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NESA Number

CHERRYBROOK TECHNOLOGY HIGH SCHOOL



2023

YEAR 12

AP4

MATHEMATICS ADVANCED

General

Instructions

- Reading time – 10 minutes
- Working time – 3 hours
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided
- For questions in Section II, show relevant mathematical reasoning and/or calculations
- Write your NESA Number on the Question Booklet, Multiple Choice sheet and any spare writing paper used

Total marks:

100

Section I – 10 marks (pages 1 – 4)

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section

Section II – 90 marks (pages 5 – 31)

- Attempt Questions 11 – 35
- Allow about 2 hours and 45 minutes for this section

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Section I

10 marks

Attempt Questions 1 – 10

Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1 – 10.

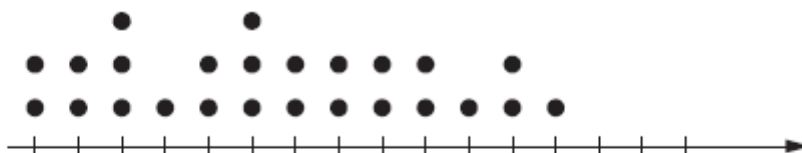
- 1 What is the amplitude and period of the function $f(x) = 3 \tan\left(\frac{x}{5}\right)$?
- A. Amplitude = 3 , Period = 10π
 - B. No amplitude , Period = 5π
 - C. Amplitude = 3 , Period = 5π
 - D. No Amplitude , Period = 10π
- 2 The circle $x^2 + y^2 = 9$ is transformed to $(x - 2)^2 + (y + 3)^2 = 9$.
What are the horizontal and vertical translations?
- A. Horizontally left 3 units, vertically up 2 units
 - B. Horizontally right 3 units, vertically down 2 units
 - C. Horizontally left 2 units, vertically up 3 units
 - D. Horizontally right 2 units, vertically down 3 units
- 3 What amount (to the nearest dollar) must be invested now at 4% per annum, compounded quarterly, so that in five years it will have grown to \$60 000?
- A. \$27 383
 - B. \$49 173
 - C. \$49 316
 - D. \$57 088

4 Which interval gives the domain of $f(x) = \frac{2}{\sqrt{4-x^2}}$?

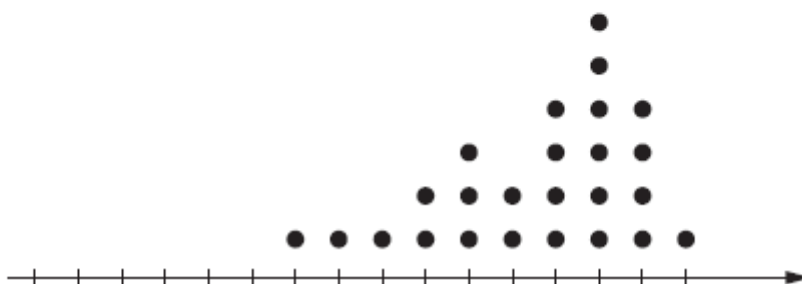
- A. $(-\infty, 2)$
- B. $(2, \infty)$
- C. $(-2, 2)$
- D. $[-2, 2]$

5 The dot plots below show the class scores in the same topic test before and after a unit of work was completed. They are drawn on the same scale.

Before



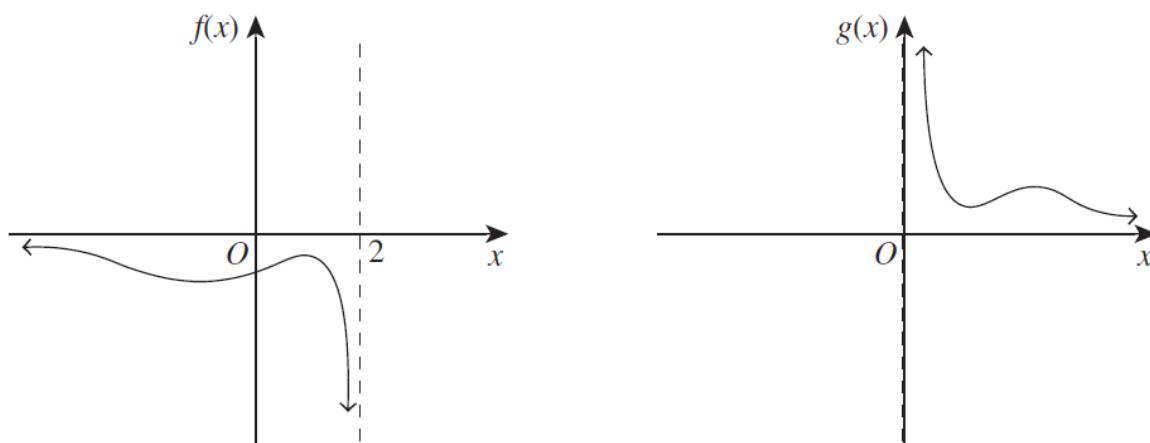
After



Which statement about the change in scores from before to after is correct?

- A. The mean increased and the standard deviation increased.
- B. The mean increased and the standard deviation decreased.
- C. The mean decreased and the standard deviation increased.
- D. The mean decreased and the standard deviation decreased.

- 6 The function $y = f(x)$ is transformed to $y = g(x)$, as shown in the diagram.



Which of the following equations best represents the transformed function?

- A. $g(x) = f(x - 2)$
 - B. $g(x) = f(2 - x)$
 - C. $g(x) = -f(x - 2)$
 - D. $g(x) = -f(2 - x)$
- 7 The acceleration of a particle moving in a straight line is given by $a = 4t + 4$. Initially the particle has a velocity of -30 m/s. When is the particle at rest?
- A. $t = 0$
 - B. $t = 2$
 - C. $t = 3$
 - D. $t = 5$

8 Consider the information about events A and B .

- $P(B) = 0.6$
- $P(A|B) = 0.4$
- $P(A|\bar{B}) = 0.8$

What is the value of $P(B|A)$?

- A. $\frac{1}{5}$
- B. $\frac{2}{5}$
- C. $\frac{3}{7}$
- D. $\frac{4}{5}$

9 Let $\log_7 10 = x$. Which expression is equal to $\log_7 \left(\frac{1}{70} \right)$?

- A. $-(1 + x)$
- B. $(1 + x)^{-1}$
- C. $\frac{x}{10}$
- D. $\frac{1}{10x}$

10 If $e^y = \sin x$, which of the following is $\frac{dy}{dx}$?

- A. $\cot x$
- B. $\tan x$
- C. $-\cot x$
- D. $-\tan x$

End of Section I

Section II

90 marks

Attempt Questions 11 – 35

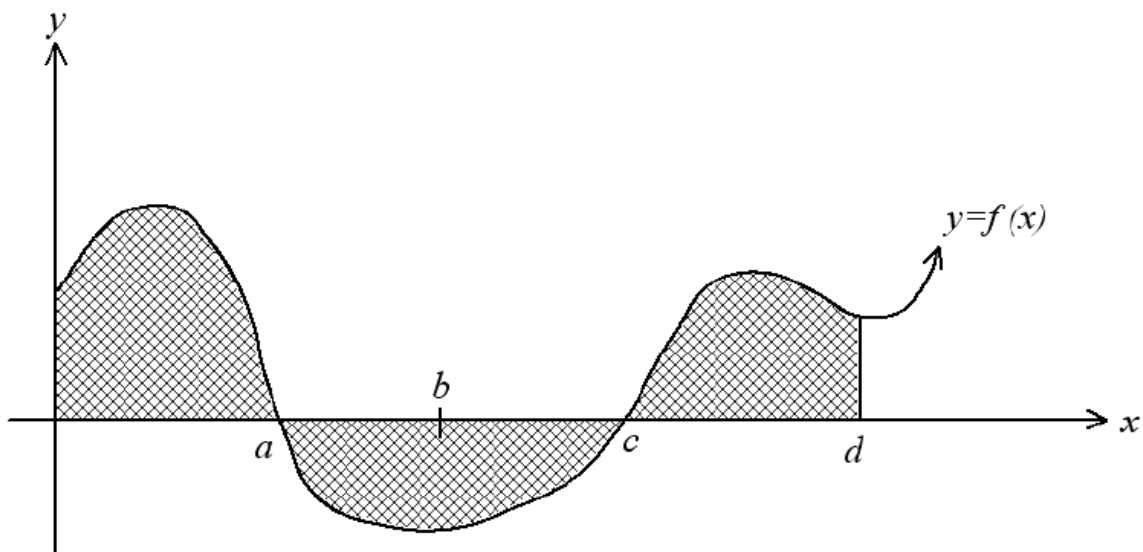
Allow about 2 hours and 45 minutes for this section

- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
 - For questions in Section II, your responses should include relevant mathematical reasoning and/or calculations.
 - Extra writing space is provided at the back of this booklet.
If you use this space, clearly indicate which question you are answering.
-

Question 11 (1 mark)

Write an expression for the shaded area shown in the diagram below.

1



Question 12 (2 marks)

The probability distribution of a discrete random variable X is given in the table below.

x	3	5	9	a
$P(X = x)$	0.1	0.3	0.2	0.4

- (a) Show that $a = 30$ given that $E(X) = 15.6$ **1**

- (b) Hence, find the variance of X . **1**

Question 13 (1 mark)

- Simplify $\frac{1 - \sin^2 x}{\cot x}$. **1**

Question 14 (4 marks)

Consider the geometric series $64 - 16a + 4a^2 - \dots$

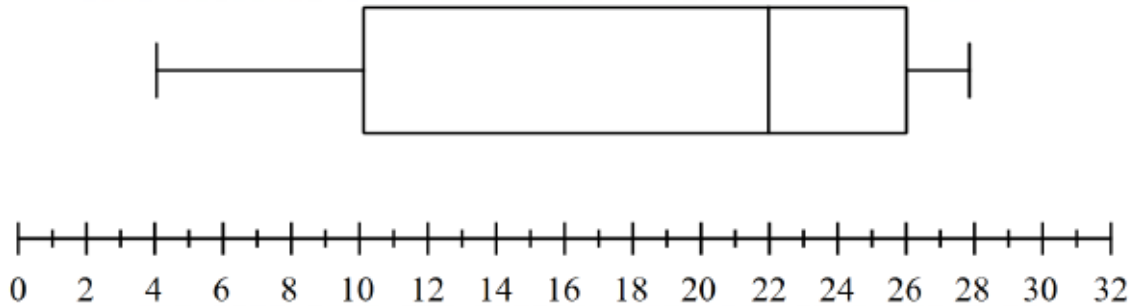
- (a) For what values of a does this series have a limiting sum? 2

- (b) It is known that the limiting sum of the series is 256. Find the value of a . 2

Question 15 (3 marks)

Jimmy recorded the number of cars in the carpark each day.

He displayed the data in the boxplot below:



- (a) Find the interquartile range given that $Q_1 = 10$. **1**

- (b) One day, Jimmy recorded 49 cars in the carpark. Is this recording an outlier? **1**
Justify your answer.

- (c) Comment on the shape of the distribution of this boxplot. **1**

Question 16 (3 marks)

A circle is divided into n sectors in such a way that the angles of the sectors are in arithmetic progression. The smallest two angles are 3° and 5° . Find the value of n .

3

This image shows a single sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

Question 17 (2 marks)

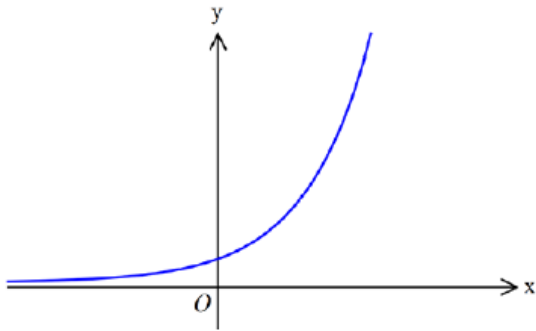
Find $\int \frac{1}{\sqrt{1+2x}} dx$.

