



2022

HIGHER SCHOOL CERTIFICATE TRIAL EXAMINATION

Mathematics Advanced

General

Instructions:

- Reading Time – 10 mins
- Working time – 3 hours
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided as on a separate document
- For questions in Section II, show relevant mathematical reasoning and/or calculations

Total marks: 100

Section I – 10 marks (pages 2–6)

- Attempt Questions 1–10
- Allow about 15 minutes for this section

Section II – 90 marks (pages 8–32)

- Attempt Questions 11–31
- Allow about 2 hours and 45 minutes for this section

Questions	Mark	Out of
Q1-10		/10
Q11-15		/15
Q16-19		/14
Q20-23		/14
Q24-26b		/15
Q26c-28		/16
Q29-31		/16
Total		/100

Section I

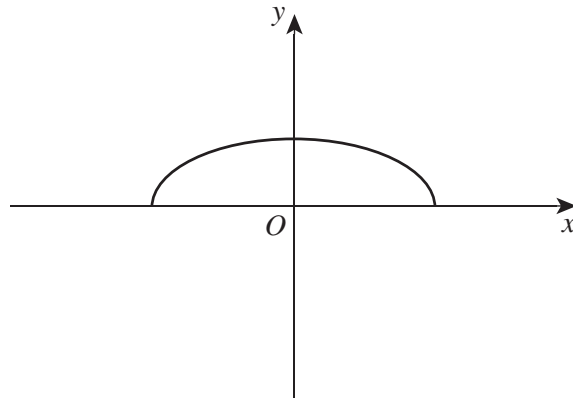
10 marks

Attempt Questions 1–10

Allow about 15 minutes for this section

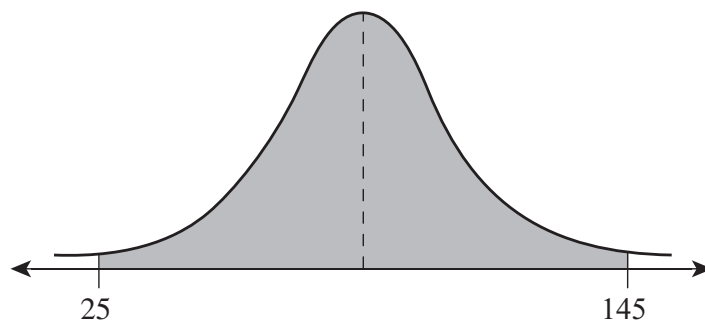
Use the multiple-choice answer sheet for Questions 1–10.

- 1 The diagram shows a curve.



Which type of relationship best describes the curve?

- A. one-to-one
 - B. one-to-many
 - C. many-to-one
 - D. many-to-many
- 2 The graph shows a normal distribution. Approximately 99.7% of the area under the curve is bounded by $x = 25$ and $x = 145$.



What are the values of μ and $\text{Var}(X)$?

- A. $\mu = 60$ and $\text{Var}(X) = 20$
- B. $\mu = 60$ and $\text{Var}(X) = 400$
- C. $\mu = 85$ and $\text{Var}(X) = 20$
- D. $\mu = 85$ and $\text{Var}(X) = 400$

- 3 A circle has the equation $(x + 2)^2 + (y + 3)^2 = d$.
What is the value of d such that the x -axis is a tangent to the circle?
- A. 2
B. 3
C. 4
D. 9

- 4 Phoebe wants to borrow money from the bank to buy a caravan. The table is used to calculate the monthly repayments for the loan.

Monthly repayments					
Amount borrowed	Loan term length				
	10 years	15 years	20 years	25 years	30 years
\$100 000	\$1213.28	\$955.65	\$836.44	\$771.82	\$733.76
\$110 000	\$1334.60	\$1051.22	\$920.08	\$849.00	\$807.14
\$120 000	\$1455.96	\$1146.78	\$1003.37	\$926.18	\$880.52
\$130 000	\$1577.26	\$1242.35	\$1087.37	\$1003.36	\$953.89
\$140 000	\$1698.59	\$1337.91	\$1171.02	\$1080.54	\$1027.27
\$150 000	\$1819.91	\$1433.48	\$1254.66	\$1157.72	\$1100.65
\$160 000	\$1941.24	\$1529.04	\$1338.30	\$1234.91	\$1174.02

Phoebe is considering two options.

- Option 1: Borrow \$100 000 for 20 years.
- Option 2: Borrow \$110 000 for 30 years.

Phoebe calculates that she will pay more interest by choosing option 2 than choosing option 1.

Calculate the difference in interest between the two options.

