NORMANHURST BOYS HIGH SCHOOL

# MATHEMATICS ADVANCED

2025 Year 12 Course Assessment Task 4 (Trial HSC Examination)
Friday, 1 August 2025

**General instructions**

- Working time – 3 hours. (plus 10 minutes reading time)
- Write using blue or black pen. Where diagrams are to be sketched, these may be done in pencil.
- NESA approved calculators may be used.
- Attempt **all** questions.

**SECTION I**

- Mark your answers on the answer grid provided (on page 32)

**SECTION II**

- All necessary working should be shown in every question. Marks may be deducted for illegible or incomplete working.

**NESA STUDENT #:** ...........................

**Class** (please ✔)

◯ 12MAA.1 – Mr Tran

◯ 12MAX.1 – Miss J. Kim

◯ 12MAX.2 – Miss Chaussivert

◯ 12MAX.3 – Miss Lee

◯ 12MAX.4 – Mr Lam

Marker's use only.

| **QUESTION** | 1-10 | 11-14 | 15-17 | 18-21 | 22-24 | 25-27 | 28-29 | **Total** |
|---|---|---|---|---|---|---|---|---|
| **MARKS** | /10 | /15 | /14 | /16 | /14 | /17 | /14 | /100 |

# Section I

**10 marks**
**Attempt Question 1 to 10**
**Allow approximately 15 minutes for this section**

Mark your answers on the answer grid provided (labelled as page 32).

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**Questions** **Marks**

**1.** What is the period of the function $f(x) = 2\cot(3x)$? **1**

(A) $\dfrac{\pi}{3}$

(B) $\dfrac{\pi}{2}$

(C) $\dfrac{2\pi}{3}$

(D) $\pi$

**2.** For a function $f(x)$, what type of dilation is required to sketch $y = f(4x)$? **1**

(A) Vertical dilation by a factor of $\dfrac{1}{4}$

(B) Horizontal dilation by a factor of $\dfrac{1}{4}$

(C) Vertical dilation by a factor of $4$

(D) Horizontal dilation by a factor of $4$

**3.** Which of the following are the asymptote(s) of the graph of $y = \log(x-1) + 2$? **1**

(A) $x = -1$ only

(B) $x = 1$ only

(C) $y = 2$ only

(D) $x = 1$ and $y = 2$

**4.** What is the solution of $3^x = 5$? **1**

(A) $\dfrac{\log 5}{3}$

(B) $\dfrac{3}{\log 5}$

(C) $\dfrac{\log 5}{\log 3}$

(D) $\log\left(\dfrac{5}{3}\right)$

**5.** Which of the following is the domain of the function **1**

$$f(x) = \frac{1}{\sqrt{x^2 - 4}}$$

(A) $[-2, 2]$

(B) $(-\infty, -2] \cup [2, \infty)$

(C) $(-2, 2)$

(D) $(-\infty, -2) \cup (2, \infty)$

**6.** Which of the following represents the range of the function **1**

$$f(x) = -x^2 - 2x - 3 \text{ for } x \in (-2, 1]$$

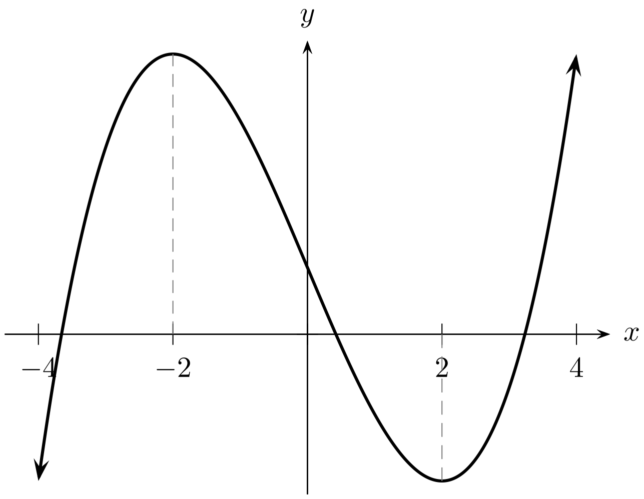
(A) $[-6, -3]$

(B) $[-6, -2]$

(C) $[-6, -3)$

(D) $[-6, -2)$

**7.** From the graph of $y = f(x)$, when is $f'(x)$ negative? **1**




(A) $x < -2$ or $x > 2$

(B) $-2 < x < 2$

(C) $x \leq -2$ or $x \geq 2$

(D) $-2 \leq x \leq 2$

**8.** Find all the values of $k$, such that the equation $x^2 + (4k+3)\,x + \left(4k^2 - \dfrac{9}{4}\right) = 0$ has two real solutions for $x$, where one of the solutions is positive and the other is negative. **1**

