Roseville COLLEGE

Student Number

2024 Trial Higher School Certificate Examination

Mathematics Advanced

General Instructions

- Reading time — 10 minutes
- Working time — 3 hours
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided at the back of this paper
- For questions in Section II, show relevant mathematical reasoning and/or calculations
- Write your Student Number at the top of this page

Total marks: 100

Section I - 10 marks (pages 2—8)

- Attempt Questions 1–10
- Allow about 15 minutes for this section

Section II - 90 marks (pages 9—36)

- Attempt Questions 11–35
- Allow about 2 hours and 45 minutes for this section

Section I

10 marks
Attempt Questions 1–10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1–10.

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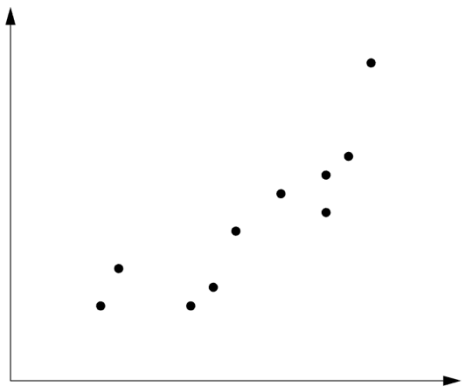
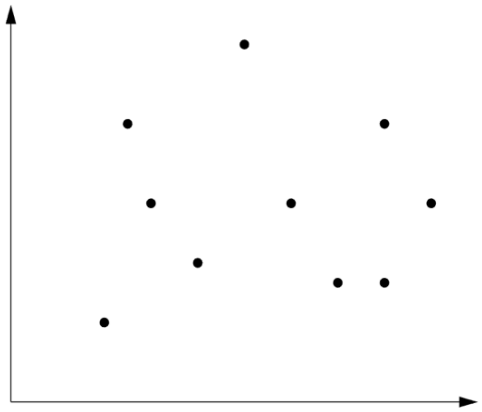
1. Which of the following scatterplots best represents a data set with a value for Pearson's correlation coefficient of $r = -0.75$?

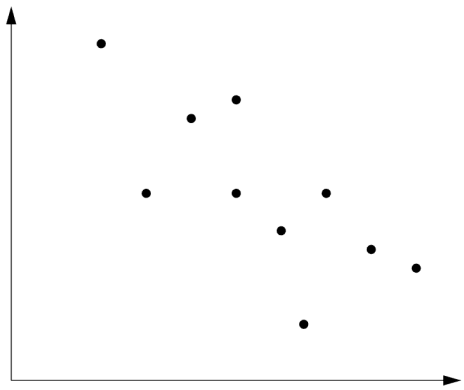
A.



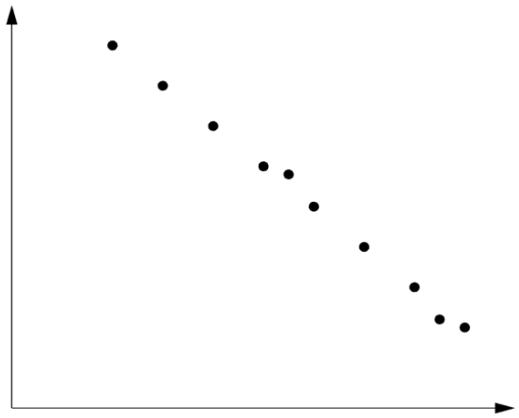
B.



C.



D.



2. The probability that it rains on any particular day in Sydney is 25%.

If three consecutive days of the year are chosen, what is the probability that it rains on all three of those days?

A. 1.56%

B. 25%

C. 42.19%

D. 75%

3. What is the value of $\int_0^4 \sqrt{16 - x^2}\,dx$?

A. 2π

B. 4π

C. 6π

D. 8π

4. What is the domain of $y = \sqrt{x} + ln\,(1 - x)$?

A. $[\,0\,,1]$

B. $(1, +\infty)$

C. $(\,0, +\infty\,)$

D. $[\,0\,,1\,)$

5. Elle recently did a Mathematics test and an English test. The class scores on each test were normally distributed.

The mean on the English test was 60%, and the standard deviation was 10%. Elle's mark was 72%.

