



Hills
Grammar

Student Number: _____

Teacher Name: _____

Trial HSC Examinations 2025

Year 12

MATHEMATICS

ADVANCED

Time Allowed: 3 hours + 10 minutes reading time

Class Teachers: Mr. Porvaznik, Mr. Seo, Mrs. Singh and Mrs. Willson

General Instructions

- Reading time – 10 minutes
- Working time – 3 hours
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided at the back of this paper
- For questions in Section II, show relevant mathematical reasoning and/or calculations
- Write your student number and teacher's name at the top of this page
- Extra writing space is provided at the back of this booklet. If you use this space, clearly indicate which question you are answering

Total marks: 100

Section I – 10 marks (pages 3 – 9)

- Attempt Questions 1 – 10
- Answer on Multiple Choice Answer Sheet provided
- Allow about 15 minutes for this section

Section II – 90 marks (pages 10 – 33)

- Attempt Questions 11 – 33
- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
- Allow about 2 hours and 45 minutes for this section

Section I	Your Mark:	10
Section II	Your Mark:	90
Total	Your Mark:	100

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Section I

10 marks

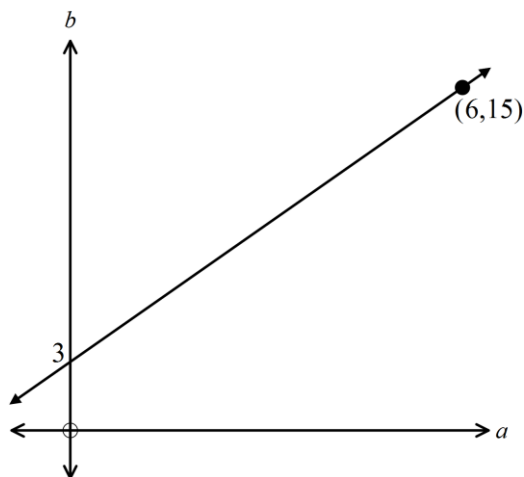
Attempt Questions 1–10

Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1–10.

- 1 Which equation describes the relationship between the two variables a and b ?

- A. $b = 2a + 3$
- B. $b = 3a + 2$
- C. $b = 6a + 3$
- D. $b = 6a + 15$



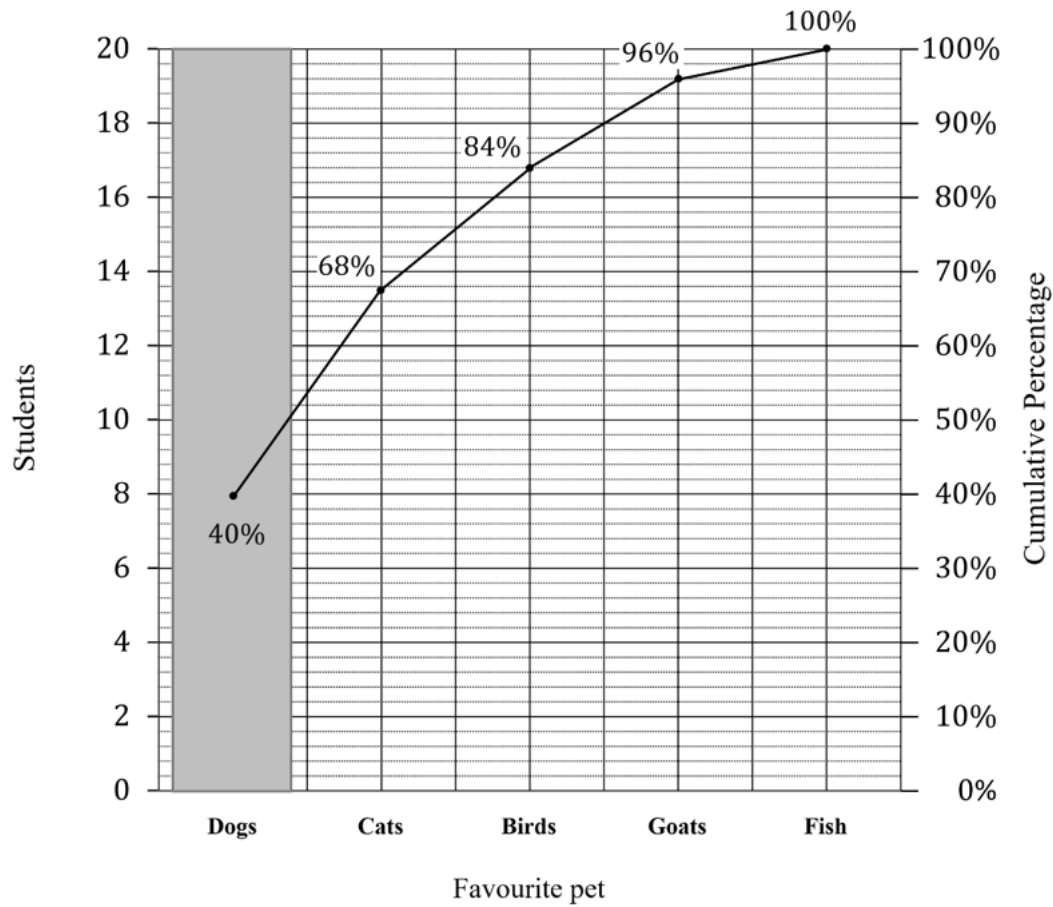
- 2 A and B are two independent events with $P(A) = \frac{5}{8}$ and $P(B) = \frac{1}{6}$.
What is the value of $P(A \cap B)$?