2

This image shows a single sheet of white paper with horizontal blue or grey ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

Question 18 (3 marks)

The diagram below shows a sketch of the curve $y = 2^{3x}$.



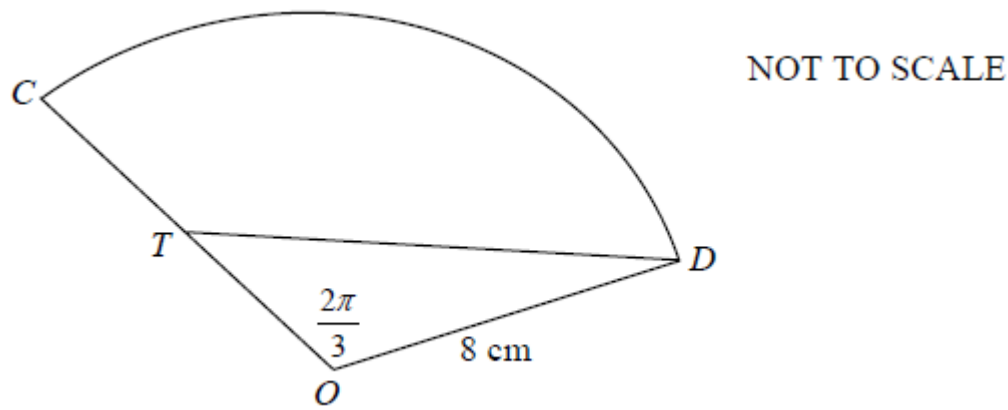
- (a) Use the trapezoidal rule with three function values to find an approximate value for $\int_0^1 2^{3x} dx$ rounded to two decimal places. 2

- (b) Is the approximate for $\int_0^1 2^{3x} dx$ an under approximation or an over approximation? 1
Justify your answer.

Question 19 (3 marks)

In the diagram, CD is an arc of a circle with radius 8 cm and centre O .
T is the midpoint of OC . It is given that $\angle COD = \frac{2\pi}{3}$.

3



Calculate the perimeter of CTD in exact form.

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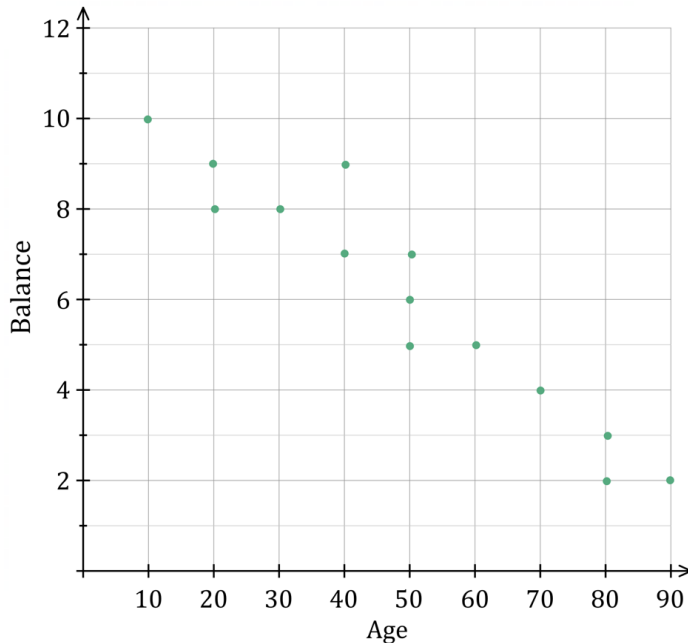
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Question 20 (4 marks)

The scatterplot below shows the relationship between age and how well you can balance with your body.



- (a) Akash uses his calculator and calculates his correlation coefficient to be $r = 1.73$. **1**
Without calculating the correlation coefficient, give ONE reason why $r = 1.73$ is incorrect.

- (b) Find the value of Pearson's correlation coefficient to two decimal places and hence describe the strength and direction of the association. **1**

- (c) Write down ONE age that would be an example of interpolation. **1**

- (d) Give ONE example of a limitation of this model. **1**

Question 21 (3 marks)

State the transformations in order that need to be applied to the hyperbola $y = \frac{1}{x}$ to obtain

$$y = \frac{1}{3x+6} - 2 .$$

This image shows a single sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

Question 22 (4 marks)

The table below shows the present value of an annuity with a contribution of \$1.

<i>Number of periods</i>	<i>Interest rate per period as a decimal</i>				
	0.001	0.00125	0.0015	0.00175	0.002
300	259.0707	250.0398	241.4379	233.2418	226.3477
330	280.9577	270.2690	260.1353	250.5239	240.7211
360	302.1982	289.7541	278.0106	266.9228	258.9154
390	325.2961	309.6290	297.0981	283.6291	261.9432

- (a) Sally pays \$2 000 into an annuity at the end of each month at 1.2% p.a. compounded monthly for 25 years. What is the present value of her annuity? 2

- (b) Thomas borrowed \$600 000 to purchase a home, with interest charged at 1.5% per annum compounding monthly. He agrees to repay the loan by making equal monthly payments over a 30-year period. 2

What is the monthly payment? Answer correct to the nearest cent.

Question 23 (6 marks)

The population of ants is growing exponentially according to the equation $P = 50e^{kt}$, where t is the time in days after the insect is first counted. After four days, it is observed that the population has doubled from the initial amount.

- (a) Find the initial number of ants and show that $k = \frac{1}{4} \ln 2$ 2

- (b) At what rate (to the nearest whole number) is the population increasing at day 15? 2

- (c) How many days will it take for the number of ants to first reach 1500? 2

Question 24 (4 marks)

The probability density function for the continuous random variable X is given by:

$$f(x) = \begin{cases} kx(4-x), & 1 \leq x \leq 4 \\ 0, & \text{otherwise} \end{cases}$$

- (a) Show that $k = \frac{1}{9}$ 2

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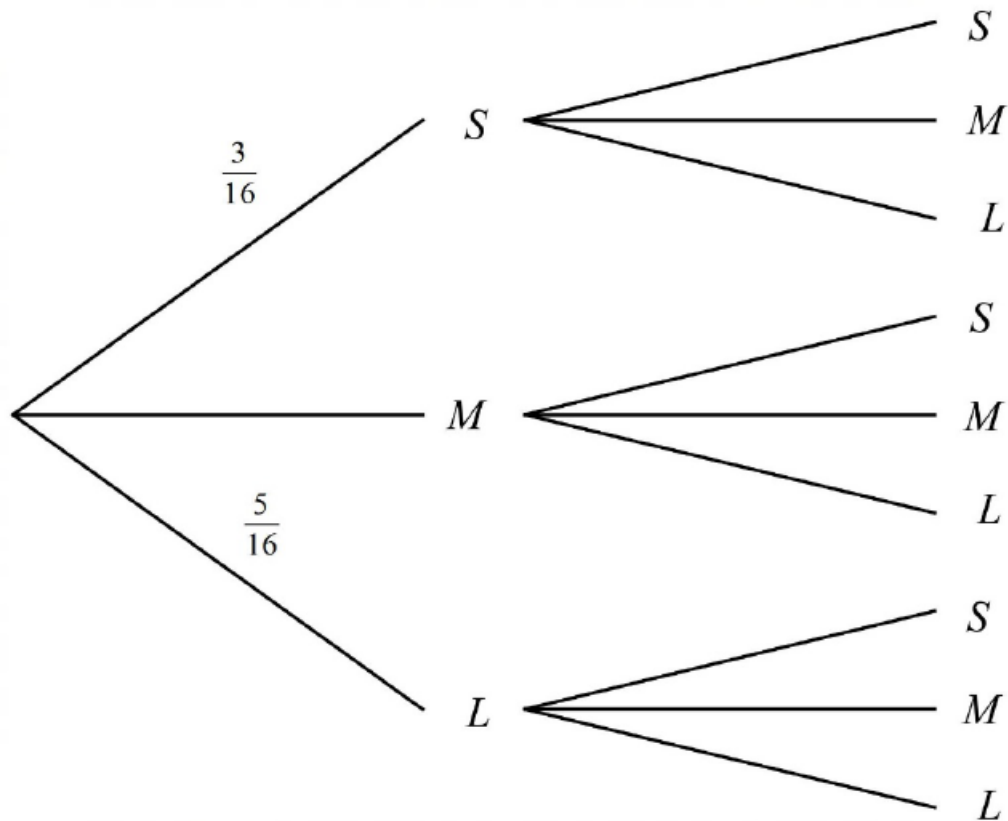
- (b) Determine the mode of X . 2

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Questions 11-24 are worth 43 marks in total

Question 25 (3 marks)

In a closet there are shirts in three sizes. 3 are small, 8 are medium and 5 are large. Johnny takes two shirts from the closet at random without replacement. The tree diagram below has been started to show the probabilities of different combinations.



- (a) Complete the tree diagram by writing the probabilities on the remaining branches. 1
- (b) Find the probability that Johnny chooses two shirts of the same size. 2

Question 26 (3 marks)

- 1**

218

Question 27 (3 marks)

The results of a Mathematics test are normally distributed. Sharon scored 64 in her Mathematics test. The mean for the test was 52 and the variance was 16.

- (a) Did Sharon do relatively well against her cohort? Justify your answer. 2

- (b) What percentage of students scored between 44 and 64? 1

Question 28 (4 marks)

The height of the tide at Manly Beach can be modelled by the function:

$H(t) = 4 + 2 \cos\left(\frac{\pi t}{6}\right), t > 0$, where $H(t)$ is the height of the tide in metres after t hours.

- (a) State the time interval between two successive high tides. **1**

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- (b) Sketch the function $H(t) = 4 + 2 \cos\left(\frac{\pi t}{6}\right)$ in the domain $0 \leq t \leq 6$, labelling **3**
important features.

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Question 29 (3 marks)

A small school has counted the number of Year 12 students in Mathematics and Physics. They found that:

3

- 12 students study Mathematics only
- 5 students study Physics only
- 3 students study neither
- x students study both.

Let M be the event that a Year 12 student studies Mathematics and let P be the event that a Year 12 student studies Physics.

By first drawing a Venn diagram, determine the value of x such that events M and P are independent.

This image shows a full page of blank handwriting practice paper. It features 20 evenly spaced, light gray horizontal lines extending across the entire width of the page. The lines are designed to help guide letter height and placement for cursive or other handwriting styles. There are no margins, text, or other markings on the page.

Question 30 (7 marks)

Consider the curve given by $f(x) = 3x^2 - x^3$.

- (a) Find any stationary points and determine their nature.

3

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- (b) Find any points of inflection.

2

This image shows a single sheet of white paper with horizontal blue or grey ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

- (c) Sketch the graph of $y = f(x)$, showing any stationary points, points of inflection and intercepts with the x and y axes.

2

This image shows a single sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

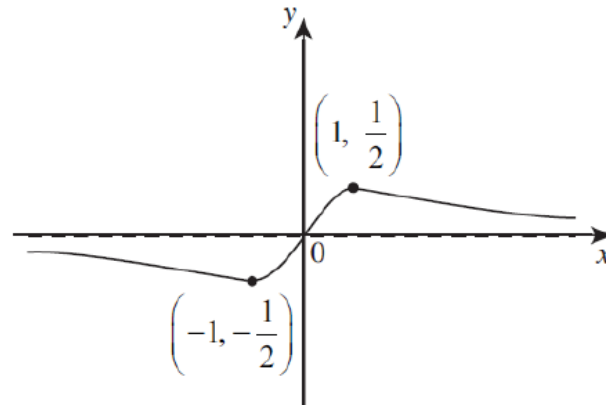
End of Question 30

Please turn over

Question 31 (3 marks)

The graph of $y = f(x)$ in the domain $(-\infty, \infty)$ is shown below.

3



There is a horizontal asymptote at the x -axis.

Sketch the graph of $y = -2 f\left(\frac{x}{4}\right) + 1$, labelling important features including any intercepts, asymptotes and stationary points.

