



PLC PRESBYTERIAN
LADIES' COLLEGE
SYDNEY
1888

Mathematics Advanced HSC Trial 2021

General Instructions

- Reading time – 10 minutes
- Working time – 3 hours
- Write using black pen
- Calculators approved by NESA may be used
- For questions in Section II, show relevant mathematical reasoning and/or calculations

Total marks: 100

Section I – 10 marks (pages 2-6)

- Attempt Questions 1-10
- Allow about 15 minutes for this section

Section II – 90 marks (pages 8-30)

- Attempt Questions 11-36
- Allow about 2 hours and 45 minutes for this section

Section I

10 marks

Attempt Questions 1-10

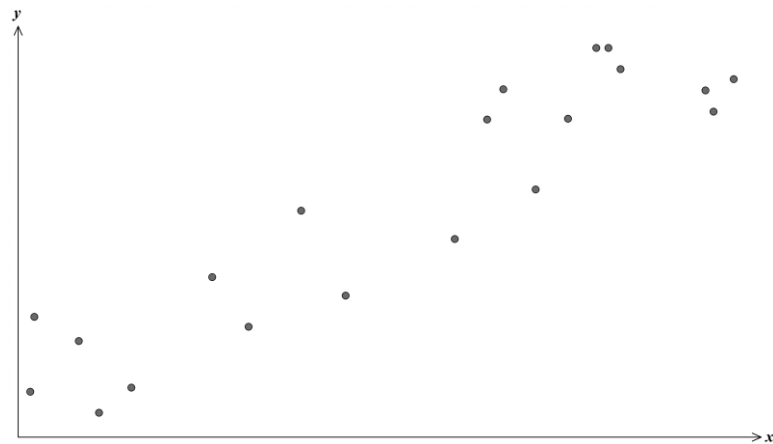
Allow about 15 minutes for this section

Write the correct alternative on your answer page

1 What is the period of $y = 2\sin(2\pi x)$?

- A. 1
- B. $\frac{\pi}{2}$
- C. 2
- D. π

2 What is the best description of the correlation between x and y for the data shown on the scatterplot?



- A. Moderate positive
- B. Moderate negative
- C. No correlation
- D. Weak positive