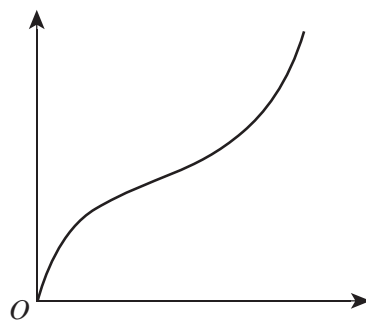
- A. \$7 485.40
B. \$79 824.80
C. \$200 745.60
D. \$290 570.40

-
- 5 Consider the two sets $A = \{5, 7, 9\}$ and $B = \{10, 11, 12\}$. A number is randomly chosen from each set.
What is the probability of selecting a number from each set that have a sum that is equal to or greater than 20?
- A. $\frac{1}{9}$
 B. $\frac{1}{6}$
 C. $\frac{2}{9}$
 D. $\frac{1}{3}$
- 6 Given that $f(x) = \int_2^x \frac{1}{1+t^3} dt$, what is the value of $f'(2)$?
- A. $-\frac{4}{27}$
 B. 0
 C. $\frac{1}{9}$
 D. $\ln 5$
- 7 The probability distribution of a discrete random variable X is shown.
- | | | | | | |
|------------|-----|-----|-----|-----|-----|
| x | 1 | 2 | 5 | 6 | 8 |
| $P(X = x)$ | 0.1 | 0.3 | 0.6 | 0.1 | 0.1 |
- If μ is the mean of X , then $P(X > \mu)$ is
- A. 0.2
 B. 0.6
 C. 4.1
 D. 19
- 8 Consider the graphs of $y = px^2$ and $y = qx^2 + r$.
The graphs will have no points of intersection for the real constants p , q and r provided that
- A. $r > 0$ and $p < q$.
 B. $r > 0$ and $p > q$.
 C. $p < 0$ and $q > 0$ for all real values of r .
 D. $p > 0$ and $q < 0$ for all real values of r .
-

- 9 The box-plot and cumulative frequency diagram for a dataset are shown.

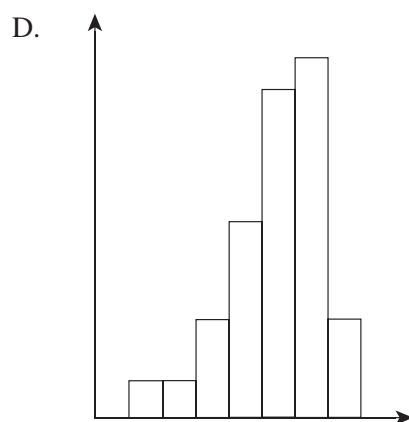
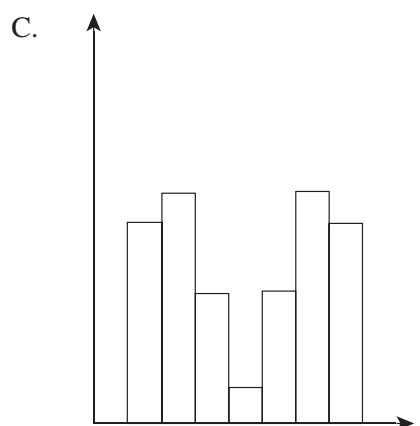
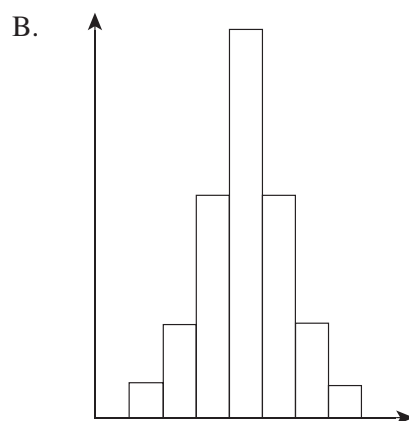
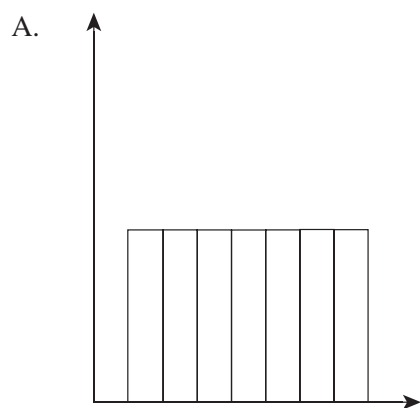


Box-plot

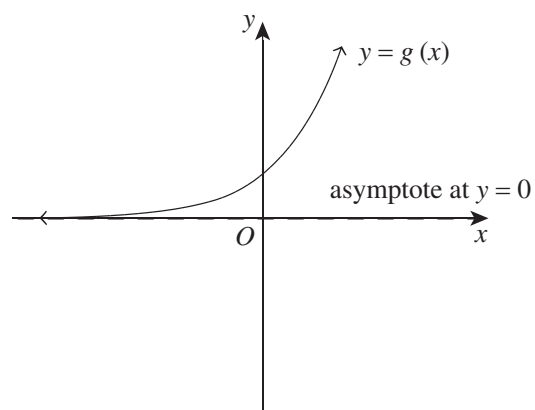
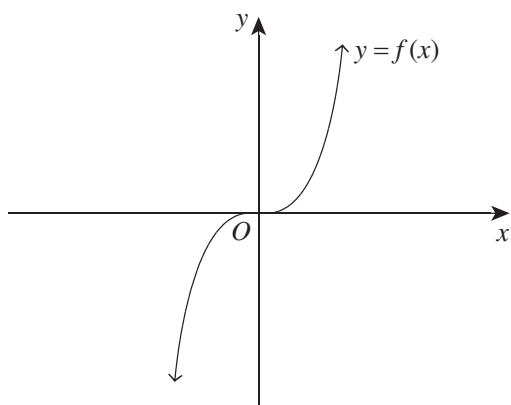


Cumulative frequency diagram

Which of the following frequency histograms best represents the dataset?