(A) $k > -\dfrac{3}{4}$

(B) $k > \dfrac{3}{4}$

(C) $-\dfrac{3}{4} < k < \dfrac{3}{4}$

(D) $k < -\dfrac{3}{4}$ or $k > \dfrac{3}{4}$

**Examination continues overleaf...**

**9.** Evaluate $\displaystyle\lim_{x \to 2} \frac{2-x}{1-\sqrt{x-1}}$ **1**

(A) $-2$ (B) $\dfrac{1}{2}$ (C) $2$ (D) $\infty$

**10.** Using the table below, what is the maximum possible value for $E(X)$? **1**

| $x$ | $-1$ | $0$ | $1$ | $2$ |
|---|---|---|---|---|
| $P(X=x)$ | $k^2$ | $3k$ | $k$ | $-k^2-4k+1$ |

(A) $\dfrac{2}{3}$ (B) $1$ (C) $2$ (D) $\dfrac{73}{12}$

# Section II

**90 marks**
**Attempt Question 11 to 29**
**Allow approximately 2 hours and 45 minutes for this section**

Write your answers in the space provided.

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**Question 11** (3 marks)

Given $f(x) = \sqrt{x}$ and $g(x) = 25 - x^2$.

(a) Find $f\Big(g\left(x\right)\Big)$. **1**

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(b) Find the domain and range of $f\Big(g\left(x\right)\Big)$. **2**

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**Question 12** (3 marks)

Solve for $x$: **3**

$$2\ln\left(x\right) - \ln\left(x+3\right) = \ln\left(\frac{1}{2}\right)$$

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**Examination continues overleaf...**

**Question 13** (6 marks)

(a) The function $y = |x|$ is subjected to the following transformations. For each of the transformations, write the correct equation that arises.

i. Vertical dilation by a factor of 2. **1**

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ii. Horizontal translation of the equation in part (i) to the left by 1 unit. **1**

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iii. Horizontal dilation of the equation in part (ii) by a factor of $\frac{1}{2}$. **1**

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(b) Hence or otherwise, find the value of $a$, where $a > 0$ such that **3**

$$\int_{-\frac{3}{4}}^{a} \left(2\,|2x+1| - 1\right)\, dx = 0$$

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**Question 14** (3 marks)

Initially there are 250 litres of water in a tank. Water starts to flow into the tank. **3**

The rate of increase of the volume of water in litres is given by

$$\frac{dV}{dt} = 250 - 7.5t$$

where $t$ is the time in hours.

Find the volume of water in the tank (correct to the nearest litre) when $\frac{dV}{dt} = 0$.

**Examination continues overleaf...**

**Question 15** (6 marks)

For the function:

$$f(x) = xe^x$$

(a) Find the coordinates of the stationary point(s) and determine the nature, the coordinates of the point(s) of inflection, and sketch the curve of $f(x)$, showing all key features. **5**

(b) Hence or otherwise, find the value(s) of $k$, where $k$ is a real number for which the equation $xe^x = k$ has 2 solutions. **1**

**Question 16** (4 marks)

A random variable $X$ has this probability distribution:

| $x$ | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $P(X = x)$ | 0.2 | 0.1 | 0.25 | 0.3 | 0.15 |

(a) Find $P(X \geq 2 | X < 4)$. **2**

(b) Find the variance of $X$, $\text{Var}(X)$. **2**

**Examination continues overleaf...**

**Question 17** (4 marks)

Joe plays soccer, a sport where a team can win, lose or draw.

Joe is prone to injury, and misses 20% of his team's games.

When Joe plays, his team has a 77% chance of winning and a 10% chance of losing.

When Joe does not play, his team has a 53% chance of winning and a 25% chance of losing.

(a) Calculate the probability that Joe's team loses any given game. **2**

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(b) Given that Joe's team does not lose in a particular match, what is the probability that Joe played in that match? **2**

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**Question 18** (4 marks)

Prove the following identity: **4**

$$\frac{\cos\theta}{\cos\left(\frac{\pi}{2}-\theta\right)}\left(1+\tan^2\theta\right)+\frac{\sin\theta}{\cos\theta}\left(1+\tan^2\left(\frac{\pi}{2}-\theta\right)\right)=2\operatorname{cosec}\theta\sec\theta$$

**Examination continues overleaf...**

**Question 19** (3 marks)

The number of concert tickets sold, $N$, is given by

$$N = 1000 \times 1.6^{kt}$$

Where $t$ is the number of weeks after being available for sale, and $k$ is a constant. Initially, 1000 tickets were sold, and after 2 weeks, the total number of tickets sold was doubled.