3 If $\tan \theta = -\frac{2}{3}$ and $\frac{\pi}{2} < \theta < \pi$. What is the value of $\sec \theta$?

A. $-\frac{3}{\sqrt{13}}$

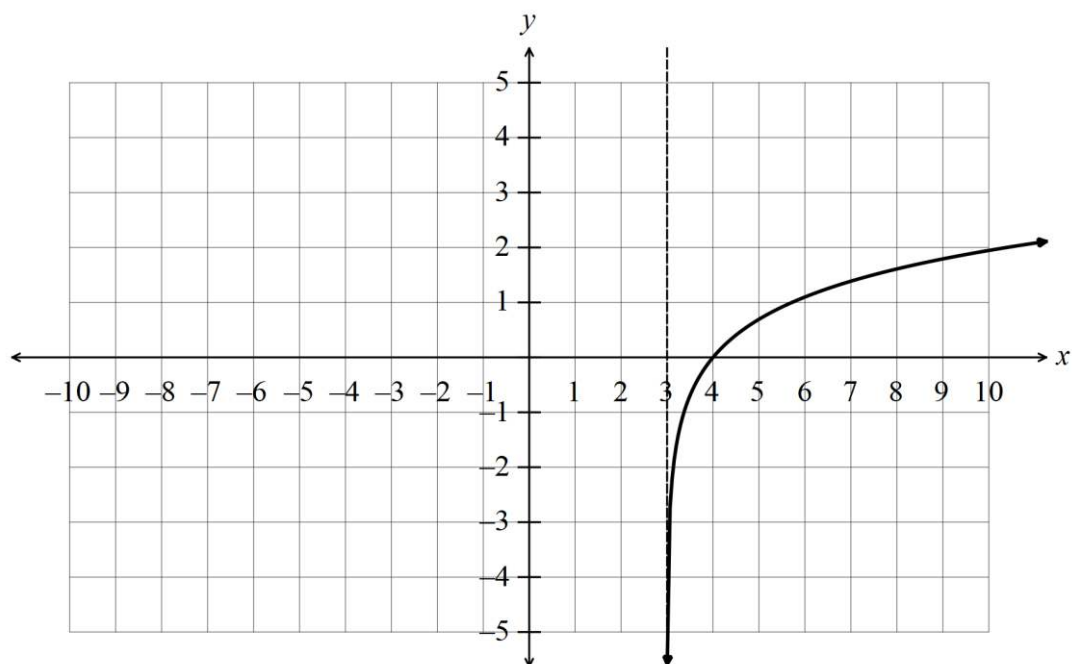
B. $-\frac{\sqrt{13}}{3}$

C. $-\frac{\sqrt{13}}{2}$

D. $-\frac{2}{\sqrt{13}}$

4 The graph of $y = f(x)$ is shown below, $x = 3$ is an asymptote.

Which inequality gives the domain of $y = -f(-x)$?



A. $D: x > 3$

B. $D: x < 3$

C. $D: x > -3$

D. $D: x < -3$

- 5 The point $P(8, -3)$ lies on the graph of $y = f(x)$.

What are the coordinates of the image of P if the function is transformed to :

$$y = -2f(x + 7) + 5 ?$$

- A. $(1, 11)$
B. $(1, -1)$
C. $(15, 11)$
D. $(15, -1)$

- 6 Marina completed tests in each of four subjects. On each test, the class scores were normally distributed.

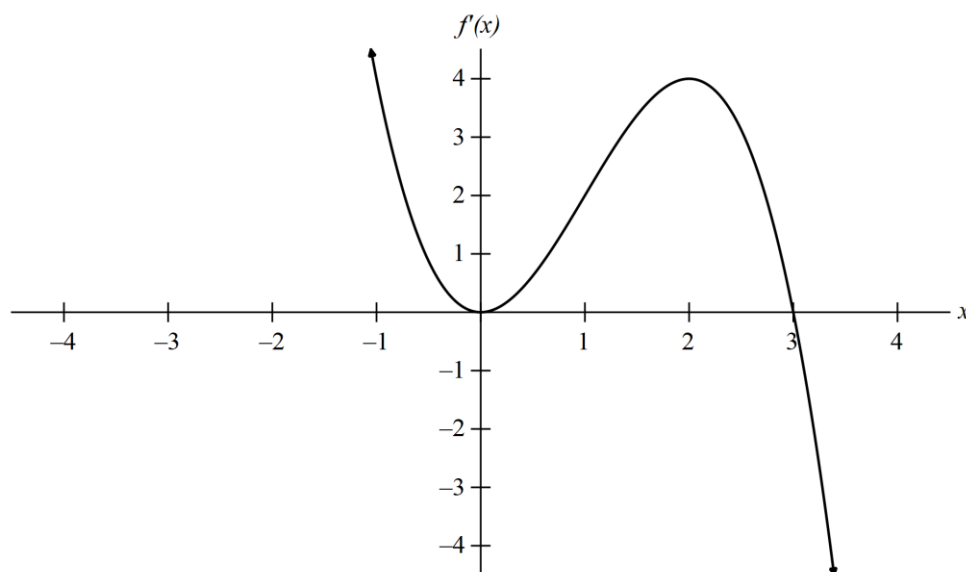
The table shows her scores in each subject as well as the mean and standard deviation of the test scores for each subject.

Subject	Mark	Class Mean	Standard Deviation
Economics	64	60	3
Mathematics	71	60	10
Art	69	60	5
Chemistry	82	60	18

Which subject is her strongest performance, relative to the rest of the class?

- A. Economics
B. Mathematics
C. Art
D. Chemistry

- 7 The graph of $f'(x)$ is shown below. For what values of x is the function $f(x)$ decreasing?



- A. $(-\infty, 3)$
- B. $(-\infty, 0) \cup (0, 2)$
- C. $(0, 2)$
- D. $(3, \infty)$
- 8 Let $g(x) = 1 + f(x)$. If $\int_1^4 f(x) \, dx = 7$. What is the value of $\int_1^4 g(x) \, dx$?
- A. -6
- B. -4
- C. 8
- D. 10

9 What is the derivative of $y = 3^{2x+1}$?

- A. $y' = 2(3^{2x+1})$
- B. $y' = \ln 3(3^{2x+1})$
- C. $y' = 2 \ln 3(3^{2x+1})$
- D. $y' = (2x+1)(3^{2x+1})$

10 A continuous random variable, X , has the following probability density function, where k is a particular positive value.

$$f(x) = \begin{cases} \sin x & \text{for } 0 \leq x \leq k \\ 0 & \text{for all other values of } x \end{cases}$$

What is the value of the median of this function?

