



**2023** TRIAL EXAMINATION

# Mathematics Advanced

---

## General Instructions

- Reading time – 10 minutes
- Working time – 3 hours
- Write using blue or black pen
- Calculators approved by NESA may be used
- A reference sheet is provided at the back of this paper
- For questions in Section II, show relevant mathematical reasoning and/or calculations
- Extra space has been provided at the back of the paper to provide any further working for questions if required. Ensure you clearly label within the question you attempt this, and in the extra space.

---

## Total Marks: 100

### Section I – 10 marks (pages 2–6)

- Attempt Questions 1–10
- Allow about 15 minutes for this section

### Section II – 90 marks (pages 7–32)

- Attempt Questions 11–31
- Allow about 2 hour and 45 minutes for this section

## Multiple Choice

10 marks

Attempt Questions 1–10

Allow about 15 minutes for this section

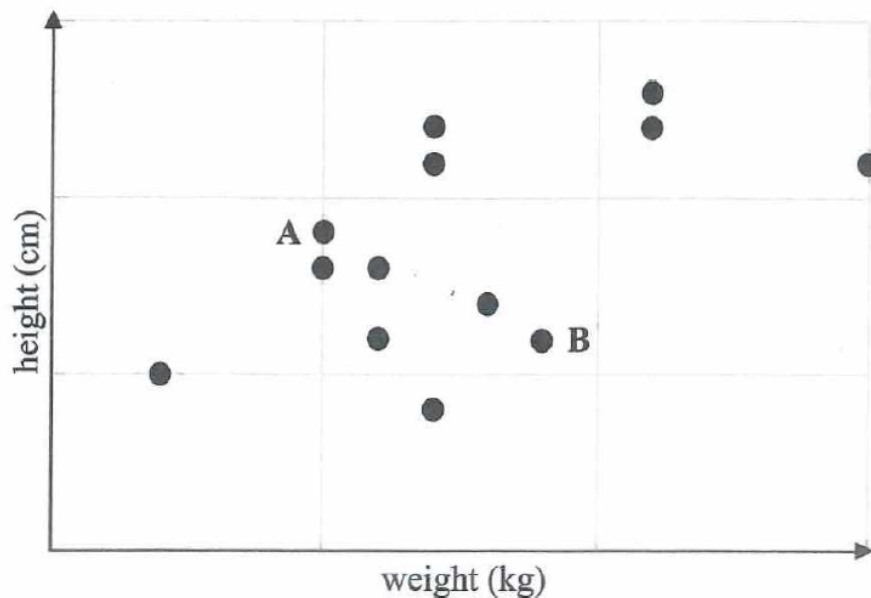
Use the multiple-choice answer sheet for Questions 1–10.

---

1 Which of the following sets of ordered pairs represent a one-to-many relationship?

- A.  $(1, 1), (1, 2), (2, 3), (3, 4)$
- B.  $(1, 1), (2, 2), (3, 2), (4, 3)$
- C.  $(1, 1), (2, 2), (3, 3), (4, 4)$
- D.  $(1, 1), (1, 2), (2, 3), (3, 3)$

2 The height and weight of a group of students is shown on the scatterplot below.



Which of the following statements is true?

- A. Student A is taller and weighs more than Student B.
- B. Student A is taller and weighs less than Student B.
- C. Student A is shorter and weighs more than Student B.
- D. Student A is shorter and weighs less than Student B.

- 3 In a mathematics examination, 25% of Students scored less than 40 marks, 75% of students scored less than 86 marks and the highest mark was 96.

Given that the distribution is symmetrical with no outliers, what is the lowest mark and the median of this data set?

- A. Lowest mark = 30, Median = 63
- B. Lowest mark = 30, Median = 68
- C. Lowest mark = 36, Median = 63
- D. Lowest mark = 36, Median = 68

- 4 In an arithmetic series, the sum of the first three terms is 21 and the sum of the first four terms is 36.

What is the sum of the first five terms of this series?

- A. 15
- B. 51
- C. 55
- D. 57

- 5 Let  $f(x) = \sqrt{x}$  and  $g(x) = \frac{1}{x-1}$ .

What is the domain and range for both  $f(g(x))$  and  $g(f(x))$ ?

