

Teacher: Blair / Mandile / Underwood (Circle)

Student Number:

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or Student Name: _____



Danebank
An Anglican School for Girls

Mathematics Advanced Year 12 Examination Block 2023 Task 4 - Written Examination

General Instructions	
Write using black or blue pen Graphs can be drawn in pencil No liquid paper or correction tape are to be used.	
Reading Time	10 minutes
Working time	3 hours

Structure of the Paper		
Section 1	/10 Marks	Allow about 15 minutes for this section
Section 2	/90 Marks	Allow about 2 hours 45 minutes for this section
Total	/100 Marks	Weighting: 30 %

Special Instructions
NESA approved Calculators may be used. A reference sheet is provided. A separate multiple-choice answer sheet is provided. Attempt all questions. This examination must not be removed from the Examination Room. In Question 11-37 show all relevant mathematical reasoning and/or calculations. Answer the questions in the space provided in the examination booklet. Additional working space is provided at the back of the paper. Clearly indicate any question number or part attempted on these pages.

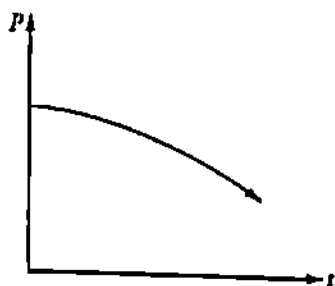
SECTION 1

10 marks

Attempt Questions 1-10

Answer these questions on the separate multiple choice answer sheet.

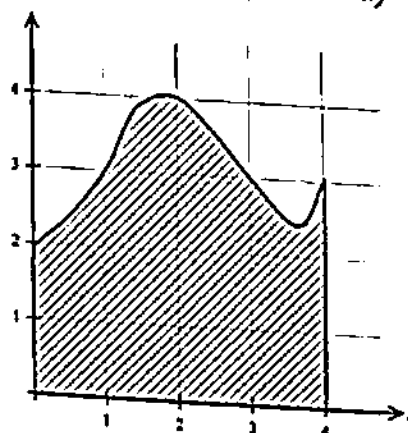
1. The population (P) of the town of Marvel, is shown over time (t) by the graph below:



Which statement best describes the population of Marvel?

- (A) The population is increasing at an increasing rate.
- (B) The population is increasing at a decreasing rate.
- (C) The population is decreasing at an increasing rate.
- (D) The population is decreasing at a decreasing rate.

2. Using two applications of the Trapezoidal rule, find an estimate for the area of the irregular shape shown below. (Each square on the grid is one square unit.)



- (A) 12 square units
- (B) 13 square units
- (C) 14 square units
- (D) 16 square units

3. The table below shows the future value of a \$1 annuity.

Future value of \$1							
Period	2%	2.5%	3%	3.5%	4%	4.5%	5%
1	1.000	1.000	1.000	1.00	1.000	1.000	1.000
2	2.020	2.025	2.030	2.035	2.040	2.045	2.050
3	3.060	3.076	3.091	3.106	3.122	3.138	3.153
4	4.122	4.153	4.184	4.215	4.246	4.277	4.310
5	5.204	5.256	5.309	5.362	5.416	5.470	5.526
6	6.308	6.388	6.468	6.55	6.633	6.716	6.802

Oscar deposits \$3 600 into an investment account every 6 months over a period of 3 years. Interest is added to Oscar's account at the rate of 5% p.a., compounded biannually. How much interest will Oscar earn on his investment?

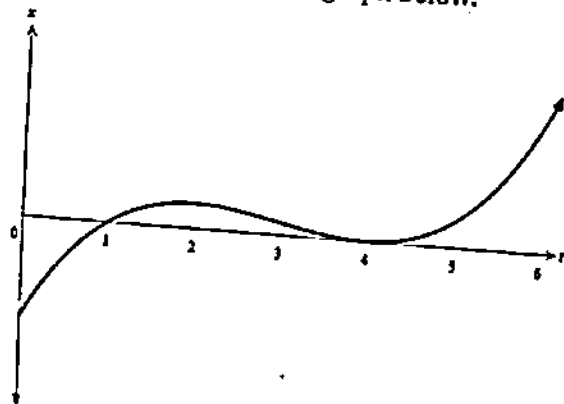
- (A) \$550.80
(B) \$1396.80
(C) \$12 196.80
(D) \$22 996.80
4. What is $\int e + e^{2x} dx$?
- (A) $ex + 2e^{2x} + C$
(B) $ex + \frac{1}{2}e^{2x} + C$
(C) $\frac{e^2}{2} + 2e^{2x} + C$
(D) $\frac{e^2}{2} + \frac{1}{2}e^{2x} + C$

5. The curve $y = \ln x$ is translated to the left by π units and then dilated horizontally by a scale factor of 3.

What is the equation which describes this new curve?

