



ABBOTSLEIGH

Student's Name: _____

Student Number:

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Teacher's Name: _____

2025 HSC ASSESSMENT TASK 4

Mathematics Advanced

General Instructions

- Reading time – 10 minutes
- Working time – 3 hours
- Write using black pen
- NESA approved calculators may be used
- A reference sheet is provided
- Diagrams are not to scale unless indicated
- Ensure your HSC candidate number is on each booklet
- For questions in Section I, answer on the sheet provided.
- For questions in Section II, show relevant mathematical reasoning and/or calculations

Total marks:
100

Section I – 10 marks (pages 3 - 7)

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section

Section II – 90 marks (pages 9 - 49)

- Attempt Questions 11 – 16
- Allow about 2 hours and 45 minutes for this section

Outcomes to be assessed:

Mathematics Advanced

Preliminary:

A student

- MA11-1** uses algebraic and graphical techniques to solve, and where appropriate, compare alternative solutions to problems
- MA11-2** uses the concepts of functions and relations to model, analyse and solve practical problems
- MA11-3** uses the concepts and techniques of trigonometry in the solution of equations and problems involving geometric shapes
- MA11-4** uses the concepts and techniques of periodic functions in the solutions of trigonometric equations or proof of trigonometric identities
- MA11-5** interprets the meaning of the derivative, determines the derivative of functions and applies these to solve simple practical problems
- MA11-6** manipulates, solves expressions using the logarithmic & index laws, uses logarithms, exponential functions to solve practical problems
- MA11-7** uses concepts and techniques from probability to present and interpret data and solve problems in a variety of contexts, including the use of probability distributions
- MA11-8** uses appropriate technology to investigate, organise, model and interpret information in a range of contexts
- MA11-9** provides reasoning to support conclusions which are appropriate to the context

HSC:

A student

- MA12-1** uses detailed algebraic and graphical techniques to critically construct, model and evaluate arguments in a range of familiar and unfamiliar contexts
- MA12-2** models and solves problems and makes informed decisions using mathematical reasoning and techniques
- MA12-3** applies calculus techniques to model and solve problems
- MA12-4** applies the concepts and techniques of arithmetic and geometric sequences and series in the solution of problems
- MA12-5** applies the concepts and techniques of periodic functions in the solution of problems involving trigonometric graphs
- MA12-6** applies appropriate differentiation methods to solve problems
- MA12-7** applies the concepts and techniques of indefinite and definite integrals in the solution of problems
- MA12-8** solves problems using appropriate statistical processes
- MA12-9** chooses and uses appropriate technology effectively in a range of contexts, models and applies critical thinking to recognise appropriate times for such use
- MA12-10** constructs arguments to prove and justify results and provides reasoning to support conclusions which are appropriate to the context

Section I (10 marks)

Attempt Questions 1 – 10

Use the multiple-choice answer sheet

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample $2 + 4 =$ (A) 2 (B) 6 (C) 8 (D) 9

(A) ☐ (B) ☒ (C) ☐ (D) ☐

If you think you've made a mistake, put a cross through the incorrect answer and fill in the new answer.

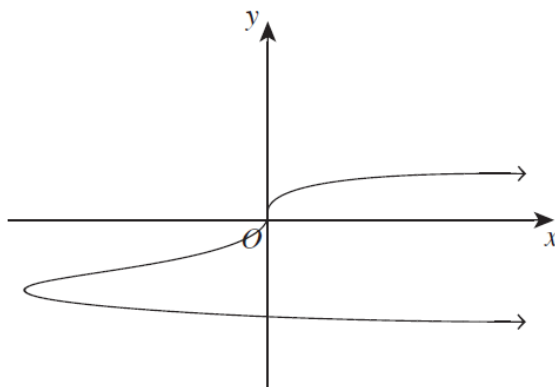
(A) ☒ (B) ☒ (C) ☐ (D) ☐

If you change your mind and have crossed out what you consider to be the correct answer, then indicate this by writing the word *correct* and drawing an arrow as follows.

(A) ☒ (B) ☒ (C) ☐ (D) ☐

correct ↖

1. The graph of a relation is shown below.



Which type of relationship does the graph represent?