- A. $\frac{\pi}{6}$
- B. $\frac{\pi}{4}$
- C. $\frac{\pi}{3}$
- D. $\frac{\pi}{2}$

Trial Higher School Certificate Examination

Mathematics Advanced

Section II Answer Book

90 marks

Attempt Questions 11-36

Allow about 2 hours and 45 minutes for this section

Instructions

- Answer the questions on your own paper. The spaces provide guidance for the expected length of response.
- Your response should include relevant mathematical reasoning and/or calculations.

START A NEW PAGE FOR YOUR ANSWERS

Question 11 (2 marks)

Differentiate $(5 - 2x^2)^7$ **2**

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Question 12 (2 marks)

The random variable X has a probability density function given by **2**

$$f(x) = \begin{cases} ax(2-x) & \text{if } 0 \leq x \leq 2 \\ 0 & \text{if } x < 0 \text{ or } x > 2 \end{cases}$$

where a is a positive constant. Find the value of a

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Question 13 (2 marks)

In an arithmetic series is $T_4 = 32$ and $T_7 = 17$. Find T_{11} . **2**

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Question 14 (4 marks)

Forty two students study at least one of either Physics or Chemistry. Of these 42 students, 27 study Physics and 33 study Chemistry.

- (a) How many students study both Physics and Chemistry? **1**

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- (b) Find the probability that a student studies Physics given that they study Chemistry. **1**

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- (c) What is the probability that 2 randomly chosen students both study only Physics? **2**

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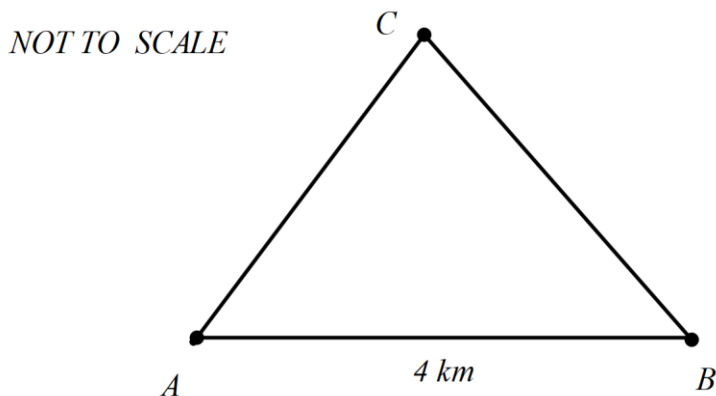
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Question 15 (3 marks)

In the following diagram, A and B represent the position of 2 campsites. Campsite A is due west of Campsite B , at a distance of 4 kilometres.



A third Campsite C , bears 035° from A and 330° from B .

- (a) Show that $\angle CBA = 60^\circ$ and $\angle BAC = 55^\circ$ **1**

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- (b) Find the distance from Campsite A to Campsite C . Give your answer correct to one decimal place. **2**

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Question 16 (2 marks)

The curve $y = f(x)$ has gradient function $\frac{dy}{dx} = 3x^2 - 2x - 1$. Find the equation of the curve if $f(1) = 5$. **2**

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Question 17 (3 marks)

Find $\frac{d}{dx} \left(\frac{\cos 2x}{e^{3x}} \right)$. Write your answer in simplified form. **3**

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Question 18 (2 marks)

Evaluate $\int_2^3 \frac{2x+1}{x^2+x} dx$, leaving your answer in simplified, exact form.

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Question 19 (3 marks)

Given that $0 < x < 4$ and $\log_5(4-x) - 2\log_5 x = 1$, find the value of x .