- A. Domain:  $[-1, 1]$ , Range  $(-\infty, \infty)$
- B. Domain:  $(0, \infty)$ , Range  $(-\infty, 0) \cup (0, \infty)$
- C. Domain:  $[0, \infty]$ , Range  $(0, \infty)$
- D. Domain:  $(1, \infty)$ , Range  $(0, \infty)$

- 6 The limiting sum of a geometric series is  $\frac{10}{9}$  and the sum of its first three terms is  $\frac{26}{25}$ .

What is the common ratio of this series?

- A. 1.39
- B. 1.25
- C. 0.40
- D. 0.25

- 7 In a bag of  $n$  coloured discs, exactly 7 are red.

Two discs are selected at random, without replacement.

The probability that both discs are red is  $\frac{21}{55}$ .

Which quadratic equation could be used to find the value of  $n$ ?

- A.  $n^2 + n + 110 = 0$
- B.  $n^2 - n - 110 = 0$
- C.  $n^2 + n - 110 = 0$
- D.  $n^2 - n + 110 = 0$

- 8 A particle is moving in a straight line such that its displacement  $x$  metres at time  $t$  seconds from the origin is given by the equations  $x = k \cos(at + b)$ , where  $a$ ,  $b$  and  $k$  are constants.

The particle is stationary when  $t = \frac{5\pi}{12}, \frac{11\pi}{12}, \frac{17\pi}{12}, \dots$

The initial acceleration of the particle is  $6\sqrt{3} \text{ ms}^{-2}$ .

What could be the values of  $a, b$  and  $k$ ?

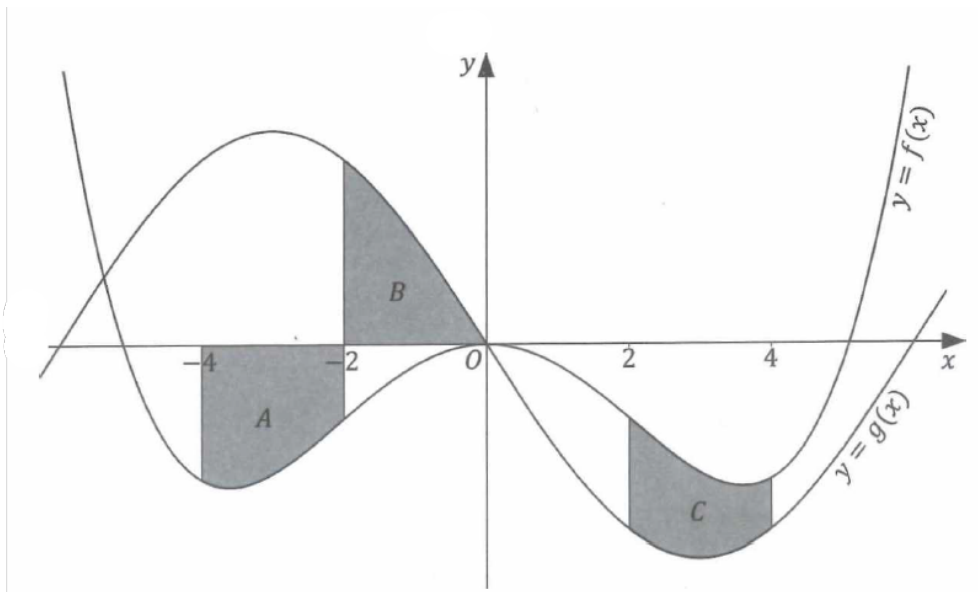
- A.  $a = 4, b = -\frac{5\pi}{6}, k = 3$
- B.  $a = 2, b = -\frac{5\pi}{6}, k = -3$
- C.  $a = 2, b = \frac{\pi}{6}, k = 3$
- D.  $a = 2, b = \frac{\pi}{6}, k = -3$

- 9 Consider the equation  $\left(\sin\theta - \frac{\sqrt{2}}{2}\right)\left(\tan\theta - \frac{\sqrt{3}}{3}\right) = 0$

How many solutions are there in the interval  $0 \leq \theta \leq \pi$ ?

- A. 4
- B. 3
- C. 2
- D. 1

- 10 The graph of an even function,  $f(x)$ , and an odd function,  $g(x)$  are shown below.