- A. One-to one
- B. One-to-many
- C. Many-to-one
- D. Many-to-many

Section I continues on page 4

2. Consider the function $f(x) = 4^{x+2}$. What is the derivative of $f(x)$ with respect to x ?

- A. $2 \times 4^{x+2}$
- B. $\ln 2 \times 4^{x+2}$
- C. $\ln 4 \times 4^{x+2}$
- D. $(x+2) \times 4^{x+2}$

3. A and B are two independent events such that $P(\bar{A} \cap B) = 0.4$ and $P(B) = 0.5$.

What is the value of $P(A)$?

- A. 0.1
- B. 0.2
- C. 0.3
- D. 0.4

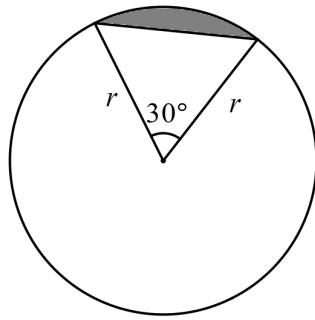
4. Gemma scored 70 on a test in which the marks were normally distributed with mean 50 and standard deviation 15.

What new mark would Gemma be given if the marks were adjusted to have mean 60 and standard deviation 12?

- A. 74
- B. 76
- C. 78
- D. 80

Section I continues on page 5

5. The diagram below shows a circle with radius r cm and a sector with an internal angle of 30° .



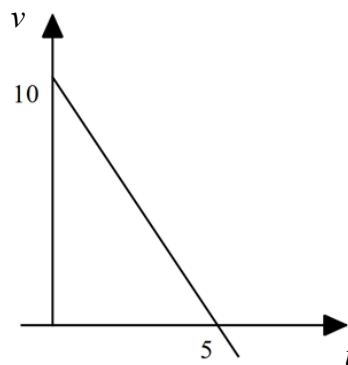
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If the area of the shaded segment is 1.7 cm^2 , which of the following is closest to the radius of the circle?

- A. 0.12 cm
- B. 0.34 cm
- C. 12.00 cm
- D. 144.08 cm
6. A particle is moving along the x -axis. The graph below shows its velocity, v metres per second, at a time, t seconds.

When $t = 0$, the displacement x is equal to 3 metres.

What is the maximum value of the displacement x ?



- A. 10 m
- B. 13 m
- C. 28 m
- D. 53 m

Section I continues on page 6

7. A circular Ferris wheel has a diameter of 40 metres and rotates at a constant rate, completing one full revolution every 2 minutes.
The height of a certain passenger above ground as a function of time can be modelled by the equation:

$$h(t) = 20 \sin\left(\frac{\pi}{60}(t-15)\right) + 30,$$

where $h(t)$ represents the height in metres and t is the time in seconds.

What does the parameter 30 in the equation $h(t) = 20 \sin\left(\frac{\pi}{60}(t-15)\right) + 30$, represent in the context of the Ferris wheel?

- A. the phase shift in time of the passenger on the Ferris wheel.
 - B. The amplitude of the motion of the passenger on the Ferris wheel.
 - C. The period of rotation of the Ferris wheel.
 - D. The vertical shift of the centre of the Ferris wheel above the ground.
8. Given that $\int_1^6 f(x) \, dx = 6$ and $\int_3^6 f(x) \, dx = -4$, what is the value of $\int_1^3 (f(x) + x) \, dx$?
- A. 2
 - B. 6
 - C. 10
 - D. 14

Section I continues on page 7

9. The strength of an earthquake is calculated using Richter's formula:

$$M = \log_{10}(I),$$

where M is the magnitude of the earthquake and I is the intensity.

An earthquake with a magnitude of 5.4 was detected in the desert.

A second earthquake occurred in the same location a few years later and was measured to be twice as intense as the first earthquake.

What was the magnitude of the second earthquake?

- A. $2 \times 10^{5.4}$
- B. $10^{5.4}$
- C. 10.8
- D. 5.7
10. What is the limiting sum of the geometric series $\log_e 2 - (\log_e 2)^2 + (\log_e 2)^3 - (\log_e 2)^4 + \dots$?
- A. $\log_{\frac{e}{2}} 2$
- B. $\log_{\frac{2}{e}} 2$
- C. $\log_{2+e} 2$
- D. $\log_{2e} 2$

End of Section I

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2025 HSC Course

Assessment Task 4

Mathematics Advanced

Section II

90 marks

Attempt Questions 11 - 16

Allow about 2 hour and 45 minutes for this section

Answer the questions in the spaces provided.