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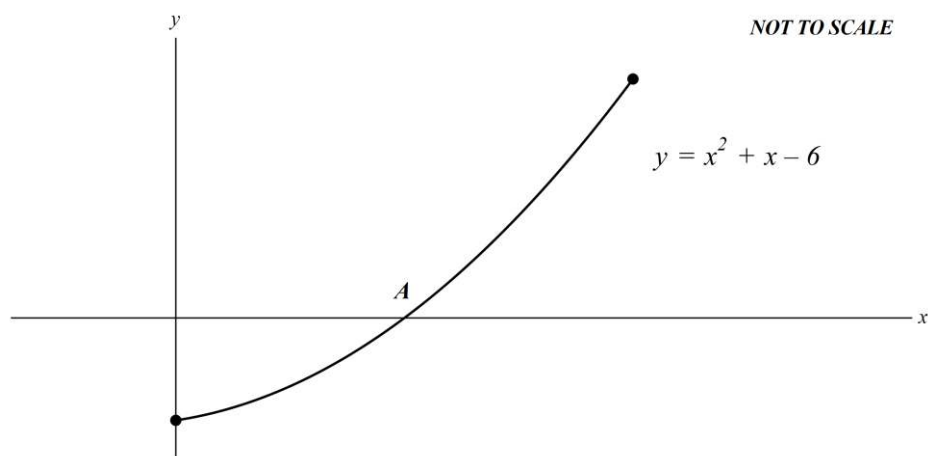
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Question 20 (3 marks)

The diagram below shows the graph of $y = x^2 + x - 6$ for $0 \leq x \leq 4$.



- (a) Find the value of A.

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- (b) Hence find the area bounded by the x -axis, the curve $y = x^2 + x - 6$, $x = 0$ and $x = 4$.

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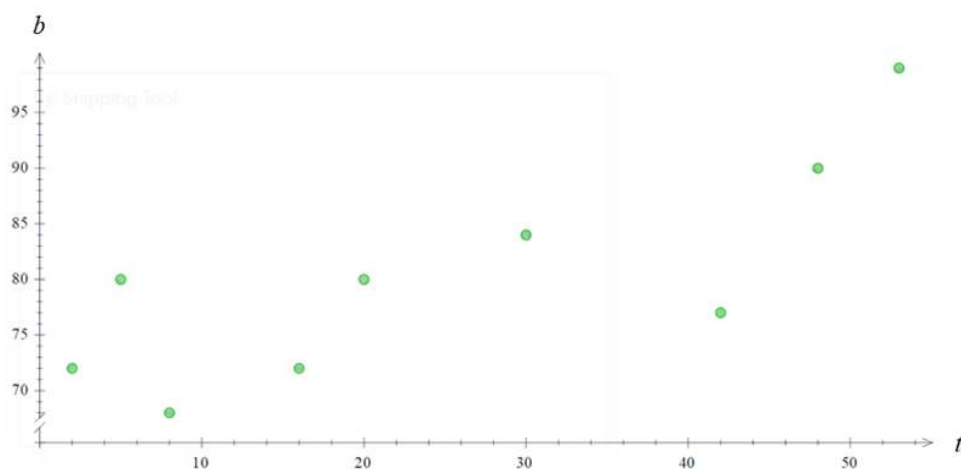
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Question 21 (4 marks)

Yuki and Jerry are watching China's Lin Dan play against Malaysia's Lee Chong Wei in the Final of the World Badminton Championships. It is a very exciting match.

Yuki is worried that Jerry will faint with excitement. She takes his pulse at various intervals during the game. The information is recorded in the table and graphed on the scatterplot below.

t = time since the match started (minutes)	2	5	8	16	20	30	42	48	53
b = heart rate (beats / minute)	72	80	68	72	80	84	77	90	99



- (a) Find the value of r , the Pearson's correlation coefficient, correct to 2 decimal places. 1

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- (b) Find the equation of the least squares regression line, correct to 2 decimal places, using the variables given in the table. 2

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- (c) Use the least squares regression line to estimate Jerry's pulse rate at the 35th minute. 1

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START A NEW PAGE FOR YOUR ANSWERS

Question 22 (1 mark)

The following table shows the distribution of the number of children of some families:

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Number of children	0	1	2	3	4
Number of families	k	2	9	6	7

It is given that k is a positive integer.

If the mode of the distribution is 2, write down the range of values of k .