- A. $\frac{5}{48}$
- B. $\frac{11}{24}$
- C. $\frac{11}{20}$
- D. $\frac{15}{4}$

- 3 What is the solution of the equation $x = \log_2 2\sqrt{2} + \log_2 \sqrt{2}$?
- A. $x = 2$
- B. $x = 3$
- C. $x = 4$
- D. $x = 5$
- 4 What is the gradient of the normal to the curve $f(x) = 3x^3 - 4x + 2$ at the point $(-1, 3)$?
- A. 5
- B. -5
- C. $-\frac{1}{5}$
- D. $-\frac{1}{3}$
- 5 What is $\int (e^{2x} + e^{2x+1})dx$?
- A. $\frac{e^{2x}(3-2e)}{2} + c$
- B. $\frac{e^{2x}(3+2e)}{2} + c$
- C. $\frac{e^{2x}(1-e)}{2} + c$
- D. $\frac{e^{2x}(1+e)}{2} + c$

- 6 A teacher asked fifty students to identify their favourite type of pet.

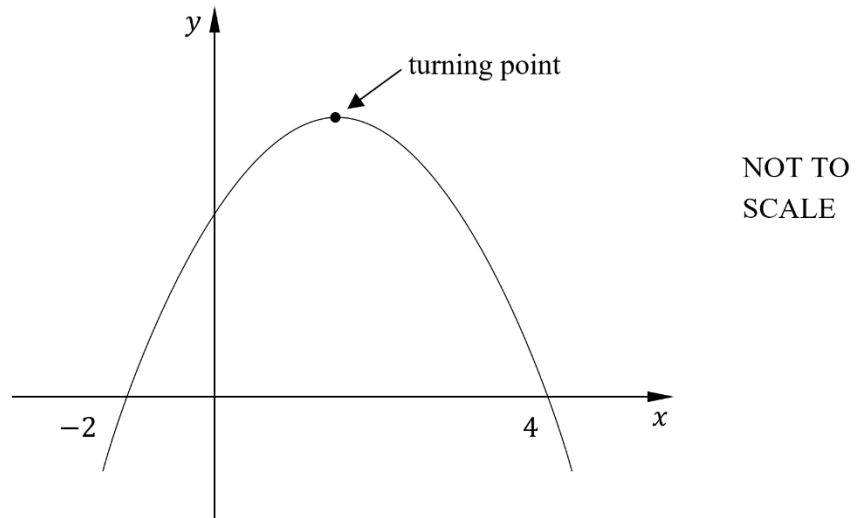
A partially completed Pareto chart shows some of the data collected.



How many students picked cats as their favourite pet?

- A. 12 students
- B. 14 students
- C. 28 students
- D. 34 students

- 7 Consider the diagram of the parabola $y = -x^2 + 2x + 8$.



What are the coordinates of the turning point?

- A. (1, 8)
 - B. (2, 8)
 - C. (1, 9)
 - D. (2, 9)
- 8 Consider the function $f(x) = \ln(x - 2) + \sqrt{3 - x}$.

What is the domain of the function?

- A. $x \in (-\infty, \infty)$
- B. $x \in (0, \infty)$
- C. $x \in (3, 2]$
- D. $x \in (2, 3]$

- 9 The stem and leaf plot below shows the number of lollies that fourteen students in the class have eaten this week.

0	0	0	2	5	
1	1	2	2	6	9
2	0	4	4	7	
3	3				

Two students were absent when this data was collected.