(a) Calculate the value of $k$, leaving your answer in exact form. **1**

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(b) At what rate was the number of tickets sold increasing after 4 weeks? Give your answer to the nearest whole number. **2**

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**Question 20** (6 marks)

A fisherman's boat is trying to reach the open sea, however, it is heavily affected by the ocean tides. To reach the open sea, the boat must cross a sandbar which requires the water to be at least 4 metres deep. On a particular day, high tide occurred at midnight, and the depth, $D$, of water over the sandbar on the ensuing day is given by $D = 2\cos\left(\frac{4\pi}{25}t\right) + 3$, where $t$ is the number of hours after midnight.

(a) Sketch the graph of $D = 2\cos\left(\frac{4\pi}{25}t\right) + 3$, for $0 \leq t \leq 25$, showing all key features. **3**

(b) Find the times when the fisherman would not be able to cross the sandbar in his boat. **3**

**Examination continues overleaf...**

**Question 21** (3 marks)

Show that $\displaystyle\int_{-1}^{1} \frac{1}{1+e^{-x}}\,dx = 1$. **3**

**Question 22** (5 marks)

It is known that $T_1 = 1 + \cos A$, $T_2 = \dfrac{1}{2}$ and $T_3 = 1 - \cos A$ form the first three consecutive terms of a geometric sequence, with $T_1 > T_2 > T_3$.

(a) Find the value of $A$, given $\angle A$ is an angle within a triangle. **3**

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(b) Using the value of $A$ found in part (a), show that the series **2**

$$\left(1 + \cos A\right) + \frac{1}{2} + \left(1 - \cos A\right) + \cdots$$

has a limiting sum.

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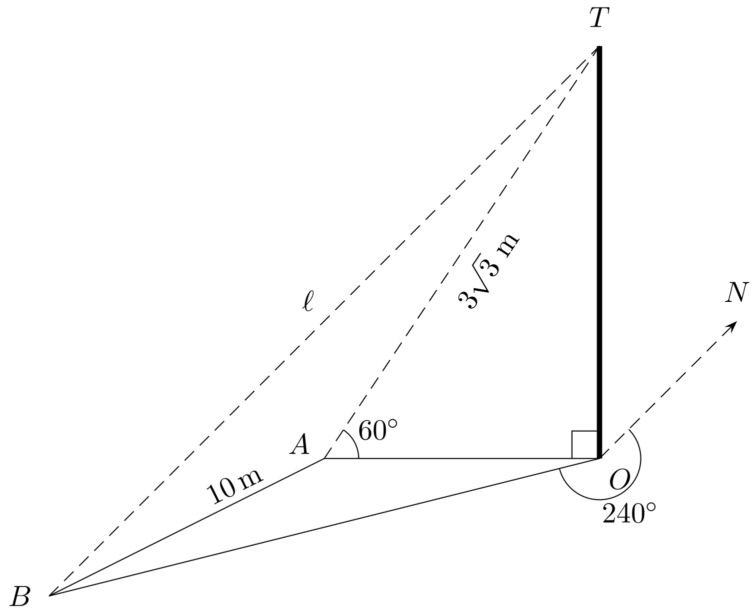
**Examination continues overleaf**...

**Question 23** (3 marks)

If $y^2 = (2m - y)(2n - y)$, show that $\dfrac{1}{m}, \dfrac{1}{y}, \dfrac{1}{n}$ (in this order) forms the terms of an arithmetic sequence. **3**

**Question 24** (6 marks)

A tall pole $OT$ is held in place vertically by ropes anchored by two pegs in the ground 10 metres apart at $A$ and $B$ as shown. $A$ is due west of $O$, and $B$ is at a bearing of 240°T from $O$. The rope connecting the top of the pole $T$ to the peg at $A$ is $3\sqrt{3}$ metres long and makes an angle of 60° with the ground.