Let  $A = \int_{-2}^{-4} f(x) dx$ ,  $B = \int_{-2}^0 g(x) dx$  and  $C = \int_2^4 f(x) - g(x) dx$ .

Which of the following expression equals  $\int_0^4 g(x) dx$ ?

- A.  $A + B + C$
- B.  $A - B + C$
- C.  $-A + B - C$
- D.  $-A - B - C$

## Section II

90 marks

Attempt Questions 11-32

Allow about 2 hours and 45 minutes for this section

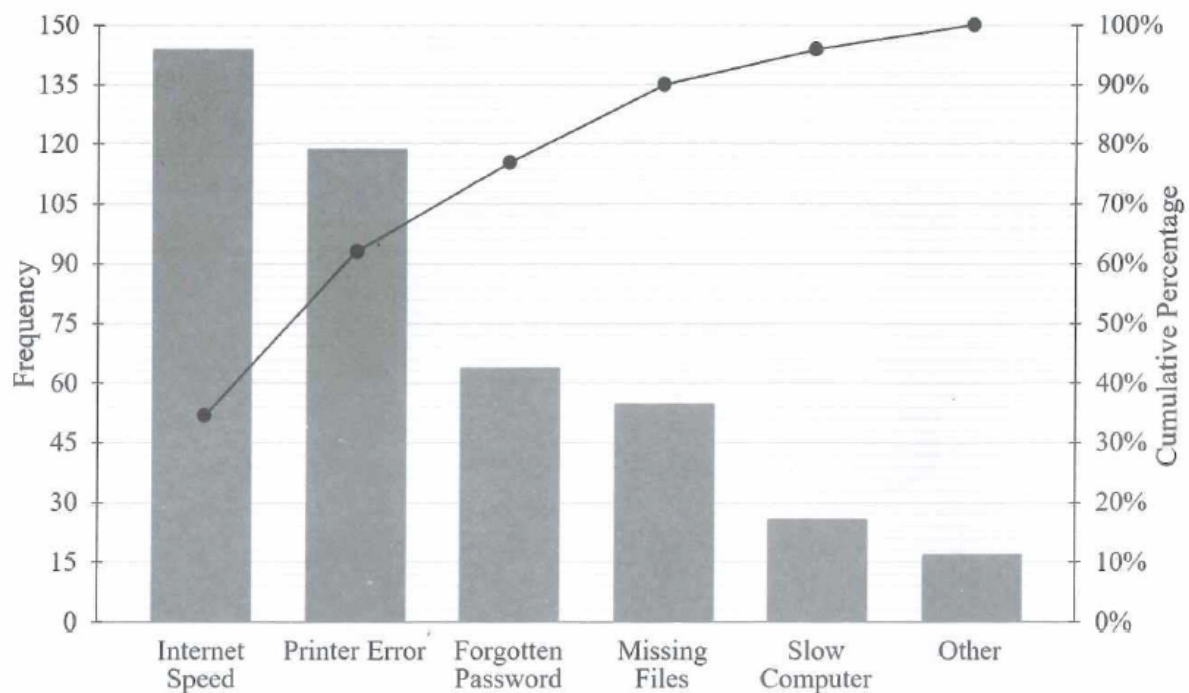
---

### Question 11 (2 marks)

A company records the type of issues for IT support over a period of time.

2

The results are shown in the Pareto chart below.



What percentage of issues were due to "Printer Error" or "Forgotten Password"

.....

.....

.....

.....

**Question 12** (2 marks)

Find the equation of the normal to the curve  $y = x^3 - 2x$  at the point  $(-1, 1)$ . **2**

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

**Question 13** (2 marks)

The time it takes to complete a task varies inversely with the number of people assigned. **2**

It takes 5 people to complete a task in 4 hours.

Find the amount of time it would take 8 people to complete the same task.

.....

.....

.....

.....

.....

.....

.....

.....

.....

**Question 14** (3 marks)

(a) Find  $\frac{dy}{dx}$ , given  $y = (\ln x)^2$ . **1**

.....

.....

(b) Hence or otherwise, find  $\int_1^e \frac{\ln(x^2)}{2x}$  **2**

.....

.....

.....

.....

.....

.....

**Question 15** (3 marks)

The graph of the primitive function of  $f(x) = \tan^2 x$  crosses the x axis at  $x = \frac{\pi}{3}$ . **3**

Find the primitive function, leaving all values in exact form.