Once their data was added to the data of the rest of the class, the grouped frequency distribution table below was completed.

The cumulative frequency for the 10 – 19 class is marked with an **X**.

Class	Frequency	Cumulative frequency
0 – 9		
10 – 19		X
20 – 29		
30 – 39		

Which of the following values for **X** could be true?

- A. **X** = 10
- B. **X** = 12
- C. **X** = 14
- D. **X** = 16

- 10** The function $y = 2\sin x + 1$, is translated up m units and then vertically dilated by a factor of n from the x -axis, where m and n are positive numbers.

What is the range of the new curve?

- A. $[mn - 2n, mn + n]$
- B. $[mn - n, mn + 3n]$
- C. $[mn + n, mn + 3n]$
- D. $[mn + 2n, mn + n]$

End of Section I

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Section II

90 marks

Attempt Questions 11–35

Allow about 2 hours and 45 minutes for this section

Answer the questions in the spaces provided. These spaces provide guidance for the expected length of responses. There is extra writing space at the end of this booklet.

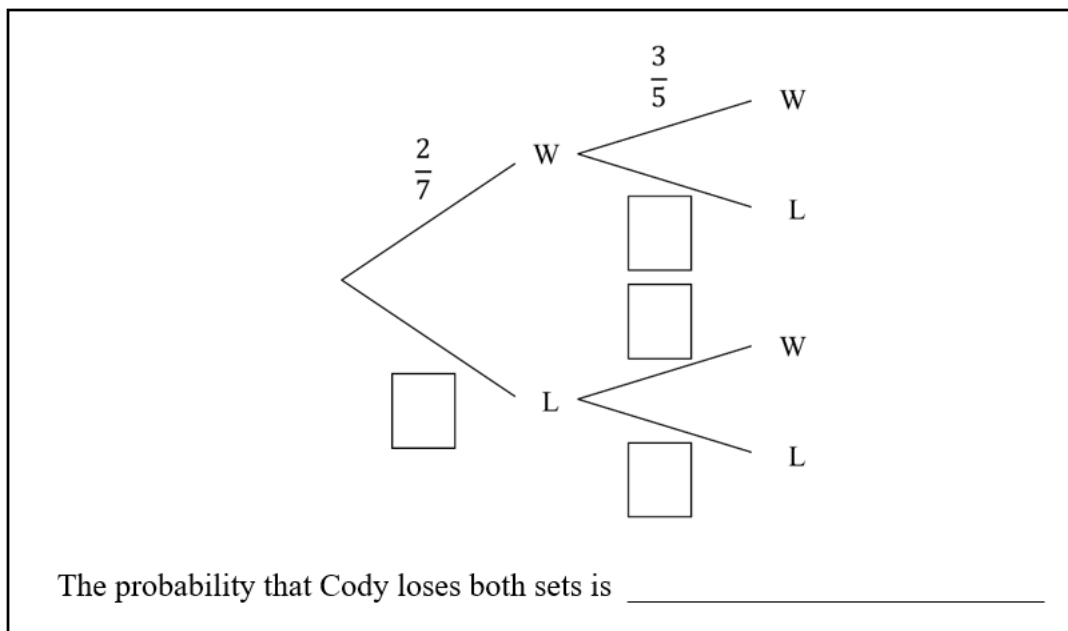
Your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (3 marks)

Cody wins the first set against Sofia in two out of every seven tennis matches they play.

If he wins the first set, he wins the second set three out of every five times, but if he loses the first set, he wins the second set one out of every five times.

- (a) Complete the probability tree below and find the probability that Cody loses both sets. **2**



- (b) Suppose Cody and Sofia play 70 matches in a year. **1**

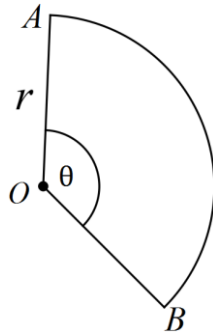
In how many of those matches would Cody expect to win at least one set?

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Question 12 (3 marks)

The sector OAB has an area of $\frac{25\pi}{6}$ square units. The arc AB is $\frac{5\pi}{3}$ units long.

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Find the exact values of r and θ .

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Question 13 (2 marks)

Find the gradient of the tangent to the curve $y = \ln x$ when $x = 2$.