(a) Show that $OB = \dfrac{9 + 11\sqrt{13}}{4}$ metres. **4**

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(b) Hence or otherwise, find the length of the rope, $\ell$, connecting $T$ and $B$. Give your answer to the nearest metre. **2**

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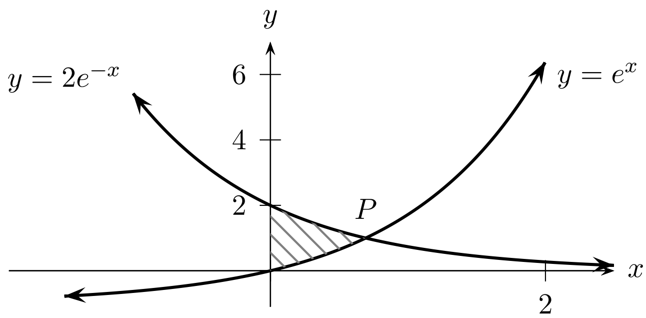
**Examination continues overleaf...**

**Question 25** (3 marks)

The line $y = mx$ is a tangent to the curve $y = \ln(kx)$, where $k > 0$. **3**
Find the value of $m$, in terms of $k$.

**Question 26** (6 marks)

Two curves $y = 2e^{-x}$ and $y = e^x - 1$ intersect at point $P$.



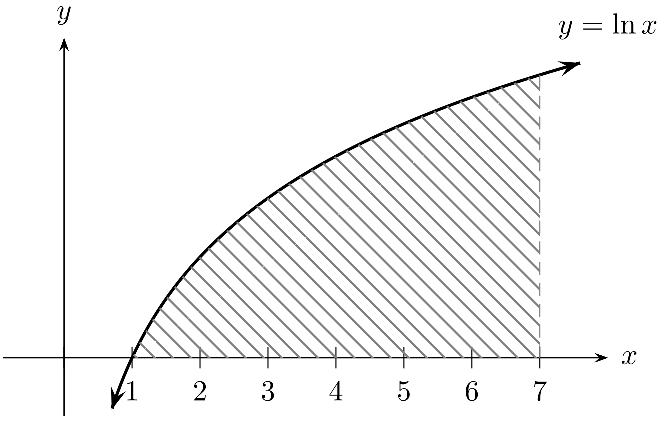

(a) Show that the coordinates of $P$ are $(\ln 2, 1)$. **3**

(b) Hence, calculate the shaded area in simplest exact form. **3**

**Examination continues overleaf...**

**Question 27** (8 marks)

The diagram below shows the graph of $y = \ln x$. The region bounded by the graph, the $x$-axis and the line $x = 7$ has been shaded.




(a) Using the trapezoidal rule with 4 function values, find the approximate shaded area in exact form. **2**

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(b) Find $\dfrac{d}{dx}(x \ln x)$. **2**

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(c) Hence, or otherwise, find $\int_1^7 \ln x \, dx$. **3**

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(d) Deduce that $\ln 7 \approx \frac{1}{3}(\ln 3 + \ln 5) + 1$. **1**

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**Examination continues overleaf...**

**Question 28** (7 marks)

The acceleration of a particle moving along the $x$-axis is given by $\ddot{x} = 4\sin 2t$, where $x$ is the displacement of the particle from the origin in metres, and $t$ is the time in seconds. Initially, the particle is at the origin moving to the left at 1 metre per second.

(a) Show that the velocity of the particle is given by $\dot{x} = 1 - 2\cos 2t$. **2**

(b) Find the time when the particle first comes to rest. **2**

(c) Find the distance travelled by the particle in the first $\dfrac{\pi}{2}$ seconds. **3**

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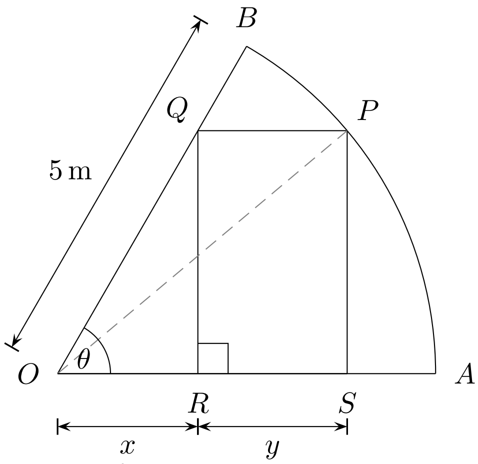
**Examination continues overleaf**. . .

**Question 29** (7 marks)

The diagram shows a rectangle $PQRS$ drawn in a sector $OAB$ with radius 5 metres.
It is given that $\angle AOB = \theta$, $OR = x$, $RS = y$ and $\tan\theta = 2$.




