TRINITY GRAMMAR SCHOOL

Mathematics Department

(NESA Student Number | Year 12 only)

2024

Year 12

Mathematics Advanced

TRIAL EXAMINATION

ASSESSMENT TASK 4

Date of Assessment Task: Tuesday, 27 August 2024

General Instructions

- Reading time – 10 minutes
- Working time – 3 hours
- Write using black pen
- Erasable pens and all electronic devices (or timekeepers), unless otherwise advised, are not permitted in this Assessment
- Calculators approved by NESA may be used
- A reference sheet is provided
- For questions in Section II, show relevant mathematical reasoning and/or calculations
- *Write your NESA Student Number (Year 12) at the top of this*

Total marks: 100

Section I – 10 marks (pages 2 – 6)

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section

Section II – 90 marks (pages 7 – 30)

- Attempt Questions 11 – 31
- Allow about 2 hours and 45 minutes for this section

- **Year 12 Assessment Weighting:** 30%

Section I

10 marks
Attempt Questions 1–10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1–10.

---

1 What values of x satisfy $3-2x \geq 8$?

A. $x \leq -\frac{11}{2}$

B. $x \geq -\frac{11}{2}$

C. $x \leq -\frac{5}{2}$

D. $x \geq \frac{5}{2}$

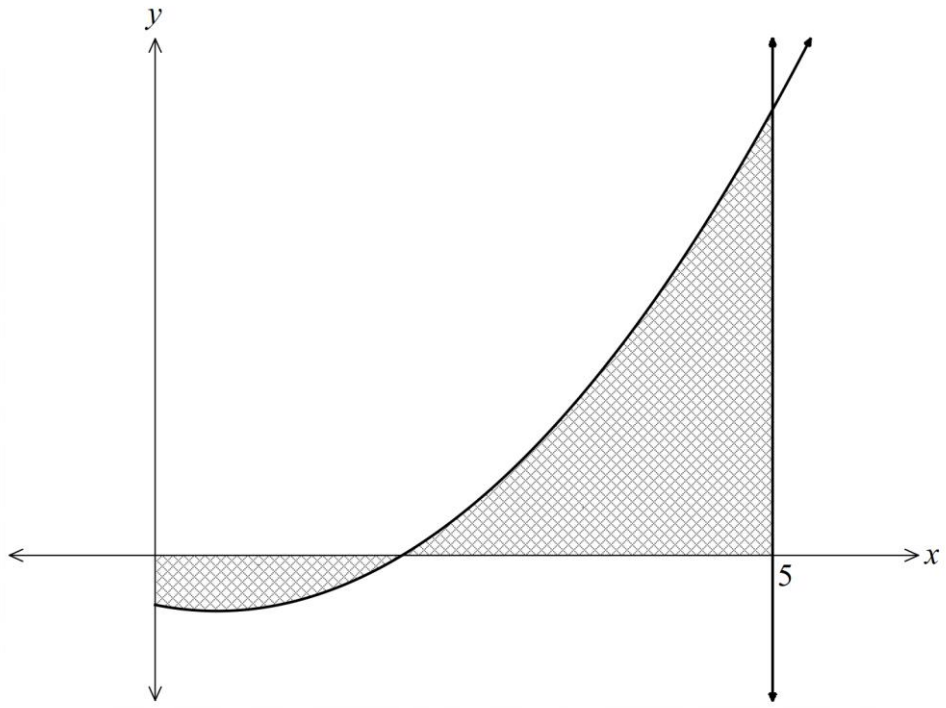
2 Which of the following represents the domain of the function $f(x) = \ln(x+1)$?

A. $[-\infty, -1)$

B. $(-\infty, 1)$

C. $[-1, \infty)$

D. $(-1, \infty)$



3 The shaded region in the diagram above shows the area, *A*, bounded by the curve $y = x^2 - x - 2$, the $x-$axis and the lines $x = 0$ and $x = 5$?

Which expression will give the area, *A*, of the shaded region?

A. $A = \left| \int_0^1 \left(x^2 - x - 2 \right) dx \right| + \int_1^5 \left(x^2 - x - 2 \right) dx$

B. $A = \int_0^2 \left(x^2 - x - 2 \right) dx + \left| \int_2^5 \left(x^2 - x - 2 \right) dx \right|$

C. $A = \int_2^0 \left(x^2 - x - 2 \right) dx + \int_2^5 \left(x^2 - x - 2 \right) dx$

D. $A = \int_1^5 \left(x^2 - x - 2 \right) dx$

4 Which of the following represent respectively the radius and coordinates of the centre of the circle $x^2 - 6x + y^2 + 8y + 21 = 0$?

A. Radius of 2 **and** centre $(-3, 4)$

B. Radius of 2 **and** $(3, -4)$

C. Radius of 4 **and** $(-3, 4)$

D. Radius of 4 **and** $(3, -4)$

5 What is the period of $y = \tan 3x$?

A. $\frac{\pi}{3}$

B. $\frac{\pi}{6}$

C. $\frac{2\pi}{3}$

D. 3π

6 The values of x for which the geometric series $2 + 6x + 18x^2 + \ldots$ has a limiting sum are:

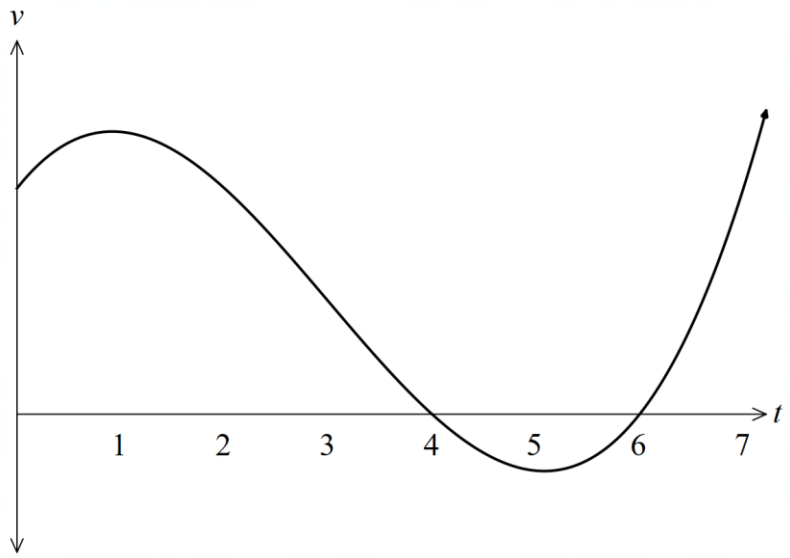
A. $x < \frac{1}{3}$

B. $x \leq \frac{1}{3}$

C. $-\frac{1}{3} < x < \frac{1}{3}$

D. $-\frac{1}{3} \leq x \leq \frac{1}{3}$

7 A particle is moving along a straight line. The graph below shows the velocity, v, of the particle at any time t seconds.



For what value of t is the displacement, x, a maximum?

A. 1

B. 4

C. 5

D. 6

8 Consider the following probability distribution table, where a is a constant.

x	**1**	**2**	**3**	**4**	**5**
$P(X = x)$	a	$3a$	$5a$	$2a$	a

What is the mean, $E(X)$, for this probability distribution?

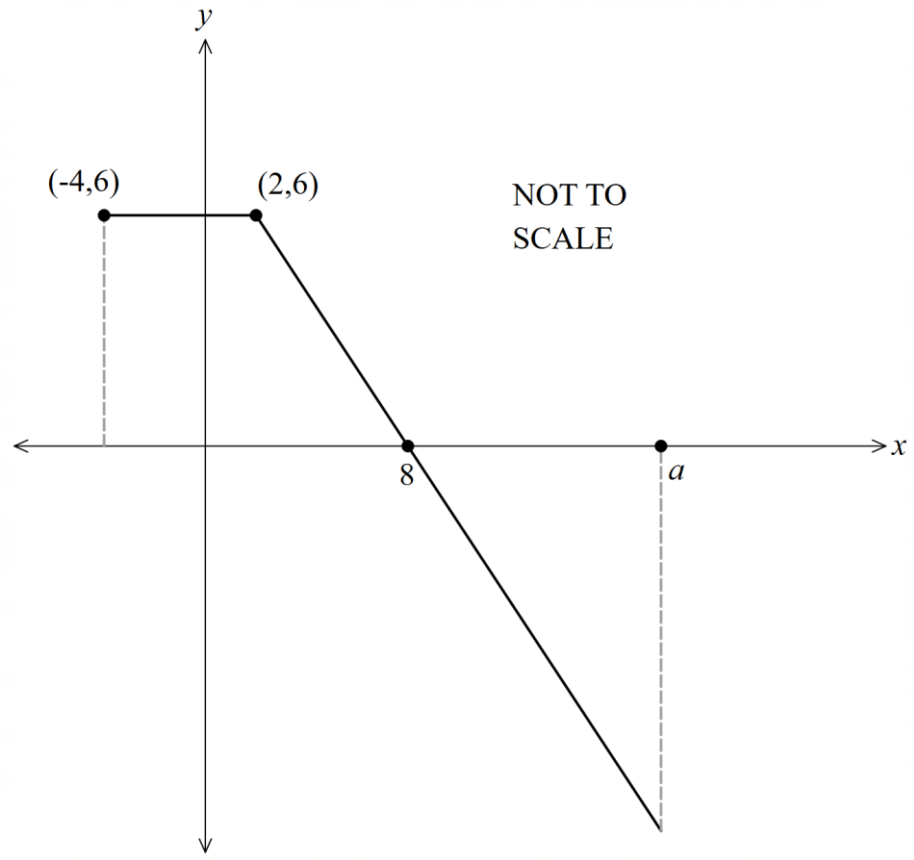
A. $\frac{1}{12}$

B. $1\frac{3}{12}$

C. $2\frac{11}{12}$

D. 3

9 The graph of $y = f(x)$ is a piecewise function with a domain of $[-4, a]$, where $a > 8$.



Find the value of a given that $\int_{-4}^{a} f(x)\,dx = 4$.