.....

.....

.....

.....

.....

.....

.....

.....

**Question 16** (3 marks)

Consider the function  $f(x) = -\sin\left(\frac{1}{2}(x - \pi)\right)$ .

- (a) Show that  $f(x)$  is even. **1**

.....

.....

.....

.....

.....

- (b) Hence, or otherwise, find the area bounded by the curve  $y = f(x)$  and the x-axis in the interval  $[-\pi, \pi]$ . **2**

.....

.....

.....

.....

.....

.....

.....

.....

**Question 17** (5 marks)

Jason buys a ticket in a lottery, where the first prize is \$10,000.

Tickets cost \$20 each with the probability of winning a prize shown in the discrete distribution table below.

|            |      |     |      |       |       |        |
|------------|------|-----|------|-------|-------|--------|
| Prize (\$) | 0    | 20  | 100  | 500   | 5000  | 10 000 |
| $P(X = x)$ | 0.75 | $m$ | 0.03 | 0.002 | 0.001 | 0.0001 |

(a) Find the value of  $m$ . **1**

.....

.....

.....

(b) Show that Jason is expected to lose \$5.66 every time he plays this lottery. **2**

.....

.....

.....

.....

.....

.....

.....

(c) Find the variance of the discrete distribution. **2**

.....

.....

.....

.....

.....

.....

.....

**Question 18** (7 marks)

Ben is cooking a pizza. The temperature of the pizza oven is modelled by the equation

$$T = 175 - 150 \times (0.9)^{0.2t},$$

where  $T$  is the temperature of the oven ( $^{\circ}\text{C}$ ) at a time  $t$  minutes since the oven was switched on.

- (a) What was the initial temperature of the oven? **1**

.....  
.....

- (b) Calculate the average rate at which the temperature changes per minute from  $t = 25$  to  $t = 75$ , correct to two decimal places. **3**

.....  
.....  
.....  
.....  
.....

- (c) Ben can place the pizza in the oven when the instantaneous rate of change of temperature is less than or equal to  $1^{\circ}\text{C}$  per minute. Given that Ben turned the oven on at 10:30am, find the earliest time that Ben can place the pizza in the oven, correct to the nearest minute. **3**

.....  
.....  
.....  
.....  
.....  
.....  
.....  
.....  
.....

**Question 19** (3 marks)

A group of students were surveyed on the type of pets they own. The results are summarised in the following incomplete two-way table.

|        | Cat | No Cat |
|--------|-----|--------|
| Dog    | A   | 32     |
| No Dog | 45  | 35     |

Given that a student owns a dog, the probability that they also own a cat is 60%.

Find the probability that a student chosen at random owns both a cat and a dog.

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

**Question 20** (2 marks)

The relative level of a noise compared to a reference level is calculated by the formula

**2**

$$R = 20 \log_{10} \left( \frac{N}{N_0} \right),$$

where  $R$  is the relative noise level measure in decibels (dB),  $N$  is the level of a noise and  $N_0$  is the maximum noise level permitted.

The relative noise level at a construction site when operating machinery was found to be 8 dB higher than the maximum noise level permitted.

Show that the level of noise produced by the machinery was approximately 2.5 times greater than the maximum noise level permitted.

.....

.....

.....

.....

.....

.....

.....

.....