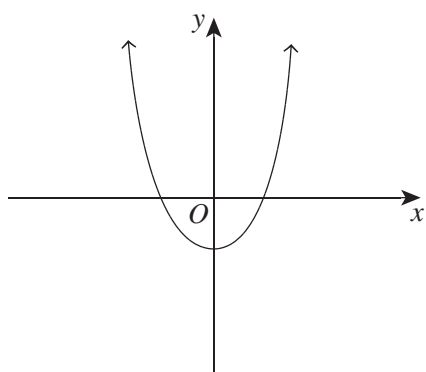


- 10 The graphs of $f(x)$ and $g(x)$ are shown.

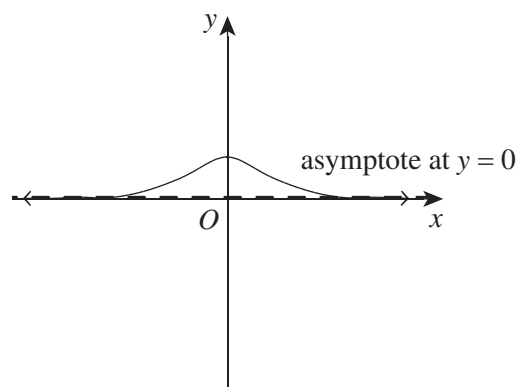


Which of the following best represents the graph of $y = f(g(-x))$?

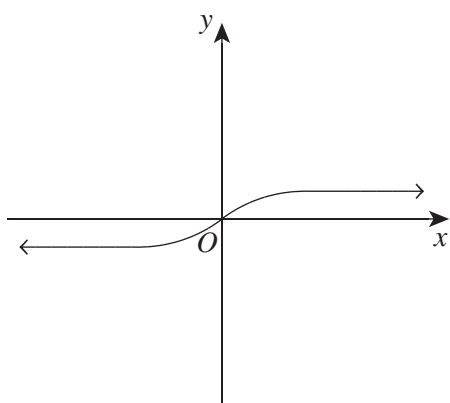
A.



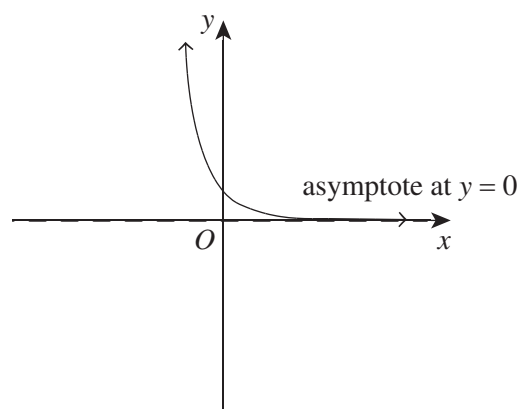
B.



C.



D.



Section II

90 marks

Attempt Questions 11–31

Allow about 2 hours and 45 minutes for this section

Question 11 (2 marks)

Solve $|2x - 3| = 6$

2

.....

.....

.....

.....

.....

.....

Question 12 (2 marks)

The function $y = f(x)$ is defined as

2

$$f(x) = \begin{cases} 1 & \text{for } -2 \leq x \leq 0 \\ -x^2 + 1 & \text{for } x \geq 0 \end{cases}$$

By sketching $y = f(x)$ or otherwise, write the domain and range of $y = f(x)$ using interval notation.

.....

.....

.....

.....

.....

Question 13 (3 marks)

Prove that $\frac{\sin A}{1 + \cos A} + \frac{1 + \cos A}{\sin A} = 2 \operatorname{cosec} A$.

3

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

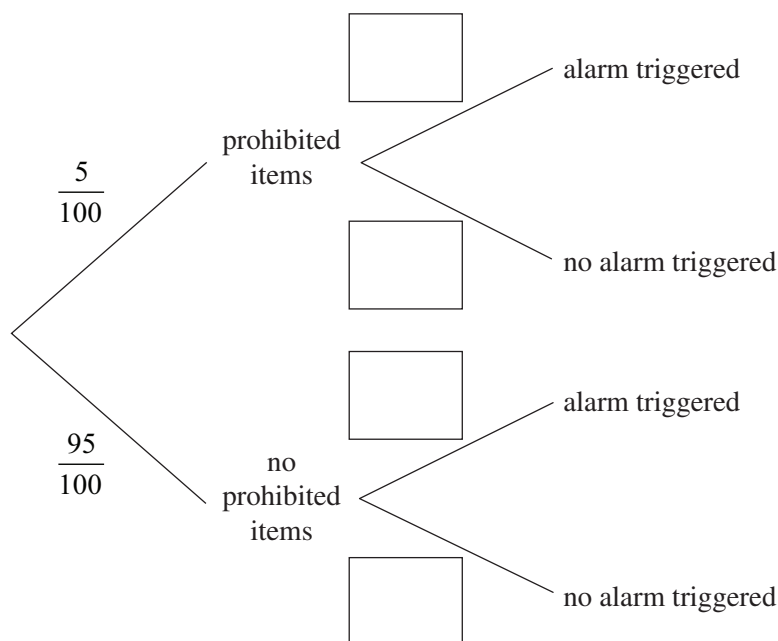
Question 14 (4 marks)

An airport scanner screens bags for prohibited items. An alarm is supposed to be triggered when a prohibited item is detected.