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Question 23 (3 marks)

A city is suffering through a drought and adopts a plan to import water from another city. The volume of water imported in the 1st year is $1.8 \times 10^7 \text{ m}^3$ and in subsequent years, the volume of water imported each year is 9% less than the volume of water imported in the previous year.

(a) Find how much water was imported in the 3rd year.

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(b) Find the total volume of water imported in the first 8 years.

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Question 24 (3 marks)

Let F be a cumulative distribution function shown below.

- (a) Sketch the graph of $F(x)$.

$$F(x) = \begin{cases} 0 & x < 1 \\ 0.25(x-1) & 1 \leq x < 3 \\ 0.5 + 0.125(x-3) & 3 \leq x < 7 \\ 1 & x \geq 7 \end{cases}$$

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- (b) Find $P(3 \leq X \leq 5)$.

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Question 25 (5 marks)

A curve is defined by $f(x) = \frac{1}{4}x^4 - 2x^2 + 1$.

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Sketch the curve for the domain $[-3, 3]$, showing all key features including classifying the nature of stationary points and finding any inflections. Also state where the curve is concave down. You **do not** need to find the x – intercepts.

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Question 26 (2 marks)

The heights of a group of students are normally distributed with a mean of 168 cm and a standard deviation of 12 cm.

- (a) A student is chosen at random. Find the probability that the students height is greater than 180 cm. **1**

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- (b) In this group of students, 2.5% have heights less than d . Find the value of d . **1**

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Question 27 (2 marks)

Whilst performing some experiments, researchers found that the rate of change of the distance (in cm/s) travelled by a virus particle is given by 2

$$A(t) = 50 \log_e(1 + 5t),$$

where t is the number of seconds elapsed since the start of the research.

Use the trapezoidal rule with 4 sub-intervals to find an approximate distance, D cm, the virus particle travelled in the time from $t = 0.1$ to $t = 0.5$, correct to 1 decimal place.

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Question 28 (2 marks)

Solve $2 \sin^2\left(\frac{x}{3}\right) = 1$ for the domain $[-\pi, \pi]$ 2

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Question 29 (3 marks)

Use the table below to answer the following questions for a standard normal distribution.

- (a) Find $P(z \leq 1.2)$. 1

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- (b) Find $P(z \geq 2.84)$ 1

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- (c) Find $P(1.4 \leq z \leq 2.1)$. 1

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STANDARD NORMAL DISTRIBUTION: Table Values Represent AREA to the LEFT of the Z score.

Z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.50000	.50399	.50798	.51197	.51595	.51994	.52392	.52790	.53188	.53586
0.1	.53983	.54380	.54776	.55172	.55567	.55962	.56356	.56749	.57142	.57535
0.2	.57926	.58317	.58706	.59095	.59483	.59871	.60257	.60642	.61026	.61409
0.3	.61791	.62172	.62552	.62930	.63307	.63683	.64058	.64431	.64803	.65173
0.4	.65542	.65910	.66276	.66640	.67003	.67364	.67724	.68082	.68439	.68793
0.5	.69146	.69497	.69847	.70194	.70540	.70884	.71226	.71566	.71904	.72240
0.6	.72575	.72907	.73237	.73565	.73891	.74215	.74537	.74857	.75175	.75490
0.7	.75804	.76115	.76424	.76730	.77035	.77337	.77637	.77935	.78230	.78524
0.8	.78814	.79103	.79389	.79673	.79955	.80234	.80511	.80785	.81057	.81327
0.9	.81594	.81859	.82121	.82381	.82639	.82894	.83147	.83398	.83646	.83891
1.0	.84134	.84375	.84614	.84849	.85083	.85314	.85543	.85769	.85993	.86214
1.1	.86433	.86650	.86864	.87076	.87286	.87493	.87698	.87900	.88100	.88298
1.2	.88493	.88686	.88877	.89065	.89251	.89435	.89617	.89796	.89973	.90147
1.3	.90320	.90490	.90658	.90824	.90988	.91149	.91309	.91466	.91621	.91774
1.4	.91924	.92073	.92220	.92364	.92507	.92647	.92785	.92922	.93056	.93189
1.5	.93319	.93448	.93574	.93699	.93822	.93943	.94062	.94179	.94295	.94408
1.6	.94520	.94630	.94738	.94845	.94950	.95053	.95154	.95254	.95352	.95449
1.7	.95543	.95637	.95728	.95818	.95907	.95994	.96080	.96164	.96246	.96327
1.8	.96407	.96485	.96562	.96638	.96712	.96784	.96856	.96926	.96995	.97062
1.9	.97128	.97193	.97257	.97320	.97381	.97441	.97500	.97558	.97615	.97670
2.0	.97725	.97778	.97831	.97882	.97932	.97982	.98030	.98077	.98124	.98169
2.1	.98214	.98257	.98300	.98341	.98382	.98422	.98461	.98500	.98537	.98574
2.2	.98610	.98645	.98679	.98713	.98745	.98778	.98809	.98840	.98870	.98899
2.3	.98928	.98956	.98983	.99010	.99036	.99061	.99086	.99111	.99134	.99158
2.4	.99180	.99202	.99224	.99245	.99266	.99286	.99305	.99324	.99343	.99361
2.5	.99379	.99396	.99413	.99430	.99446	.99461	.99477	.99492	.99506	.99520
2.6	.99534	.99547	.99560	.99573	.99585	.99598	.99609	.99621	.99632	.99643
2.7	.99653	.99664	.99674	.99683	.99693	.99702	.99711	.99720	.99728	.99736
2.8	.99744	.99752	.99760	.99767	.99774	.99781	.99788	.99795	.99801	.99807
2.9	.99813	.99819	.99825	.99831	.99836	.99841	.99846	.99851	.99856	.99861