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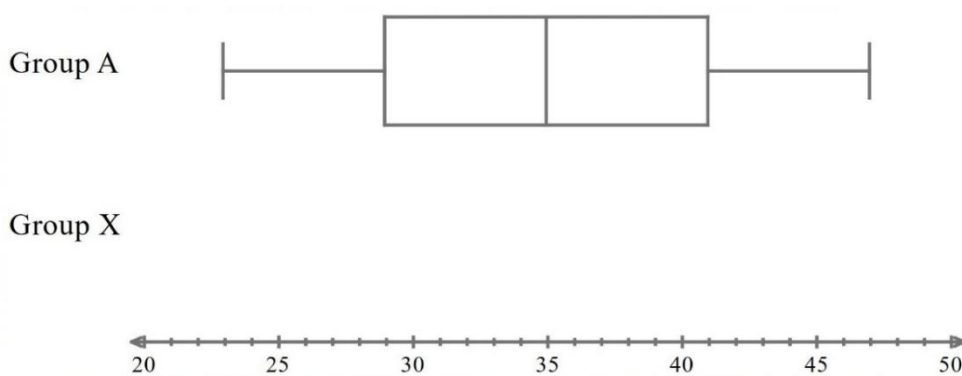
Question 14 (4 marks)

Sets of data on the attention span of participants (in seconds) are collected for two groups of people.

The stem and leaf plot below shows the data for Group X.

2	3	4	9						
3	1	2	5	7					
4	1	2	3	3	4	5	6	7	

The box plot below shows the data for Group A.



- (a) In the space provided above, and using the same scale, draw a box plot for Group X. 2

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- (b) Compare the skew and spread of the two sets of data, quoting appropriate results from the parallel box plots. 2

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Question 15 (3 marks)

Use two applications of the trapezoidal rule to find an approximate value, correct to two decimal places, for

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$$\int_1^3 \ln(x^2 + 1) dx$$

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Question 16 (4 marks)

Differentiate the following with respect to x , leaving your answer in factorised surd form.

(a) $\sqrt{e^{2x} - 2x}$

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(b) $\frac{\sin x}{\sqrt{x}}$

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Question 17 (3 marks)

A table of future values for an annuity of \$1 invested at the start of each period is shown.

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Future Value of \$1 invested at the start of each period						
Periods	1%	2%	3%	4%	5%	6%
5	5.1520	5.3081	5.4684	5.6330	5.8019	5.9753
10	10.5668	11.1687	11.8078	12.4864	13.2068	13.9716
15	16.2579	17.6393	19.1569	20.8245	22.6575	24.6725
20	22.2392	24.7833	27.6765	30.9692	34.7193	38.9927

Liv plans to save \$150,000 over the next 5 years to use as a deposit for buying a house.

She intends to make equal quarterly investments at the start of each quarter for 5 years, and the investments will earn 4% per annum compounded quarterly.

What is the amount of her quarterly investment?

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Question 18 (3 marks)

Find all the values of θ , where $0^\circ \leq \theta \leq 360^\circ$, such that

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$$\sin(2\theta - 30^\circ) = -\frac{1}{2}$$

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Question 19 (3 marks)

Determine the global maximum and minimum values of y for the curve:

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$$y = e^2 + e^x - x e^2 \text{ in the domain } 0 \leq x \leq 5.$$

(Give your answers as exact values.)

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Question 20 (4 marks)

- (a) Show that $\frac{d}{dx}(e^x \tan x) = e^x(\tan^2 x + \tan x + 1)$. **2**

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- (b) Hence evaluate $\int_0^{\frac{\pi}{4}} e^x(\tan^2 x + \tan x + 1)dx$. **2**

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Question 21 (3 marks)

Using log laws or otherwise, differentiate $y = \ln\left(\frac{x^2}{\sqrt{x+1}}\right)$, leaving your answer in simplest form.

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Question 22 (2 marks)

Show $\frac{\sin x}{1 + \cos x} + \frac{\sin x}{1 - \cos x} = 2\operatorname{cosec} x$

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Question 23 (2 marks)

Find $\int \frac{6x^2 + 1}{2x^3 + x + 1} dx$.

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Question 24 (3 marks)

The time it takes for a cupcake at a picnic to be eaten by ants (t minutes) varies inversely with the number of ants eating the cupcake (N ants). When there are 10 ants the cupcake is eaten in 120 minutes.

- (a) The equation relating t and N is $t = \frac{k}{N}$. What is the value of k ?