On a given day, 5% of bags contain prohibited items. If a bag contains prohibited items, there is a 95% chance that it triggers the alarm. If a bag does not contain prohibited items, there is a 2% chance that it triggers the alarm.

- (a) Complete the probability tree using the information provided.

2



- (b) Given that a randomly selected bag triggers the alarm, what is the probability that it contains a prohibited item?

2

.....

.....

.....

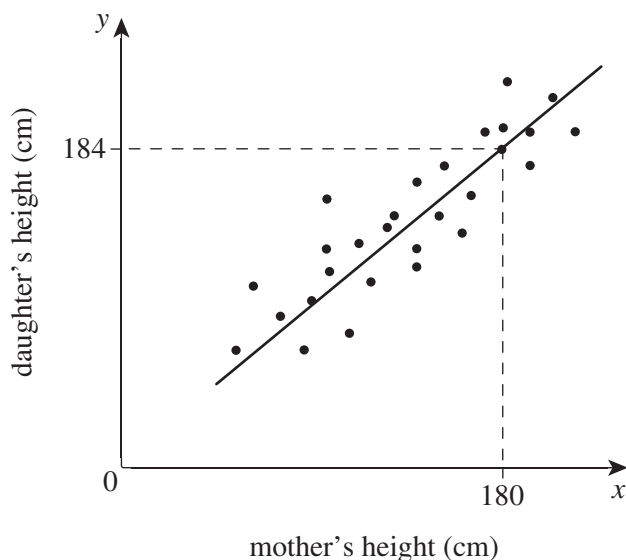
.....

.....

.....

Question 15 (4 marks)

The heights of 30 mothers and daughters were recorded. The results are shown in the scatterplot. The least-squares regression line on the scatterplot passes through the point (180, 184).



- (a) Katarina calculated that the correlation coefficient for this data set is $r = -2.5$. 2
Without calculating the correlation coefficient, give TWO reasons why Katarina must be incorrect.

.....

.....

.....

.....

- (b) The equation of the least-squares regression line for this data set is in the form 2
$$y = 0.8077x + A,$$

where A is a constant. Katarina is 160 cm tall.

Use the equation of the least-squares regression line to estimate the height of Katarina's mother. Give your answer correct to the nearest whole number.

.....

.....

.....

.....

.....

.....

.....

Question 16 (3 marks)

(a) Evaluate $f'(\pi)$ given that $f(x) = \frac{x^2}{\cos x}$.

3

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

Question 17 (3 marks)

Let $y = x \ln x$.

- (a) Show that $\frac{dy}{dx} = 1 + \ln x$. **1**

.....

.....

.....

.....

- (b) Hence, or otherwise, evaluate $\int_1^e \ln x \, dx$. **2**

.....

.....

.....

.....

.....

.....

Question 18 (5 marks)

- (a) Find the exact value of $\int_{\frac{1}{3}}^{\frac{1}{2}} \sec^2\left(\frac{\pi x}{2}\right) dx$.

3

.....

.....

.....

.....

.....

.....

.....

.....

.....

- (b) Find the primitive function y given that $\frac{dy}{dx} = \frac{2x}{3x^2 + 1}$.

2

.....

.....

.....

.....

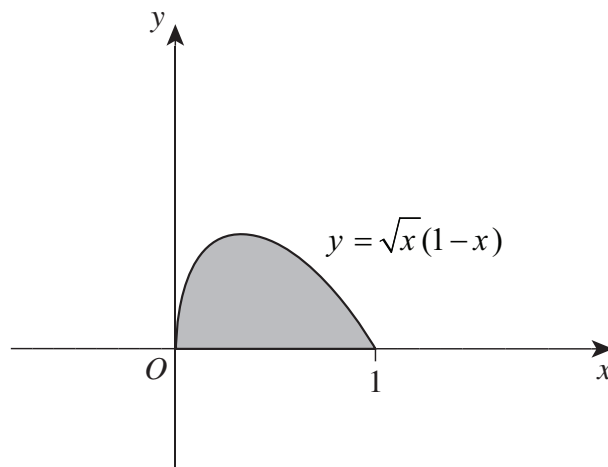
.....

.....

Question 19 (3 marks)

The graph of $y = \sqrt{x}(1-x)$ in the domain $[0, 1]$ is shown.

3



Find the area of the shaded region.

.....