Question 30 (7 marks)

A heated metal ball is dropped into a liquid. As the ball cools, its temperature, T (in degrees Celsius), t minutes after it enters the liquid is given by,

$$T = 400e^{-0.05t} + 25 \text{ where } t \geq 0.$$

- (a) Find the initial temperature of the balls as it enters the liquid. **1**

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- (b) Find the value of t for which $T = 200$, giving your answer to 3 decimal places. **2**

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- (c) Find the rate at which the temperature of the ball is decreasing after 50 minutes, giving your answer to 3 decimal places. **2**

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Question 30 continues on page 22

Question 30 (continued)

- (d) As it cools the ball will eventually approach what temperature? **1**

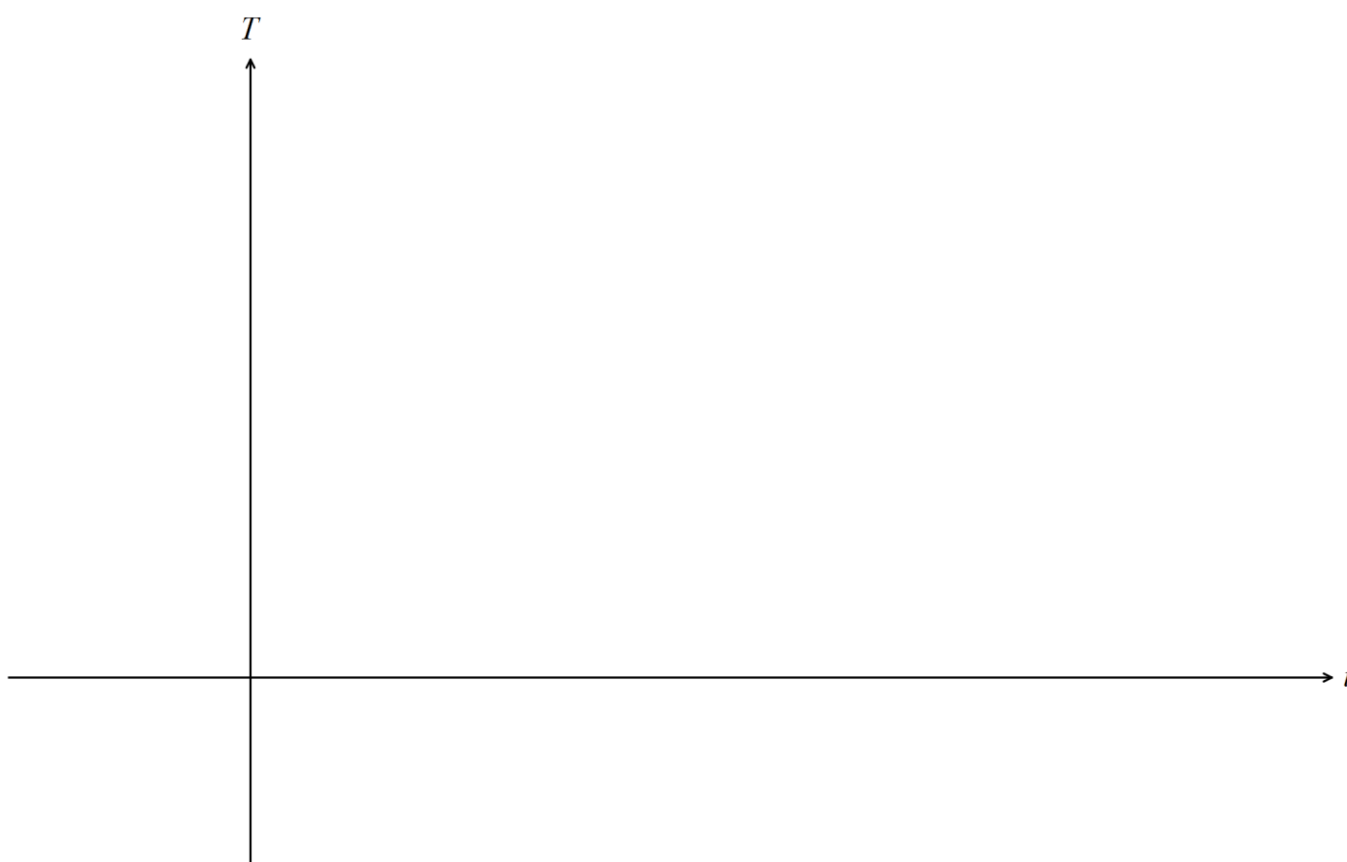
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- (e) Sketch the graph of T . **1**