The mean on the Mathematics test was 73%, and the standard deviation was 5%.

What was Elle's mark in the Mathematics test, if her z-scores in English and Maths were equal?

A. 72%

B. 78%

C. 79%

D. 83%

6. What is $\int x^2 (x^3 + 1)^4 dx$?

A. $\dfrac{(x^3 + 1)^3}{3} + c$

B. $\dfrac{(x^3 + 1)^3}{9} + c$

C. $\dfrac{(x^3 + 1)^5}{5} + c$

D. $\dfrac{(x^3 + 1)^5}{15} + c$

7. The point $A(4, 2)$ lies on the graph of $y = f(x)$.

What are the coordinates of the point A', the image point of A, on the graph of $y = g(x)$ if $g(x) = -f\left(\frac{x}{2}\right)$?

A. $(-4, -2)$

B. $(-4, 2)$

C. $(8, 2)$

D. $(8, -2)$

8. A particle is moving along the x-axis. The graph shows its velocity, v metres per second, at a time, t seconds.

When $t = 0$, the displacement x is equal to 3 metres.

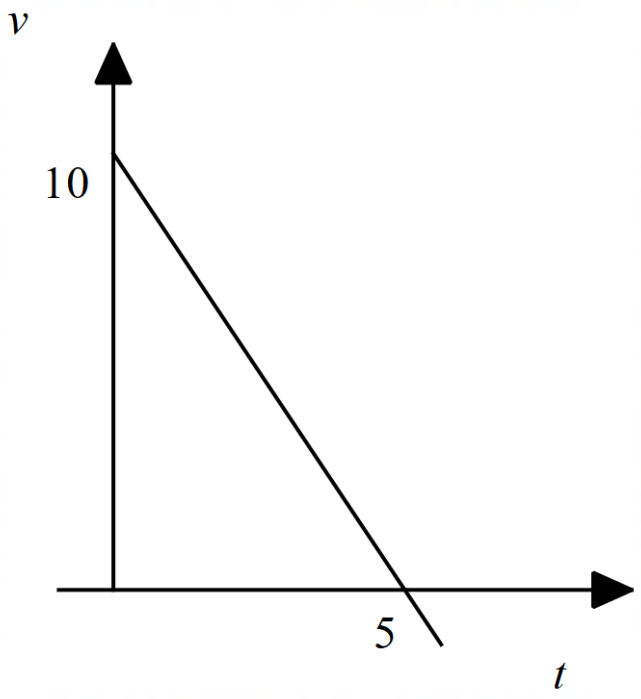
What is the maximum value of the displacement x ?