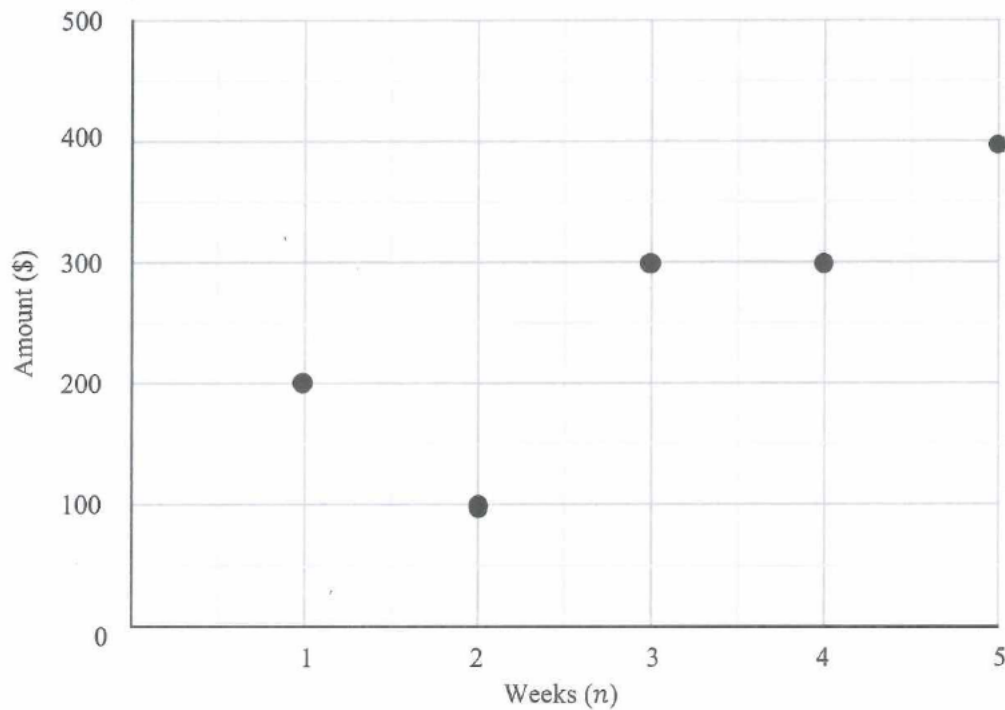
.....

.....

**Question 21** (5 marks)

Ivy has plotted the income that she has received each week for the past five weeks on the graph below.

**5**



Over the same five weeks, Ivy's expenses which were originally \$200 per week, increase by \$20 each week.

Write the equation of the least-squares regression line for Ivy's income, determine the equation of Ivy's expenses and draw both lines neatly on the graph above.

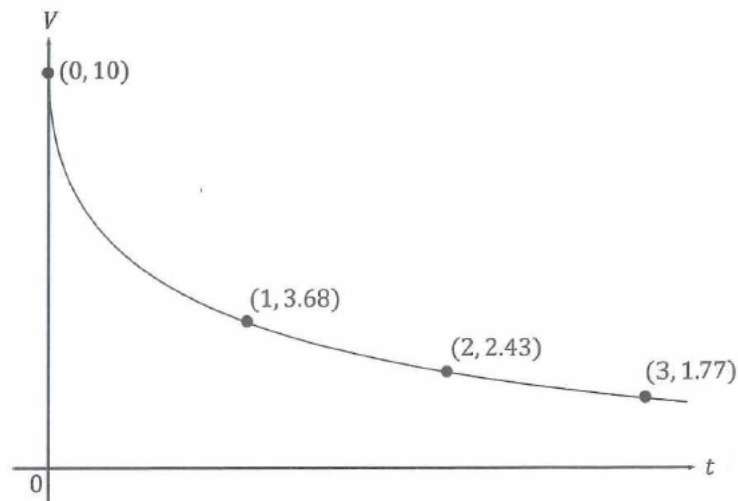
Income = .....

Expenses = .....

**Question 22** (3 marks)

A deflated weather balloon at a scientific research station is being inflated such that the rate of air flow can be modelled by the equation  $V = 10e^{-\sqrt{t}}$ , where  $V$ , is the rate of air flow at time  $t$ .

The graph of  $V = 10e^{-\sqrt{t}}$  is shown below with the flow rates at hourly intervals.



Using three applications of the Trapezoidal rule, estimate the volume of air in the balloon at time  $t = 3$ .

**2**

.....

.....

.....

.....

A safety shutoff valve is programmed to be activated before the balloon reaches its maximum volume, after which the balloon will burst.

**1**

By referring to the graph, or otherwise, explain why the Trapezoidal rule is a valid method of estimation in this context.

.....

.....

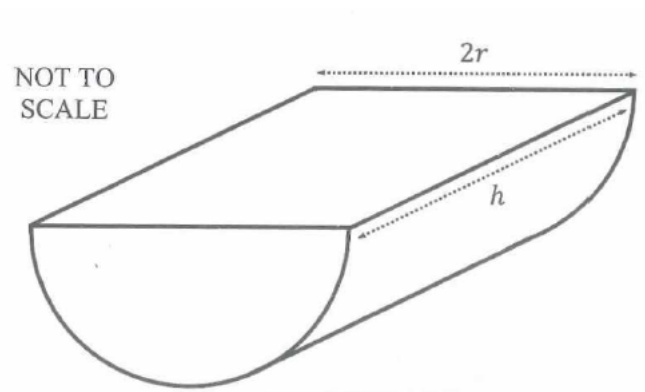
.....