A. 16

B. 18

C. 20

D. 22

10 How many solutions does the equation $x = 10\sin x$ have, given $x \in [-3\pi, 3\pi]$?

A. 4

B. 5

C. 6

D. 7

TRINITY GRAMMAR SCHOOL

Mathematics Department

(NESA Student Number | Year 12 only)

2024

Mathematics Advanced

Section II Answer Booklet

90 marks
Attempt Questions 11-31
Allow about 2 hours and 45 minutes for this section

Instructions

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- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
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Please turn over

Question 11 (4 marks)

The first three terms of an arithmetic sequence are 4, 9 and 14.

(a) Find the 20^{th} term. **2**

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(b) Find the sum of the first 20 terms. **2**

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Question 12 (4 marks)

Find the value(s) of x such that $25^{x^2-2x} = \left(\frac{1}{125}\right)^{4x+2}$. **4**

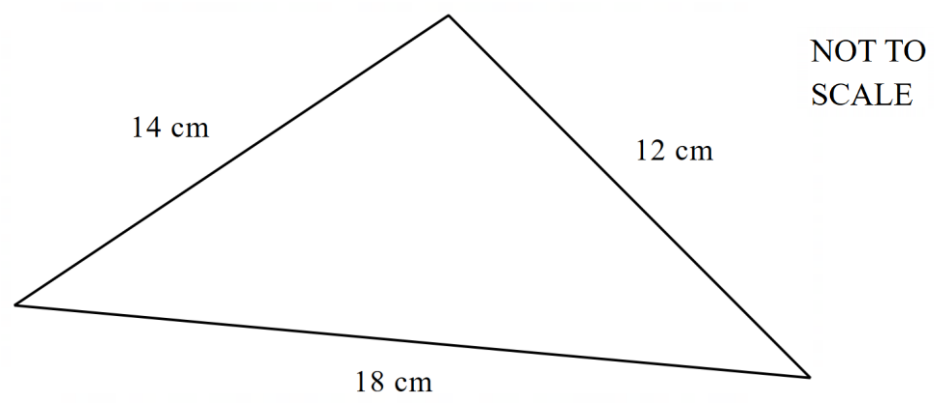
Question 13 (3 marks)

Find the equation of the tangent to the curve $y = \frac{1}{\sqrt{x}}$ at the point $\left(4, \frac{1}{2}\right)$.

Give your final answer in the form $ax + by + c = 0$, where $a, b, c \in \mathbb{Z}$. **3**

Question 14 (3 marks)

Find the area of the triangle below. Round your answer to 1 decimal place. **3**



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Question 15 (2 marks)

Find $\int \frac{5x}{x^2+1}\,dx$. **2**

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Question 16 (5 marks)

Differentiate each of the following expressions with respect to x, leaving your answer in simplest form where possible:

(a) $\left(2x^3 - 1\right)^5$ **1**

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(b) $x \cos x$ **2**

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(c) $\dfrac{2x}{\ln x}$ **2**

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Question 17 (2 marks)

Find all the values of θ, where $0 \le \theta \le \pi$, such that $\sin\left(2\theta - \frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$ **2**

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Question 18 (3 marks)

A and B are independent events with $P(A) = 0.28$ and $P(A \cup B) = 0.75$.

Find $P(B)$. **3**

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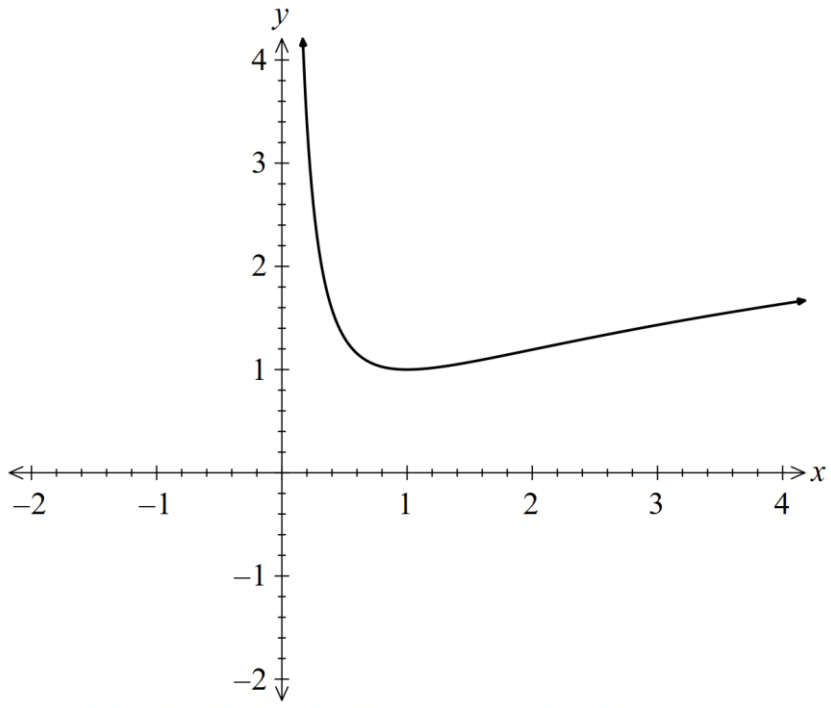
Question 19 (2 marks)

The graph of $y = \frac{1}{x} + \ln(x)$ is shown over part of its domain.



Use **three** applications of the trapezoidal rule to find the approximate area between the curve, the $x-$axis and the lines $x=1$ and $x=4$. Give your answer correct to 2 decimal places. **2**

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Question 20 (5 marks)

A table of future value interest factors for an annuity of $1000 is shown.

Rate / *Period*	*Interest rate per period*				
	1%	1.5%	3%	4.5%	6%
3	3030.10	3045.23	3090.90	3137.03	3183.60
6	6152.02	6229.55	6468.41	6716.89	6975.32
12	12682.50	13041.21	14192.03	15464.03	16869.94
24	26973.46	28633.52	34426.47	41689.20	50815.58

(a) Taliah is saving for a new car. She considers contributing $7 500 each year for the next 6 years.

If the interest rate is 6% per annum, compounding annually, how much money will Taliah have saved after 6 years? **2**

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(b) Taliah requires $85 000 to buy her dream car.

She decides to make regular payments into an account earning 6% per annum, compounding **quarterly** for the next 6 years.

Given that Taliah will make her contribution every 3 months, how much more will she pay per year, than in part (a)? **3**

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Question 21 (3 marks)

Solve the equation $2\log_5(4) - \log_5(2) + \log_5(x) = 2$ for x. **3**

Question 22 (3 marks)

Consider the function f such that $f'(x) = \frac{8x}{x^2+4}$. The graph of $y = f(x)$ passes through the point $(2, 9\ln 2)$.

Find $f(x)$. **3**

Question 23 (5 marks)

Let $f(x) = xe^x$ and $g(x) = 4x - x^2$.

Solve $(f \circ g)(x) = 0$. **5**

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Question 24 (9 marks)

The curve $y = x^3 + ax^2 + 7x$ has a stationary point at $x = 1$.

(a) Find the value of a. **2**

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(b) Find the coordinates of all stationary points **and** determine their nature. **3**

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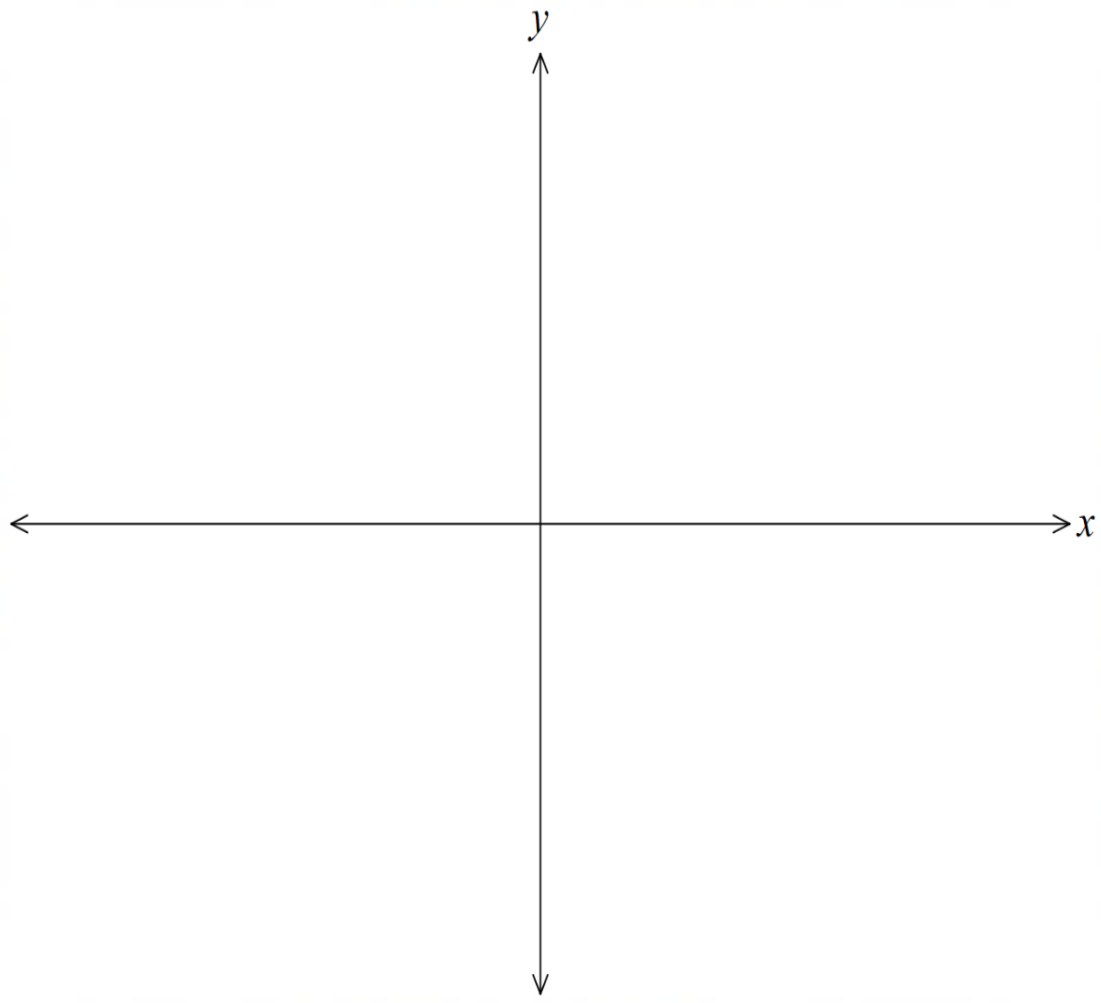
Question 24 continues on page 19