These spaces provide guidance for the expected length of response.

Your responses should include relevant mathematical reasoning and/or calculations.

Extra writing space is provided at the end of each question. If you use this space, clearly indicate which question you are answering.

Question 11 (15 marks)

Marks

- (a) Given $f(x) = \log_{10}(3x^2 - 2x)$, find the exact value of $f'(2)$.

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Question 11 continues on page 10

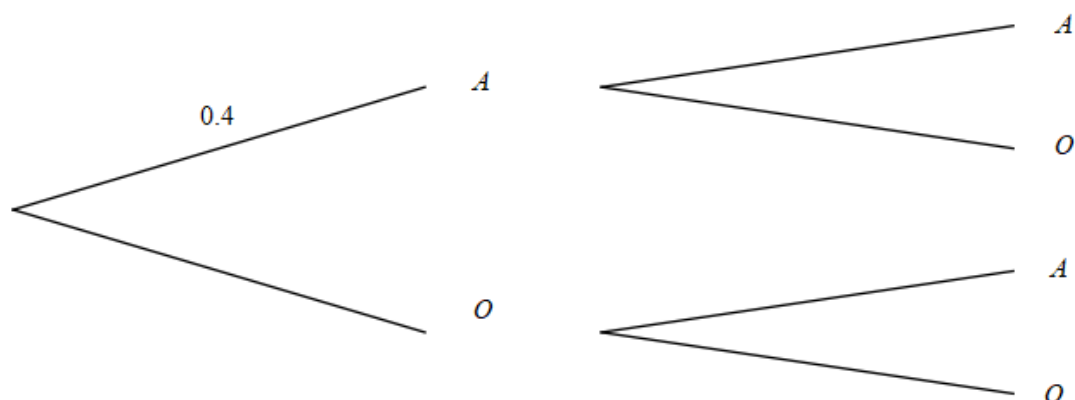
Question 11 (continued)

- (b) In a population 60% are of blood type O , and 40% are of blood type A .

Two people are chosen at random from this population.

- (i) Complete the tree diagram representing this situation.

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- (ii) Calculate the probability that both people are of blood type A .

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- (iii) Calculate the probability that at least one person is of blood type O .

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Question 11 continues on page 11

Question 11 (continued)

(c) Let $f(x) = 4x^2 - 3x$.

Use the definition $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ to find the derivative of $f(x)$.

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Question 11 (continued)

(d) Solve $\log_5 9 = 2\log_5 12 - \log_5 x$. **2**

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(e) Consider the functions $f(x) = \frac{1}{x} - 2$ and $g(x) = x + 3$. **3**

Let $h(x) = f(g(x))$.

Find the domain and range of $h(x)$.

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Question 11 continues on page 13

Question 11 (continued)

(f) (i) Find the derivative of the function $y = \tan(x^2)$. **1**

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(ii) By using an appropriate trigonometric identity, find $\int x \tan^2(x^2) dx$. **2**

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End of Question 11

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Question 12 (16 marks)

- For questions in Question 12, your responses should include relevant reasoning and/or calculations.
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(a) Show that $\log_e(1 + \sin x) - \log_e(1 - \sin x) = 2 \log_e(\sec x + \tan x)$.

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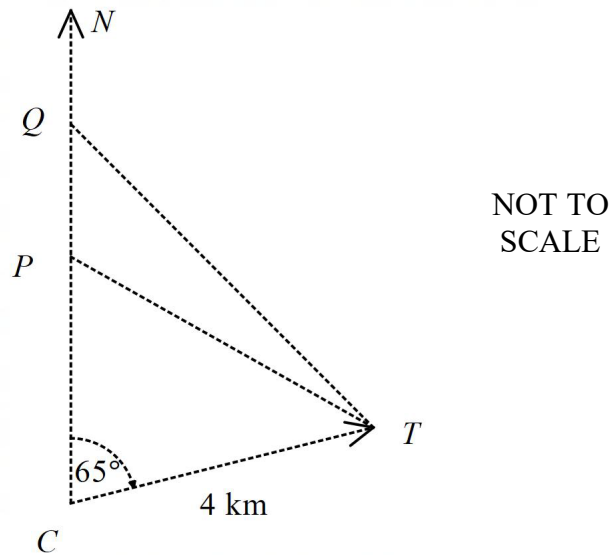
Question 12 continues on page 16

Question 12 (continued)

- (b) Two mountains, P and Q lie directly north of a campground C .