(a) Show that the perimeter of the rectangle is given by: **3**

$$2x + 2\sqrt{25 - 4x^2}$$

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(b) Find the maximum perimeter of the rectangle. Leave your answer in exact form. **4**

**End of paper.**

# Suggested Band 6 Responses

## Section I

**1.** (A) **2.** (B) **3.** (B) **4.** (C) **5.** (D)
**6.** (B) **7.** (B) **8.** (C) **9.** (C) **10.** (C)

## Section II

**Question 11** (3 marks) (Tran)

(a) (1 mark)

- ✓ [1] for correct answer

$$f\left(g\left(x\right)\right) = \sqrt{25 - x^2}$$

(b) (2 marks)

- ✓ [1] for correct domain
- ✓ [2] for correct range

$$\begin{aligned} D_{f(g(x))} &= \{x : \quad x \in [-5, 5]\} \\ R_{f(g(x))} &= \{y : \quad y \in [0, 5]\} \end{aligned}$$

**Question 12** (3 marks) (Tran)

- ✓ [1] for equating argument of logarithms
- ✓ [2] for obtaining all possible solutions
- ✓ [3] for correct answer

$$\begin{aligned} \ln\left(x^2\right) - \ln\left(x+3\right) &= \ln\left(\frac{1}{2}\right) \\ \ln\left(\frac{x^2}{x+3}\right) &= \ln\left(\frac{1}{2}\right) \\ \frac{x^2}{x+3} &= \frac{1}{2} \\ 2x^2 &= x + 3 \\ 2x^2 - x - 3 &= 0 \\ \left(2x-3\right)\left(x+1\right) &= 0 \\ x &= \frac{3}{2} \text{ or } \; -1 \end{aligned}$$

Since $x > 0$:

$$x = \frac{3}{2}$$

**Question 13** (6 marks) (Tran)

(a) i. (1 mark)

✓ [1] for correct answer

$$2f(x) = 2|x|$$

ii. (1 mark)

✓ [1] for correct answer

$$2f(x+1) = 2|x+1|$$

iii. (1 mark)

✓ [1] for correct answer

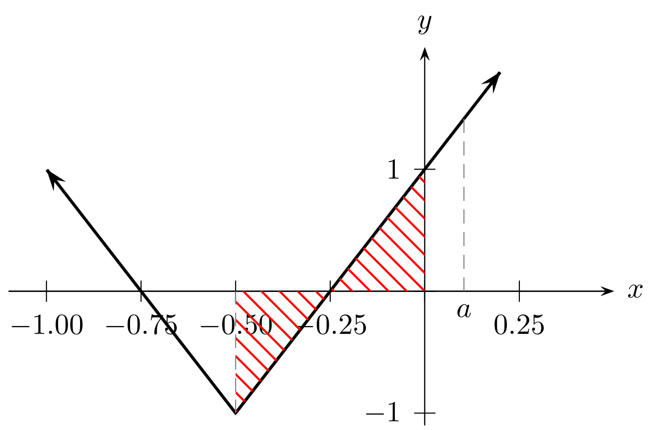
$$2f(2x+1) = 2|2x+1|$$

(b) (3 marks)

✓ [1] for obtaining the correct integral or equivalent merit

✓ [2] for obtaining $16a^2 + 8a - 1 = 0$ or equivalent merit

✓ [3] for correct answer



$$\begin{aligned}
\int_{-0.75}^{-0.5} 2|2x+1|-1\,dx &= \frac{1}{2}\times\frac{1}{4}\times 1 \\
&= \frac{1}{8} \\
\int_{0}^{a} 2|2x+1|-1\,dx &= \frac{1}{8} \\
\int_{0}^{a} 4x+1\,dx &= \frac{1}{8} \\
\Big[2x^2+x\Big]_0^a &= \frac{1}{8} \\
2a^2+a &= \frac{1}{8} \\
16a^2+8a-1 &= 0 \\
a &= \frac{-8\pm\sqrt{8^2-4\times 16}}{2\times 16} \\
&= \frac{-8\pm 8\sqrt{2}}{32} \\
&= -\frac{1}{4}+\frac{\sqrt{2}}{4}\ \ (a>0)
\end{aligned}$$