Question 24 (continued)

(c) Sketch the graph of $y = f(x)$ showing any stationary points and its $y-$intercept. **2**



(d) Find any values of x such that the curve is concave up. **2**

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End of Question 24

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Question 25 (3 marks)

(a) Find $\frac{d}{dx}\left(e^{\tan x}\right)$. **1**

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(b) Hence, or otherwise, find $\int \frac{2e^{\tan x}}{3-3\sin^2 x}\,dx$. **2**

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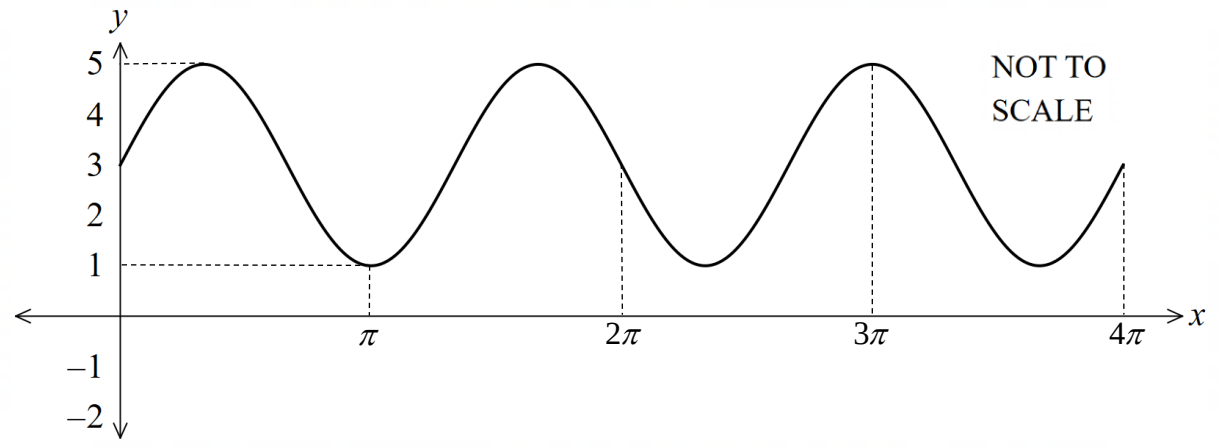
Question 26 (4 marks)

A trigonometric function $y = f(x)$ is drawn below in the domain $[0, 4\pi]$.



(a) State the amplitude of $y = f(x)$. **1**

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(b) Determine the equation of the function $y = f(x)$.

Express your final answer in the form $y = a\sin(bx) + d$. **3**

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Question 27 (7 marks)

The following table shows values of x and y.

x	4	13	26	30	35	50
y	1.8	2.9	3.8	4.2	5.7	5.1

(a) Calculate Pearson's correlation coefficient for the data represented in the table above, correct to 2 decimal places. **1**

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(b) Identify the direction **and** strength of the linear association between x and y. **2**

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(c) State the equation of the least-squares regression line in the form $y = ax + b$. **2**

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(d) Use the regression equation from part (c) to estimate the value of x when $y = 4.8$ to 2 decimal places. **2**

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Question 28 (5 marks)

A continuous random variable X has a function given by

$$f(x) = \begin{cases} \pi \sin(2\pi x) & \text{if } 0 \le x \le \frac{1}{2} \\ 0 & \text{elsewhere} \end{cases}$$

(a) Show that the function $f(x)$ is a probability density function. **3**

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(b) Find the mode of X. **2**

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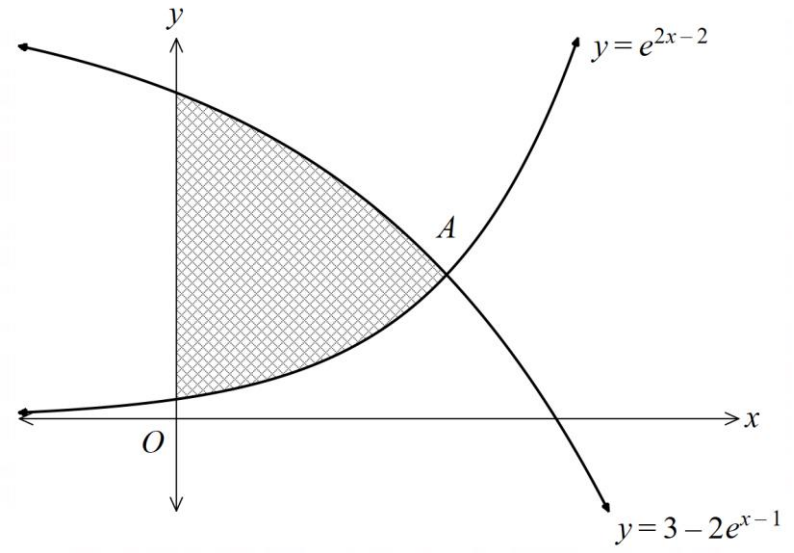
Question 29 (4 marks)

The shaded region is bound by the functions $y = e^{2x-2}$, $y = 3 - 2e^{x-1}$ and the $y-$axis, as shown in the diagram. Point A is the intersection of the two functions.



(a) Show that the $x-$coordinate of point A is $x = 1$. **2**

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Question 29 continues on page 25

Question 29 (continued)

(b) Find the **exact** area of the shaded region. **2**

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End of Question 29

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Question 30 (8 marks)

Kareem borrows $\$140\ 000$ to start his own business. He plans to repay the loan in equal monthly instalments of $\$M$ at 6% per annum reducible interest. Interest is calculated and charged just before each repayment is made.

Let A_n be the amount owing after n repayments (in dollars).

(a) Show that $A_n = 140\ 000(1.005)^n - 200M\left(1.005^n - 1\right)$. **3**

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Question 30 continues on page 27

Question 30 (continued)

(b) Kareem would like to repay the loan in exactly 15 years. Calculate the amount of each instalment. **2**

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(c) Kareem can afford to repay $1 850$ per month. How many years would it take him to repay the loan? **3**

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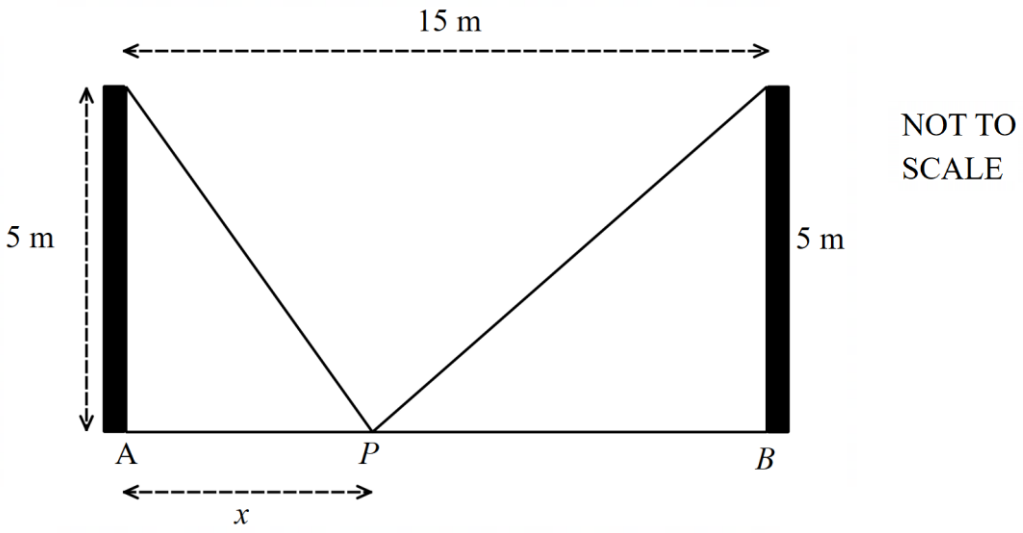
End of Question 30

Please turn over

Question 31 (6 marks)

The diagram shows two 5 m poles, A and B, that are 15 metres apart. The ends of a single length of wire are attached to the top of each pole. The length of wire is then attached to a stake in the ground between them at a moveable point, P, as shown in the diagram below.