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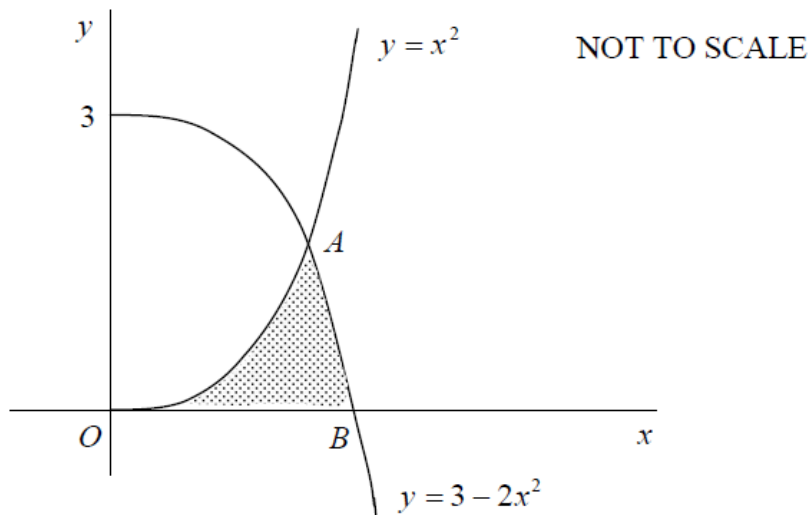
Question 32 (3 marks)

The curve $y = px^3 + qx$ passes through the point $(1, 4)$. The tangent at this point is parallel to the line $y = 2x - 6$. Find the values of p and q .

3

This image shows a single sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

Question 33 (4 marks)



The shaded region OAB is bounded by the parabolas $y = x^2$, $y = 3 - 2x^2$ and the x -axis. It is given that point A has the coordinates $(1, 1)$ and O is the origin.

- (a) Find the coordinates of B .

1

- (b) Show that the exact area of the shaded region OAB is given by:

3

$$\left(2\sqrt{\frac{3}{2}} - 2\right) \text{ square units.}$$

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End of Question 33

Please turn over

Question 34 (6 marks)

John borrowed an amount $\$P$ at a rate of 3.6% per annum compounded monthly on a reducing balance. At the end of each month, he repays an amount of $\$M$.

- (a) Show that the amount owing at the end of the second month is 1
 $A_2 = P(1.003)^2 - M(1.003) - M$.

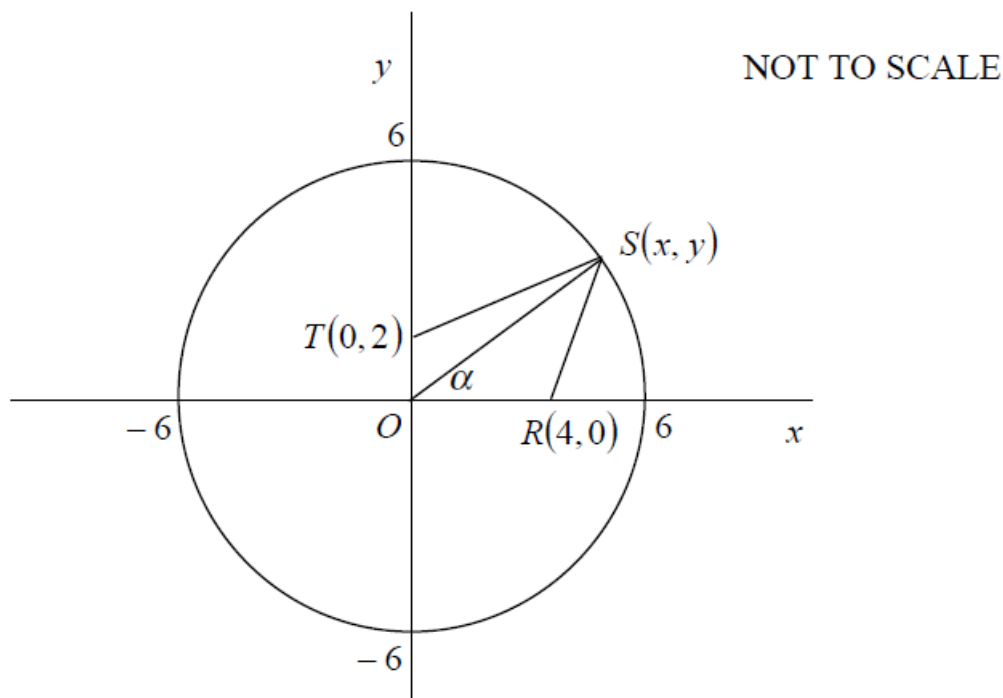
- (b) It is now given that the amount borrowed is \$50 000 and his monthly repayment is \$350. Show that the amount owing after one year is approximately \$47 560. 2

- (c) Find how many months it will take for John to fully repay the loan.

3

[illegible]

Question 35 (8 marks)



The diagram above shows the circle $x^2 + y^2 = 36$. The point $S(x, y)$ lies on the circle in the first quadrant. $R(4, 0)$ lies on the x -axis and $T(0, 2)$ lies on the y -axis and O is the origin. The size of $\angle SOR = \alpha$ where $0 < \alpha < \frac{\pi}{2}$.

- (a) Show that the area of $\triangle SOR$ is $12 \sin \alpha$. **1**

- (b) Hence, show that the area of the quadrilateral $ORST$ is given by: **2**
 $A = 6 \cos \alpha (2 \tan \alpha + 1)$

- (c) Find the value of $\tan \alpha$ for which the area A is a maximum.

3

This image shows a single sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

- (d) Hence, or otherwise, when the area is a maximum, find the coordinates of point S in exact form.

2

This image shows a single sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

End of Paper

Section II Extra writing space

If you use this space, clearly indicate which question you are answering.

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[illegible][illegible]

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NESA Number

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample: $2 + 4 =$ (A) 2 (B) 6 (C) 8 (D) 9
 A ☐ B ☒ C ☐ D ☐

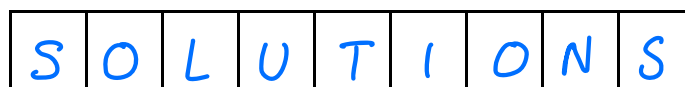
If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ☒ B ☒ C ☐ D ☐

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word correct and drawing an arrow as follows.

A ☒ B ☒ C ☐ D ☐
 correct
 ↖

1. A ☐ B ☐ C ☐ D ☐
2. A ☐ B ☐ C ☐ D ☐
3. A ☐ B ☐ C ☐ D ☐
4. A ☐ B ☐ C ☐ D ☐
5. A ☐ B ☐ C ☐ D ☐
6. A ☐ B ☐ C ☐ D ☐
7. A ☐ B ☐ C ☐ D ☐
8. A ☐ B ☐ C ☐ D ☐
9. A ☐ B ☐ C ☐ D ☐
10. A ☐ B ☐ C ☐ D ☐



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Section I

10 marks

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A. Amplitude = 3 , Period = 10π

☒ B. No amplitude , Period = 5π

C. Amplitude = 3 , Period = 5π

D. No Amplitude , Period = 10π

$$\text{Period} = \frac{\pi}{1/5} \\ = 5\pi$$

- 2 The circle $x^2 + y^2 = 9$ is transformed to $(x - 2)^2 + (y + 3)^2 = 9$.
What are the horizontal and vertical translations?

A. Horizontally left 3 units, vertically up 2 units

B. Horizontally right 3 units, vertically down 2 units

C. Horizontally left 2 units, vertically up 3 units

☒ D. Horizontally right 2 units, vertically down 3 units

- 3 What amount (to the nearest dollar) must be invested now at 4% per annum, compounded quarterly, so that in five years it will have grown to \$60 000?

A. \$27 383

☒ B. \$49 173

C. \$49 316

D. \$57 088

$$60000 = P(1.01)^{20} \quad r = \frac{4\%}{4} \\ = 0.01 \\ \text{quarterly} \\ P = \frac{60000}{1.01^{20}} \\ = 49172.66 \dots \quad n = 5 \times 4 \\ = 20$$

4 Which interval gives the domain of $f(x) = \frac{2}{\sqrt{4-x^2}}$?

A. $(-\infty, 2)$

B. $(2, \infty)$

☒ C. $(-2, 2)$

D. $[-2, 2]$

$$4 - x^2 > 0$$

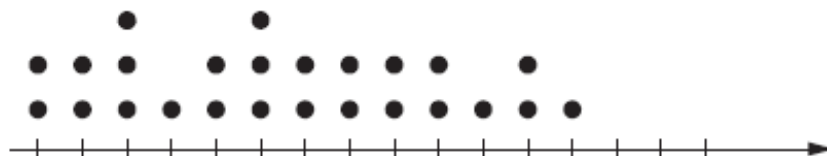
$$(2+x)(2-x) > 0$$



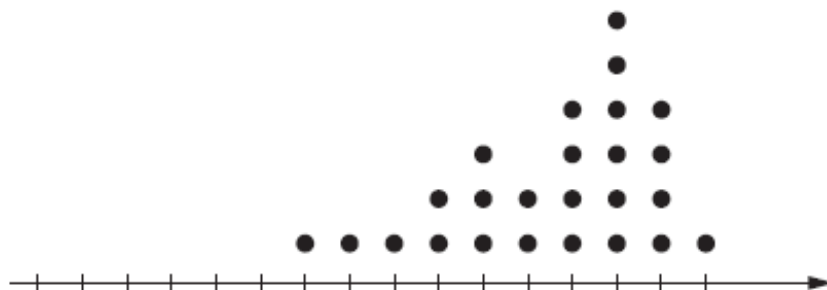
5 The dot plots below show the class scores in the same topic test before and after a unit of work was completed. They are drawn on the same scale.

-8

Before



After



Which statement about the change in scores from before to after is correct?

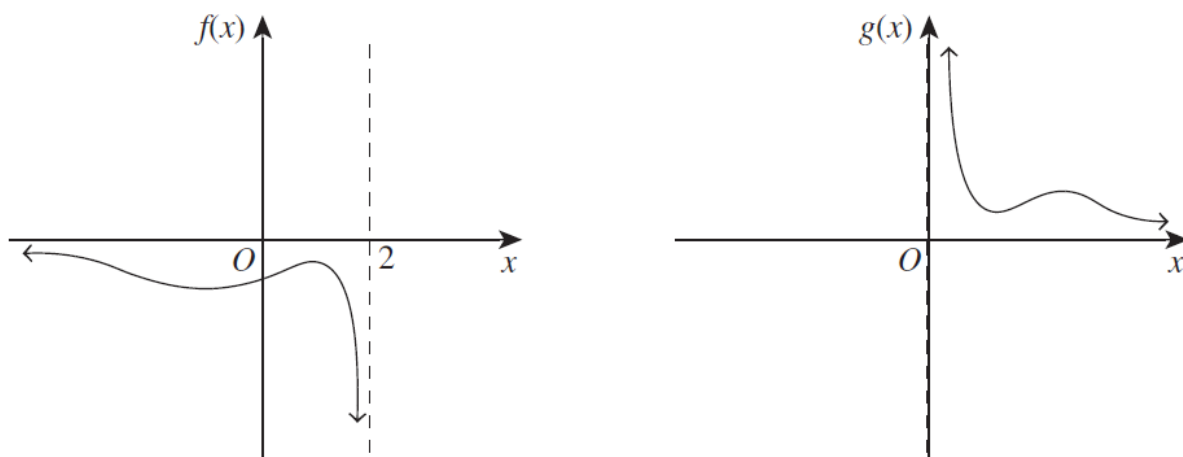
A. The mean increased and the standard deviation increased.

☒ B. The mean increased and the standard deviation decreased.