.....

.....

.....

.....

.....

.....

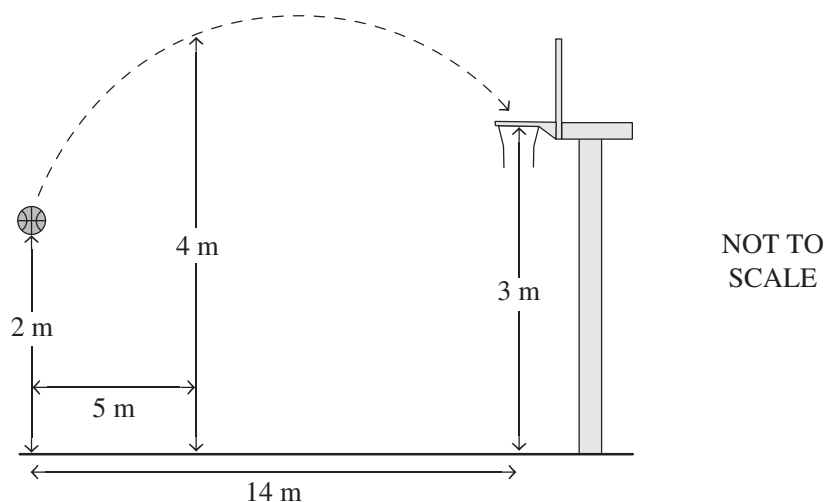
.....

.....

Question 20 (3 marks)

A basketball player shoots a ball from a height of 2 metres into a hoop 14 metres away. The hoop is 3 metres from the ground. After moving 5 metres horizontally, the ball reaches a height of 4 metres from the ground. This information is shown in the diagram.

3



The path of the ball can be modelled by the quadratic function

$$y = ax^2 + bx + c,$$

where y represents the height of the ball from the ground, x is the horizontal displacement from the basketball player, and a , b and c are constants.

Find the values of a , b and c that best model the path of the ball.

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

Question 21 (4 marks)

A company made a revenue of \$200 000 in 2021. To maintain sustainable growth of the company, the executive team is considering two models to increase its revenue.

- Model A would increase the yearly revenue by 10% of the previous year's revenue.
- Model B would increase the yearly revenue by a fixed amount, \$ M .

- (a) If the company chooses model A, show that the company would make a revenue of \$292 820 in 2025. 2

.....

.....

.....

.....

- (b) If the company chooses model B, find the value of M such that the company's revenue is \$292 820 in 2025. 2

.....

.....

.....

.....

.....

.....

.....

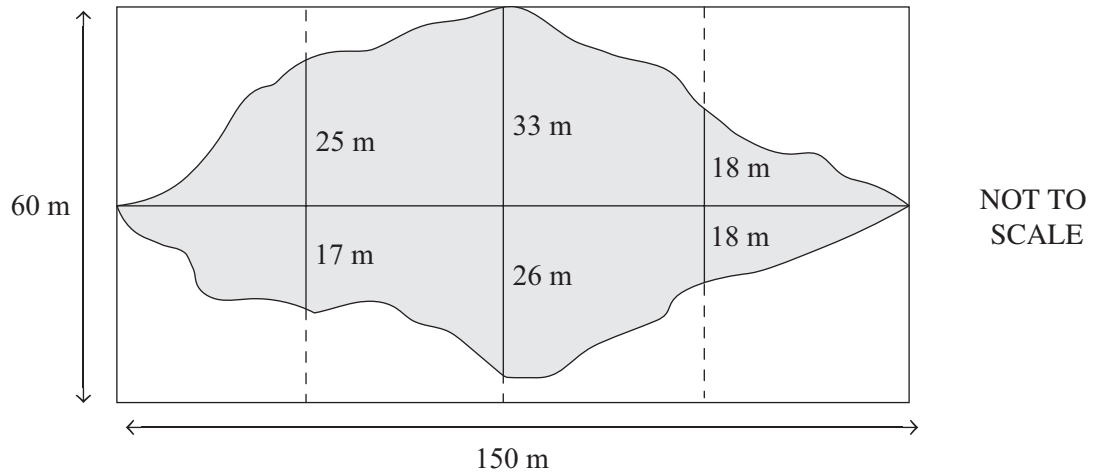
If you use this space, clearly indicate which question you are answering.

[illegible]

Question 22 (2 marks)

Archaeologists are excavating a rectangular site with dimensions of 150 metres by 60 metres.

The shaded region indicates the portion of the site that has been excavated.



Use the trapezoidal rule to best approximate the total area that has been excavated. Give your answer correct to the nearest whole number.

2

.....

.....