Let x be the distance between A and P.



(a) Show that the length of the wire, L, is given by $L = \sqrt{x^2 + 25} + \sqrt{x^2 - 30x + 250}$. **1**

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Question 31 continues on page 29

Question 31 (continued)

(b) Show that $\frac{dL}{dx} = \frac{x}{\sqrt{x^2+25}} + \frac{x-15}{\sqrt{x^2-30x+250}}$. **2**

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(c) Find the value of x needed to minimise the length of wire, L used.

Justify your answer. **3**

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End of Question 31
End of paper

Section II extra writing space

If you use this space, clearly indicate which question you are answering

Do NOT write anything, or make any marks below this line or outside borders

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Section I

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Use the multiple-choice answer sheet for Questions 1–10.

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1 What values of x satisfy $3-2x \geq 8$?

A. $x \leq -\frac{11}{2}$

B. $x \geq -\frac{11}{2}$

C. $x \leq -\frac{5}{2}$

D. $x \geq \frac{5}{2}$

$3-2x \geq 8-3$

$-2x \geq 5$

$x \leq -\frac{5}{2}$

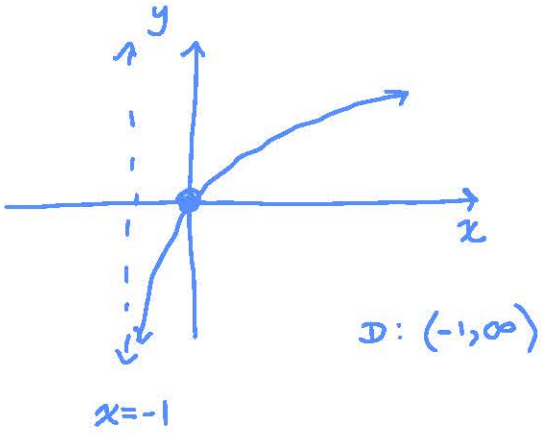
2 Which of the following represents the domain of the function $f(x)=\ln(x+1)$?

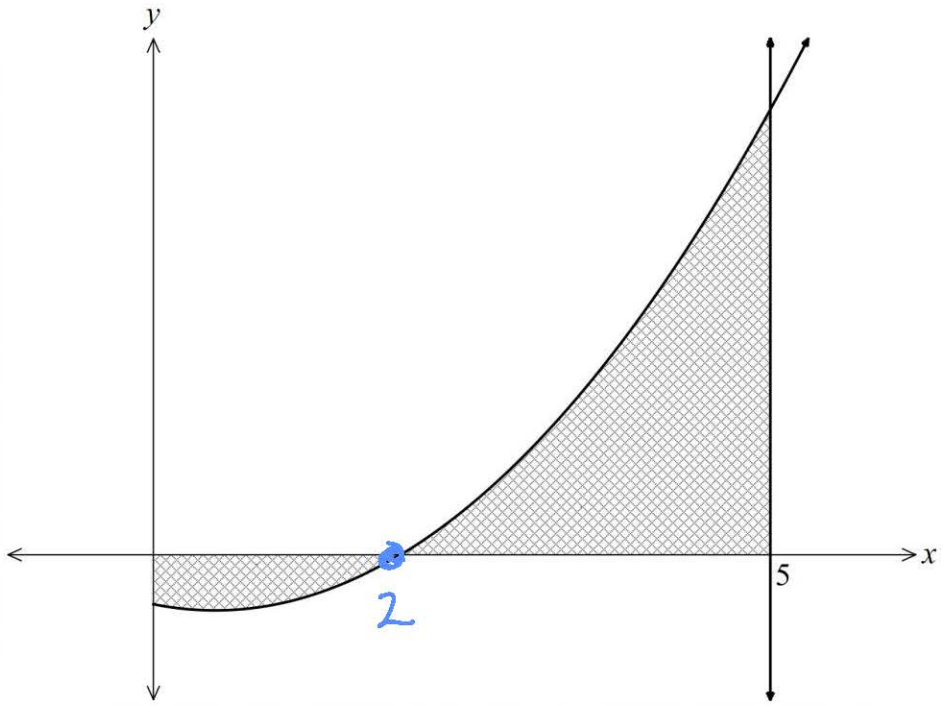
A. $[-\infty,-1)$

B. $(-\infty,1)$

C. $[-1,\infty)$

D. $(-1,\infty)$





3 The shaded region in the diagram above shows the area, A, bounded by the curve $y = x^2 - x - 2$, the $x-$axis and the lines $x = 0$ and $x = 5$?

Which expression will give the area, A, of the shaded region?

A. $A = \left| \int_0^1 \left(x^2 - x - 2\right) dx \right| + \int_1^5 \left(x^2 - x - 2\right) dx$

B. $A = \int_0^2 \left(x^2 - x - 2\right) dx + \left| \int_2^5 \left(x^2 - x - 2\right) dx \right|$

C. $A = \int_2^0 \left(x^2 - x - 2\right) dx + \int_2^5 \left(x^2 - x - 2\right) dx$

D. $A = \int_1^5 \left(x^2 - x - 2\right) dx$

$x^2 - x - 2 = 0$

$(x-2)(x+1) = 0$

$x = 2, \; x = -1.$

$\left| \int_0^2 (x^2 - x - 2)\, dx \right| + \int_2^5 (x^2 - x - 2)\, dx$

4 Which of the following represent respectively the radius and coordinates of the centre of the circle $x^2-6x+y^2+8y+21=0$?

A. Radius of 2 **and** centre $(-3,4)$

B. Radius of 2 **and** $(3,-4)$

C. Radius of 4 **and** $(-3,4)$

D. Radius of 4 **and** $(3,-4)$

$(x-3)^2-9+(y+4)^2-16+21=0$

$(x-3)^2+(y+4)^2=4$

Centre : $(3,-4)$ radius $=2$

5 What is the period of $y=\tan 3x$?

A. $\frac{\pi}{3}$

B. $\frac{\pi}{6}$

C. $\frac{2\pi}{3}$

D. 3π

period $=\frac{\pi}{b}=\frac{\pi}{3}$

6 The values of x for which the geometric series $2+6x+18x^2+\ldots$ has a limiting sum are:

A. $x<\frac{1}{3}$

B. $x\leq\frac{1}{3}$

C. $-\frac{1}{3}<x<\frac{1}{3}$

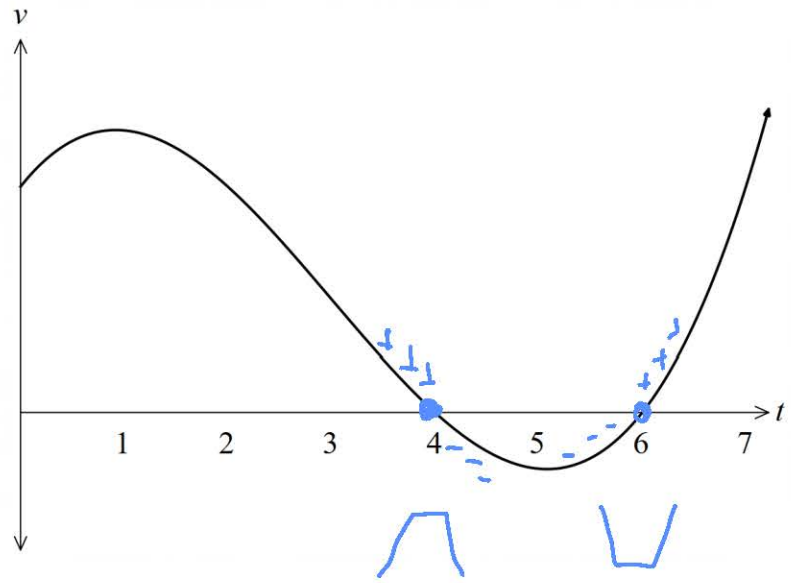
D. $-\frac{1}{3}\leq x\leq\frac{1}{3}$

$-1<r<1$ $\quad r=\frac{6x}{2}=3x$

$-1<3x<1$

$-\frac{1}{3}<x<\frac{1}{3}$

7 A particle is moving along a straight line. The graph below shows the velocity, v, of the particle at any time t seconds.



For what value of t is the displacement, x, a maximum?

A. 1

B. 4

C. 5

D. 6

8 Consider the following probability distribution table, where a is a constant.

x	1	2	3	4	5
$P(X=x)$	a	$3a$	$5a$	$2a$	a

What is the mean, $E(X)$, for this probability distribution?