A. 10 m

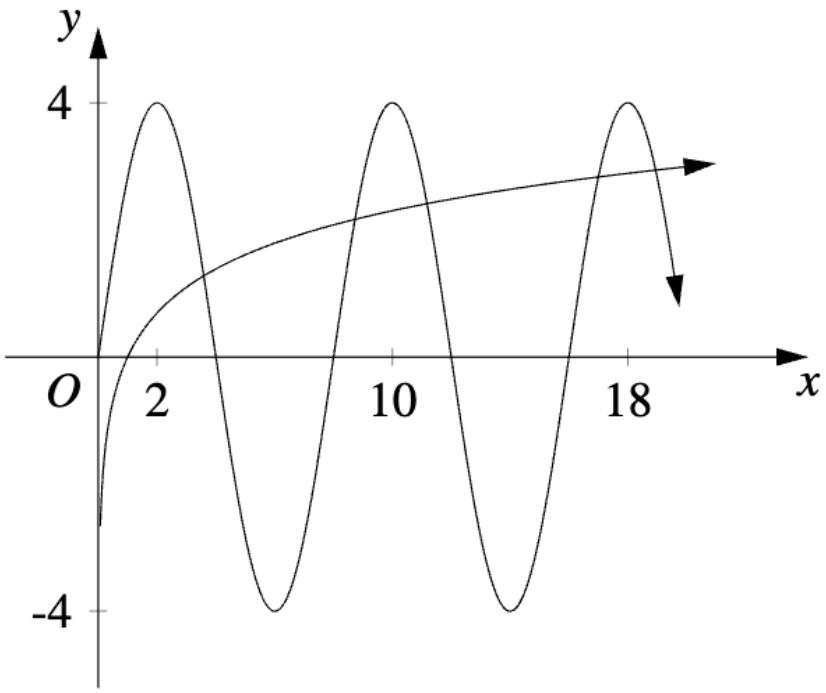
B. 13 m

C. 28 m

D. 53 m



9. The graphs of $y = 4sin\left(\frac{\pi}{4}x\right)$ and $y = lnx$ are shown on the number plane below.



How many solutions exist to the equation $4sin\left(\frac{\pi}{4}x\right) = lnx$ in the domain $0 \leq x \leq 50$?

A. 10

B. 11

C. 12

D. 13

10. Let $h(x) = f\big(g(x)\big)$, with $f(4) = 1,\ f'(4) = 6,\ g(2) = 4$ and $g'(2) = 4$.

What is the equation of the tangent to the graph of $y = h(x)$ at $x = 2$?

A. $y = x - 6$

B. $y = 4x - 2$

C. $y = 6x - 11$

D. $y = 24x - 47$

2024 TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Roseville COLLEGE

Student Number

Mathematics Advanced

Section II Answer Booklet

90 marks
Attempt Questions 11-35
Allow about 2 hours and 45 minutes for this section

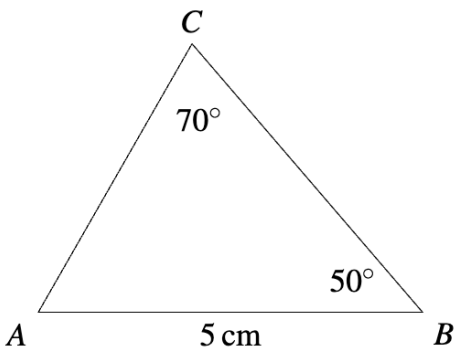
Instructions

- Write your Student Number at the top of this page.
- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided at the back of the booklet. If you use this space, clearly indicate which question you are answering.

Please turn over

Question 11 (4 marks)

The diagram shows triangle ABC. It is known that $AB = 5$ cm and $\angle ABC = 50°$ and $\angle BCA = 70°$.



(a) Find the length of AC, correct to two decimal places. **2**

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(b) Hence, find the area of triangle ABC, correct to the nearest square centimetre. **2**

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Question 12 (2 marks)

Find the exact gradient of the tangent to the curve $y = e^{2x+3}$ at the point where $x = 0$. **2**

Question 13 (2 marks)

Evaluate $\int_1^4 \sqrt{3x-2}\, dx$, correct to two decimal places. **2**

Question 14 (2 marks)

Evaluate $17 + 14 + 11 + \cdots - 106$. **2**

Question 15 (2 marks)

Solve $|3x - 1| = 5$. **2**

Question 16 (3 marks)

Ballet and karate are two clubs that students can do after school, on different days of the week. For the 60 students in Year 7 the following is known.

- 23 student do neither ballet nor karate
- 28 students do ballet
- 22 students do karate

(a) Find the number of students who do both ballet and karate and then draw a Venn Diagram to accurately to represent above information. **2**

(b) If a Year 7 student is chosen at random, what is the probability that they aren't one of the students who does both ballet and karate? **1**

Question 17 (2 marks)

The five-number summary for the dataset representing the time taken to run 100 metres by a group of Year 12 students is given. **2**

Lowest score $= 12.3$ seconds
Lower quartile $(Q_1) = 13.1$ seconds
Median $(Q_2) = 14.5$ seconds
Upper quartile $(Q_3) = 16.3$ seconds
Highest score $= 27.4$ seconds

Karson is a slower runner, and his time to run 100 metres is only just fast enough to avoid being classed as an outlier.

What is the maximum possible time Karson could have taken to run the 100 metres, correct to one decimal place?