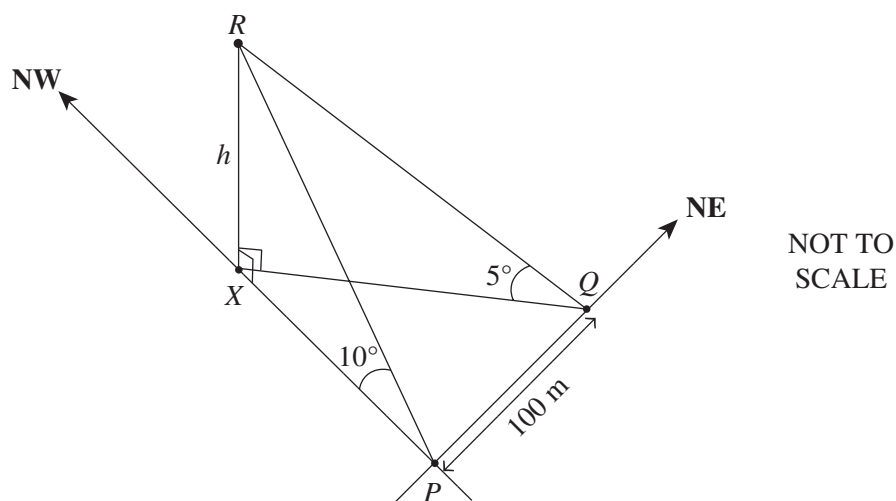
.....

.....

.....

Question 23 (5 marks)

A hiker starts from point P and walks 100 metres along a straight path to the north-east, arriving at point Q . A peak is located at point R , which is directly above point X . From point P , the angle of elevation to peak R is 10° . At point Q , the angle of elevation to the peak is 5° .



- (a) Show that $XP = h \cot 10^\circ$ and find a similar expression for XQ . 2

.....

.....

.....

.....

.....

.....

- (b) Hence, find the value of h . Give your answer correct to the nearest metre. 3

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

Question 24 (7 marks)

On a given day, the height of the water in a river is modelled by the function

$$h(t) = 5 + 3 \sin\left(\frac{\pi t}{4}\right),$$

where h is the height of the water, in metres, and t is the time, in hours, after 12 am.

- (a) What is the height of the water at 12 am?

1

.....

.....

.....

- (b) Sketch the graph of $h(t) = 5 + 3 \sin\left(\frac{\pi t}{4}\right)$ in the domain $[0, 24]$.

3

.....

.....

.....

.....



Question 24 continues on page 21

Question 24 (continued)

- (c) A family decides to go on a picnic by the river from 12 pm to 2 pm. It is only safe to swim in the river if the height of the water is less than 4 metres.

3

When is the earliest time the family can swim in the river after 12 pm? Give your answer correct to the nearest minute.

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

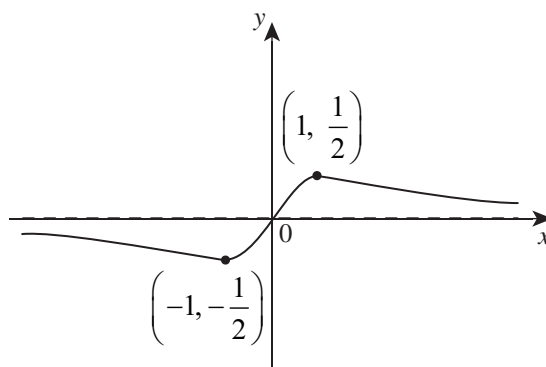
.....

End of Question 24

Question 25 (4 marks)

The graph of $y = f(x)$ in the domain $(-\infty, \infty)$ is shown.

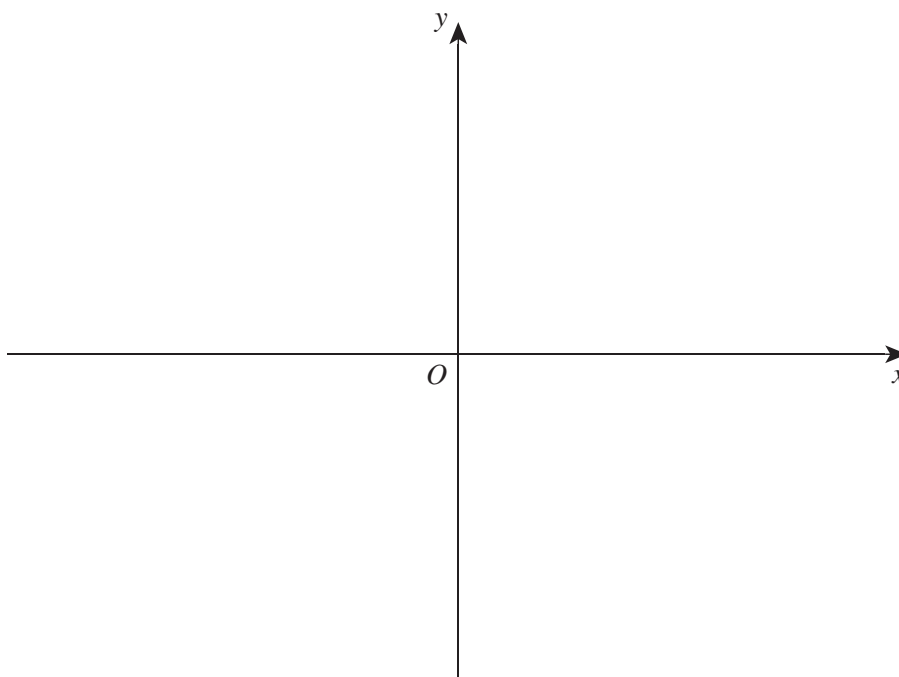
4



$y = f(x)$ has the following key features.