Cathy walks from C on a bearing of 065° , to a point T , a distance of 4km.

The diagram below shows the relative positions, when viewed from above.



- (i) From point T , the base of mountain P is on a bearing of 285° while the base of mountain Q is on a bearing of 315° .

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Show that $\angle QTC = 70^\circ$.

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Question 12 continues on page 17

Question 12 (continued)

(ii) Calculate the distance CQ , correct to 1 decimal place. 2

(iii) Hence or otherwise calculate the distance PQ . 2

Question 12 continues on page 18

Question 12 (continued)

- (c) Tables of future value and present value interest factors for an annuity of \$1 are shown below.

Future Value Interest Factors for an Annuity of \$1				
Period	Rate 2%	Rate 4%	Rate 6%	Rate 8%
1	1.000	1.000	1.000	1.000
2	2.020	2.040	2.060	2.080
3	3.060	3.122	3.184	3.246
4	4.122	4.246	4.375	4.506
5	5.204	5.416	5.637	5.867
6	6.308	6.633	6.975	7.336
7	7.434	7.898	8.394	8.923
8	8.583	9.214	9.897	10.637

Present Value Interest Factors for an Annuity of \$1				
Period	Rate 2%	Rate 4%	Rate 6%	Rate 8%
1	0.9804	0.9615	0.9434	0.9091
2	1.9416	1.8861	1.8334	1.7355
3	2.8839	2.7751	2.6730	2.4869
4	3.8077	3.6299	3.4651	3.1699
5	4.7135	4.4518	4.2124	3.7908
6	5.6014	5.2421	4.9173	4.3553
7	6.4720	6.0021	5.5824	4.8684
8	7.3255	6.7327	6.2098	5.3349

- (i) Kim is saving to go on a holiday. She plans to deposit \$720 at the end of each quarter into an annuity paying 8% p.a. compounding quarterly. What is the future value of her investment after 2 years? 2

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Question 12 continues on page 19

Question 12 (continued)

- (ii) Sophie has decided to join Kim on her trip but needs to borrow \$6000 to cover her costs. What is the present value that Sophie will need to pay each year to repay the loan at 6.0% p.a. compounding annually over 2 years?

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Question 12 continues on page 20

Question 12 (continued)

- (d) A semi-circle on the number plane is defined by the following equation:

$$y = \sqrt{-x^2 + 10x - 16}$$

- (i) Find the centre and radius of the semi-circle.

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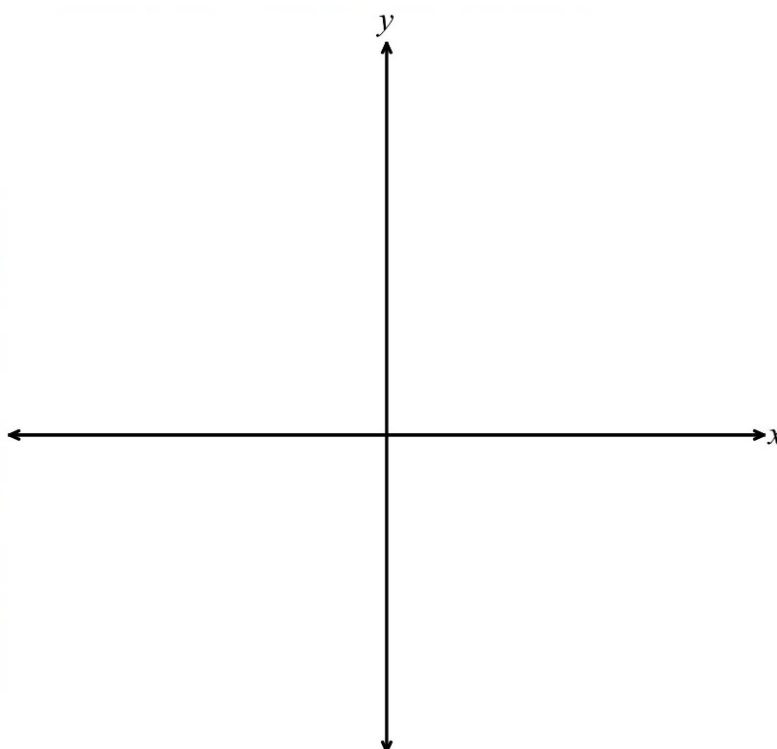
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- (ii) Sketch the curve on the axes below