A. $\frac{1}{12}$

B. $1\frac{3}{12}$

C. $2\frac{11}{12}$

D. 3

$a+3a+5a+2a+a=1$

$12a=1$

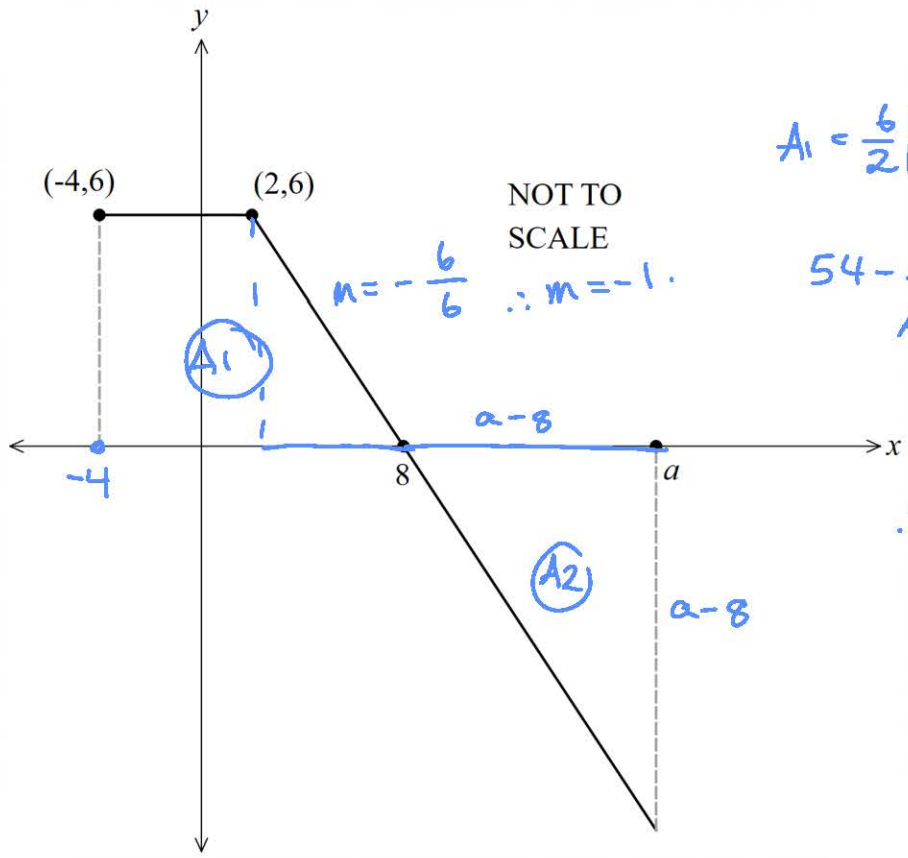
$a=\frac{1}{12}$

$\therefore E(X)=a+6a+15a+8a+5a$

$=35a$

$=35\times\frac{1}{12}=\frac{35}{12}=2\frac{11}{12}$

9 The graph of $y = f(x)$ is a piecewise function with a domain of $[-4, a]$, where $a > 8$.



$A_1 = \frac{6}{2}[6+12] = 3 \times 18 = 54$

$54 - A_2 = 4$

$A_2 = 50$

$m = -\frac{6}{6} \therefore m = -1.$

$\therefore \frac{1}{2}(a-8)^2 = 50$

$(a-8)^2 = 100$

$a - 8 = 10$

$a = 18$

Find the value of a given that $\int_{-4}^{a} f(x)\,dx = 4$.

A. 16

B. 18

C. 20

D. 22

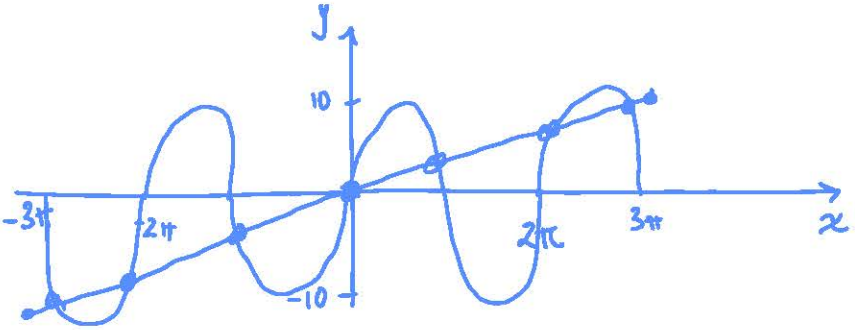
10 How many solutions does the equation $x = 10\sin x$ have, given $x \in [-3\pi, 3\pi]$?

A. 4

B. 5

C. 6

D. 7



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Please turn over

Question 11 (4 marks)

The first three terms of an arithmetic sequence are 4, 9 and 14.

(a) Find the 20th term. **2**

$T_{20} = 4 + (20-1)\times 5$ 1mk

$T_{20} = 99$ 1mk

2mk all correct
1mk using n^{th} term rule

(b) Find the sum of the first 20 terms. **2**

$S_{20} = \frac{20}{2}[4 + 99]$ 1mk

$= 1030$ 1mk

2mk all correct
1mk using S_n Rule

Alternate: $S_{20} = \frac{20}{2}[2\times 4 + (20-1)\times 5]$

$S_{20} = 1030$

Question 12 (4 marks)

Find the value(s) of x such that $25^{x^2-2x} = \left(\frac{1}{125}\right)^{4x+2}$. **4**

$(5^2)^{x^2-2x} = (5^{-3})^{4x+2}$ 1mk

$5^{2x^2-4x} = 5^{-12x-6}$

$\therefore 2x^2 - 4x = -12x - 6$ 1mk

$2x^2 + 8x + 6 = 0$

$x^2 + 4x + 3 = 0$

$(x+3)(x+1) = 0$

$x = -3, \quad x = -1$ 1mk each

4mks all correct
3mks one correct solution
2mks equating powers
1mk attempt common base

Question 13 (3 marks)

Find the equation of the tangent to the curve $y = \frac{1}{\sqrt{x}}$ at the point $\left(4, \frac{1}{2}\right)$.

Give your final answer in the form $ax + by + c = 0$, where $a, b, c \in \mathbb{Z}$. **3**

$y = x^{-\frac{1}{2}}$

$y' = -\frac{1}{2}x^{-\frac{3}{2}}$ 1mk

1mk

$\therefore x = 4, \quad m = -\frac{1}{16}$ @ $\left(4, \frac{1}{2}\right)$

$\therefore y - \frac{1}{2} = -\frac{1}{16}(x-4)$

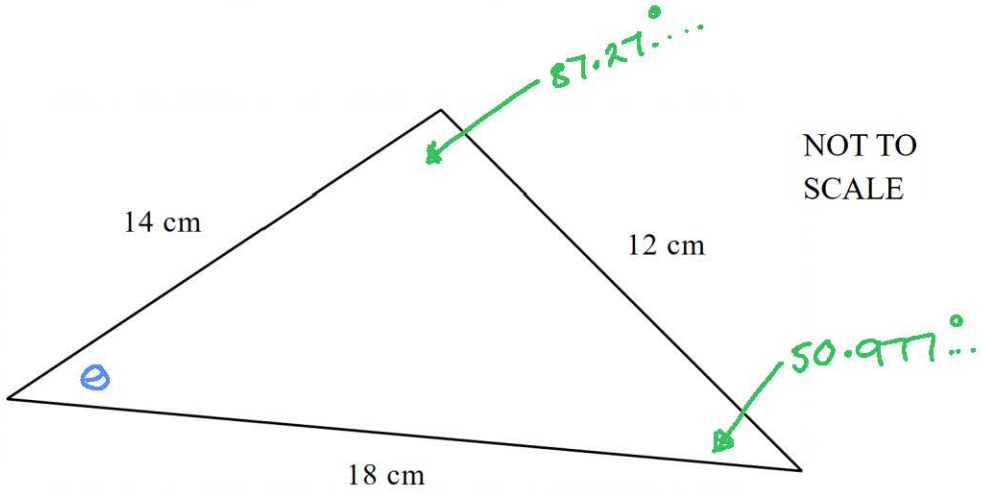
$16y - 8 = -x + 4$

$x + 16y - 12 = 0$ 1mk

3mks all correct
2mks correct gradient
1mk derivative

Question 14 (3 marks)

Find the area of the triangle below. to 1dp. **3**



$\cos\theta = \frac{14^2+18^2-12^2}{2\times14\times18}$ 1mk

$\theta = 41.7522052\ldots$ 1mk

$\therefore A = \frac{1}{2}\times14\times18\times\sin 41.7522052\ldots$

$A = 83.9\ cm^2$ 1mk

3mks all correct
2mks a correct angle
1mk cosine attempt

Question 15 (2 marks)

Find $\int \frac{5x}{x^2+1}\,dx$. **2**

$= 5\int \frac{x}{x^2+1}\,dx$

$= \frac{5}{2}\ln(x^2+1)\,dx$

2mks all correct
1mk $\ln(x^2+1)$

Question 16 (5 marks)

Differentiate each of the following expressions with respect to x, leaving your answer in simplest form where possible:

(a) $\left(2x^3-1\right)^5$ **1**

$\frac{d}{dx} = 5(2x^3-1)^4 \cdot 6x^2$

$= 30x^2(2x^3-1)^4$

1mk correct

(b) $x\cos x$ **2**

$\frac{d}{dx} = vu' + uv'$ $\quad u = x \quad v = \cos x$ 1mk

$= \cos x \cdot 1 + x \cdot (-\sin x)$ $\quad u' = 1 \quad v' = -\sin x$

$= \cos x - x\sin x$

1mk

2mks all correct

1mk correct derivatives

(c) $\frac{2x}{\ln x}$ **2**

$\frac{d}{dx} = \frac{vu' - uv'}{v^2}$ $\quad u = 2x \quad v = \ln x$ 1mk

$= \frac{\ln x \cdot 2 - 2x \cdot \frac{1}{x}}{(\ln x)^2}$ $\quad u' = 2 \quad v' = \frac{1}{x}$

$= \frac{2\ln x - 2}{2\ln x}$

2 mks all correct

1mk correct derivatives

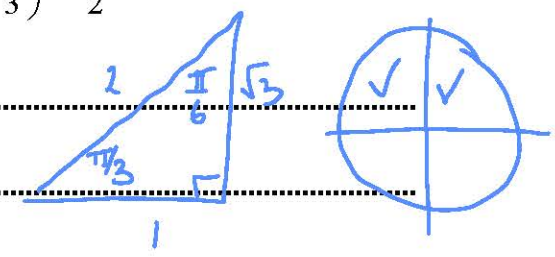
$= \frac{\ln x - 1}{\ln x}$ 1mk

$0 \le 2\theta < 2\pi$

Question 17 (2 marks)

$-\frac{\pi}{3} \le 2\theta - \frac{\pi}{3} \le \frac{5\pi}{3}$

Find all the values of θ, where $0 \le \theta \le \pi$, such that $\sin\left(2\theta - \frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$ **2**



1mk

$2\theta - \frac{\pi}{3} = \frac{\pi}{3}, \frac{2\pi}{3}$

$2\theta = \frac{2\pi}{3}, \pi$

$\therefore \theta = \frac{\pi}{3}, \frac{\pi}{2}$ 1mk

2mks all correct
1mk $\frac{\pi}{3}$ seen.