- (A) $y = \ln\left(\frac{x}{3} + \pi\right)$
(B) $y = \ln\left(\frac{x}{3} - \pi\right)$
(C) $y = 3 \ln(x + \pi)$
(D) $y = \ln\left(\frac{x + \pi}{3}\right)$

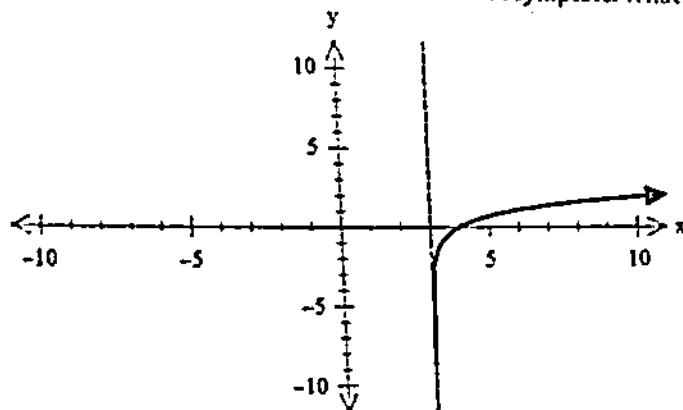
6. The displacement, x metres, from the origin of a particle moving in a straight line at any time, t seconds, is shown in the graph below.



When was the particle at rest?

- (A) $t = 0$
(B) $t = 1$ and $t = 4$
(C) $t = 2$ and $t = 4$
(D) $t = 1, t = 2$ and $t = 4$

7. The graph of $f(x)$ is shown below, $x = 3$ is an asymptote. What is the domain for $-f(-x)$?



- (A) $D: x > 3$
(B) $D: x < 3$
(C) $D: x > -3$
(D) $D: x < -3$
8. Differentiate $x \sin\left(\frac{1}{x}\right)$

- (A) $\sin\left(\frac{1}{x}\right) - \frac{1}{x} \cos\left(\frac{1}{x}\right)$
(B) $\cos\left(\frac{1}{x}\right) - \frac{1}{x} \sin\left(\frac{1}{x}\right)$
(C) $\sin\left(\frac{1}{x}\right) + \frac{1}{x} \cos\left(\frac{1}{x}\right)$
(D) $x \cos\left(\frac{1}{x}\right)$

9. $\int \left(\frac{1}{5}\right)^x dx =$

(A) $\frac{1}{\ln 5 \cdot 5^x} + c$

(B) $\frac{-1}{\ln 5 \cdot 5^x} + c$

(C) $\frac{-1}{\ln \frac{1}{5} \cdot \left(\frac{1}{5}\right)^x} + c$

(D) $\frac{1}{5 \ln x} + c$

10. Find the range of $y = \frac{1}{2} - \frac{3}{2} \sin \left(2x + \frac{\pi}{3} \right)$

(A) $\left[-\frac{3}{2}, \frac{3}{2} \right]$

(B) $[-1, 2]$

(C) $[-1, 1]$

(D) $\left[-\frac{1}{2}, \frac{1}{2} \right]$

END OF SECTION 1

SECTION 2

Attempt all questions – Allow about 2 hours 45 minutes for this section.
Show all necessary working in the space provided.
Additional space may be found at the end of the paper.

Question 11 (3 marks)

Given that $f(x) = x^2 - x$ and $h(x) = 2x + 1$, find:

a) $f(-2)$

1

b) $f(h(x))$

1

c) x where $h(x) = 0$

1

Question 12 (3 marks)

Find the equation of the normal to the curve $y = 2(5x - 4)^4$ at $x = 1$.
Express your answer in general form $ax + by + c = 0$.

3

Question 13 (2 marks)

An investment is modelled by the recurrence relation: $V_{n+1} = V_n(1 + r) + D$

Where V_{n+1} is the value of the investment after $(n + 1)$ time periods

V_n is the value of the investment after n time periods

r is the rate of interest

D is the deposit made per compounding period.