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Question 12 continues on page 21

Question 12 (continued)

- (iii) Hence or otherwise find $\int_2^8 \sqrt{-x^2 + 10x - 16}$. Leave your answer in exact form. **1**

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End of Question 12

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Question 13 (15 marks)

- For questions in Question 13, your responses should include relevant reasoning and/or calculations.
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(a) Evaluate $\int_1^3 \frac{24x^2 - 14}{4x^3 - 7x + 5} dx$ leaving your answer in simplest exact form.

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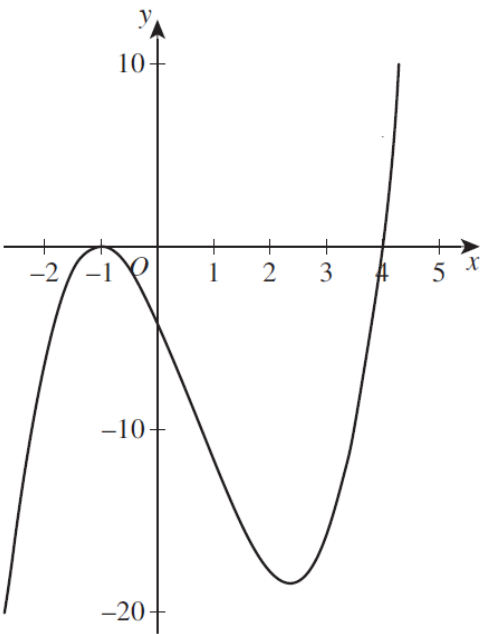
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Question 13 continues on page 24

Question 13 (continued)

(b) The graph of $y = f'(x)$ is shown below:



- (i) Determine the nature of the point on the curve $y = f(x)$ at $x = -1$. Justify your answer with appropriate mathematical reasoning and reference to the graph.
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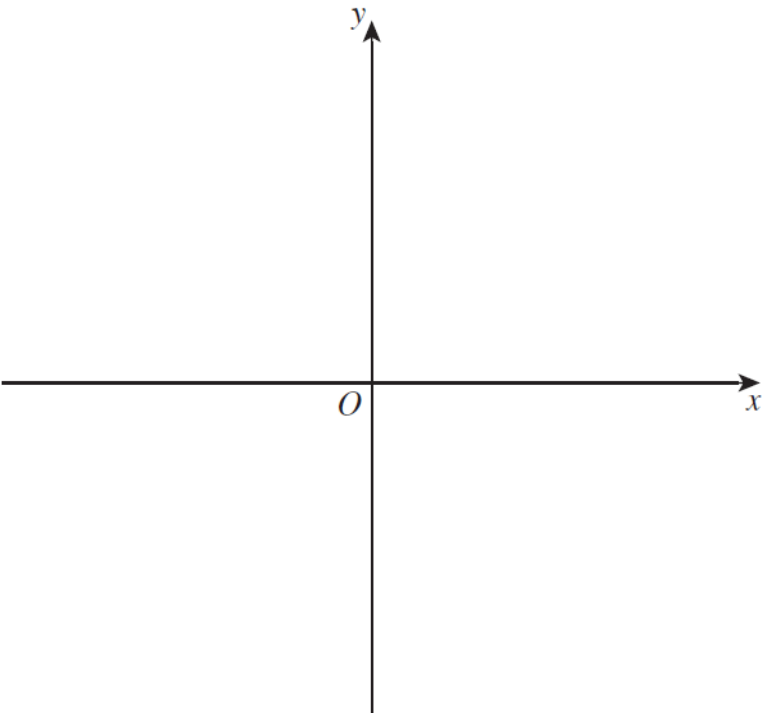
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Question 13 (continued)

- (ii) Sketch a possible graph for $y = f(x)$, showing any stationary points or turning points. **2**



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Question 13 continues on page 26

Question 13 (continued)

- (c) Find the equation of the normal to the curve $y = \frac{\log_e(x+2)}{x+2}$ at the point $(-1, 0)$.3