Question 18 (3 marks)

Solve the equation $2log_2(x-6) = log_2 x$. **3**

Question 19 (3 marks)

Solve the equation $2\cos 2x = \sqrt{3}$ where $0 \leq x \leq 2\pi$. **3**

Question 20 (2 marks)

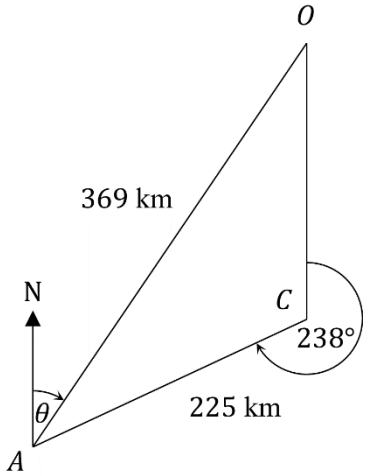
Use two applications (three function values) of the trapezoidal rule to find an approximate value of **2**

$$\int_1^2 \ln x \, dx.$$

Give your answer correct to 2 decimal places.

Question 21 (3 marks)

Orange, O, is due north of Canberra, C, while Albury, A, is 225 kilometres from Canberra at a bearing of 238°. **3**



What is the bearing, θ, of Orange from Albury, correct to the nearest degree?

Question 22 (2 marks)

A scientist is studying the growth of bacteria. The scientist models the number of bacteria, N, by the equation **2**

$$N(t) = 20\,000e^{0.03t},$$

where t is the number of days after starting the experiment.

After how many days will the number of bacteria be closest to twice the number of bacteria that were present at the start of the experiment? Answer correct to the nearest whole day.

Question 23 (3 marks)

A particle is shot vertically upwards from flat ground. The position of the particle, y metres above the ground after t seconds, is given by $y = 40t - 5t^2$. **3**

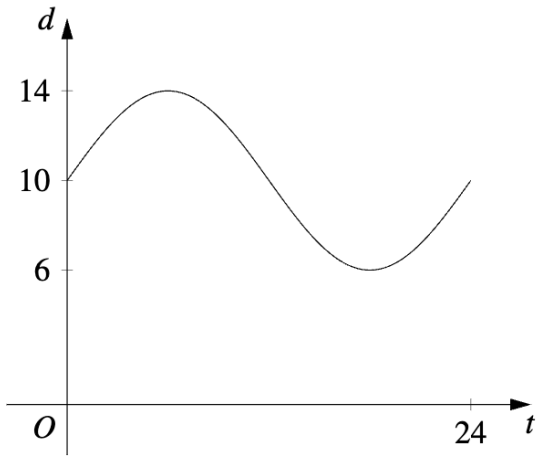
At what time(s) will the particle have a speed of 10 m/s?

Question 24 (5 marks)

The depth, d metres, of water in a reservoir is shown below. The depth is modelled using the function

$$d(t) = c + ksin(bt)$$

where $0 \leq t \leq 24$ is the time in days.



(a) Find the values of c and k. **2**

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(b) Find the value of b. **1**

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Questions 24 continues on page 21

Question 24 (continued)

(c) Find the duration when the depth of water in the reservoir is 11 metres or more, correct to one decimal place. **2**