George initially invests \$1 200, earning compound interest annually at a rate of 4% p.a. and he also makes a deposit of \$150 at the end of every year. How many years will it take for the George's investment to accumulate to \$1 818 ?

Question 14 (2 marks)

For what values of x , in the interval $0 \leq x \leq \frac{\pi}{2}$, does the line $y = 1$ intersect the graph of $y = 2\sin 3x$?

2

Question 15 (2 marks)

2

Evaluate $\int_{-3}^0 \sqrt{3x+9} dx$

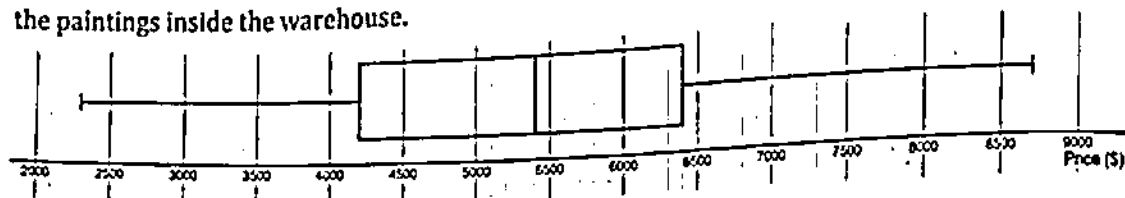
Question 16 (3 marks)

3

Solve the equation $2 \ln(x+2) - \ln x = \ln(2x+1)$ where $x > 0$

Question 17 (5 marks)

There are 30 paintings in a warehouse. The box-and-whisker diagram below shows the prices of the paintings inside the warehouse.



- a) Find the Interquartile range.

1

- b) Show there are no outliers.

2

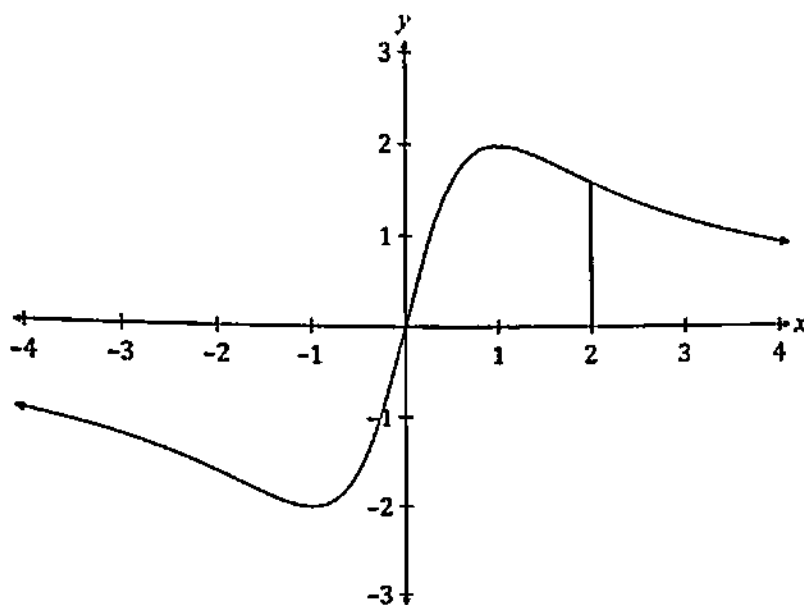
- c) It is given that the mean is \$5 500 dollars. Four paintings of respective prices \$3 400, \$3 500, \$5 700 and \$6 800 are now donated to an art gallery. Find the mean and the median of the prices of the remaining paintings in the warehouse.

2

Question 18 (3 marks)

The diagram below shows the graph of $y = \frac{4x}{x^2 + 1}$.

3



The region enclosed by the graph, the x axis and the line $x = 2$ is shaded. Calculate the exact value of the area of the shaded region.

[illegible]

Question 19 (4 marks)

The table below shows the present value of a \$1 annuity.

Number of periods	Interest rate per period as a decimal				
	0.001	0.00125	0.0015	0.00175	0.002
300	259.0707	250.0398	241.4379	233.2418	226.3477
330	280.9577	270.2690	260.1353	250.5239	240.7211
360	302.1982	289.7541	278.0106	266.9228	258.9154
390	325.2961	309.6290	297.0981	283.6291	261.9432

- a) What would be the present value of a \$2 000 per month annuity at 1.2% per annum for 25 years, with interest compounding monthly?