- The graph passes through the origin.
- There is a maximum turning point at $(1, \frac{1}{2})$.
- There is a minimum turning point at $(-1, -\frac{1}{2})$.
- The x -axis is a horizontal asymptote.

Sketch the graph of $y = -2f\left(\frac{x}{3}\right) + 1$, labelling any intercepts, asymptotes and turning points.



Question 26 (11 marks)

Consider the function $y = \frac{2x}{e^x}$.

It is known that as $x \rightarrow \infty$, $y \rightarrow 0$ and as $x \rightarrow -\infty$, $y \rightarrow -\infty$.

- (a) Find the x - and y -intercepts.

1

.....

.....

.....

.....

.....

- (b) Find and classify any turning points on $y = \frac{2x}{e^x}$.

3

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

Question 26 continues on page 24

Question 26 (continued)

- (c) Show that a point of inflection occurs at $\left(2, \frac{4}{e^2}\right)$. **3**

.....

.....

.....

.....

.....

.....

.....

.....

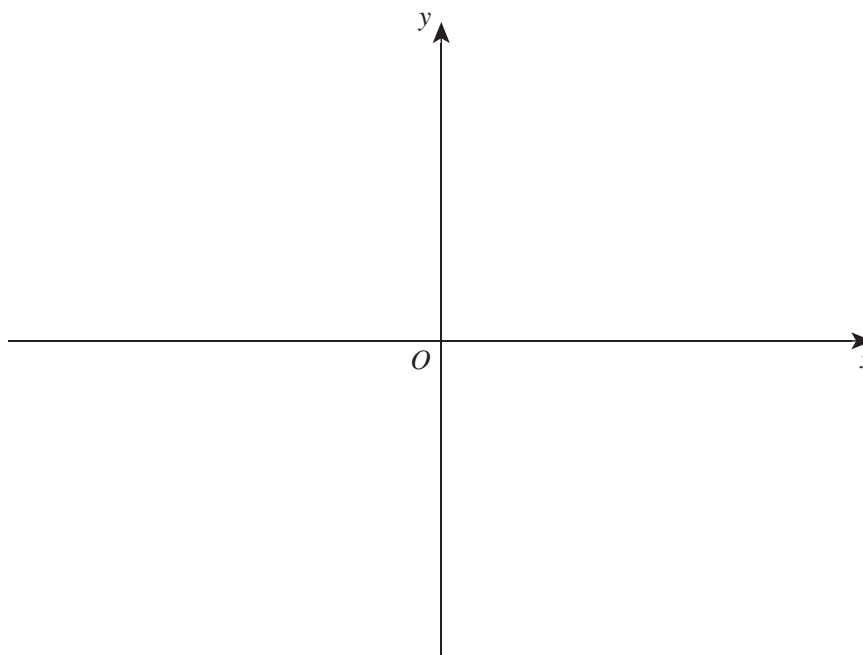
.....

.....

.....

.....

- (d) Sketch the graph of $y = \frac{2x}{e^x}$, labelling all important features. **4**



End of Question 26

Question 27 (3 marks)

The time, in minutes, between the arrival of customers at an Automated Teller Machine (ATM) is represented by the variable, t , which is a continuous random variable modelled by the probability density function

3

$$f(t) = \begin{cases} \frac{1}{3}e^{-\frac{t}{3}} & t \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

Find the median time between the arrival of customers at the ATM. Give your answer correct to the nearest minute.

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

Question 28 (6 marks)

A particle moves in a straight line such that its displacement is given by the function

$$x = 1 - \frac{3}{t+1},$$

where x is the particle's displacement from the origin, in metres, and t is the time, in seconds.

- (a) What is the initial displacement of the particle? 1

.....
.....

- (b) Find the expressions for the velocity and acceleration of the particle in terms of t . 2

.....
.....
.....
.....
.....
.....

- (c) As t increases indefinitely, describe the motion of the particle with reference to its displacement and velocity. 2

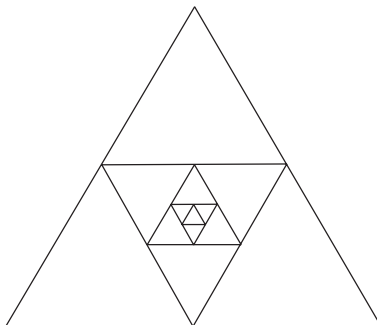
.....
.....
.....
.....

- (d) Sketch the graph of displacement as a function of time, labelling all important features. 1



Question 29 (2 marks)

An equilateral triangle has a perimeter of p cm. The midpoints of each side are connected to form another triangle. This pattern can continue for an infinite number of times, as shown in the diagram.



- (a) Show that the perimeter of the n th triangle is $\frac{p}{2^{n-1}}$. 1

.....

.....

.....

.....

.....

.....