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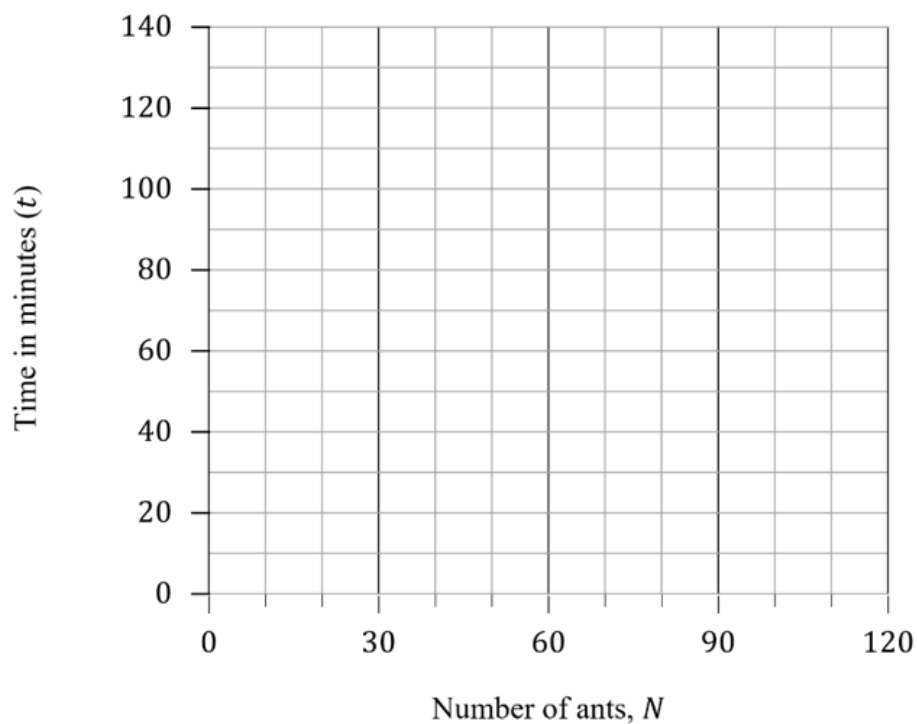
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- (b) By first completing this table of values, graph the curve representing the relationship between the time taken for the cupcake to be eaten and the number of ants eating the cupcake from $N = 10$ to $N = 120$.

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N	10	30	60	120
t	120			



Question 25 (5 marks)

Finn accidentally damages the bottom of his rainwater tank, which initially contains 10 200 litres of water.

- In the first hour, 200 litres leak out.
- In the second hour, a further 250 litres leak out.

Finn suspects the amount leaking each hour may follow an arithmetic or geometric sequence. He wonders how the leak will continue.

- (a) If the amount of water that leaks out of the tank each hour forms an arithmetic sequence, how many hours will it take until the tank is empty? **3**

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Question 25 continues on the next page

Question 25 (continued)

- (b) If the amount of water that leaks out of the tank each hour forms a geometric sequence, show that eight hours after the leak, over 60% of the water remains in the tank. 2

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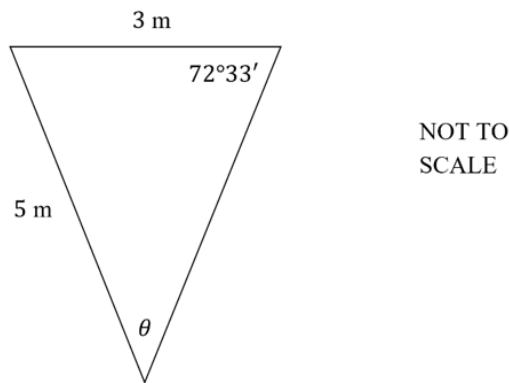
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Question 26 (2 marks)

A triangle has one side that is 5 metres in length with the angle opposite being $72^\circ 33'$ correct to the nearest minute. A second side is 3 metres in length. 2



Show that the size of θ , the acute angle opposite the side of length 3 metres, is $34^\circ 55'$ correct to the nearest minute.