C. The mean decreased and the standard deviation increased.

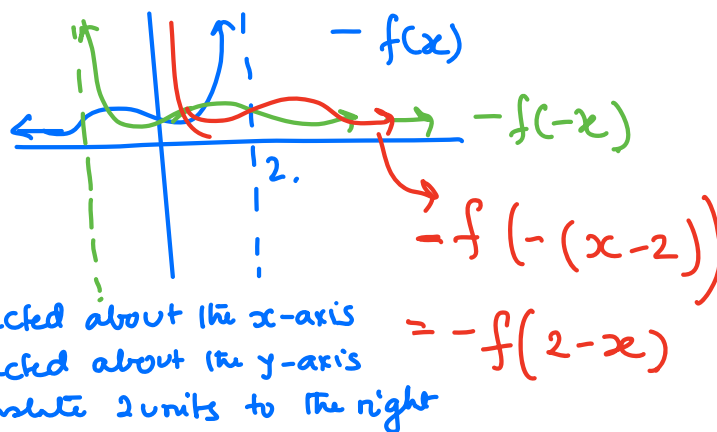
D. The mean decreased and the standard deviation decreased.

- 6 The function $y = f(x)$ is transformed to $y = g(x)$, as shown in the diagram.



Which of the following equations best represents the transformed function?

- A. $g(x) = f(x - 2)$
 B. $g(x) = f(2 - x)$
 C. $g(x) = -f(x - 2)$
 D. $g(x) = -f(2 - x)$



- 7 The acceleration of a particle moving in a straight line is given by $a = 4t + 4$. Initially the particle has a velocity of -30 m/s. When is the particle at rest?

- A. $t = 0$
 B. $t = 2$
 C. $t = 3$
 D. $t = 5$

$$V = 2t^2 + 4t + C$$

$$t = 0, \quad v = -30$$

$$\therefore C = -30$$

$$V = 2t^2 + 4t - 30$$

$$= 2(t^2 + 2t - 15)$$

$$= 2(t + 5)(t - 3)$$

$$t \geq 0$$

$$\therefore t = 3$$

- 8 Consider the information about events A and B .

- $P(B) = 0.6$
- $P(A|B) = 0.4$
- $P(A|\bar{B}) = 0.8$

What is the value of $P(B|A)$?

A. $\frac{1}{5}$

B. $\frac{2}{5}$

☒ C. $\frac{3}{7}$

D. $\frac{4}{5}$

$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$

$$P(A|\bar{B}) = \frac{P(A \cap \bar{B})}{P(\bar{B})}$$

$$= \frac{P(A \cap \bar{B})}{1 - 0.6}$$

$$\text{since } \frac{P(A \cap \bar{B})}{0.4} = 0.8$$

$$\therefore P(B|A) = \frac{6/25}{14/25} = \frac{3}{7}$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$= \frac{P(A \cap B)}{0.6}$$

$$\text{since } \frac{P(A \cap B)}{0.6} = 0.4$$

$$\therefore P(A \cap B) = \frac{6}{25}$$

- 9 Let $\log_7 10 = x$. Which expression is equal to $\log_7 \left(\frac{1}{70} \right)$?

☒ A. $-(1 + x)$

B. $(1 + x)^{-1}$

C. $\frac{x}{10}$

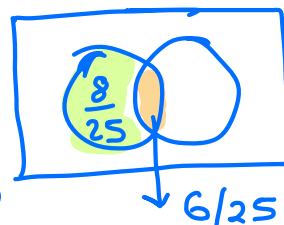
D. $\frac{1}{10x}$

$$= \log_7 1 - \log_7 70$$

$$= -\log_7 7 \times 10$$

$$= -(\log_7 7 + \log_7 10)$$

$$= -(1 + x)$$



- 10 If $e^y = \sin x$, which of the following is $\frac{dy}{dx}$?

☒ A. $\cot x$

B. $\tan x$

C. $-\cot x$

D. $-\tan x$

$$y = \ln(\sin x)$$

$$\frac{dy}{dx} = \frac{\cos x}{\sin x}$$

$$= \cot x$$

End of Section I

Section II

90 marks

Attempt Questions 11 – 35

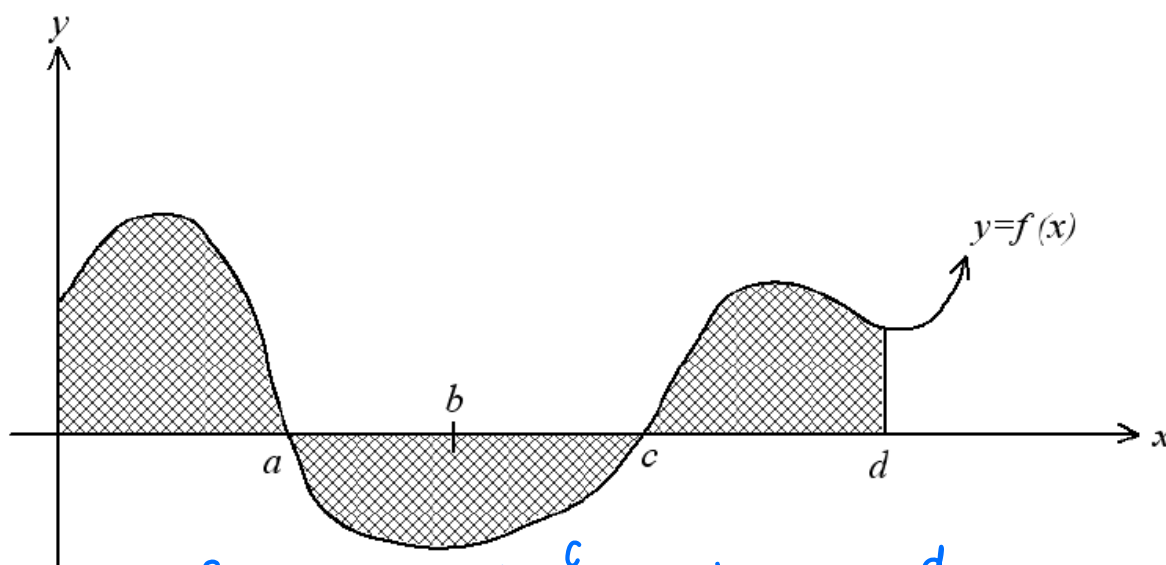
Allow about 2 hours and 45 minutes for this section

- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
- For questions in Section II, your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided at the back of this booklet.
If you use this space, clearly indicate which question you are answering.

Question 11 (1 mark)

Write an expression for the shaded area shown in the diagram below.

1



$$\int_0^a f(x) dx + \left| \int_a^c f(x) dx \right| + \int_c^d f(x) dx$$

$$\text{or } \int_0^a f(x) dx - \int_a^c f(x) dx + \int_c^d f(x) dx$$

$$\text{or } \int_0^a f(x) dx + \int_c^a f(x) dx + \int_c^d f(x) dx$$

Question 12 (2 marks)

The probability distribution of a discrete random variable X is given in the table below.

x	3	5	9	a
$P(X = x)$	0.1	0.3	0.2	0.4

- (a) Show that $a = 30$ given that $E(X) = 15.6$

1

$$E(X) = 3 \times 0.1 + 5 \times 0.3 + 9 \times 0.2 + a \times 0.4$$

$$= 0.3 + 1.5 + 1.8 + 0.4a$$

$$= 3.6 + 0.4a$$

$$3.6 + 0.4a = 15.6$$

$$a = \frac{15.6 - 3.6}{0.4} = 30$$

- (b) Hence, find the variance of X .

1

$$\text{Var}(X) = \sum x^2 p(x) - [E(X)]^2$$

$$= 3^2 \times 0.1 + 5^2 \times 0.3 + 9^2 \times 0.2 + 30^2 \times 0.4 - [15.6]^2$$

$$= 141.24$$

Question 13 (1 mark)

Simplify $\frac{1 - \sin^2 x}{\cot x}$.

1

$$= \cos^2 x \times \frac{\sin x}{\cos x}$$

$$= \sin x \cos x$$

Question 14 (4 marks)

Consider the geometric series $64 - 16a + 4a^2 - \dots$

- (a) For what values of a does this series have a limiting sum?

2

$$r = \frac{-16a}{64} = \frac{4a^2}{-16a}$$

$$r = -\frac{a}{4} \quad |r| < 1$$

$$-1 < r < 1$$

$$-1 < -\frac{a}{4} < 1$$

i.e.

$$-4 < a < 4$$

$$-4 < a < 4$$

$$4 > a > -4$$

- (b) It is known that the limiting sum of the series is 256. Find the value of a .

2

$$S_{\infty} = \frac{a}{1-r}$$

$$= \frac{64}{1-r}$$

$$\frac{64}{1-r} = 256$$

$$\frac{64}{256} = 1-r$$

$$r = 1 - \frac{64}{256}$$

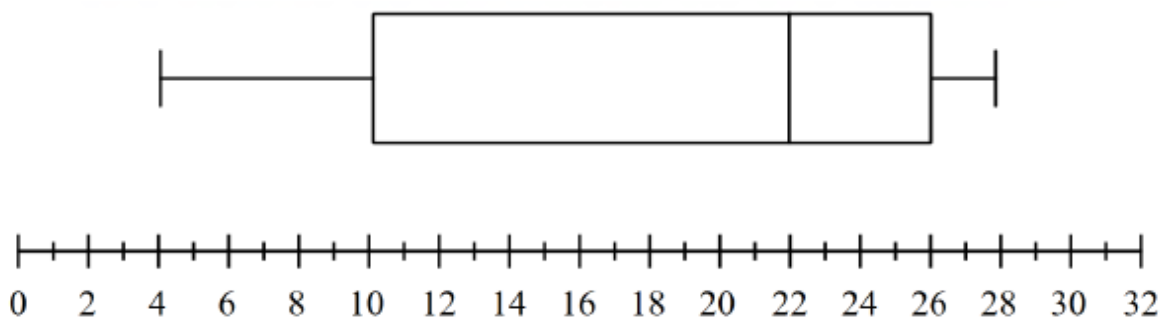
$$= \frac{3}{4}$$

$$-\frac{a}{4} = \frac{3}{4} \Rightarrow a = -3$$

Question 15 (3 marks)

Jimmy recorded the number of cars in the carpark each day.

He displayed the data in the boxplot below:



- (a) Find the interquartile range given that $Q_1 = 10$.

1

$$\begin{aligned} Q_3 - Q_1 &= 26 - 10 \\ &= 16 \end{aligned}$$

- (b) One day, Jimmy recorded 49 cars in the carpark. Is this recording an outlier?

1

Justify your answer.

$$Q_3 + 1.5 \times IQR$$

$$= 50$$

Since $49 < 50 \therefore$ This recording is not an outlier

- (c) Comment on the shape of the distribution of this boxplot.

1

Negatively skewed distribution

Question 16 (3 marks)

A circle is divided into n sectors in such a way that the angles of the sectors are in arithmetic progression. The smallest two angles are 3° and 5° . Find the value of n .

3

$$3^\circ, 5^\circ, \dots$$

$$a = 3, d = 2$$

$$S_n = 360^\circ$$

$$360 = \frac{n}{2} [6 + (n-1)2]$$

$$720 = 6n + 2n^2 - 2n$$

$$2n^2 + 4n - 720 = 0$$

$$n^2 + 2n - 360 = 0$$

$$(n+20)(n-18) = 0$$

$$\underline{\underline{n = 18}}$$

$$n > 0$$

Question 17 (2 marks)

Find $\int \frac{1}{\sqrt{1+2x}} dx$.

2

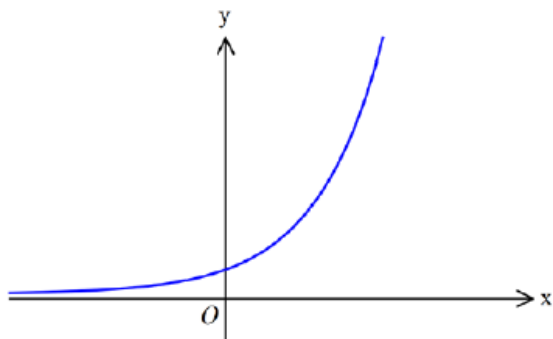
$$\frac{1}{2} \int 2(1+2x)^{-1/2} dx$$

$$= \frac{1}{2} \frac{(1+2x)^{1/2}}{1/2} + C$$

$$= \sqrt{1+2x} + C$$

Question 18 (3 marks)

The diagram below shows a sketch of the curve $y = 2^{3x}$.