End of Question 30

START A NEW PAGE FOR YOUR ANSWERS

Question 31 (3 marks)

The rise and fall in sea level in a channel, due to tides, can be modelled by the cosine function $h = -3\cos\left(\frac{\pi t}{6}\right) + 13$, where h , in metres, is the water level in the channel at time t , in hours.

Considering how the graph of $h = \cos t$ has been transformed into the graph of

$$h = -3\cos\left(\frac{\pi t}{6}\right) + 13,$$

- (a) How deep is the channel at high tide? 1

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- (b) How many hours is there between consecutive high tides? 2

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Question 32 (7 marks)

In lockdown, Grant has invented a game for one person. He throws two ordinary coloured dice, one orange and one green. He notes the result.

Grant wins if the number on the green die is **greater** than the number on the orange die.

Grant loses if the number on the green die is **smaller** than the number on the orange die.

If neither of these events occur, Grant continues to throw the dice until it does.

- (a) Show that the probability that Grant wins on the first roll of the dice is $\frac{5}{12}$. **1**

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- (b) What is the probability that a second throw is needed? **2**

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- (c) Calculate the probability that Grant wins on his first, second or third throw. **2**

Leave your answer in unsimplified form.

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Question 32 continues on page 25

Question 32 (continued)

(d) What is the probability that Grant wins the game?

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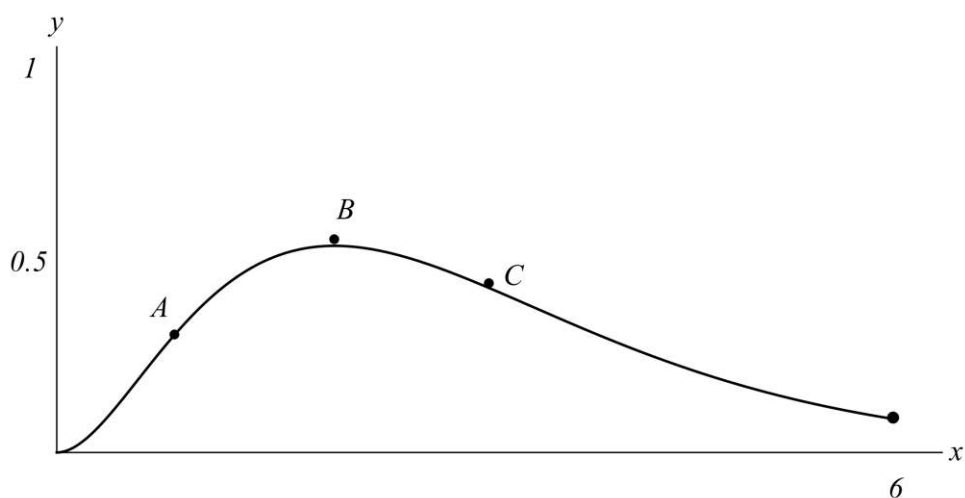
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End of Question 32