2

- b) Thomas borrowed \$600 000 to purchase a home, with interest charged at 1.5% per annum compounding monthly. He agrees to repay the loan by making equal monthly payments over a 30-year period.
What is the monthly payment? Answer correct to the nearest cent.

2



Question 20 (6 marks)

Differentiate with respect to x:

a) $\frac{3x+1}{x+4}$

2

Handwritten student work for part a, showing the quotient rule formula and the resulting derivative expression.

b) $\ln\sqrt{4x^2-1}$

2

Handwritten student work for part b, showing the chain rule and the final derivative.

c) $\sec^2 x$

2

Handwritten student work for part c, showing the derivative of sec^2 x.

Question 21 (2 marks)

Let $f(x)$ be a function such that $\int_0^3 f(x) dx = 8$.

a) Find the value of $\int_0^3 2f(x) dx$

1

b) Find the values of c and d if

$$\int_c^d f(x-2) dx = 8$$

1

Question 22 (2 marks)

The positive numbers $3, k, \frac{3}{2}$, form the start of a geometric sequence.

a) Show that $k = \frac{3}{\sqrt{2}}$

1

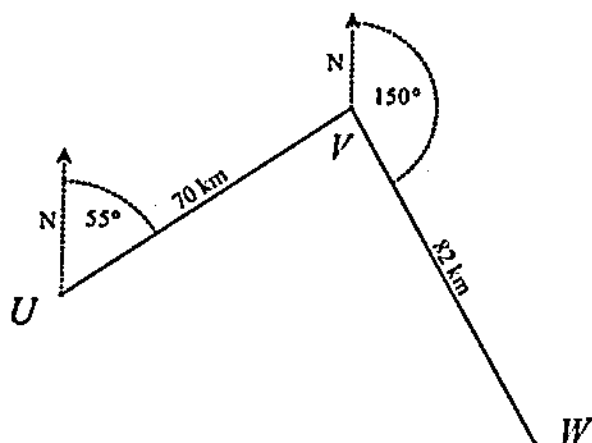
b) Find the 7th term of this sequence

1

Question 23 (4 marks)

A helicopter leaves Utopia and flies 70 km on a bearing of 055° to Vanna Beach. It then flies 82 km on a bearing of 150° to Weston.

The diagram below illustrates the journey.



- a) The helicopter then plans to fly back to Utopia. Calculate, to the nearest km, the distance that it will fly from Weston to Utopia. 2

- b) Determine the bearing from Weston on which it should fly to return to Utopia. Give your answer to the nearest degree. 2

Question 24 (2 marks)

Daniel has a bag that contains 5 red, and 8 black balls. Two balls are drawn randomly from the bag.

- a) Find the probability that the first ball drawn is red.

1

- b) Find the probability that both balls drawn are the same colour.

1

Question 25 (3 marks)

The gradient of a curve is given by $y' = 1 - 6 \sin 3x$

3

The curve passes through the point $(0, 7)$. Find the equation of the curve?

Question 26 (1 mark)

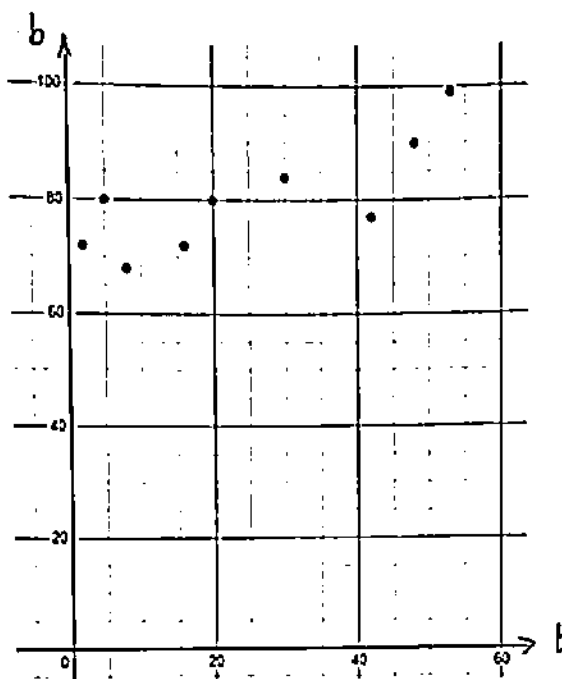
The Pearson's correlation coefficient between two variables x and y is 0.10.
What is the meaning of this correlation?