- (a) Use the trapezoidal rule with three function values to find an approximate value for $\int_0^1 2^{3x} dx$ rounded to two decimal places. 2

$$\begin{aligned}\int_0^1 2^{3x} dx &\approx \frac{1/2}{2} \left[1 + 2^3 + 2 \left(2^{3 \times \frac{1}{2}} \right) \right] \\ &= \frac{1}{4} \left[1 + 8 + 2 \times 2^{\frac{3}{2}} \right] \\ &= 3.66421356... \\ &\approx 3.66 \text{ (2dp.)}\end{aligned}$$

- (b) Is the approximate for $\int_0^1 2^{3x} dx$ an under approximation or an over approximation? 1
Justify your answer.

It is over approximation

as the graph from 0 to 1 is concave up

$\therefore$ trapeziums fit over the curve i.e.

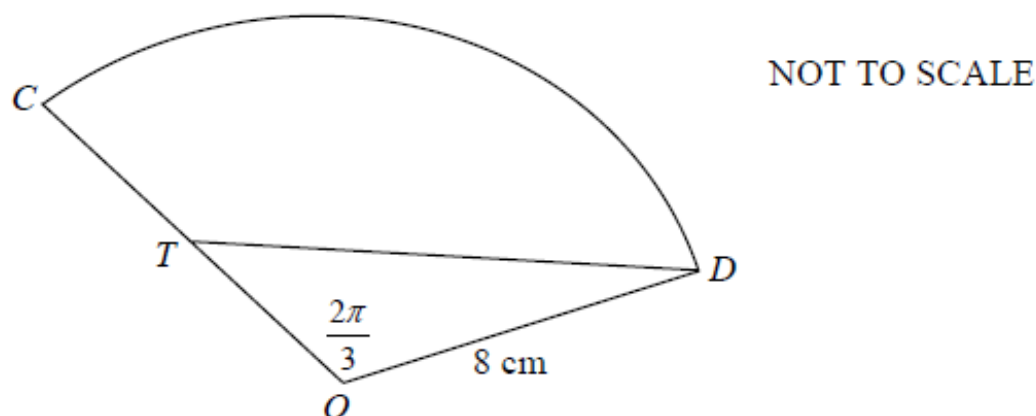
 area of trapezium is more than the area under the curve.

Question 19 (3 marks)

In the diagram, CD is an arc of a circle with radius 8 cm and centre O .

3

T is the midpoint of OC . It is given that $\angle COD = \frac{2\pi}{3}$.



Calculate the perimeter of CTD in exact form.

$$\begin{aligned}\widehat{CD} &= r\theta \\ &= 8 \times \frac{2\pi}{3} \\ &= \frac{16\pi}{3}\end{aligned}$$

$$CT = \frac{1}{2} \times 8 = 4$$

$$\begin{aligned}TD &= \sqrt{4^2 + 8^2 - 2 \times 4 \times 8 \times \cos \frac{2\pi}{3}} \\ &= \sqrt{80 - 64(-\cos \frac{\pi}{3})}\end{aligned}$$

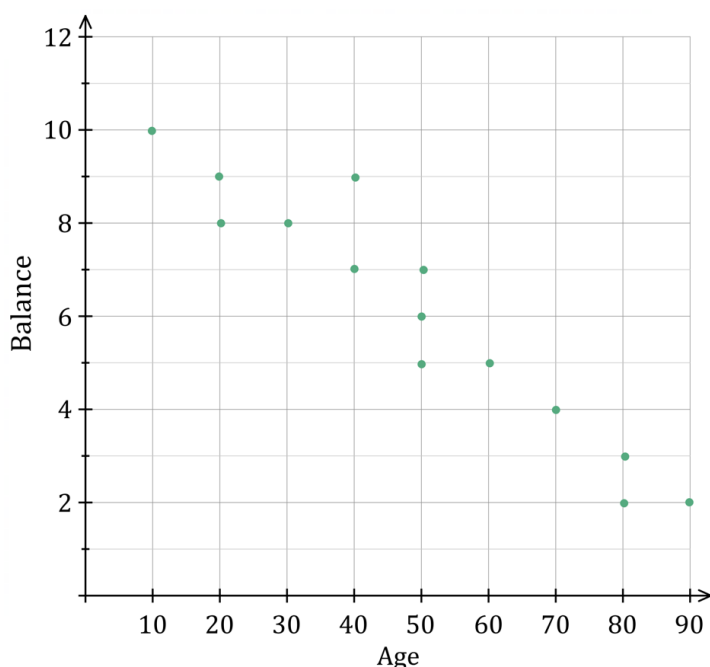
$$= \sqrt{80 + 64 \times \frac{1}{2}}$$

$$\begin{aligned}&= \sqrt{112} \text{ cm} \quad \therefore P = \frac{16\pi}{3} + 4 + \sqrt{112} \text{ cm} \\ &= 4\sqrt{7}\end{aligned}$$

$$\text{or } P = \frac{16\pi}{3} + 4 + 4\sqrt{7}$$

Question 20 (4 marks)

The scatterplot below shows the relationship between age and how well you can balance with your body.



- (a) Akash uses his calculator and calculates his correlation coefficient to be $r = 1.73$. Without calculating the correlation coefficient, give ONE reason why $r = 1.73$ is incorrect.

1

r lies between -1 and 1
or Since the above association is negative $\therefore -1 < r < 0$

- (b) Find the value of Pearson's correlation coefficient to two decimal places and hence describe the strength and direction of the association.

1

$r = -0.955$
Strong negative association

- (c) Write down ONE age that would be an example of interpolation.

1

Many possible answers! $10 < \text{age} < 90$

- (d) Give ONE example of a limitation of this model.

1

- Babies under 1 yr old won't have good body balance
- With age balance reduces but it may not be zero.

Question 21 (3 marks)

State the transformations in order that need to be applied to the hyperbola $y = \frac{1}{x}$ to obtain

3

$$y = \frac{1}{3x+6} - 2.$$

$$y = \frac{1}{3(x+2)} - 2$$

- Horizontally Dilated by a scale factor $\frac{1}{3}$ from the y-axis

- Translated to the left by 2 units

Vertically

- Translated down by 2 units.

OR

- Vertically dilated by a scale factor $\frac{1}{3}$

- Vertically translated down by 2

- Horizontally translated to the left by 2.

OR

- Horizontally translated to the left by 6

- Horizontally dilated by a scale factor of $\frac{1}{3}$

- Vertically translated down by 2 units.

Question 22 (4 marks)

The table below shows the present value of an annuity with a contribution of \$1.

Number of periods	Interest rate per period as a decimal				
	0.001	0.00125	0.0015	0.00175	0.002
300	259.0707	250.0398	241.4379	233.2418	226.3477
330	280.9577	270.2690	260.1353	250.5239	240.7211
360	302.1982	289.7541	278.0106	266.9228	258.9154
390	325.2961	309.6290	297.0981	283.6291	261.9432

- (a) Sally pays \$2 000 into an annuity at the end of each month at 1.2% p.a. compounded monthly for 25 years. What is the present value of her annuity? 2

$$\begin{aligned}
 & r = \frac{1.2}{12} \% \\
 & = 0.001 \\
 & 25 \times 12 = 300 \\
 & 2000 \times 259.0707 \\
 & = \$518141.40
 \end{aligned}$$

- (b) Thomas borrowed \$600 000 to purchase a home, with interest charged at 1.5% per annum compounding monthly. He agrees to repay the loan by making equal monthly payments over a 30-year period. 2

What is the monthly payment? Answer correct to the nearest cent.

$$\begin{aligned}
 & 600000 = M \times 289.7541 \quad \frac{1.5\%}{12} = 0.00125 \\
 & M = \frac{600000}{289.7541} \quad n = 30 \times 12 \\
 & \quad \quad \quad = 360 \\
 & = \$2070.72
 \end{aligned}$$

Question 23 (6 marks)

The population of ants is growing exponentially according to the equation $P = 50e^{kt}$, where t is the time in days after the insect is first counted. After four days, it is observed that the population has doubled from the initial amount.

- (a) Find the initial number of ants and show that $k = \frac{1}{4} \ln 2$

2

$$\begin{array}{l}
 t = 0 \\
 \text{Initial } P_0 = 50 \times e^0 \\
 P_0 = 50 \\
 \therefore \text{Double of initial} = 100 \\
 100 = 50 e^{k \times 4} \\
 e^{4k} = 2 \\
 4k = \ln 2 \\
 k = \frac{1}{4} \ln 2
 \end{array}$$

- (b) At what rate (to the nearest whole number) is the population increasing at day 15?

2

$$\begin{array}{l}
 \frac{dP}{dt} = 50 \times k e^t \\
 \frac{1}{4} \ln 2 \times 15 \\
 \text{At } t = 15 = 50 \times \frac{1}{4} \ln 2 \times e \\
 \approx 117 \text{ ants / day}
 \end{array}$$

- (c) How many days will it take for the number of ants to first reach 1500?

2

$$\begin{array}{l}
 \frac{1}{4} \ln 2 \times t \\
 1500 = 50 \times e \\
 30 = e^{(\frac{1}{4} \ln 2)t} \\
 (\frac{1}{4} \ln 2)t = \ln 30 \\
 t = \frac{\ln 30}{\frac{1}{4} \ln 2} \approx 20 \text{ days.}
 \end{array}$$

Question 24 (4 marks)

The probability density function for the continuous random variable X is given by:

$$f(x) = \begin{cases} kx(4-x), & 1 \leq x \leq 4 \\ 0, & \text{otherwise} \end{cases}$$

- (a) Show that $k = \frac{1}{9}$

2

$$\int_1^4 kx(4-x) dx = 1$$

$$k \int_1^4 (4x - x^2) dx = 1$$

$$k \left[2x^2 - \frac{x^3}{3} \right]_1^4 = 1$$

$$k \left(\left(32 - \frac{64}{3} \right) - \left(2 - \frac{1}{3} \right) \right) = 1$$

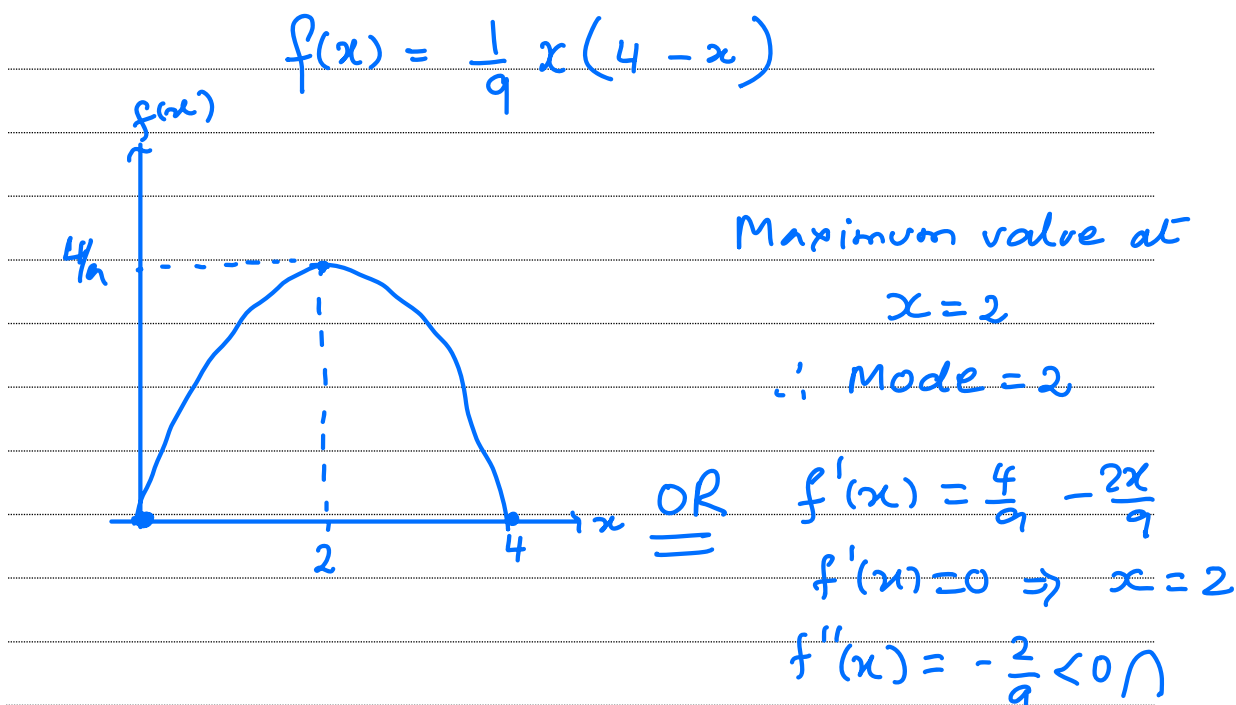
$$k \times 9 = 1$$

$$k = \frac{1}{9}$$

2

- (b) Determine the mode of X .