Question 18 (3 marks)

A and B are independent events with $P(A) = 0.28$ and $P(A \cup B) = 0.75$.

Find $P(B)$. **3**

$P(A \cap B) = P(A) \times P(B)$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$ 1mk

$\therefore$ $0.75 = 0.28 + P(B) - 0.28\,P(B)$ 1mk

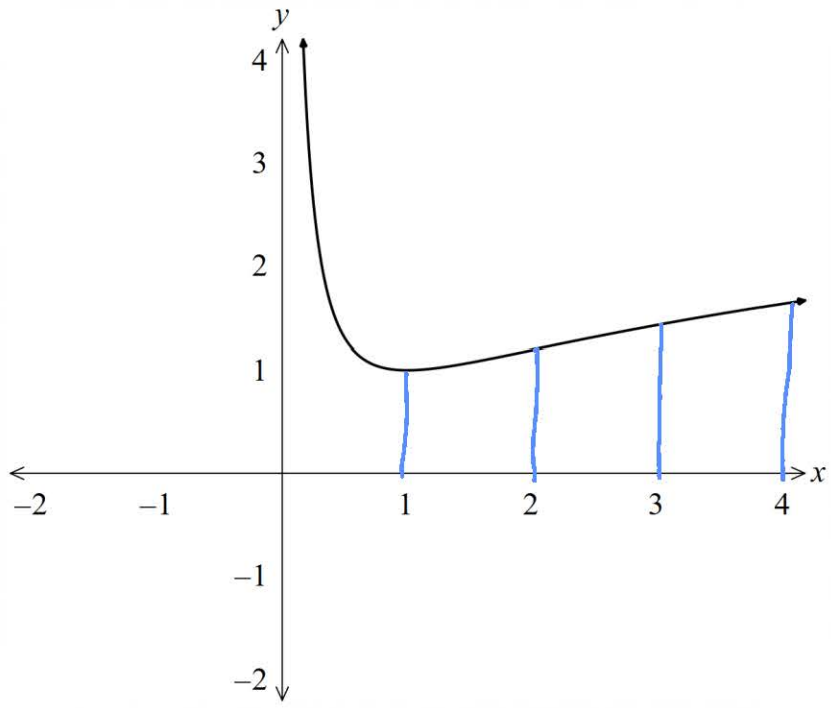
$0.47 = 0.72\,P(B)$

$\therefore P(B) = \frac{47}{72}$ 1mk

3 mks all correct
2 mks correct substitution
1mk either formula seen

Question 19 (2 marks)

The graph of $y = \frac{1}{x} + \ln(x)$ is shown over part of its domain.



Use **three** applications of the trapezoidal rule to find the approximate area between the curve, the $x-$axis and the lines $x=1$ and $x=4$. Give your answer correct to 2 decimal places. **2**

$A \doteq \frac{1}{2}\left[1 + \ln 1 + \frac{1}{4} + \ln 4 + 2\left(\frac{1}{2} + \ln 2 + \frac{1}{3} + \ln 3\right)\right]$

$A \doteq 3.94 u^2$

2 mks all correct

1 mk substitution into correct formula

Question 20 (5 marks)

A table of future value interest factors for an annuity of $1000 is shown.

Rate / Period	*Interest rate per period*				
	1%	1.5%	3%	4.5%	6%
3	3030.10	3045.23	3090.90	3137.03	3183.60
6	6152.02	6229.55	6468.41	6716.89	6975.32
12	12682.50	13041.21	14192.03	15464.03	16869.94
24	26973.46	28633.52	34426.47	41689.20	50815.58

(a) Taliah is saving for a new car. She considers contributing $7 500 each year for the next 6 years.

If the interest rate is 6% per annum, compounding annually, how much money will Taliah have saved after 6 years? **2**

$FV = PV \times IF$

$FV = 7.5 \times 6975.32$

$FV = \$52314.90$

2mks all correct
1mk for Interest Factor

(b) Taliah requires $85 000 to buy her dream car.

She decides to make regular payments into an account earning 6% per annum, compounding **quarterly** for the next 6 years.

Given that Taliah will make her contribution every 3 months, how much more will she pay per year, than in part (a)? **3**

$FV = PV \times IF$

$85000 = PV \times 28633.52$

$PV = 2.9685\ldots$

$PV = \$2968.55 \times 4$

$= \$11874.195 - 7500$

$= \$4374.19$ more each year

3mks all correct
2mks $2968.55
1mk Interest Factor

Accept $4374.20 for early round

Question 21 (3 marks)

Solve the equation $2\log_5(4)-\log_5(2)+\log_5(x)=2$ for x. **3**

$\log_5 16 - \log_5 2 + \log_5 x = 2$ 1mk

$\log_5(8x) = 2\log_5 5$

$\log_5 8x = \log_5 25$ 1mk

$\therefore 8x = 25$

$x = \frac{25}{8}$ 1mk

3mks all correct
2mks equating sides same base
1mk any correct log conversion

Question 22 (3 marks)

Consider the function f such that $f'(x)=\frac{8x}{x^2+4}$. The graph of $y=f(x)$ passes through the point $(2, 9\ln 2)$.

Find $f(x)$. **3**

$f(x) = \int \frac{8x}{x^2+4}\, dx$

$= 8\int \frac{x}{x^2+4}\, dx$

1mk

$f(x) = 4\ln(x^2+4) + c$ 1mk

$9\ln 2 = 4\ln(8) + c$

$c = 9\ln 2 - 4\ln 2^3$

$c = 9\ln 2 - 12\ln 2$

$c = -3\ln 2$

$\therefore f(x) = 4\ln(x^2+4) - 3\ln 2$ 1mk

3mks all correct
2mks $4\ln(x^2+4)+c$
1mk $\ln(x^2+4)$ seen

Question 23 (5 marks)

Let $f(x)=xe^x$ and $g(x)=4x-x^2$.

Solve $(f\circ g)(x)=0$. **5**

$(f\circ g)(x) = (4x-x^2)\cdot e^{4x-x^2}$

$0 = (4x-x^2)e^{4x-x^2}$

$\therefore 4x-x^2=0 \qquad e^{4x-x^2} \neq 0 \therefore$ No solution

$x(4-x)=0$

$x=0, x=4.$

5mks all correct
4mks $x=0, x=4$ and no exclusion
3mks correct composite $=0$
2mks correct composite
1mk any correct substitution

Please turn over

Question 24 (9 marks)

The curve $y = x^3 + ax^2 + 7x$ has a stationary point at $x = 1$.

(a) Find the value of a. **2**

$y' = 3x^2 + 2ax + 7$ 1mk

$y' = 0$ for stat pt.

$0 = 3(1)^2 + 2a(1) + 7$

$0 = 3 + 2a + 7$

$2a = -10$ $\therefore a = -5$ 1mk

2mks all correct
1mk correct derivative

(b) Find the coordinates of all stationary points **and** determine their nature. **3**