.....

.....

.....

- (b) Find an expression in terms of p that represents the total perimeter of all the triangles. 1

.....

.....

.....

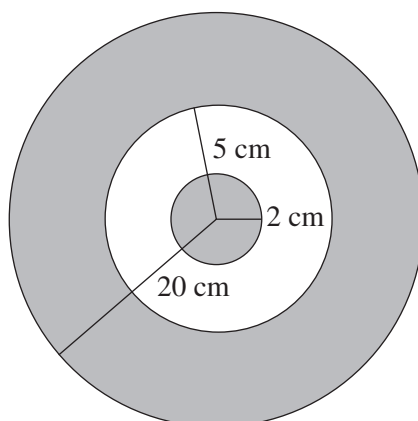
.....

.....

.....

Question 30 (7 marks)

A simple dartboard consists of three sections: the centre circle, middle section and outer section. The sections have a radius of 2 cm, 5 cm and 20 cm respectively, as shown in the diagram.



NOT TO
SCALE

After a player has thrown their dart, they are awarded a score according to the landing position of their dart on the dartboard. The table shows the scores awarded for each landing position.

<i>Landing position</i>	<i>Score</i>
centre circle	100
middle section	20
outer section	5

Assume that all darts thrown will hit the board and no darts will land on the lines.

- (a) Paula, who is new to playing darts, throws a dart so that it lands somewhere on the board.

Show that the probability that Paula's dart will land in the centre circle is $\frac{1}{100}$.

1

.....
.....
.....

- (b) Paula has two attempts at throwing a dart at the board. On one of her attempts, Paula's dart lands in the centre circle. On another attempt, her dart lands in the outer section.

2

Show that the probability that this occurs, in any order, is $\frac{3}{160}$.

.....
.....
.....
.....
.....
.....

Question 30 continues on page 29

Question 30 (continued)

Let X be a discrete random variable that represents the number of points scored after two attempts at throwing a dart at the board. The probability distribution table is shown.

x	10	25	40	105	120	200
$P(X = x)$	$\frac{225}{256}$	$\frac{63}{640}$	$\frac{441}{160\,000}$	$\frac{3}{160}$	$\frac{21}{20\,000}$	$\frac{1}{10\,000}$

An amateur dart player is throwing two darts to achieve a score. Assume the probability distribution table models the amateur player's likelihood of landing a dart in a specific section.

- (c) Calculate the expected value of the probability distribution. 1

.....

.....

.....

- (d) Calculate the standard deviation of the probability distribution. Give your answer correct to two decimal places. 3

.....

.....

.....

.....

.....

.....

.....

.....

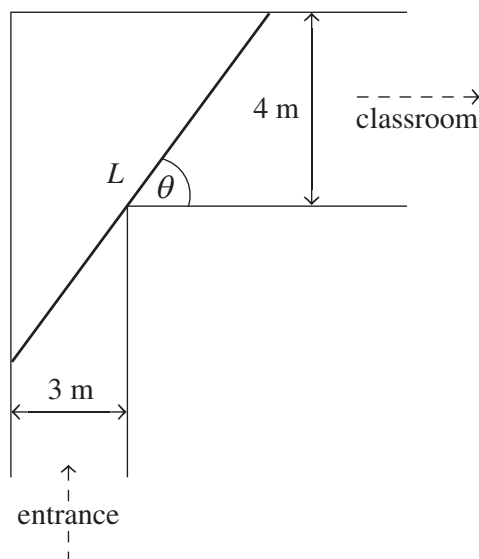
.....

.....

End of Question 30

Question 31 (7 marks)

A new whiteboard is being moved into a classroom. The whiteboard must be taken from the entrance, through the school's corridors and into the classroom. Two of the corridors are perpendicular to each other. The first corridor is 3 metres wide and the second corridor is 4 metres wide, as shown in the diagram. The length of the whiteboard is shown using L .



The whiteboard makes an angle θ to the horizontal on the corner of the corridors such that $0^\circ < \theta < 90^\circ$.

- (a) Show that the length of the whiteboard, L , is $\frac{3}{\cos \theta} + \frac{4}{\sin \theta}$.

1

.....

.....

.....

.....

.....

.....

.....

.....

Question 31 continues on page 31

Question 31 (continued)

- (b) Find the value of θ that minimises the function of L .

4

This image shows a full page of primary-ruled paper. It features multiple sets of horizontal dotted lines spaced evenly down the page, providing a guide for handwriting practice. The background is white, and there are no margins or additional markings.

Question 31 continues on page 32

- 2

This image shows a full page of white paper with horizontal dashed lines, typical of primary-ruled notebook paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

End of paper

Section II extra writing space

If you use this space, clearly indicate which question you are answering.

This image shows a full page of white paper with horizontal dotted lines, typical of primary school writing paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

Section II extra writing space

If you use this space, clearly indicate which question you are answering.

This image shows a full page of primary-ruled paper. It features approximately 20 horizontal dotted lines spaced evenly down the page, providing a guide for handwriting practice. The paper is otherwise blank, with no margins, text, or other markings.