2

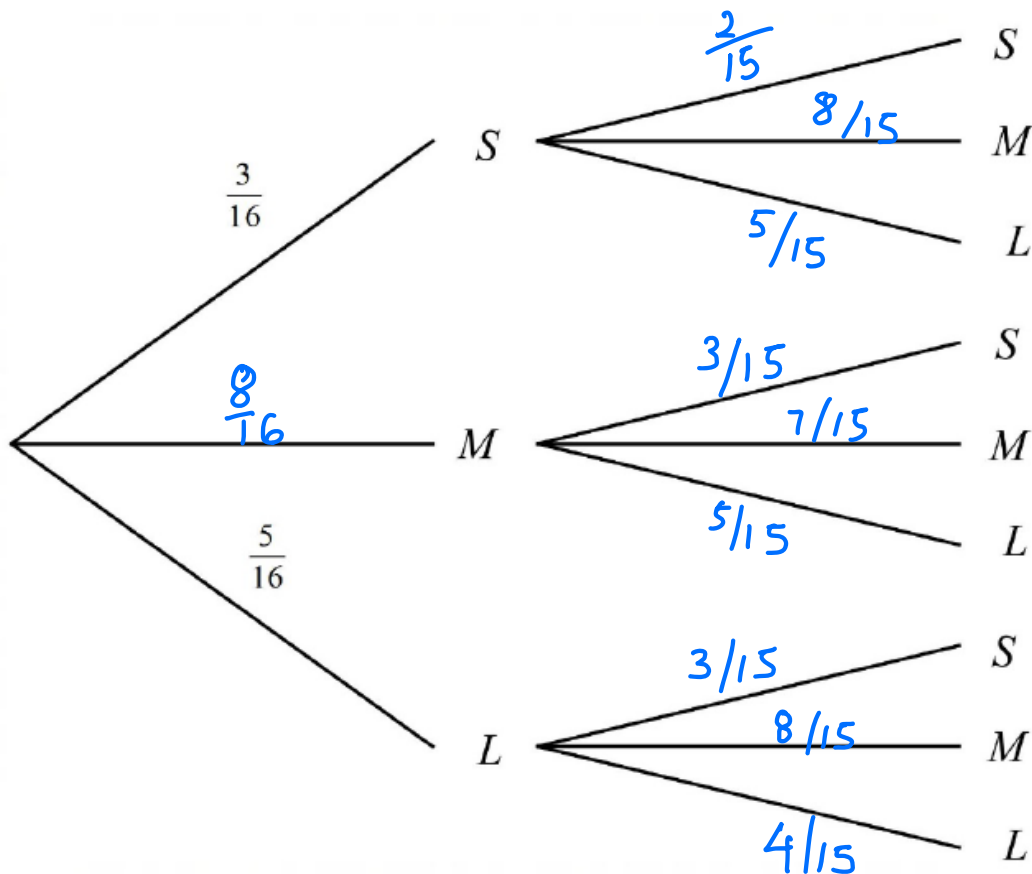


$\therefore x=2$ is mode.

Questions 11-24 are worth 43 marks in total

Question 25 (3 marks)

In a closet there are shirts in three sizes. 3 are small, 8 are medium and 5 are large. Johnny takes two shirts from the closet at random without replacement. The tree diagram below has been started to show the probabilities of different combinations.



- (a) Complete the tree diagram by writing the probabilities on the remaining branches. 1
- (b) Find the probability that Johnny chooses two shirts of the same size. 2

$$P(SS) + P(MM) + P(LL)$$

$$= \frac{3}{16} \times \frac{2}{15} + \frac{8}{16} \times \frac{7}{15} + \frac{5}{16} \times \frac{4}{15}$$

$$= \frac{41}{120}$$

Question 26 (3 marks)

- (a) Show that $\frac{d}{dx}(x \ln x - x) = \ln x$

1

$$\therefore \frac{d}{dx}(x \ln x - x)$$

$$= 1 \times \ln x + \cancel{x \times \frac{1}{x}} - 1$$

$$= \ln x + 1 - 1$$

$$= \ln x = \text{RHS.}$$

$$u = x$$

$$u' = 1$$

$$v = \ln x$$

$$v' = \frac{1}{x}$$

- (b) Hence, evaluate $\int_1^e \ln x^2 dx$.

2

$$\frac{d}{dx}(x \ln x - x) = \ln x$$

Take integral of both sides.

$$\int_1^e \ln x dx = [x \ln x - x]_1^e$$

$$\int_1^e \ln x^2 dx = 2 \int_1^e \ln x dx$$

$$= 2 [x \ln x - x]_1^e$$

$$= 2 [(e \ln e - e) - (1 \ln 1 - 1)]$$

$$= 2 [\cancel{e} - \cancel{e} - 0 + 1]$$

$$= 2$$

Question 27 (3 marks)

The results of a Mathematics test are normally distributed. Sharon scored 64 in her Mathematics test. The mean for the test was 52 and the variance was 16.

- (a) Did Sharon do relatively well against her cohort? Justify your answer.

2

$$Z = \frac{64 - 52}{4}$$

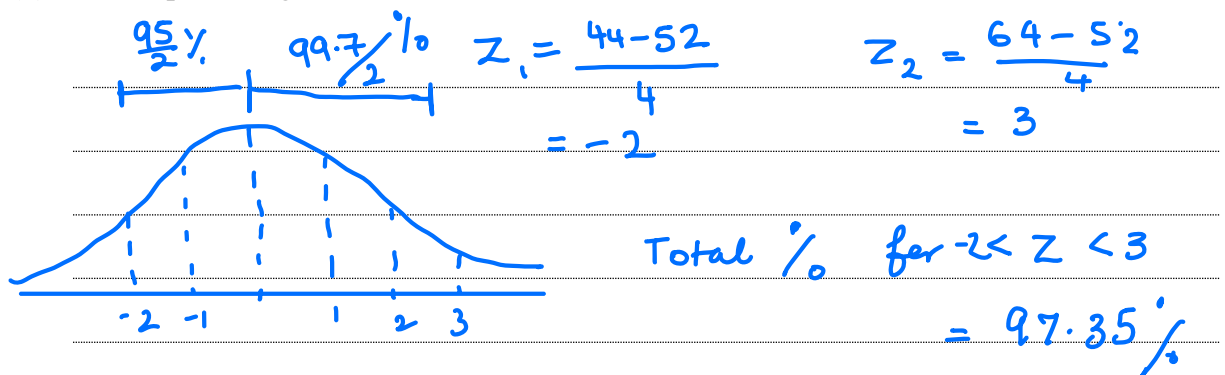
$$= \frac{12}{4}$$

$$= 3$$

Yes Sharon did really well as her score is 3 S.D. above the mean.

- (b) What percentage of students scored between 44 and 64?

1



97.35% scored between 44 and 64.

Question 28 (4 marks)

The height of the tide at Manly Beach can be modelled by the function:

$H(t) = 4 + 2 \cos\left(\frac{\pi t}{6}\right)$, $t > 0$, where $H(t)$ is the height of the tide in metres after t hours.

- (a) State the time interval between two successive high tides.

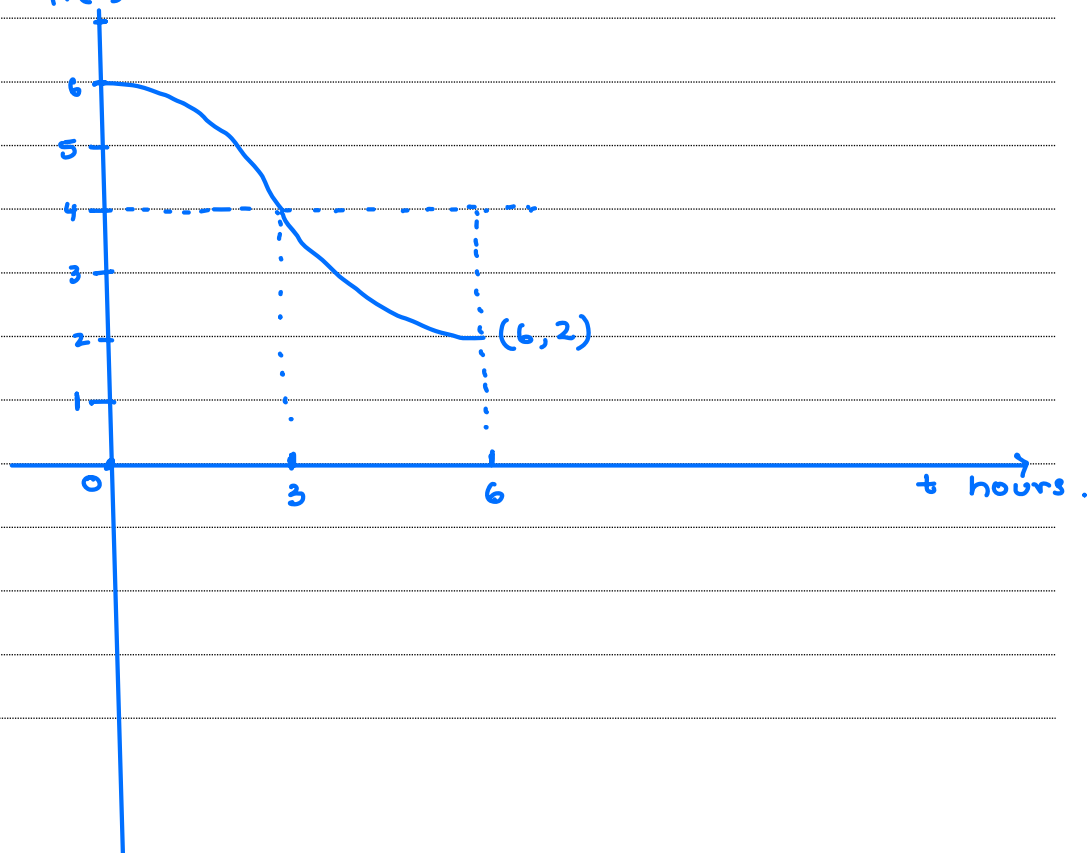
1

$$\text{Period} = \frac{2\pi}{\pi/6} = 12 \text{ hours.}$$

- (b) Sketch the function $H(t) = 4 + 2 \cos\left(\frac{\pi t}{6}\right)$ in the domain $0 \leq t \leq 6$, labelling important features.

3

$H(t)$ in metres.



Question 29 (3 marks)

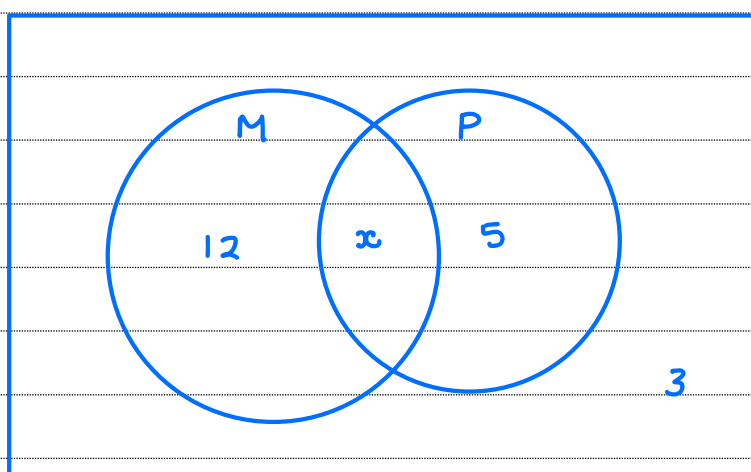
A small school has counted the number of Year 12 students in Mathematics and Physics. They found that:

3

- 12 students study Mathematics only
- 5 students study Physics only
- 3 students study neither
- x students study both.