Question 33 (5 marks)

The diagram below shows the graph of $f(x) = x^2 e^{-x}$ for $0 \leq x \leq 6$. There are points of inflexion at A and C and there is a maximum at B .



- (a) Find $f'(x)$. 1

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- (b) Find the exact y coordinate of B . 2

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- (c) The second derivative of $f(x)$ is $f''(x) = (x^2 - 4x + 2)e^{-x}$. 2
Using this result, or otherwise, find the exact value of the x coordinate at C .

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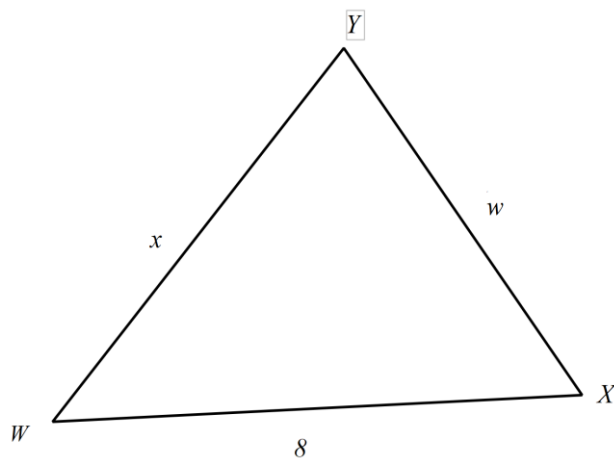
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START A NEW PAGE FOR YOUR ANSWERS

Question 34 (8 marks)

The triangle WXY has $WX = 8$, $YW = x$, $XY = w$ as shown below.

The perimeter of the triangle WXY is 22.



(a) Show that $\cos W = \frac{7x-33}{4x}$.

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Question 34 continues on page 28

Question 34 (continued)

- (b) Let the area of triangle be A . Show that $A^2 = -33x^2 + 462x - 1089$. 3

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- (c) Hence, calculate the maximum area for triangle and state what type of triangle with the maximum area. 3

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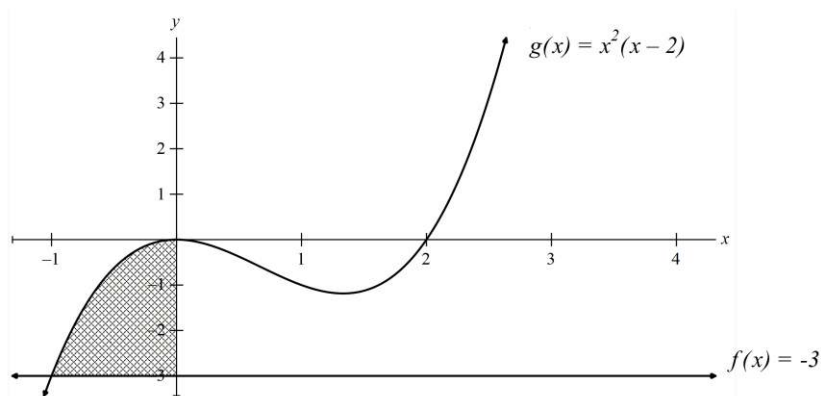
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End of Question 34

Question 35 (3 marks)

The graph below shows functions $f(x) = -3$ and $g(x) = x^2(x - 2)$. The curves intersect at the point $(-1, -3)$. Find the area of the shaded region.

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Question 36 (6 marks)

Let X be a **uniform** discrete random variable with possible values $X = 1, 2, 3, \dots, n$.

- (a) Explain why $P(X = x) = \frac{1}{n}$ **1**

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- (b) Given $1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}$. [DO NOT PROVE THIS], **5**

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Show that $Var(X) = \frac{n^2 - 1}{12}$

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End of paper