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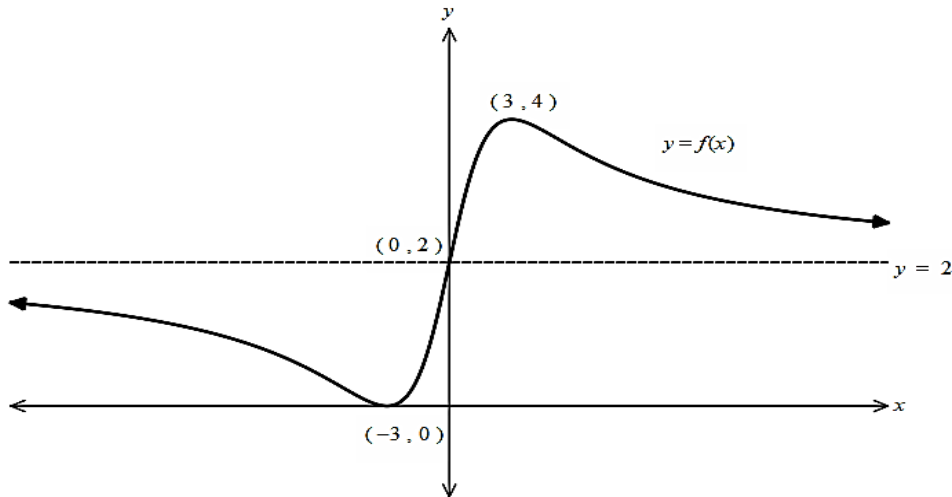
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Question 27 (3 marks)

The figure below shows the graph of a curve with equation $y = f(x)$. The curve meets the x -axis at $(-3, 0)$ and the y -axis at $(0, 2)$. The curve has a maximum at $(3, 4)$ and a minimum at $(-3, 0)$.

The line $y = 2$ is a horizontal asymptote to the curve.



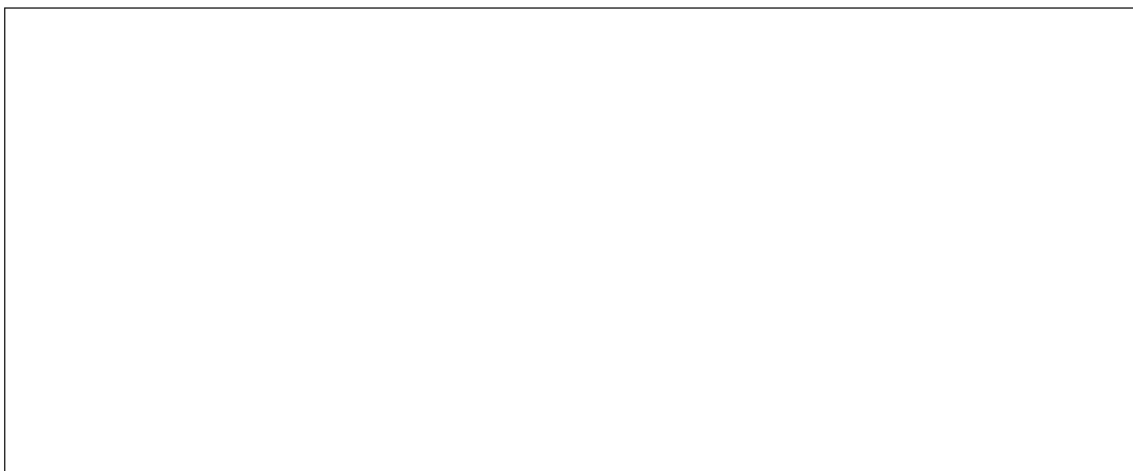
- (a) What is the range of $\frac{1}{2}f(x)$?

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- (b) Sketch the graph of $g(x) = f(3x) - 2$, labelling all points and asymptotes.

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Question 28 (3 marks)

A discrete random variable X has the probability distribution shown below, where r is a constant.

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x	1	2	3	4	5
$P(X=x)$	r	$2r$	$3r$	$2r$	r