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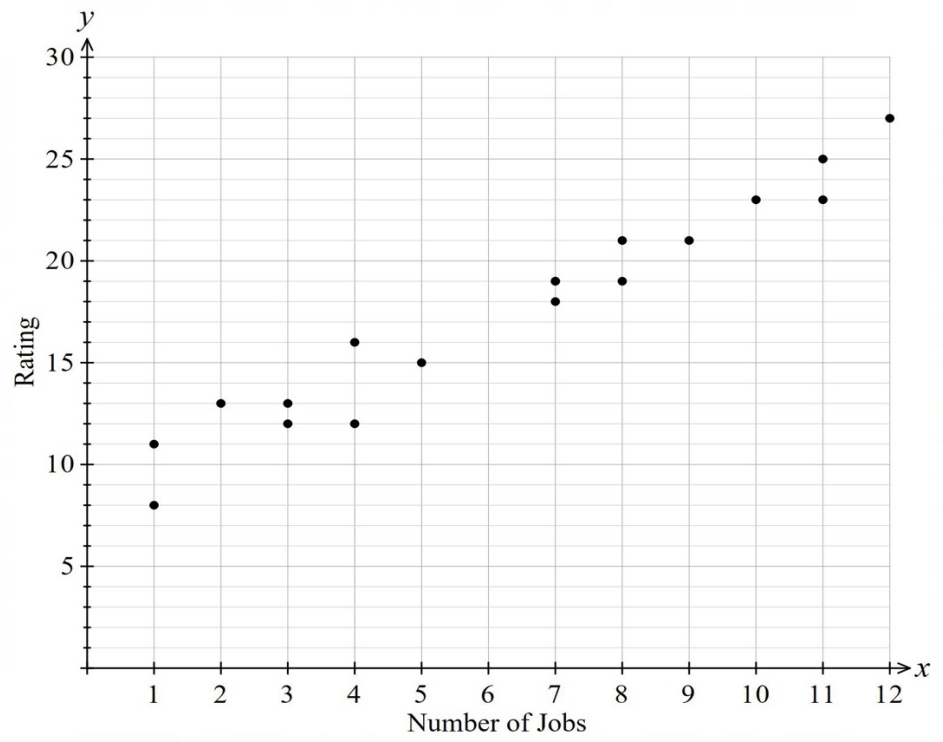
End of Question 24

Please turn over

Question 25 (5 marks)

The scatterplot below shows the number of jobs that each apprentice in a mechanical engineering course has submitted, compared to the rating (out of 40) awarded to their most recent job.



Using x as the number of jobs and y as the most recent rating, the teacher finds the gradient of the least-squares regression line to be 1.5.

In any bivariate dataset, the least-squares regression line passes through the point $(\bar{x},\ \bar{y})$, where $\bar{x}$ is the sample mean of the x-values and $\bar{y}$ is the sample mean of the y-values.

For this dataset, $\bar{x} = 6$ and $\bar{y} = 17$.

(a) Using the information above, determine the equation of the least-squares regression line and **draw the line on the graph above**. **3**

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Questions 25 continues on page 23

Question 25 (continued)

(b) Use the least-squares regression line to estimate the most recent rating for an apprentice who has submitted 16 jobs. **1**

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(c) Outline the limitations of using the least-squares regression line to estimate ratings for apprentices who have submitted much larger numbers of jobs. **1**

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End of Question 25

Please turn over

Question 26 (4 marks)

Consider the geometric series

$1 + \left(\sqrt{5} - 2\right) + \left(\sqrt{5} - 2\right)^2 + ...$

(a) Show with calculations why this geometric series has a limiting sum. **2**

(b) Find the exact value of the limiting sum. **2**

Question 27 (4 marks)

A discrete probability distribution is given by

$P(X = x) = \frac{x^2 - x}{20}$, where $x = 2, 3$ and 4 .

(a) Show the expected value $E(X) = 3\frac{1}{2}$. **2**

(b) Calculate the variance and the standard deviation, correct to two decimal places. **2**

Question 28 (4 marks)

(a) Show that $\frac{sec^2x}{tan^2x} = \text{cosec}^2x$. **2**

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(b) Hence, or otherwise evaluate $\int \text{cosec}^2x \ dx$. **2**

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Question 29 (4 marks)

A company is analysing the productivity of its employees during a particular project.

The probability density function (PDF) represents the number of productive hours in the day of a random employee, and is modelled by the quadratic function:

$$f(x) = -\frac{3}{1\,372}(x^2 - 16x + 15)$$

where x is measured in hours.