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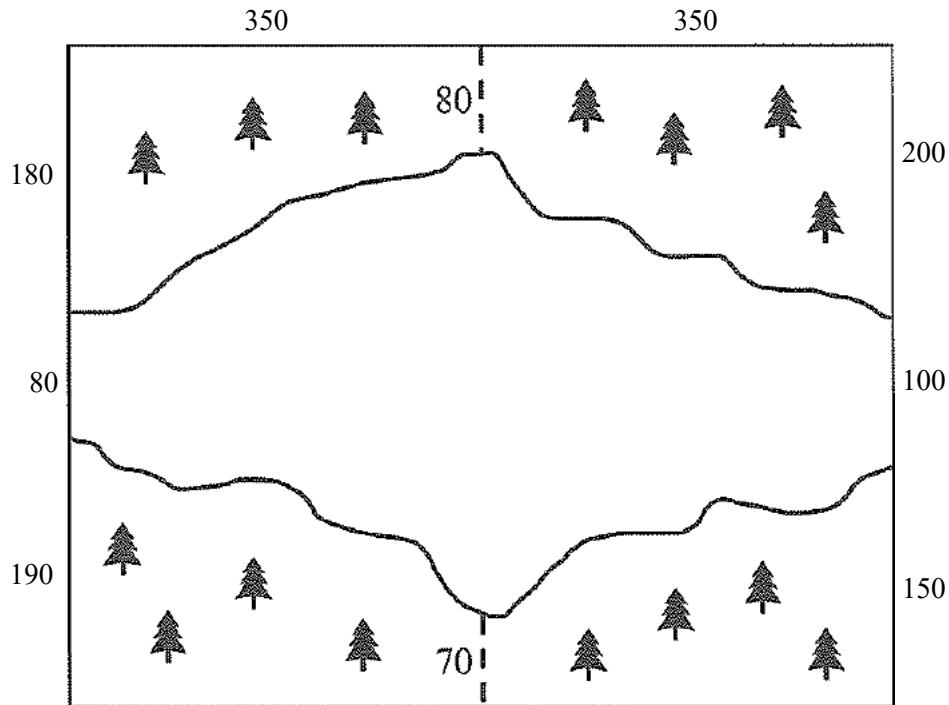
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Question 13 continues on page 27

Question 13 (continued)

- (d) A plane is flying through clouds to drop a water quality monitoring system onto a lake. The lake is surrounded by forest as shown below, with all measurement in metres.

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Estimate the probability that the plane will drop the monitoring system onto the lake.
Use the trapezoidal rule in your calculations.

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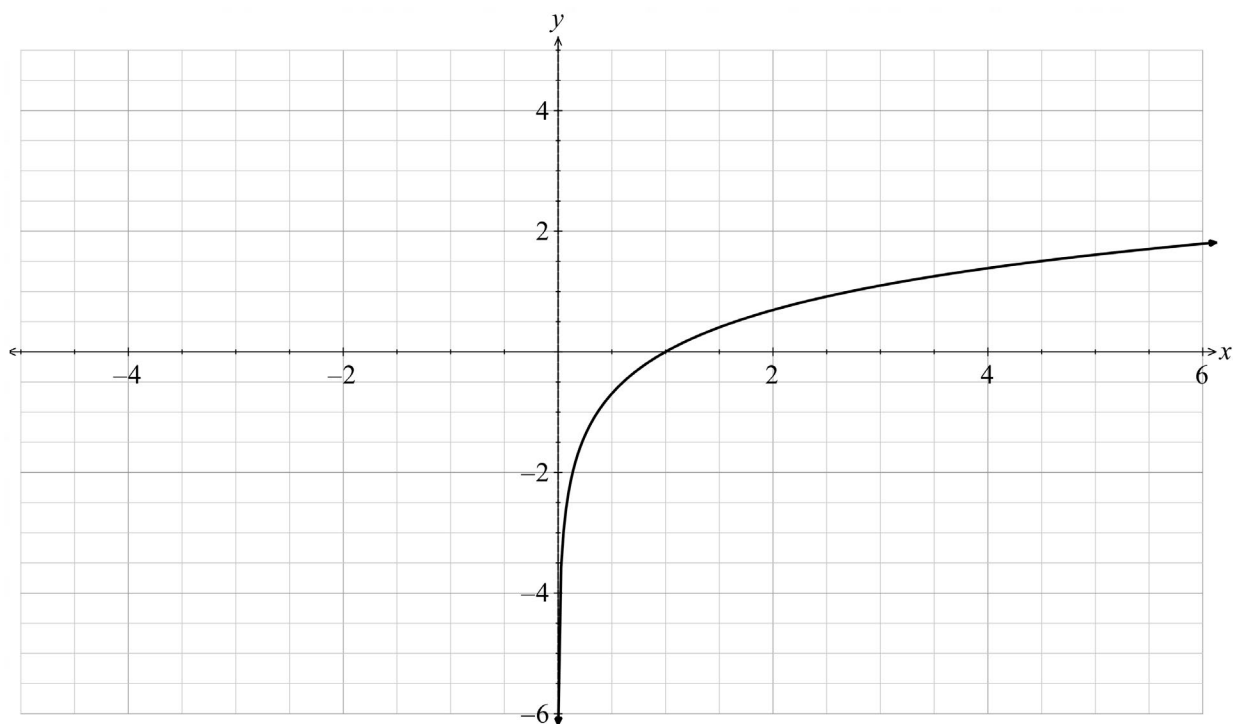
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Question 13 continues on page 28