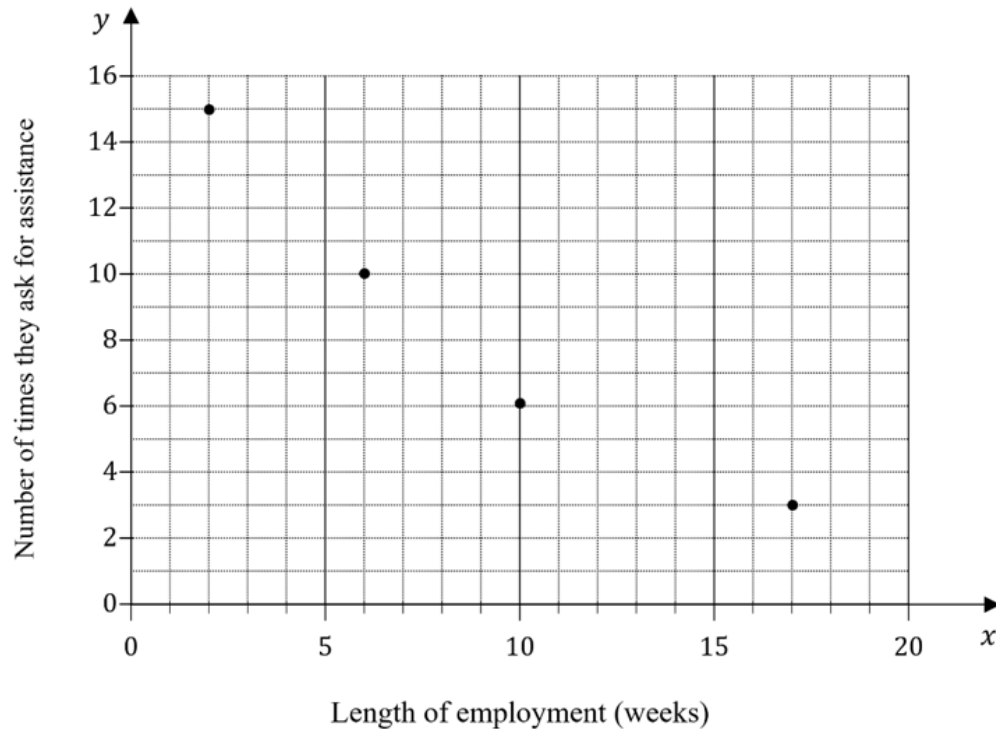
Find $Var(X)$.

This image shows a full page of white paper with horizontal dashed lines, typical of primary school writing paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

Question 29 (4 marks)

A manager is in charge of four teenage employees working in a fast-food restaurant.

The number of weeks they have been working at the restaurant, x , and the number of times they need to ask the manager for assistance during a four-hour shift, y , are shown on the graph below.



- (a) Use a calculator to find the equation of the least-squares regression line for the dataset, finding the gradient and y -intercept to 2 decimal places.

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Question 29 continues on the next page

Question 29 (Continued)

- (b) Using the least-squares regression line for these four employees as a guide, on average how many full weeks will an employee have been working at the restaurant before they require assistance less than 8 times during a four-hour shift?

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Question 30 (5 marks)

A particle moves along the x -axis, and after t seconds its displacement, x metres, is given by the equation $x = (t^2 - 1)^2$ metres.

- (a) Show that the acceleration of the particle is given by the equation $\ddot{x} = 12t^2 - 4 \text{ m/s}^2$. **2**

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- (b) The particle initially moves left, before turning around and moving to the right. **3**
Find the particle's acceleration when it returns to its initial position.

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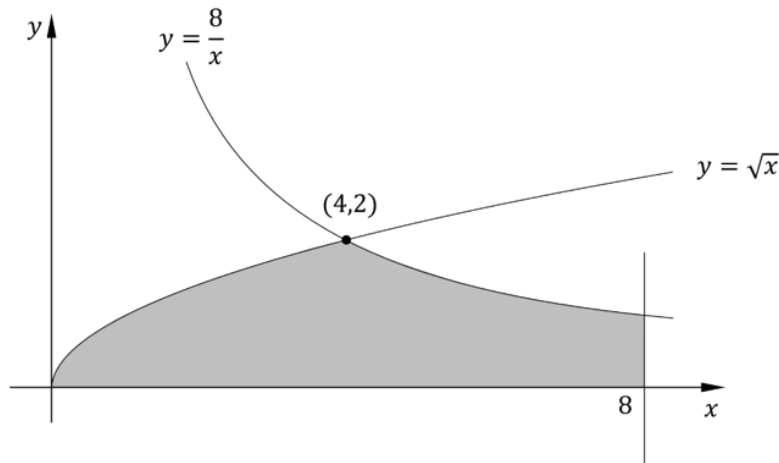
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Question 31 (3 marks)

The curve $y = \sqrt{x}$ intersects the curve $y = \frac{8}{x}$ at $(4,2)$.

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The region bounded by the curve $y = \sqrt{x}$, the curve $y = \frac{8}{x}$, the line $x = 8$ and the x -axis is shaded in the diagram.



Show that the exact area of the shaded region is $\frac{8(2 + 3 \ln 2)}{3}$ units².

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Question 32 (5 marks)

On Arian's first day of work life, he deposited \$10 000 into a superannuation account that earned 4.5% per annum compound annually.

At the beginning of each year after that, he deposited the same amount into the account.

Let A_n be the amount in the account at the end of Arian's n th year of work.

