

.....

.....

.....

Question 27 (3 marks)

Show that $\sin x + 1 + \cos x \cot x - \operatorname{cosec} x = 1$

3

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

Question 28 (2 marks)

2

The graph of $y = \sqrt{x}$ undergoes the following transformation:

- Translation 3 units right
- Dilation horizontally by a factor of $\frac{1}{2}$
- Translation 1 unit down
- Dilation vertically by a factor of 4

Find the rule of the resulting graph.

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

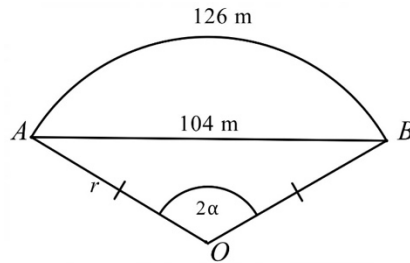
Please turn over

Question 29 (4 marks)

A sector of a circle, centre O , is shown below.

The points A and B lie on the circle, such that the length of the chord AB is 104 metres, and the length of the arc AB is 126 metres.

The radius is r metres and the angle subtended at the centre by the arc is 2α radians.



- (a) Show that $\sin \alpha = \frac{52\alpha}{63}$

2

.....

.....

.....

.....

.....

.....

.....

- (b) If $\alpha = \frac{\pi}{3}$, find the exact radius of the circle.
Express your answer with a rational denominator.

2

.....

.....

.....

.....

.....

.....

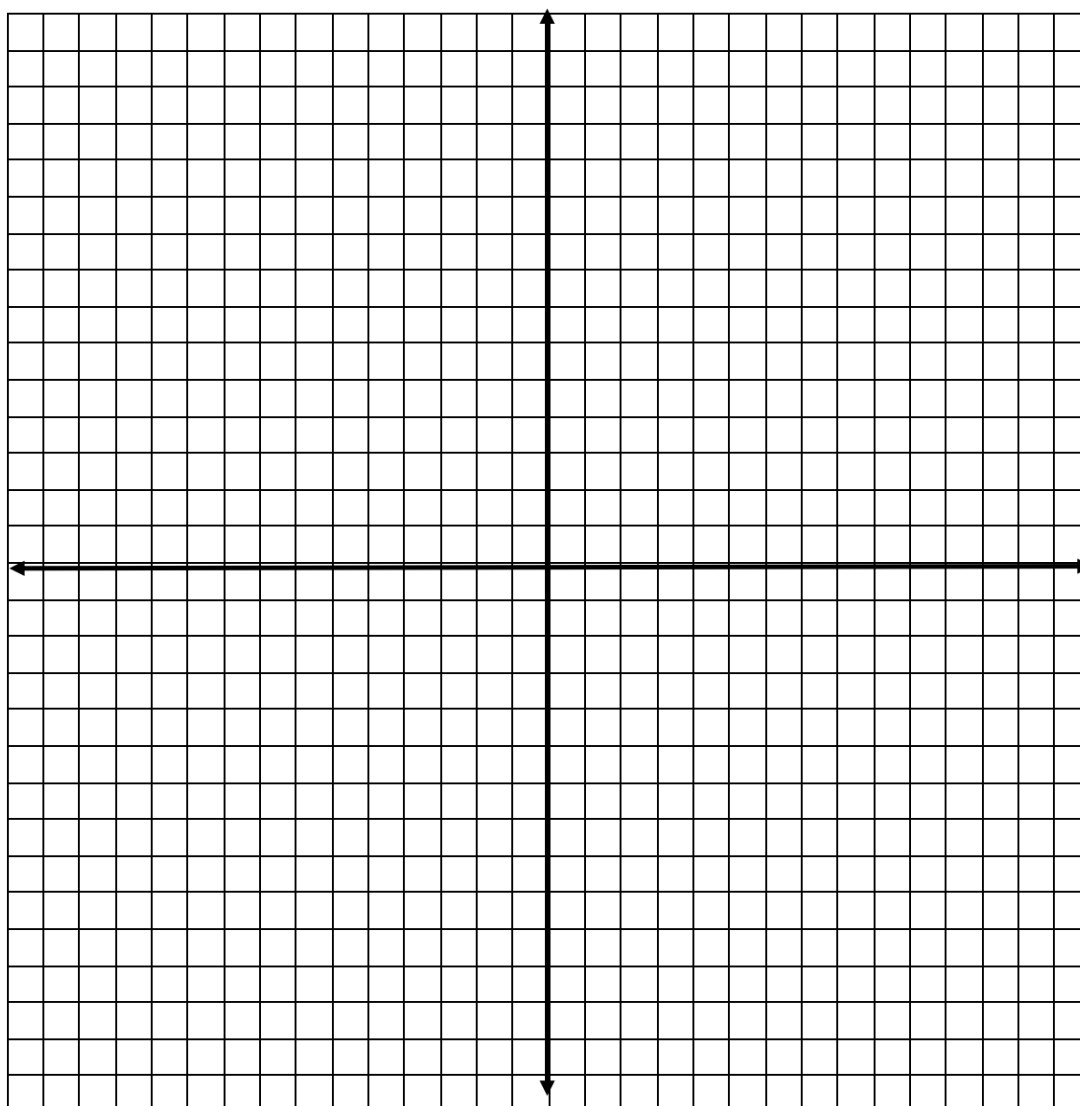
.....