$y = x^3 - 5x^2 + 7x$

$y' = 3x^2 - 10x + 7$

$0 = 3x^2 - 10x + 7$

$y'' = 6x - 10$

$x = \frac{7}{3}$, $y'' = 4$ $\therefore y'' > 0$ ∪ min

$x = 1$, $y'' = -4$ $\therefore y'' < 0$ ∩ max

p	21
5	-10

$(-7, -3)$ $\therefore 0 = 3x^2 - 7x - 3x + 7$

$0 = x(3x - 7) - 1(3x - 7)$

$0 = (3x - 7)(x - 1)$

$x = \frac{7}{3}$, $x = 1$

$y = \frac{49}{27}$, $y = 3$

$\therefore \left(\frac{7}{3}, \frac{49}{27}\right)$ minimum

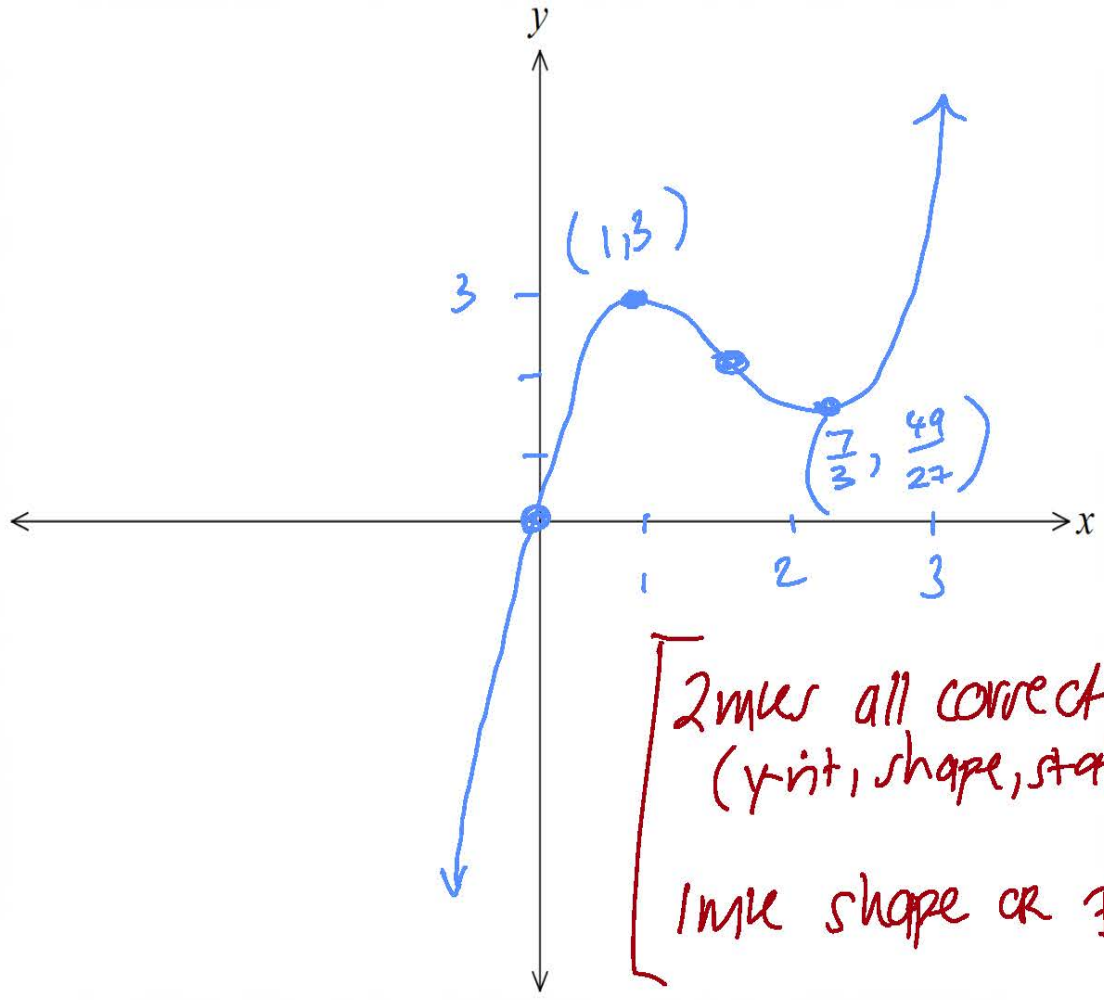
and $(1, 3)$ maximum

3mks all correct
2mks coordinates, no nature
1mk letting $y' = 0$ correctly

Question 24 continues on page 19

Question 24 (continued)

(c) Sketch the graph of $y = f(x)$ showing any stationary points and its y – intercept. **2**



2mks all correct
(y-int, shape, stat pts)

1mk shape or points

(d) Find any values of x such that the curve is concave up. **2**

$y'' = 6x - 10$

$0 = 6x - 10$

$6x = 10$

$x = \frac{5}{3}$

$x > \frac{5}{3}$

2mks all correct
1mk $y'' = 0$

End of Question 24

Please turn over

Question 25 (3 marks)

(a) Find $\frac{d}{dx}\left(e^{\tan x}\right)$. **1**

$\frac{d}{dx} = \sec^2 x \, e^{\tan x}$

[1mk only]

(b) Hence, or otherwise, find $\int \frac{2e^{\tan x}}{3-3\sin^2 x}\,dx$. **2**

$\int \frac{2e^{\tan x}}{3(1-\sin^2 x)}\,dx$

$= \frac{1}{3}\int \frac{2e^{\tan x}}{\cos^2 x}\,dx$

$= \frac{2}{3}\int \sec^2 x \cdot e^{\tan x}\,dx$

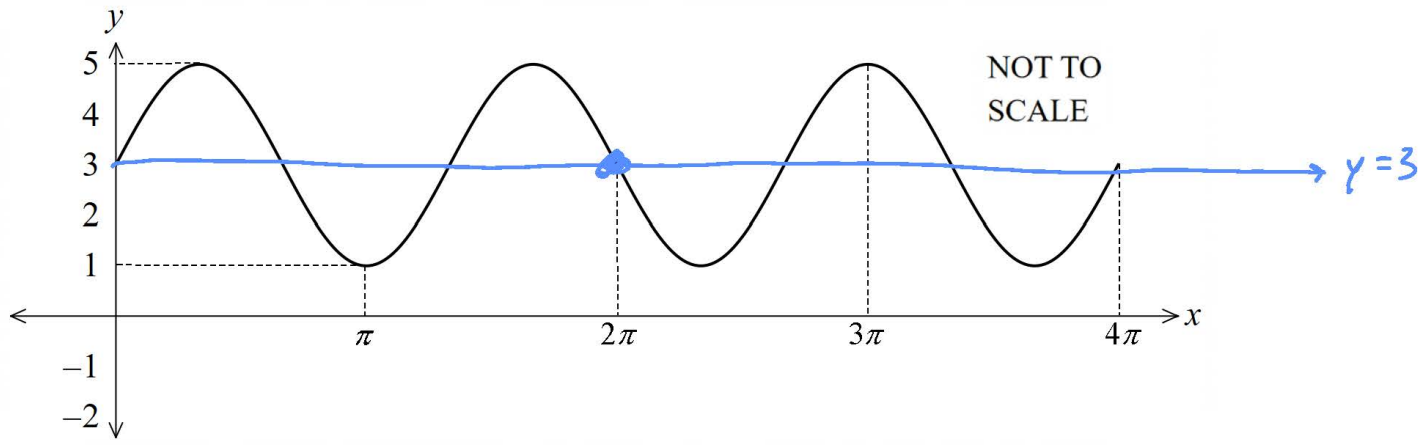
[2mks all correct
1mk working towards]

[1mk]

$= \frac{2}{3}e^{\tan x} + C$ or $\frac{2e^{\tan x}}{3} + C$ 1mk.

Question 26 (4 marks)

A trigonometric function $y = f(x)$ is drawn below in the domain $[0, 4\pi]$.



(a) State the amplitude of $y = f(x)$. **1**

amplitude = 2 [1mk only]

(b) Determine the equation of the function $y = f(x)$.

Express your final answer in the form $y = a\sin(bx) + d$. **3**

$y = 2\sin\left(\frac{3}{2}x\right) + 3$

3mks all correct
2mks 2 correct variables
1mk 1 correct variable

Question 27 (7 marks) 4+3x

The following table shows values of x and y.

x	4	13	26	30	35	50
y	1.8	2.9	3.8	4.2	5.7	5.1

(a) Calculate Pearson's correlation coefficient for the data represented in the table above, correct to 2 decimal places. **1**

0.91 [1mk only]

(b) Identify the direction **and** strength of the linear association between x and y. **2**

strong. positive correlation. [2mks all correct, 1mk either]
1mk 1mk

(c) State the equation of the least-squares regression line in the form $y = ax + b$. **2**

$y = 0.08x + 1.82$ [2mks all correct, 1mk either a or b]
1mk 1mk.

(d) Use the regression equation from part (c) to estimate the value of x when $y = 4.8$ to 2 decimal places. **2**

$4.8 = 0.08x + 1.82$ [2mks all correct, 1mk substitution]
1mk

$\therefore x = 37.25$ 1mk

Question 28 (5 marks)

A continuous random variable X has a function given by

$$f(x)=\begin{cases}\pi\sin(2\pi x) & \text{if } 0\le x\le\frac{1}{2}\\ 0 & \text{elsewhere}\end{cases}$$

(a) Show that the function $f(x)$ is a probability density function. **3**

$\therefore \int_0^{\frac{1}{2}} \pi\sin(2\pi x)\,dx = 1$

$-\pi\left[\frac{\cos(2\pi x)}{2\pi}\right]_0^{\frac{1}{2}} = 1$

$-\frac{1}{2}\left[\cos(2\pi x)\right]_0^{\frac{1}{2}} = 1$

$-\frac{1}{2}\left[\cos\pi - \cos 0\right] = 1$

$-\frac{1}{2}\left[-1-1\right] = 1$

$1 = 1$ $\therefore$ LHS = RHS it is a PDF

3mks all correct
2mks correct integral
1mk let integral = 1

(b) Find the mode of X. **2**

Mode: $f'(x) = 2\pi\cdot\pi\cos(2\pi x)$

Let $f'(x)=0$ for turning point

$0 = 2\pi^2\cos(2\pi x)$

$0 = \cos(2\pi x)$

$(2\pi x) = \frac{\pi}{2}, \frac{3\pi}{2}$

$x = \frac{1}{4}, \frac{3}{4}$

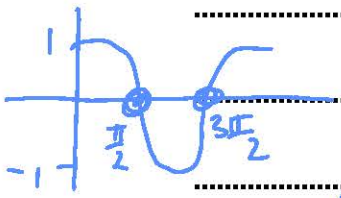
$\therefore x = \frac{1}{4}$

$f''(x) = -4\pi^3\sin(2\pi x)$

$x=\frac{1}{4}, f''(x) = -124$ max

$x=3.14, f''(x) = 124$ min

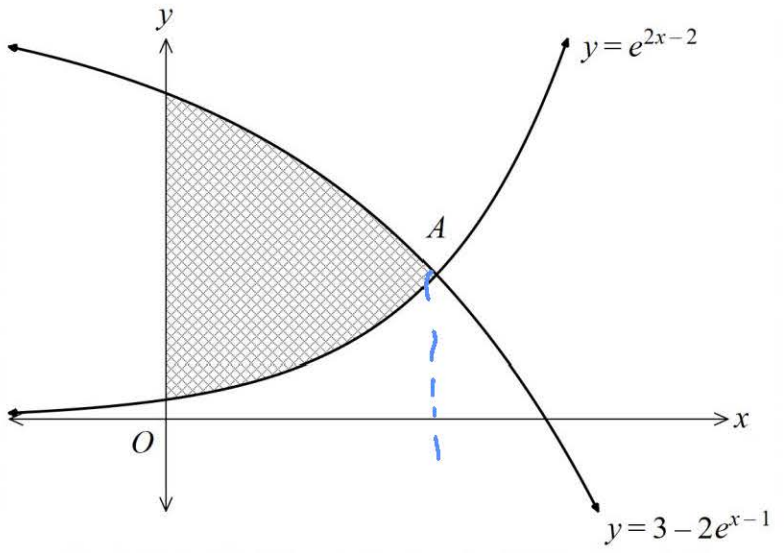
2mks all correct
1mk $x=\frac{1}{4}, x=\frac{3}{4}$ with no test



3.14 -3.14

Question 29 (4 marks)

The shaded region is bound by the functions $y = e^{2x-2}$, $y = 3 - 2e^{x-1}$ and the $y-$axis, as shown in the diagram. Point A is the intersection of the two functions.