.....

.....

**Question 23** (6 marks)

A closed water tank in the shape of a half-cylinder with diameter  $2r$  metres and height  $h$  metres is to be made from sheet metal as shown below.



The surface area of the tank is  $A$  square metres and the volume of the tank is  $V$  cubic metres.

(a) Show  $A = \pi r^2 + \frac{2V}{\pi} \left( \frac{2 + \pi}{r} \right)$ , given that  $V = \pi r^2 h$ .

2

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(b) Show that the amount of sheet metal used is minimised when  $r = \sqrt[3]{\frac{V(2+\pi)}{\pi^2}}$

3

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(c) Hence find the height of the tank, given that the volume of the tank is 10 cubic metres.

1

.....

.....

.....

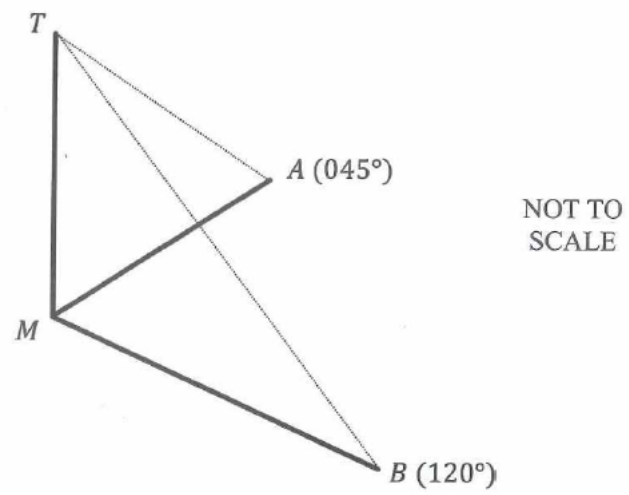
.....

.....

**Question 24** (5 marks)

Michael installs a vertical radio mast on level ground at  $M$ . Two pegs are installed on the ground at  $A$  and  $B$  on bearings of  $045^\circ$  and  $120^\circ$  respectively from  $M$ .

The distance from  $A$  to  $M$  is 100 m and the area of the triangle  $AMB$  is  $10\,432\text{ m}^2$ .



- (a) Find the distance between the base of the mast,  $M$ , and the peg at  $B$ . 2

.....

.....

.....

.....

.....

.....

.....

.....

.....

- (b) Supporting wires are connected from each peg to the top of the mast at  $T$ .

3

By considering triangle  $AMT$  and triangle  $BMT$ , find the height of the mast, given that the total length of the supporting wires is 380 m.

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

**Question 25** (4 marks)

Consider the function  $f(x) = (x + 1)^2$ .

**4**

The following transformations were applied to the graph of  $y = f(x)$  in the order shown below.

- Horizontal translation to the right by 4 units
- Horizontal dilation by a factor of  $\frac{4}{3}$
- Vertical dilation by a factor of  $\frac{1}{3}$
- Vertical translation down by 3 units
- Reflection in the  $x$ -axis

Find the equation of the transformed function and sketch its graph, showing the location of the vertex and the axis intercepts.

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

**Question 26** (7 marks)

A particle is moving along the  $x$ -axis so that its position, in metres, at time  $t$  seconds is given by

$$x = 2\ln(t^2 + 3) - t$$

- (a) Determine the times when the particle is at rest. **2**

.....

.....

.....

.....

- (b) Find the distance travelled by the particle in the first second. Leave your answer in the form  $\ln\left(\frac{a}{b}\right) - c$ , where  $a$ ,  $b$  and  $c$  are integers. **2**

.....

.....

.....

.....

- (c) Find the maximum velocity of the particle. **3**

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

**Question 27** (7 marks)

Consider the function  $f(x) = 3x\left(1 - \frac{x}{4}\right)^3 + 1$ .