Alternative Method:

$$\begin{aligned}
\int_{-0.75}^{-0.25} 2|2x+1|-1\,dx &= \frac{1}{2}\times\frac{1}{2}\times 1 \\
&= \frac{1}{4} \\
\int_{-0.25}^{a} 2|2x+1|-1\,dx &= \frac{1}{4} \\
\frac{1}{2}\times\left(a+\frac{1}{4}\right)\times\left(2\left(2a+1\right)-1\right) &= \frac{1}{4} \\
\left(a+\frac{1}{4}\right)\times(4a+1) &= \frac{1}{2} \\
4a^2+2a+\frac{1}{4} &= \frac{1}{2} \\
16a^2+8a-1 &= 0 \\
a &= \frac{-8\pm\sqrt{8^2-4\times 16}}{2\times 16} \\
&= \frac{-8\pm 8\sqrt{2}}{32} \\
&= -\frac{1}{4}+\frac{\sqrt{2}}{4}\ \ (a>0)
\end{aligned}$$

**Question 14** (3 marks) (Tran)

✓ [1] for obtaining $V(t)$ equation or equivalent merit
✓ [2] for finding the value of $t$ or equivalent merit
✓ [3] for correct answer

$$\begin{aligned}
V(t) &= 250t - 3.75t^2 + C \\
\text{Since } V(0) &= 250 \\
C &= 250 \\
V(t) &= 250t - 3.75t^2 + 250 \\
\text{When } \frac{dV}{dt} &= 0 \\
250 - 7.5t &= 0 \\
t &= \frac{100}{3} \\
V\left(\frac{100}{3}\right) &= 250\left(\frac{100}{3}\right) - 3.75\left(\frac{100}{3}\right)^2 + 250 \\
&= 4416.66\cdots \\
&= 4417 \text{ Litres}
\end{aligned}$$

**Question 15** (6 marks) (J. Kim)

(a) (5 marks)

✓ [1] for finding coordinates of stationary point
✓ [2] for showing stationary point is local minimum or equivalent merit
✓ [3] for proving there exist a point of inflection or equivalent merit
✓ [4] for correct shape/scale of graph of $f(x)$ or equivalent merit
✓ [5] for correct solution, all key features were labeled the graph of $f(x)$

$$\begin{aligned}
f'(x) &= e^x + xe^x \\
&= e^x(x+1)
\end{aligned}$$

Let $f'(x) = 0$

$$\begin{aligned}
x &= -1 \ \ (e^x \neq 0) \\
f(-1) &= -\frac{1}{e}
\end{aligned}$$

| $x$ | $-2$ | $-1$ | $0$ |
|---|---|---|---|
| $f'(x)$ | $-0.14$ | $0$ | $1$ |
| $f(x)$ | $\searrow$ | $-\frac{1}{e}$ | $\nearrow$ |

$$\therefore \left(-1, -\frac{1}{e}\right) \quad \text{is a local minimum}$$

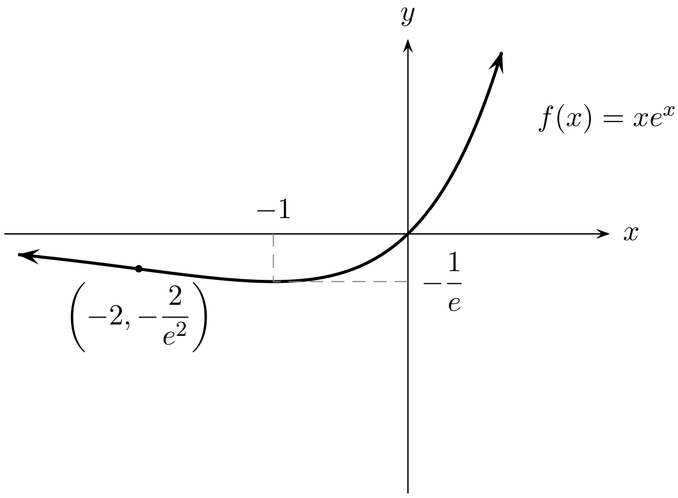
$$\begin{aligned}
f''(x) &= e^x + e^x + xe^x \\
&= e^x(x+2)
\end{aligned}$$

Let $f''(x) = 0$

$$x = -2 \ \ (e^x \neq 0)$$
$$f(-2) = -\frac{2}{e^2}$$

| $x$ | $-3$ | $-2$ | $0$ |
|---|---|---|---|
| $f'(x)$ | $-0.05$ | $0$ | $2$ |
| $f(x)$ | $\frown$ | $-\frac{2}{e^2}$ | $\smile$ |

$$\therefore \left(-2, -\frac{2}{e^2}\right) \quad \text{is a point of inflection}$$




(b) (1 mark)

✓ [1] for correct answer

$$-\frac{1}{e} < k < 0$$

**Question 16** (4 marks) (J. Kim)

(a) (2 marks)