(a) Determine the modal number of the productive hours, denoted by M. **2**

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(b) Calculate the probability that an employee's productivity level is between 5 and 8 hours. **2**

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Question 30 (4 marks)

Consider the graph of $y = af(x+b) + c$ shown below, where $f(x) = \frac{1}{x}$. **4**

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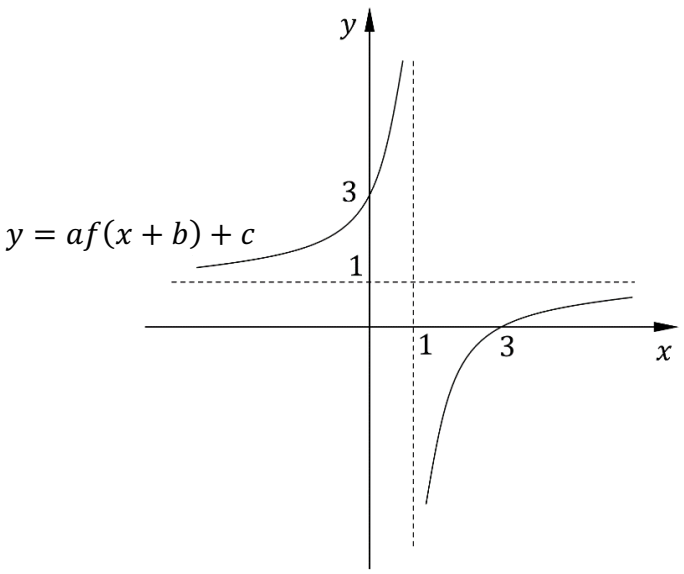


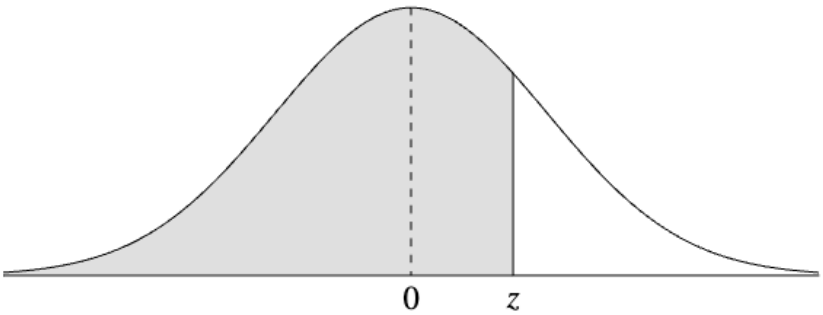
Find the value of a, b, c, then describe in order the transformations to the graph of $y = f(x)$ that would produce the graph of $y = af(x+b) + c$.

Question 31 (3 marks)

The table gives the probability that a random variable lies below some z-score, where the random variable is normally distributed. **3**

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549

The probability values given in the table below represent the shaded area in the diagram.



The results of a university entrance test sat by Year 12 students is normally distributed with a mean of 35 and a standard deviation of 5. Students who score in the 67th percentile and above are offered a place at the university.

What is the minimum mark students need to achieve in the entrance test in order to be offered a place at the university?

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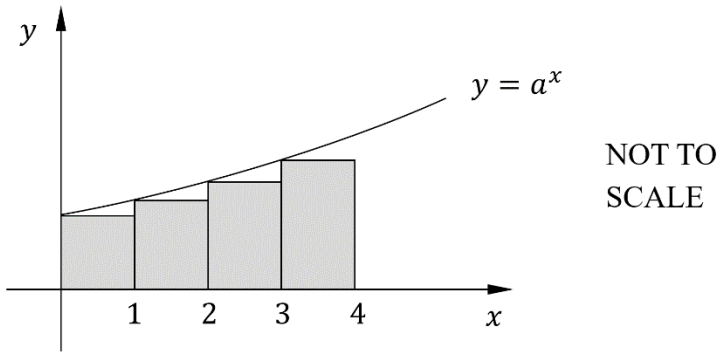
Question 32 (5 marks)

The diagram shows the graph of $y = a^x$, where $a > 1$. Also shown on the diagram are 4 inner rectangles of width 1 unit.



(a) Use the sum of geometric series to show the area of the 4 rectangles is $\frac{a^4-1}{a-1}$ units^2 **2**

Questions 32 continues on page 31

Question 32 (continued)

(b) By finding the area under the curve $y = a^x$ between $x = 0$ and $x = 4$, and using the result in part (a), prove that $\ln a < a - 1$. **3**

Question 33 (7 marks)

Nick constructs a swimming pool in the shape below. Nick's pool design is made up of a sector and a semi-circle, the radius(r) of the sector is equal to the diameter of the semi-circle.