Let M be the event that a Year 12 student studies Mathematics and let P be the event that a Year 12 student studies Physics.

By first drawing a Venn diagram, determine the value of x such that events M and P are independent.



$$P(M \cap P) = P(M)P(P)$$

$$\frac{x}{20+x} = \frac{12+x}{20+x} \times \frac{5+x}{20+x}$$

$$x(20+x) = (12+x)(5+x)$$

$$20x + \cancel{x^2} = 60 + 17x + \cancel{x^2}$$

$$3x = 60$$

$$x = 20$$

Question 30 (7 marks)

Consider the curve given by $f(x) = 3x^2 - x^3$.

- (a) Find any stationary points and determine their nature.

3

$$f'(x) = 6x - 3x^2$$

$$f'(x) = 0$$

$$\Rightarrow 3x(2 - x) = 0$$

$$x = 0, \quad x = 2$$

$$y = 0 \quad y = 4$$

$$f''(x) = 6 - 6x$$

$$f''(0) = 6 > 0 \quad \cup \quad \therefore (0, 0) \text{ is a local minimum t.p.}$$

$$f''(2) = 6 - 12 = -6 < 0 \quad \cap \quad \therefore (2, 4) \text{ is a local maximum t.p.}$$

- (b) Find any points of inflection.

2

$$f''(x) = 0$$

$$6 - 6x = 0$$

$$6x = 6$$

$$x = 1$$

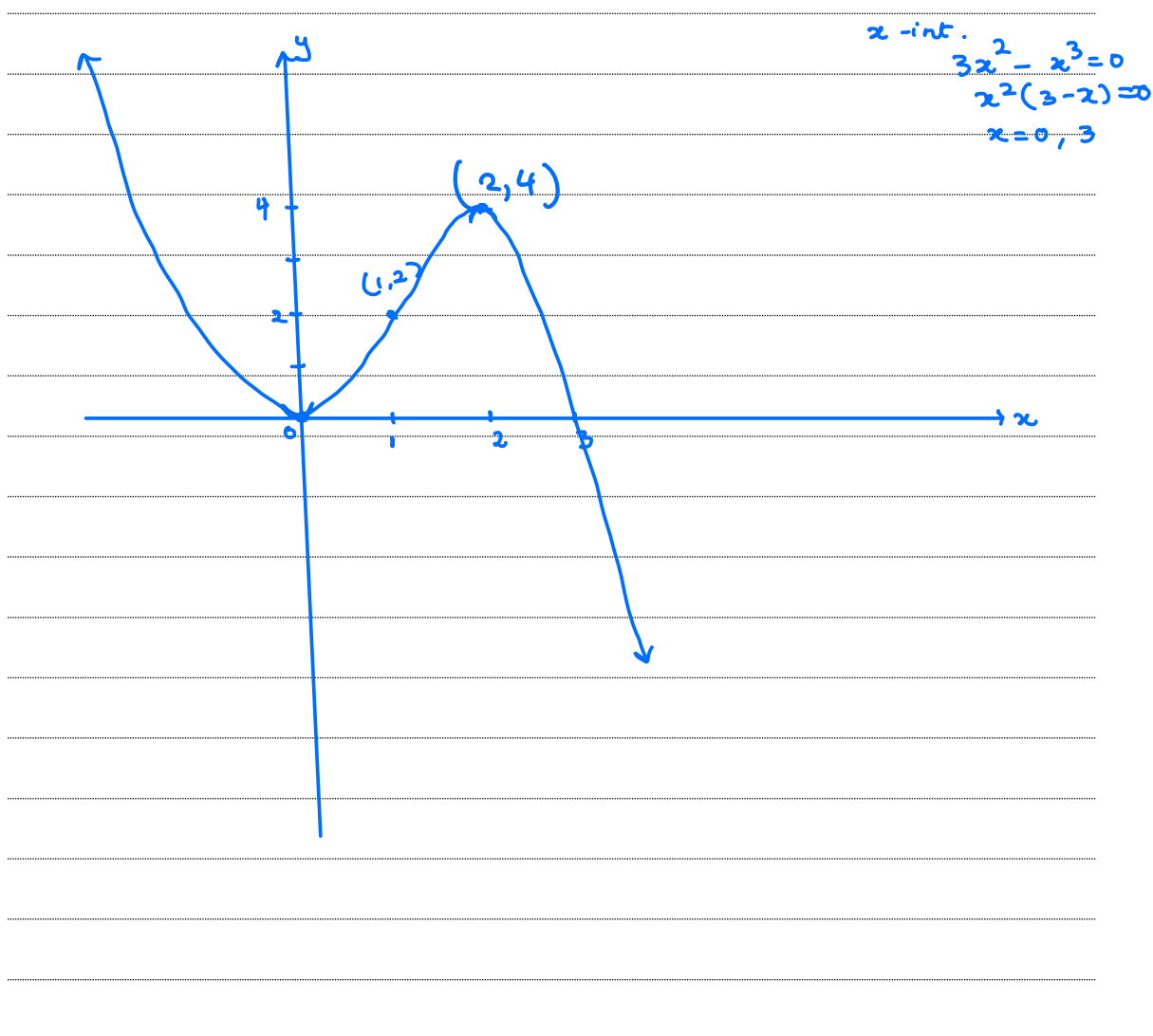
$$y = 2 \quad (1, 2)$$

x	0.9	1	1.1
$f''(x)$	$0.6 > 0$	0	$-0.6 < 0$
	$\cup$		$\cap$

Since the concavity changes
 $\therefore (1, 2)$ is a point of inflection.

- (c) Sketch the graph of $y = f(x)$, showing any stationary points, points of inflection and intercepts with the x and y axes.

2



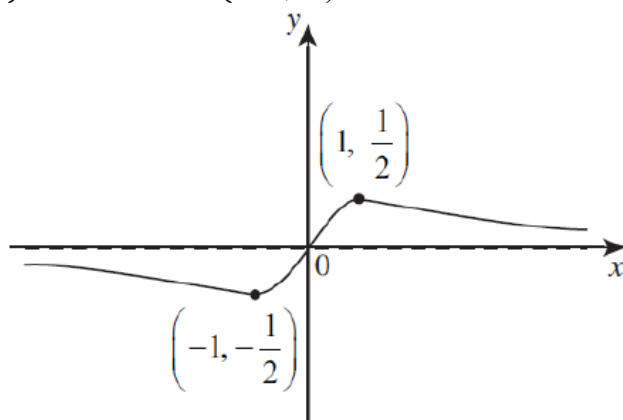
End of Question 30

Please turn over

Question 31 (3 marks)

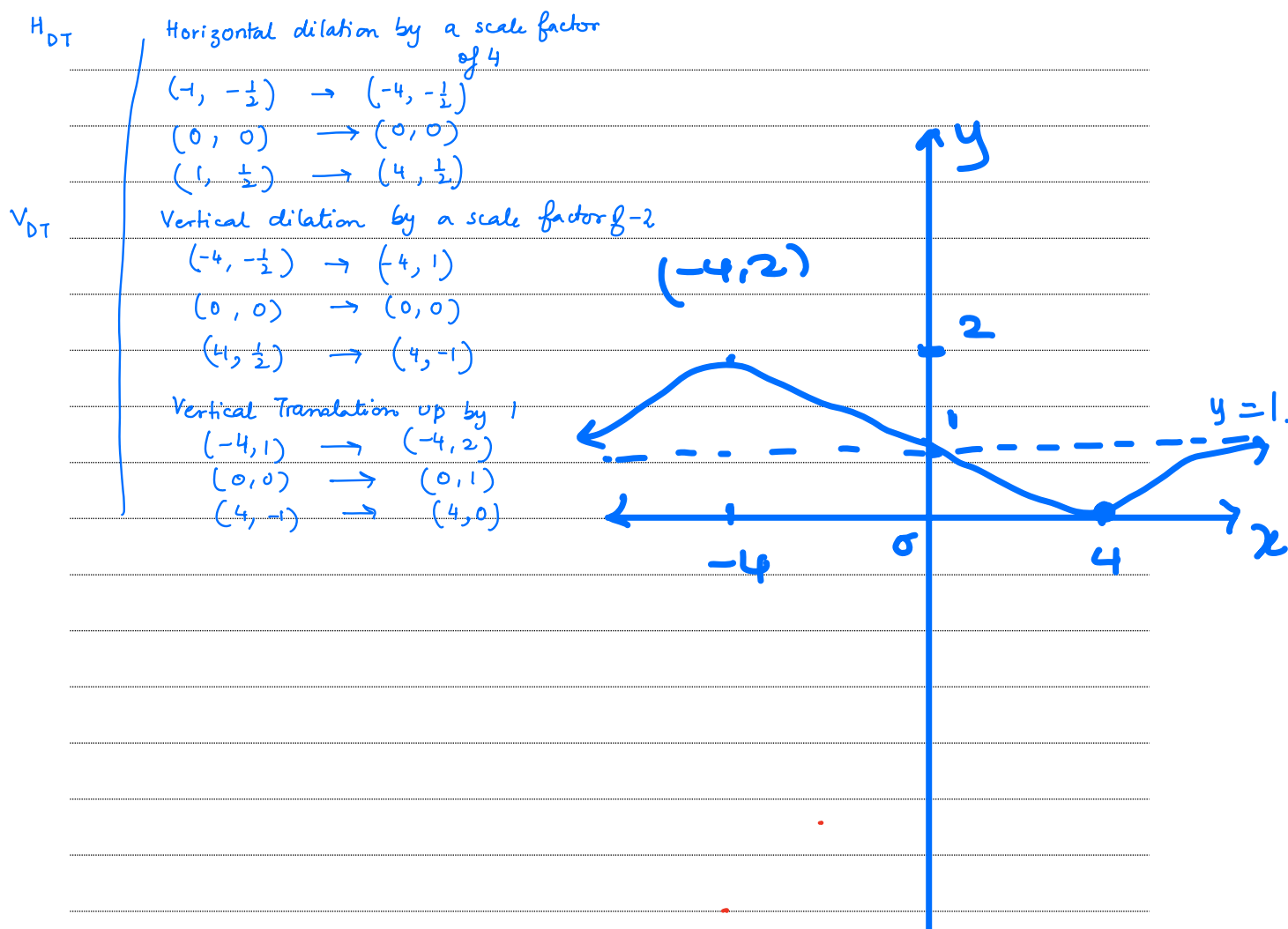
The graph of $y = f(x)$ in the domain $(-\infty, \infty)$ is shown below.

3



There is a horizontal asymptote at the x -axis.

Sketch the graph of $y = -2f\left(\frac{x}{4}\right) + 1$, labelling important features including any intercepts, asymptotes and stationary points.



Question 32 (3 marks)

The curve $y = px^3 + qx$ passes through the point $(1, 4)$. The tangent at this point is parallel to the line $y = 2x - 6$. Find the values of p and q .