Question 30 (3 marks)

By drawing graphs on the number plane, determine how many solutions there are to the equation

$$\sin x = \frac{3}{4} \left| \frac{x}{7} \right| \text{ in the domain } (-\infty, \infty).$$

3



.....

.....

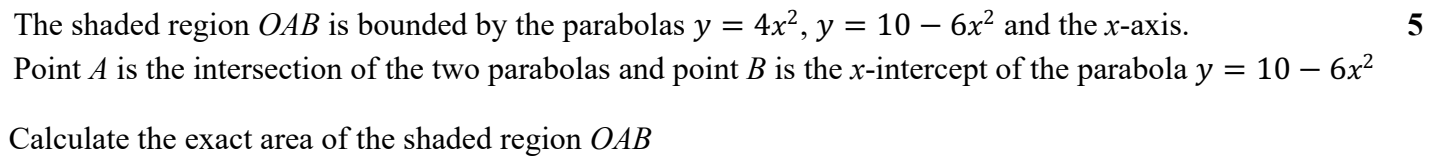
.....

.....

.....

.....

.....

This image shows a full page of white paper with horizontal dashed lines, typical of primary school writing paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

Question 32 (continued)

(b) Find the median value of X .

2

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

Question 33 (2 marks)

By first differentiating $\ln(\cos x)$, find $\int_0^{\frac{\pi}{3}} \tan x \, dx$.

2

.....

.....

.....

.....

.....

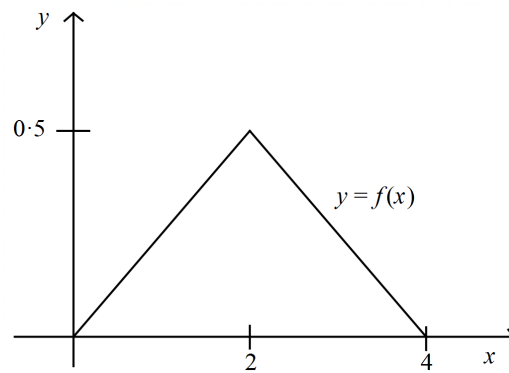
.....

.....

.....

Question 34 (3 marks)

The graph of the probability density function of a random variable is shown below, where $y = f(x)$ exists for all x .



Explain why $y = f(x)$ could be a probability density function and write down the piece-wise equation that describes the probability density function.

This image shows a full page of white paper with horizontal dashed lines, typical of primary school writing paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

Question 35 (continued)

- (b) Show that the total surface area $A \text{ cm}^2$, of the box, is given by

2

$$A = \frac{1200\sqrt{3}}{x} + \frac{\sqrt{3}x^2}{2}.$$

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

- (c) Find the value of x that gives the minimum total surface area.

3

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

Question 36 (5 marks)

A curve is defined by $f(x) = \frac{1}{4}x^4 - 2x^2 + 1$.

Sketch the curve for the domain $[-3, 3]$, showing all key features including classifying the nature of stationary points and finding any inflections. You **do not** need to find the x – intercepts.

5

Space to draw the graph is provided on the next page.

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

Answer space for Question 36 continues next page

[illegible]

L

Additional space to draw graph

END OF EXAMINATION

Section II Extra writing space

If you use this space, clearly indicate which question you are answering.

[illegible]

Section II Extra writing space

If you use this space, clearly indicate which question you are answering.

[illegible]

Section II Extra writing space

If you use this space, clearly indicate which question you are answering.

[illegible]

Section II Extra writing space

If you use this space, clearly indicate which question you are answering.

[illegible]

Section II Extra writing space

If you use this space, clearly indicate which question you are answering.

[illegible]