1

Question 27 (4 marks)

At various times during her walk, Sarah took her pulse and recorded the results in a table and created a scatter graph.

t (time since walk started in minutes)	2	5	8	16	20	30	42	48	53
b (heart rate in beats / minute)	72	80	68	72	80	84	77	90	99



- Draw a line of best fit by eye on this scatter graph. 1
- Find the value of r , the correlation coefficient, correct to 2 decimal places. 1
- Using the variables in the table, find the equation of the line of best fit. (round your calculations to 3 decimal places) 2

Question 28 (3 marks)

3

A rational function $f(x)$ has the following properties,

- The horizontal asymptote of its graph is $y = 0$
- The vertical asymptotes of its graph are $x = -2$ and $x = 2$
- The table below shows the first and second derivatives at various points.

	$x < -2$	$-2 < x < 0$	$x = 0$	$0 < x < 2$	$x > 2$
$f(x)$			1		
$f'(x)$	< 0	< 0	0	> 0	> 0
$f''(x)$	< 0	> 0	> 0	> 0	< 0

Sketch the graph of $y = f(x)$ in the space below, using the properties in the table above.

Question 29 (3 marks)

Let $f(x) = e^{-kx} + 3x$ where k is a positive rational number.

- a) Find, in terms of k , the x -coordinate of the stationary point of the graph of $y = f(x)$.

2

- b) State the values of k such that the x -coordinate of this stationary point is a positive number.

1

Question 30 (3 marks)

The population, P , of parrots, is decreasing at a rate proportional to P :

$$\frac{dP}{dt} = -kP$$

In August 2003 there were 3 000 parrots and by August 2013 the population had decreased to 2 750.

- a) Find the value the k (Answer correct to four decimal places).

2

- b) If the population continues to decrease at this rate what will be the expected population in August 2023? Answer to the nearest whole number.

1

Question 31 (4 marks)

The function $f(x) = \cos 3x - 1$ is defined in the interval $0 \leq x \leq \frac{2\pi}{3}$.

a) What is the amplitude and period of the function?

1

b) What is the range of the function?

1

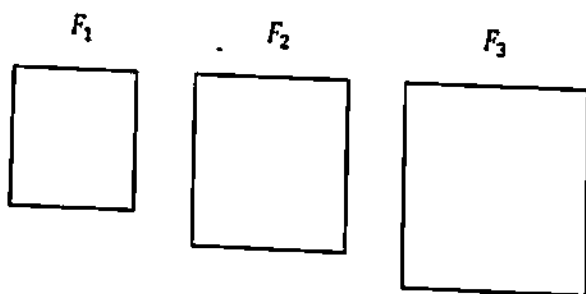
c) Sketch $f(x) = \cos 3x - 1$ in the interval $0 \leq x \leq \frac{2\pi}{3}$.

2

Question 32 (4 marks)

In the diagram below, F_1, F_2, F_3 are square frames. The perimeter of the first frame is 16cm.

The perimeter of each square frame afterwards is 4cm longer than the perimeter of the previous frame.



question continues on the next page

a) Find the perimeter of F_{12}

1

b) A thin metal wire of length 2 000cm is cut into pieces and these pieces are then bent to form the above square frames. Find the greatest number of distinct square frames that can be formed.

3

Question 33 (2 marks)

Consider the geometric series below:

2

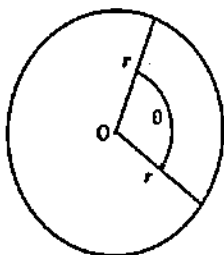
$$\left(\frac{4}{x+1}\right) + \left(\frac{4}{x+1}\right)^2 + \left(\frac{4}{x+1}\right)^3 + \dots$$

Show that the series will have a sum to infinity when $x = 4$, but not when $x = -4$

Question 34 (4 marks)

The diagram below shows a shaded sector in a circle with radius r and centre O .