- (a) Show that at the end of Arian's third year of work, his account contained \$32781.91. 2

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- (b) At the end of Arian's 30th year of work the balance of his account was \$637523.88 (Do NOT prove this). **3**

From the beginning of Year 31 to the beginning of Year 45 inclusive, Arian increased his annual deposit to \$30 000, continuing to deposit at the start of each year.

Arian retired after 45 full years of work, with the final interest added at the end of Year 45.

What was the balance in his superannuation account when he retired?

[illegible]

Question 33 (6 marks)

When a car travels at a constant speed of s kilometres per hour, the rate at which it uses fuel, R , is given by the formula

$$R = \frac{s^3}{100000} + \frac{s}{25} + \frac{1}{s} \text{ litres per hour.}$$

A car travels 100 kilometres and the total amount of fuel it uses is Q litres.

- (a) Show that $Q = \frac{s^2}{1000} + 4 + 100s^{-2}$. **2**

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- (b) Show that the speed that minimises the total fuel required for the journey is 17.8 kilometres per hour correct to 1 decimal place. **3**

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- (c) Hence, or otherwise, find the minimum amount of fuel required for the journey, correct to 1 decimal place. **1**

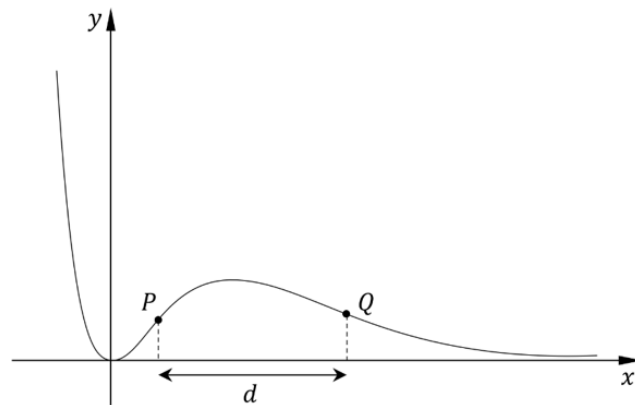
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Question 34 (3 marks)

The graph of $y = f(x)$, where $f(x) = x^2 e^{-x}$, has points of inflection at P and Q as shown below. (Do NOT prove this).

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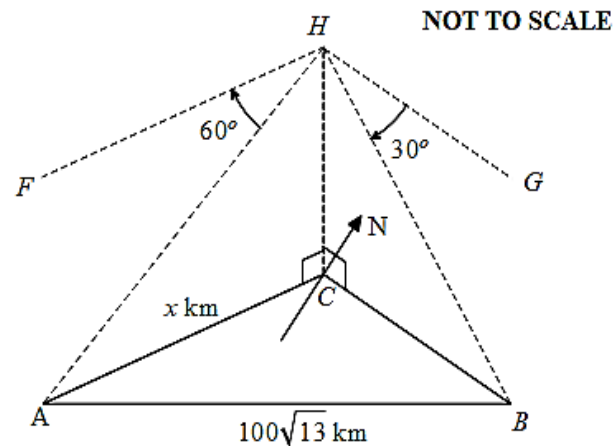
Show that the distance between the x -values of the points of inflection, d , is $2\sqrt{2}$.

[illegible]

Question 35 (9 marks)

The figure below shows three buoys at sea at A , B and C . A helicopter is flying along FH and then HG , where F, G and H are three points of the same height vertically above A, B and C respectively.

When the helicopter reaches H , the angles of depression of A and B from the helicopter are 60° and 30° respectively, while the true bearings of A and B from C are 250° and 130° respectively. The distance between A and B is $100\sqrt{13}$ km.



- (a) Show that the distance between A and C is 100 km.

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Question 35 continues on the next page

Question 35 (continued)

- (c) The destroyers are known to be “on alert” when they are less than 40 km apart.
During what times will the two destroyers be “on alert?”

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End of Paper

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Section I – Multiple Choice Answer Sheet**Student Number:** _____

Select the alternative A, B, C or D that best answers the question. Indicate your choice by filling in the corresponding oval completely.

1 A ○ B ○ C ○ D ○

2 A ○ B ○ C ○ D ○

3 A ○ B ○ C ○ D ○

4 A ○ B ○ C ○ D ○

5 A ○ B ○ C ○ D ○

6 A ○ B ○ C ○ D ○

7 A ○ B ○ C ○ D ○

8 A ○ B ○ C ○ D ○

9 A ○ B ○ C ○ D ○

10 A ○ B ○ C ○ D ○

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Section II Extra writing space

If you use this space, clearly indicate which question you are answering.

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Section II Extra writing space

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