- (a) By showing that  $f'(x) = 3x\left(1 - \frac{x}{4}\right)^2(1 - x)$ , find any stationary points and determine their nature. **3**

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

- (b) Explain why the graph of  $y = f(x)$  must cross the  $x$ -axis at least once between  $x = 5$  and  $x = 6$ . **1**

.....

.....

.....

.....

.....

- (c) Sketch the graph of  $y = f(x)$  in the interval  $[0, 6]$ , showing the stationary points, the  $y$ -intercept and the approximate location of the  $x$ -intercept. **3**

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

**Question 28** (8 marks)

A continuous random variable  $X$  has a probability density function  $f(x)$  given by  $f(x) = e^{-\frac{1}{2}x}$  for  $0 \leq x \leq \log_e 4$  and  $f(x) = 0$  otherwise.

- (a) Verify that  $f(x)$  is a probability density function. **3**

.....

.....

.....

.....

.....

.....

.....

.....

.....

- (b) Find the cumulative distribution function  $F(x)$  of  $X$  for  $-\infty < x < \infty$ . **2**

.....

.....

.....

.....

.....

- (c) Find in simplest exact form the median of  $X$ . **2**

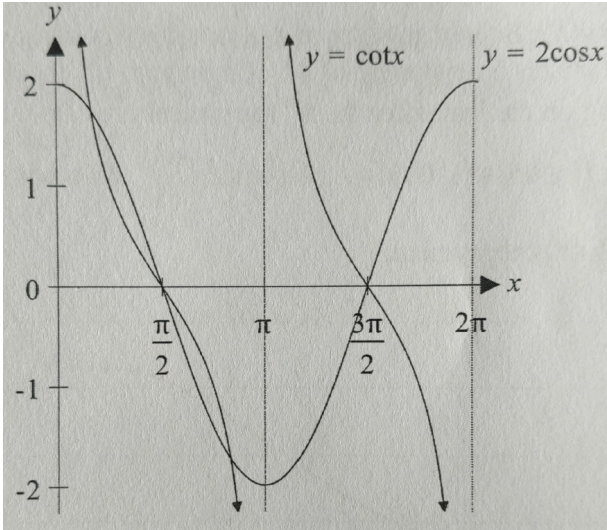
.....

.....

.....

.....

**Question 29** (7 marks)



The diagram shows the graphs of the functions  $y = 2\cos(x)$  and  $y = \cot(x)$  for  $0 \leq x \leq 2\pi$ .

- (a) Write down the periods of each of the functions  $y = 2\cos(x)$  and  $y = \cot(x)$ .

1

.....

.....

- (b) Show that the  $x$ -coordinates of the points of intersection of the two curves are the solutions of the equation  $\cos(x)(2\sin(x) - 1) = 0$  for  $0 \leq x \leq 2\pi$  then solve this equation.

3

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

- (c) Find in simplest exact form the total area of the bounded region enclosed by the two curves for  $0 < x < \pi$ .

3

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

**Question 30** (5 marks)

The time taken,  $t$  minutes, for a machine to produce car parts can be modelled by a continuous random variable with probability density function given by

$$f(t) = \begin{cases} k(t-1)^2, & 1 \leq x \leq 2 \\ k\left(2 - \frac{t}{2}\right), & 2 \leq x \leq 4 \\ 0 & \text{otherwise} \end{cases}$$

- (a) Show that the value of  $k$  is  $\frac{3}{4}$ .

**2**

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(b) Let  $F(t)$  be the cumulative distribution function of  $f(t)$ , where  $F(2.16) = 0.3652$ .

2

Find the least amount of time taken to produce 88% of the parts.

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

**Question 31** (3 marks)

A heater converts electrical power, measured in watts, into heat in such a way that the heat produced is given by the function  $h(x) = (f(x))^2$ .

The heater is plugged into a faulty socket which outputs  $x = g(t)$  watts at time  $t$  seconds.

The power output at time  $t = 1$  is 3 watts and is increasing at a rate of 4 watts per second.

By considering  $h'(x)$  and using the fact that  $f'(x) = f'(g(t)) \times g'(t)$ , find the rate of change of the heat produced at time  $t = 1$ , given that  $f(3) = 5$  and  $f'(3) = 2$ .

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

**End of paper**

[illegible]

BLANK PAGE