(a) Show that the $x-$coordinate of point A is $x = 1$. **2**

Method 1: $e^{2x-2} = 3 - 2e^{x-1}$

$(e^{x-1})^2 + 2e^{x-1} - 3 = 0$

$u^2 + 2u - 3 = 0$

$(u+3)(u-1) = 0$

$u = -3, \; u = 1$

$\therefore e^{x-1} = -3 \quad e^{x-1} = 1$

No solution $\quad x - 1 = 0$

$x = 1$

Method 2:

$e^{2(1)-2} = e^0 = 1$

$3 - 2e^{1-1} = 3 - 2 = 1$

$\therefore$ LHS = RHS = 1

[2 mks all correct
1 mk working towards]

Question 29 continues on page 25

Question 29 (continued)

(b) Find the **exact** area of the shaded region. **2**

$$A = \int_0^1 \left(3 - 2e^{x-1} - e^{2x-2}\right) dx$$

[2mk all correct
1mk correct integral]

$$A = \left[3x - 2e^{x-1} - \frac{e^{2x-2}}{2}\right]_0^1$$

$$A = \left[3 - 2 - \frac{1}{2}\right] - \left[0 - 2e^{-1} - \frac{e^{-2}}{2}\right]$$

$$A = \frac{1}{2} - \left[-\frac{2}{e} - \frac{1}{2e^2}\right]$$

$$A = \frac{1}{2} + \frac{2}{e} + \frac{1}{2e^2} \quad \text{or} \quad A = \frac{e^2 + 4e + 1}{2e^2} \; a^2$$

End of Question 29

Please turn over

Question 30 (8 marks)

Kareem borrows \$140 000 to start his own business. He plans to repay the loan in equal monthly instalments of \$$M$ at 6% per annum reducible interest. Interest is calculated and charged just before each repayment is made.

Let A_n be the amount owing after n repayments (in dollars).

(a) Show that $A_n = 140\,000(1.005)^n - 200M(1.005^n - 1)$. **3**

$A_1 = 140{,}000(1.005) - m$

$A_2 = [140{,}000(1.005) - m] \times 1.005 - m$

$A_2 = 140{,}000(1.005)^2 - m(1.005) - m$

$A_3 = [140{,}000(1.005)^2 - m(1.005) - m] \times 1.005 - m$

$A_3 = 140{,}000(1.005)^3 - m(1.005)^2 - m(1.005) - m$

$A_3 = 140{,}000(1.005)^3 - m(1 + 1.005 + 1.005^2)$

$A_n = 140{,}000(1.005)^3 - m(1 + 1.005 + 1.005^2 + \ldots + 1.005^{n-1})$

$A_n = 140{,}000(1.005)^n - m \times \dfrac{1(1.005^n - 1)}{0.005}$

$A_n = 140{,}000(1.005)^n - 200m(1.005^n - 1)$

3mks all correct
2mks for A2/A3
1mk for A1

Question 30 continues on page 27

Question 30 (continued)

(b) Kareem would like to repay the loan in exactly 15 years. Calculate the amount of each instalment. **2**

$0 = 140{,}000(1.005)^{180} - 200m(1.005^{180} - 1)$ 1mk

$200m(1.005^{180} - 1) = 140{,}000(1.005)^{180}$

$m = \dfrac{140{,}000(1.005)^{180}}{200(1.005^{180} - 1)}$

$m = \$1181.40$

1mk

2 mks all correct
1mk correct substitution

(c) Kareem can afford to repay $1 850 per month. How many years would it take him to repay the loan? **3**

$0 = 140{,}000(1.005)^{n} - 200(1850)(1.005^{n} - 1)$ 1mk

$0 = 140{,}000(1.005)^{n} - 370{,}000(1.005^{n}) + 379000$

$-370{,}000 = (1.005)^{n}(-230{,}000)$

$1.005^{n} = \dfrac{370{,}000}{230{,}000}$ 1mk

$\therefore n = \dfrac{\ln\left(\frac{37}{23}\right)}{\ln 1.005}$

3mks all correct
2mks correct exponential
1mk correct substitution

$\therefore n = 95.3222\ldots$

$\therefore n = 96$ payments

1mk

$\therefore 7.94$ years / 8 years.

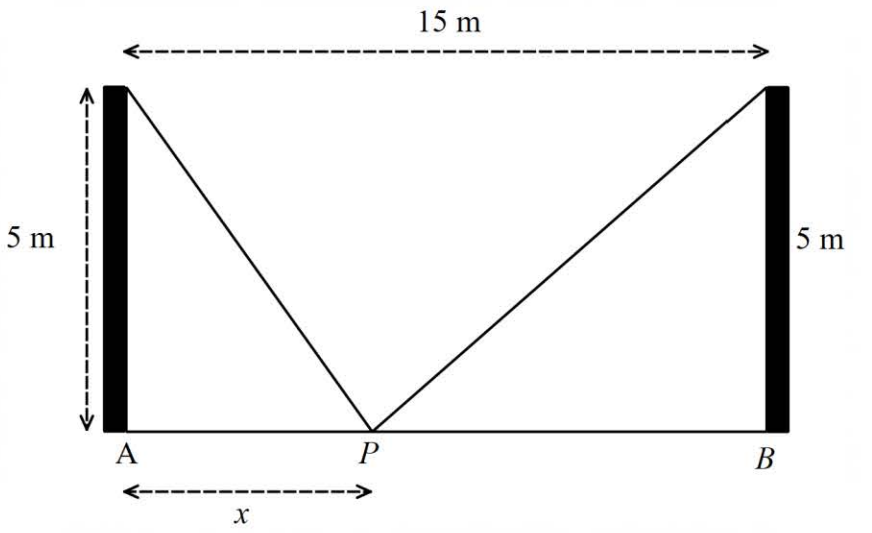
End of Question 30

Please turn over

Question 31 (6 marks)

The diagram shows two 5 m poles, A and B, that are 15 metres apart. The ends of a single length of wire are attached to the top of each pole. The length of wire is then attached to a stake in the ground between them at a moveable point, P, as shown in the diagram below.

Let x be the distance between A and P.



(a) Show that the length of the wire, L, is given by $L = \sqrt{x^2 + 25} + \sqrt{x^2 - 30x + 250}$. **1**

$L = \sqrt{x^2 + 5^2} + \sqrt{(15-x)^2 + 5^2}$

$L = \sqrt{x^2 + 25} + \sqrt{225 - 30x + x^2 + 25}$

1 mk

$L = \sqrt{x^2 + 25} + \sqrt{x^2 - 30x + 250}$

[1 mk only]

Question 31 continues on page 29

Question 31 (continued)

(b) Show that $\frac{dL}{dx} = \frac{x}{\sqrt{x^2+25}} + \frac{x-15}{\sqrt{x^2-30x+250}}$. **2**

$L = (x^2+25)^{1/2} + (x^2-30x+250)^{1/2}$ 1mk

$\frac{dL}{dx} = \frac{1}{2}(x^2+25)^{-1/2} \cdot 2x + \frac{1}{2}(x^2-30x+250)^{-1/2} \cdot (2x-30)$ 1mk

$\frac{dL}{dx} = \frac{x}{\sqrt{x^2+25}} + \frac{x-15}{\sqrt{x^2-30x+250}}$

(c) Find the value of x needed to minimise the length of wire, L used.

Justify your answer. **3**

let $\frac{dL}{dx} = 0$

$\therefore 0 = \frac{x}{\sqrt{x^2+25}} + \frac{x-15}{\sqrt{x^2-30x+250}}$ (1)

$0 = x\sqrt{x^2-30x+250} + (x-15)\sqrt{x^2+25}$ 1mk $\frac{dL}{dx} = 0$

$-(x-15)\sqrt{x^2+25} = x\sqrt{x^2-30x+250}$

$(15-x)\sqrt{x^2+25} = x\sqrt{x^2-30x+250}$

$(15-x)^2(x^2+25) = x^2(x^2-30x+250)$

$(225-30x+x^2)(x^2+25) = x^4-30x^3+250x^2$

$225x^2-30x^3+x^4+5625-750x+25x^2-x^4+30x^3-250x^2 = 0$

$\therefore 750x = 5625$

$x = 7.5$ (1)

3mks all correct
2mks $x=7.5$, no test
1mk

Test:

x	7	7.5	8
y'	-0.03	0	0.03

(1)

− 0 +

$\therefore$ minimum

End of Question 31
End of paper