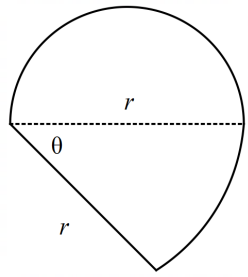


Figure not to scale

(a) Show the perimeter of the pool is $P = r + r\theta + \frac{\pi r}{2}$ **1**

(b) Nick wants to be able to use this design in several different properties over the next year, so he decides to let the area of the pool equal 1 square unit for future ease in scaling the pool sizes. **3**

Show that, if this is the case, the perimeter can be given by:

$$P = \frac{2}{r} + r\left(1 + \frac{\pi}{4}\right).$$

Question 33 (continued)

(c) Show that the least perimeter occurs when $r^2 = \dfrac{8}{\pi + 4}$ and calculate the corresponding value of θ to the nearest degree. **3**

Question 34 (8 marks)

Let $f(x) = e^{2x}(1 - x)$

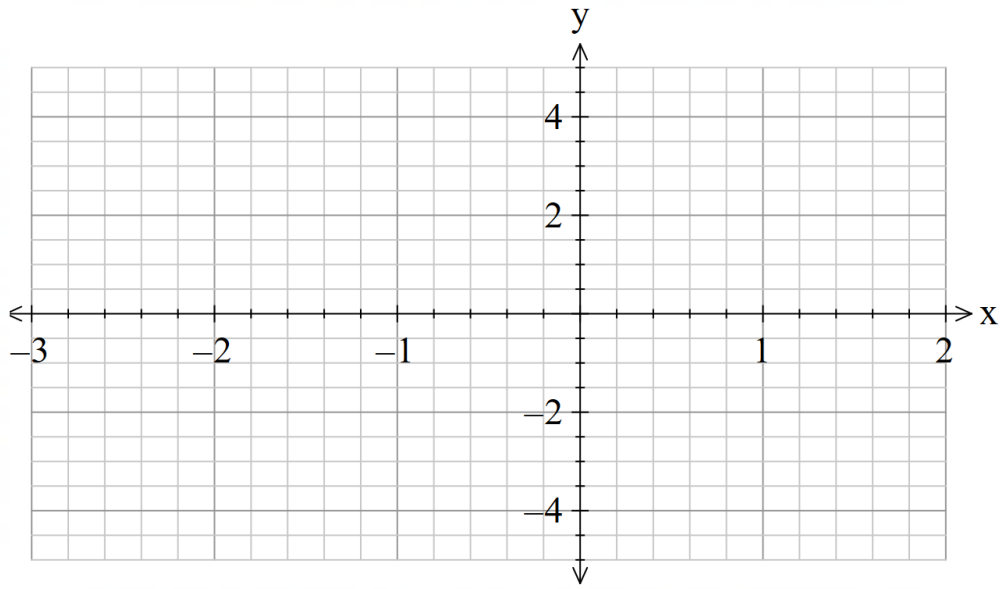
A table of values is constructed below for $[-3, 1]$

x	-3	-2	-1	0	1
f(x)	0.01	0.05	0.27	1.00	0.00

(a) Find the coordinates of any stationary points, inflections and determine their nature for $y = f(x)$. **4**

Question 34 (continued)

(b) Sketch the curve $y = e^{2x}\,(1 - x)$ for [-3, 1], showing the critical points. **2**



(c) Without any further calculus, state where, in the domain above, the curve is concave down. **1**

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(d) A student uses the trapezoidal rule with 5 function values to approximate the area under $y = f(x)$ for [-3, 1] to be 1.325 units^2. **1**

By using your diagram above, state whether this approximation will be greater or less than the exact area under this curve, giving reasons for your answer.

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Question 35 (4 marks)

E and F are two events and their complementary events are E' and F' respectively. **4**

Given that $P\ (F|E) = \frac{1}{4}$, $P\ (E|F) = \frac{2}{3}$ and $P\ (E' \cap F') = \frac{1}{4}$

Show that E and F are independent events.

End of Paper