Question 13 (continued)

- (e) Below is a sketch of the curve $y = \log_e x$.



- (i) Sketch the curve $y = \frac{\log_e (x+3)}{2}$ on the above diagram, showing the intercepts and asymptotes. **2**

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- (ii) Explain the transformations that have occurred to get this new curve. **1**

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End of Question 13

Question 13 Extra writing space

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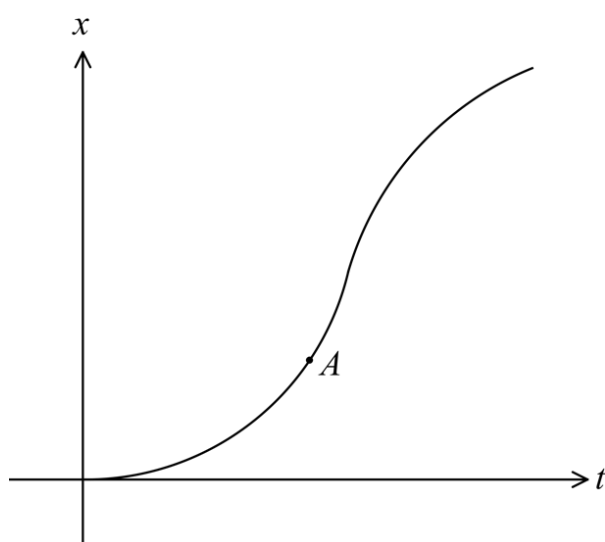
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Question 14 (15 marks)

- For questions in Question 14, your responses should include relevant reasoning and/or calculations.

- (a) This graph shows the displacement x of a particle moving along a straight line as a function of time t .

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Describe the motion of the particle at the point A : is it speeding up or slowing down?
Ensure you make reference to the velocity and acceleration of the particle.

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Question 14 continues on page 32

Question 14 (continued)

- (b) Consider the gradient function $f'(x) = 3(x - 5)(x - 1)$.
The graph of $y = f(x)$ passes through the point $(2, 16)$.

- (i) Show that $f(x) = x^3 - 9x^2 + 15x + 14$. **3**

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- (ii) Find the maximum and minimum values of $f(x)$ in the interval $-2 \leq x \leq 6$. **2**
You do not need to determine the nature of the stationary points.

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Question 14 continues on page 33

Question 14b (continued)

(iii) Show that a point of inflection exists at $(3,5)$. **2**

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(iv) Sketch the graph of $y = f(x)$ in the interval $-2 \leq x \leq 6$, showing the locations of **2**
the endpoints, the stationary points and the point of inflection.

Question 14 (continued)

(c) The discrete random variable X has the probability distribution given in the table below:

x	2	3	4	5
$P(X=x)$	0.3	a	b	0.1

(i) Find the values of a and b , given $\mu = 3.1$. **2**

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(ii) Calculate the standard deviation of X . Give your answer correct to 2 decimal places. **2**

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Question 14 Extra writing space

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Question 15 (15 marks)

- For questions in Question 15, your responses should include relevant reasoning and/or calculations.
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(a) The probability density function of a random variable, x , is given by:

$$f(x) = \begin{cases} \frac{-\sin(\ln x)}{kx} & \text{for } e^{-\pi} \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

(i) Show that $k = 2$.