3

$$\frac{dy}{dx} = 3px^2 + q$$

$$m_{\text{tangent at } (1, 4)}$$

$$= 3p + q$$

$$\begin{aligned} \text{Since // to given line} \\ = m \text{ of } y = 2x - 6 \end{aligned}$$

$$\therefore 3p + q = 2 \quad (2)$$

$$\text{From } (1) \quad q = 4 - p$$

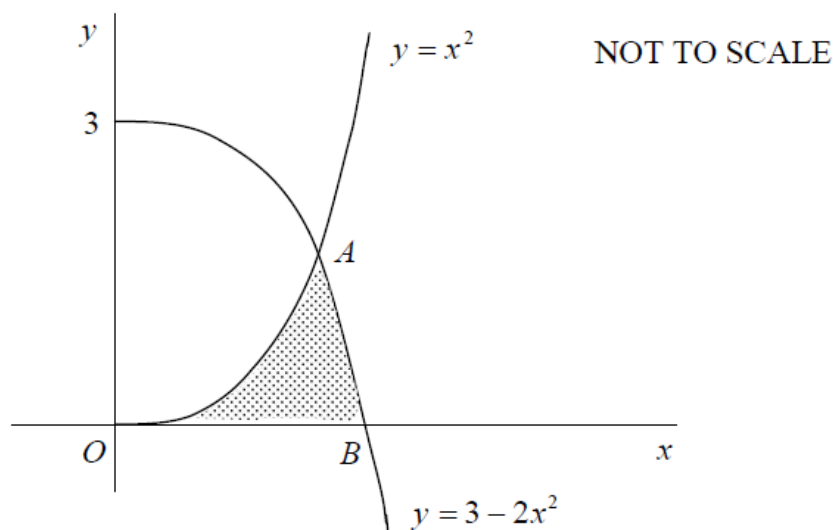
$$\therefore 3p + 4 - p = 2$$

$$2p = -2$$

$$p = -1$$

$$\begin{aligned} \therefore q &= 4 - (-1) \\ &= 5 \end{aligned}$$

Question 33 (4 marks)



The shaded region OAB is bounded by the parabolas $y = x^2$, $y = 3 - 2x^2$ and the x -axis. It is given that point A has the coordinates $(1, 1)$ and O is the origin.

- (a) Find the coordinates of B .

1

$$\text{Sub } y=0$$

$$3 - 2x^2$$

$$x = \sqrt{\frac{3}{2}} \quad B\left(\sqrt{\frac{3}{2}}, 0\right)$$

- (b) Show that the exact area of the shaded region OAB is given by:

3

$$\left(2\sqrt{\frac{3}{2}} - 2\right) \text{ square units.}$$

$$\text{Area} = \int_0^1 x^2 dx + \int_1^{\sqrt{\frac{3}{2}}} (3 - 2x^2) dx$$

$$= \frac{1}{3} [x^3]_0^1 + \left[3x - \frac{2}{3} x^3 \right]_1^{\sqrt{\frac{3}{2}}}$$

$$= \frac{1}{3} (1 - 0) + \left(3\sqrt{\frac{3}{2}} - \frac{2}{3} \times \frac{3}{2} \sqrt{\frac{3}{2}} - 3 + \frac{2}{3} \right)$$

$$= \frac{1}{3} + 2\sqrt{\frac{3}{2}} - \frac{7}{3}$$

$$= \frac{1}{3} - \frac{7}{3} + 2\sqrt{\frac{3}{2}}$$

$$= -2 + 2\sqrt{\frac{3}{2}}$$

$$= 2\sqrt{\frac{3}{2}} - 2 \text{ sq. units.}$$

End of Question 33

Please turn over

Question 34 (6 marks)

John borrowed an amount $\$P$ at a rate of 3.6% per annum compounded monthly on a reducing balance. At the end of each month, he repays an amount of $\$M$.

- (a) Show that the amount owing at the end of the second month is

1

$$A_2 = P(1.003)^2 - M(1.003) - M.$$

$$\begin{aligned} A_1 &= P(1 + 0.003) - M && \text{monthly rate of interest} \\ &= P(1.003) - M && = 0.003 \end{aligned}$$

$$\begin{aligned} A_2 &= A_1(1.003) - M \\ &= [P(1.003) - M](1.003) - M \\ &= P(1.003)^2 - M(1.003) - M \end{aligned}$$

- (b) It is now given that the amount borrowed is $\$50\,000$ and his monthly repayment is $\$350$. Show that the amount owing after one year is approximately $\$47\,560$.

2

$$\begin{aligned} A_3 &= A_2(1.003) - M \\ &= [P(1.003)^2 - M(1.003) - M](1.003) - M \\ &= P(1.003)^3 - M(1 + 1.003 + 1.003^2) \end{aligned}$$

$$\vdots$$

$$A_n = P(1.003)^n - M(1 + 1.003 + 1.003^2 + \dots + 1.003^{n-1})$$

$$\begin{aligned} \therefore A_{12} &= 50000(1.003)^{12} - 350 \underbrace{(1 + 1.003 + 1.003^2 + \dots + 1.003^{11})}_{\text{G.P. } a=1, r=1.003, n=12} \\ &= 50000(1.003)^{12} - 350 \times \frac{(1.003^{12} - 1)}{1.003 - 1} \end{aligned}$$

$$\begin{aligned} \therefore A_{12} &= 51829.99901 - 4269.997 \\ &= 47560.00131 \\ &\approx \$47560 \end{aligned}$$

(c) Find how many months it will take for John to fully repay the loan.

3

$$A_n = 0$$

$$0 = 50000 (1.003)^n - \frac{350 (1.003^n - 1)}{0.003}$$

$$= 50000 (1.003)^n - \frac{350000}{3} \times 1.003^n + \frac{350000}{3}$$

$$= 1.003^n \left[-\frac{200000}{3} \right] + \frac{350000}{3}$$

$$1.003^n \times \frac{200000}{3} = \frac{350000}{3}$$

$$1.003^n = \frac{350000}{200000}$$

$$n \log 1.003 = \log \left(\frac{35}{20} \right)$$

$$n = \frac{\log(7/4)}{\log 1.003}$$

$$= 186.818...$$

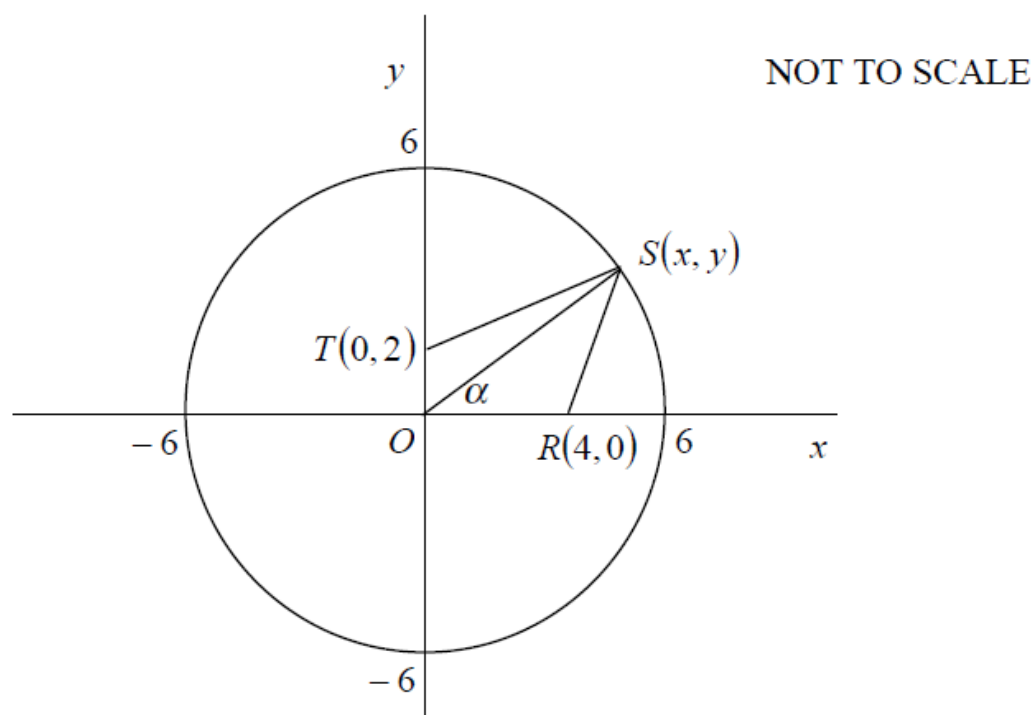
$$\approx 187 \text{ months}$$

$$15 \text{ yrs. } 7 \text{ months}$$

It will take 15 yrs and 7 months

to pay off the loan.

Question 35 (8 marks)



The diagram above shows the circle $x^2 + y^2 = 36$. The point $S(x, y)$ lies on the circle in the first quadrant. $R(4, 0)$ lies on the x -axis and $T(0, 2)$ lies on the y -axis and O is the origin. The size of $\angle SOR = \alpha$ where $0 < \alpha < \frac{\pi}{2}$.

- (a) Show that the area of $\triangle SOR$ is $12 \sin \alpha$.

1

$$\begin{aligned} \text{Area of } \triangle SOR &= \frac{1}{2} OS \times OR \times \sin \alpha \\ &= \frac{1}{2} \times \underset{\substack{\uparrow \\ \text{radius}}}{6} \times 4 \times \sin \alpha = 12 \sin \alpha \end{aligned}$$

- (b) Hence, show that the area of the quadrilateral $ORST$ is given by:
 $A = 6 \cos \alpha (2 \tan \alpha + 1)$

2

$$\begin{aligned} &\text{Area of } \triangle TOS \\ &= \frac{1}{2} \times 6 \times 2 \times \sin(90^\circ - \alpha) \\ &= 6 \cos \alpha \\ \therefore \text{Area of quad } ORST \\ A &= 12 \sin \alpha + 6 \cos \alpha \\ &= 6 \cos \alpha \left(2 \frac{\sin \alpha}{\cos \alpha} + 1 \right) \\ &= 6 \cos \alpha (2 \tan \alpha + 1) \end{aligned}$$

- (c) Find the value of $\tan \alpha$ for which the area A is a maximum.

3

$$A = 6 \cos \alpha (2 \tan \alpha + 1)$$

$$= 12 \sin \alpha + 6 \cos \alpha$$

$$\frac{dA}{d\alpha} = 12 \cos \alpha - 6 \sin \alpha$$

Maximum area may occur when $\frac{dA}{d\alpha} = 0$

$$12 \cos \alpha - 6 \sin \alpha = 0$$

$$12 \cos \alpha = 6 \sin \alpha$$

$$2 = \tan \alpha$$

$$2 = \tan^{-1} 2$$

$$\frac{d^2 A}{d\alpha^2} = -12 \sin \alpha - 6 \cos \alpha$$

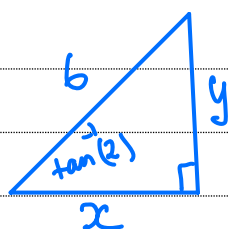
$$= -6 \sin(\tan^{-1} 2) - 6 \cos(\tan^{-1} 2)$$

$$= -9.456 < 0 \therefore$$

$\tan \alpha = 2$ gives maximum area.

- (d) Hence, or otherwise, when the area is a maximum, find the coordinates of point S in exact form.

2



$$\tan \alpha = 2$$

$$\frac{y}{x} = 2$$

$$y = 2x$$

$$\text{Also } x^2 + y^2 = 36$$

$$\therefore x^2 + 4x^2 = 36$$

$$5x^2 = 36 \quad x > 0$$

$$x = \sqrt{\frac{36}{5}} = \frac{6}{\sqrt{5}} = \frac{6\sqrt{5}}{5}$$

$$\text{and } y = \frac{12\sqrt{5}}{5} \quad S\left(\frac{6\sqrt{5}}{5}, \frac{12\sqrt{5}}{5}\right)$$

End of Paper

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NESA Number

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample: $2 + 4 =$ (A) 2 (B) 6 (C) 8 (D) 9
 A ☐ B ☒ C ☐ D ☐

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ☒ B ☒ C ☐ D ☐

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word correct and drawing an arrow as follows.

A ☒ B ☒ C ☐ D ☐
 correct
 ↖

1. A ☐ B ☒ C ☐ D ☐
2. A ☐ B ☐ C ☐ D ☒
3. A ☐ B ☒ C ☐ D ☐
4. A ☐ B ☐ C ☒ D ☐
5. A ☐ B ☒ C ☐ D ☐
6. A ☐ B ☐ C ☐ D ☒
7. A ☐ B ☐ C ☒ D ☐
8. A ☐ B ☐ C ☒ D ☐
9. A ☒ B ☐ C ☐ D ☐
10. A ☒ B ☐ C ☐ D ☐