✓ [1] for identifying all possible outcomes or equivalent merit

✓ [2] for correct answer

$$\begin{aligned} P(X \geq 2 | X < 4) &= \frac{P(X \geq 2 \cap X < 4)}{P(X < 4)} \\ &= \frac{0.1 + 0.25}{0.2 + 0.1 + 0.25} \\ &= \frac{7}{11} \end{aligned}$$

(b) (2 marks)

✓ [1] for obtaining $E(X^2)$ or equivalent merit

✓ [2] for correct answer

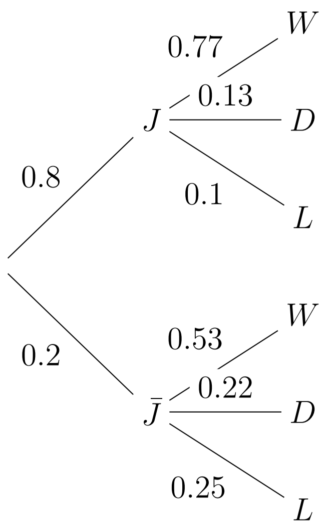
$$\begin{aligned} E(X^2) &= 0.2 + 2^2 \times 0.1 + 3^2 \times 0.25 + 4^2 \times 0.3 + 5^2 \times 0.15 \\ &= 11.4 \\ E(X) &= 0.2 + 2 \times 0.1 + 3 \times 0.25 + 4 \times 0.3 + 5 \times 0.15 \\ &= 3.1 \\ Var(X) &= 11.4 - 3.1^2 \\ &= 1.79 \end{aligned}$$

**Question 17** (4 marks) (J. Kim)

(a) (2 marks)

✓ [1] for identifying all possible outcomes or equivalent merit

✓ [2] for correct answer




$$\begin{aligned} P(L) &= P(JL) + P(\bar{J}L) \\ &= 0.8 \times 0.1 + 0.2 \times 0.25 \\ &= 0.13 \end{aligned}$$

(b) (2 marks)

- ✓ [1] for identifying all possible outcomes or equivalent merit
- ✓ [2] for correct answer

$$\begin{aligned}
P\left(J|\bar{L}\right) &= \frac{P\left(J\cap\bar{L}\right)}{P\left(\bar{L}\right)}\\
&= \frac{0.8\times(0.77+0.13)}{1-0.13}\\
&= \frac{24}{29}
\end{aligned}$$

**Question 18** (4 marks) (J. Kim)

- ✓ [1] for applying trigonometric ratios of complementary angles or equivalent merit
- ✓ [2] for applying trigonometric identities or equivalent merit
- ✓ [3] for significant progress towards answer or equivalent merit
- ✓ [4] for correct solution

$$\begin{aligned}
LHS &= \frac{\cos\theta}{\sin\theta}\times\sec^2\theta+\frac{\sin\theta}{\cos\theta}\times\sec^2\left(\frac{\pi}{2}-\theta\right)\\
&= \frac{\cos\theta}{\sin\theta}\times\frac{1}{\cos^2\theta}+\frac{\cos\theta}{\sin\theta}\times\text{cosec}^2\theta\\
&= \frac{1}{\sin\theta\cos\theta}+\frac{\sin\theta}{\cos\theta}\times\frac{1}{\sin^2\theta}\\
&= \frac{1}{\sin\theta\cos\theta}+\frac{1}{\sin\theta\cos\theta}\\
&= \frac{2}{\sin\theta\cos\theta}\\
&= 2\,\text{cosec}\theta\sec\theta\\
&= RHS
\end{aligned}$$

**Question 19** (3 marks) (J. Kim)

(a) (1 mark)

✓ [1] for correct answer

Since $t = 2$, $N = 2000$

$$\begin{aligned} 1000 \times 1.6^{2k} &= 2000 \\ 1.6^{2k} &= 2 \\ 2k &= \frac{\ln 2}{\ln 1.6} \\ k &= \frac{\ln 2}{2 \ln 1.6} \end{aligned}$$

(b) (2 marks)

✓ [1] for correct derivative or equivalent merit

✓ [2] for correct answer

$$\begin{aligned} \frac{dN}{dt} &= 1000 \times k \times 1.6^{kt} \times \ln 1.6 \\ &= 1000k \ln(1.6) \times 1.6^{kt} \end{aligned}$$

When $t = 4$:

$$\begin{aligned} \frac{dN}{dt} &= 1000 \times \frac{\ln 2}{2 \ln 1.6} \times \ln 1.6 \times 1.6^{\frac{\ln 2}{2 \ln 1.6} \times 4} \\ &= 1386.29 \cdots \\ &= 1386 \text{ tickets/week} \end{aligned}$$