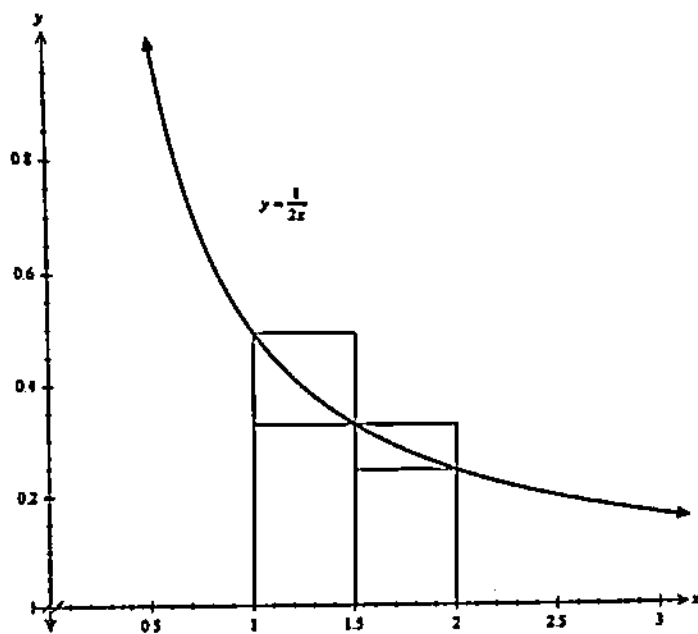
The sector subtends an angle of θ at the centre of the circle. The area of the sector is 49 cm^2 .



Show that $\theta = \frac{98}{r^2}$ and hence show that the minimum perimeter of the sector will occur when the arc length of the sector is equal to the diameter of the circle.

Question 35 (3 marks)

The graph of $f(x) = \frac{1}{2x}$ is shown below.



By considering the areas of the rectangles, show that $\frac{7}{24} < \ln \sqrt{2} < \frac{5}{12}$.

3

Question 36 (7 marks)

Ronald deposits \$2 000 at the beginning of every month for 35 years into a superannuation account, which pays an interest rate of 3.6% per annum, compounding monthly.

- a) Show that when Ronald retires, at the end of the final month, he has $A = \$1\,684\,222.89$ in his account. 2

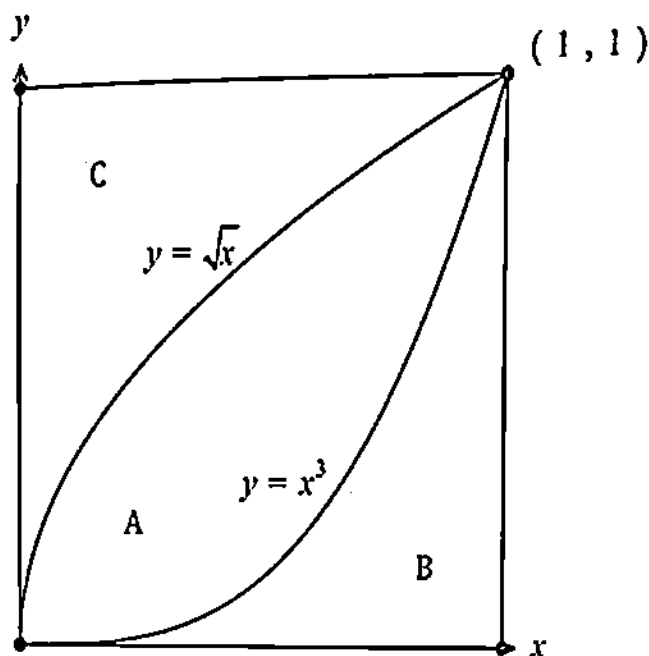
- b) After Ronald retires, he stops all deposits and now withdraws \$9 000 at the end of each month from this account. The account continues to gain interest at the same amount 3.6% pa compounded monthly. If M_n is the amount available after n withdrawals in his retirement, show that the account contains 3

$$M_n = 3000000 - (3000000 - A)r^n \quad \text{where } r = 1.003$$

c) If Ronald turns 67 the day after the final interest is paid, at what age will Ronald no longer be able to withdraw his full \$9 000. 2

Question 37 (6 marks)

The diagram below shows a unit square target for shooting on the Cartesian Plane. The target is divided into three regions, A, B and C by the curves $y = \sqrt{x}$ and $y = x^3$.



- a) Find the area of each of the three regions, A, B and C

3

b)

The scores for hitting regions A, C and B are 5, 10 and 15 points respectively.

Barry shoots randomly at the target twice and both shots hit the target.

Given Barry scored 20 points, find the probability that that both his shots hit region C.

3

END OF SECTION 2

END OF ASSESSMENT ©

Clearly Indicate the Question number or part if you answer any questions on these pages

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