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Question 15a continues on page 38

Question 15(a) (continued)

$$f(x) = \begin{cases} \frac{-\sin(\ln x)}{kx} & \text{for } e^{-\pi} \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

- (ii) Find $P\left(x < \frac{\pi}{4}\right)$. Give your answer correct to three decimal places.

2

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Question 15 continues on page 39

Question 15 (continued)

- (b) An electric battery is being recharged. The capacity, C %, of the battery while charging at time t minutes may be modelled by the equation

$$C = 100(1 - 2^{-kt}).$$

The battery is charged to 35% capacity after 50 minutes.

- (i) Show that the value of k is 0.01243, correct to 4 significant figures.

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Question 15b continues on page 40

Question 15(b) (continued)

$$C = 100(1 - 2^{-kt})$$

- (ii) To prolong the service life of the battery, the charger may be set to switch off when the battery reaches 90% capacity. 2

For how long will the battery be on charge until it reaches 90% capacity.

Give your answer in hours and minutes, correct to the nearest minute.

[illegible]

Question 15b continues on page 41

Question 15(b) (continued)

- (iii) By considering the first and second derivatives, describe the behaviour of C for $t \geq 0$ if the battery is left on charge indefinitely.
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Question 15 continues on page 42

Question 15 (continued)

- (c) A particle moves along a straight line such that its distance x , in metres, from a fixed point O is given by the equation:

$$x = 1 - 2 \sin \left(2t + \frac{\pi}{6} \right),$$

Where t is measured in seconds from $t = 0$.

- (i) Where is the particle initially. **1**

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- (ii) When and where, does the particle first come to rest? **2**

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Question 15(c) (continued)

- (iii) Calculate how far the particle has travelled at time $t = \frac{7\pi}{24}$.

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Give your answer as an exact value.

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End of Question 15

Question 15 Extra writing space

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Question 16 (14 marks)

- For questions in Question 16, your responses should include relevant reasoning and/or calculations.

(a) A quadratic has equation $y = A + Bx - x^2$.

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When the function is reflected along the y -axis, it is identical to a horizontal translation of -3 .

Given that the curve meets the y -axis at $(0,18)$, determine the area of the region bounded by the curve and the x -axis.

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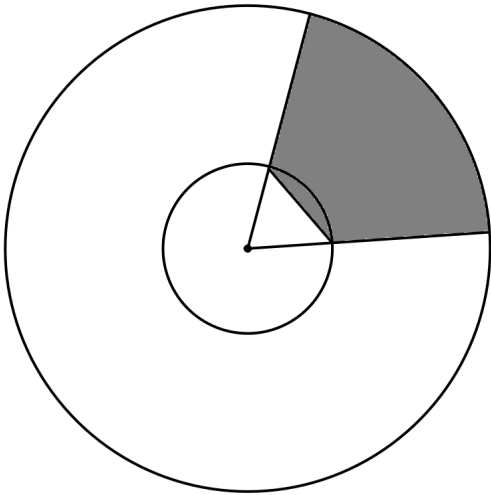
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Question 16 continues on page 46

Question 16 (continued)

- (b) The diagram shows two circles, one with radius $r\sqrt{r}$ and the other with radius r .
The shaded segment has an angle of $\frac{\pi}{3}$ subtended at the centre of the circle.



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- (i) Show that $A_{\text{shaded}} = \frac{\pi}{6}r^3 - \frac{\sqrt{3}}{4}r^2$.

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Question 16(b) (continued)

(ii) Find the value of r at which the area of the shaded segment is a minimum. **3**

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Question 16 (continued)

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- (c) Taylor is purchasing an apartment for \$1 562 500.
- She pays a 20% deposit and will borrow the remaining amount.
- The loan has an annual interest rate of 5.4% compounded monthly and a monthly account keeping fee of \$10 which is added to the balance after the interest is applied each month.
- The loan plus the account fee is to be repaid over 30 years with equal monthly repayments.
- Find the total amount that Taylor will have spent in purchasing the house and outline two things she could do to reduce this amount.

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Question 16c continues on page 49

Question 16 (continued)

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Question 16 Extra writing space

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