**Question 20** (6 marks) (Lam)

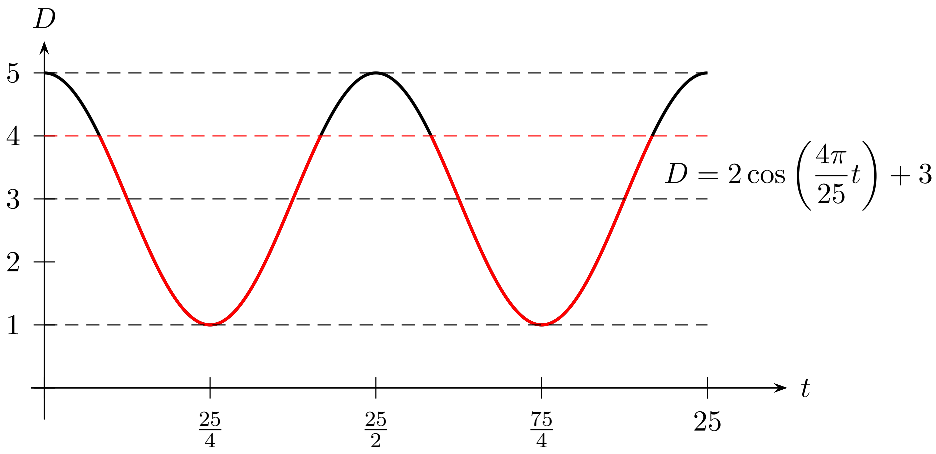
(a) (3 marks)

✓ [1] for one of the correct amplitude, or shape, or period, or equivalent merit.

✓ [2] for two of the correct amplitude, or shape, or period, or equivalent merit.

✓ [3] for correct solution.

$$\begin{aligned} T &= \frac{2\pi}{n} = \frac{2\pi}{\frac{4\pi}{25}} \\ &= \frac{25}{2} \end{aligned}$$




(b) (3 marks)

- ✓ [1] for the correct value of $\cos\left(\frac{4\pi}{25}t\right)$, or equivalent merit. Shape includes the sketching of *two* periods of the graph.
- ✓ [2] for correct values of $t$, or equivalent merit.
- ✓ [3] for correct answer (absolute time values such as 0205 hrs not necessary for this question only)

$$\begin{aligned} 2\cos\left(\frac{4\pi}{25}t\right) + 3 &= 4 \\ \cos\left(\frac{4\pi}{25}t\right) &= \frac{1}{2} \end{aligned}$$

From the graph in (a), these points are the equivalent of the $\frac{\pi}{3}$ points on a non-transformed cosine graph, i.e.

$$\begin{aligned} \frac{4\pi}{25}t &= \frac{\pi}{3}, \frac{5\pi}{3}, 2\pi + \frac{\pi}{3}, 4\pi - \frac{\pi}{3} \\ &= \frac{\pi}{3}, \frac{5\pi}{3}, \frac{7\pi}{3}, \frac{11\pi}{3} \\ 4t &= \frac{25}{3}, \frac{125}{3}, \frac{175}{3}, \frac{275}{3} \\ t &= \frac{25}{12}, \frac{125}{12}, \frac{175}{12}, \frac{275}{12} \\ \therefore \frac{25}{12} < t < \frac{125}{12} &\text{ or } \frac{175}{12} < t < \frac{275}{12} \end{aligned}$$

(Inequalities in $t$ can be read off the graph).

Hence, the ship cannot cross between 0205 hrs to 1025 hrs, and 1435 hrs to 2055 hrs, exclusive at these timing points)

**Question 21** (3 marks) (Lam)

✓ [1] for obtaining an integrand that is integrable by manipulation of the original integrand.

✓ [2] for correct primitive and substitution of limits, or equivalent merit.

✓ [3] for correct answer

$$\begin{aligned}
\int_{-1}^{1} \frac{1}{1+e^{-x}} \frac{\times e^x}{\times e^x}\, dx &= \int_{-1}^{1} \frac{e^x}{1+e^x}\, dx \\
&= \Big[\ln|1+e^x|\Big]_{-1}^{1} \\
&= \ln|1+e| - \ln\left|1+e^{-1}\right| \\
&= \ln\left|\frac{1+e}{1+\frac{1}{e}}\right| \frac{\times e}{\times e} \\
&= \ln\left|\frac{e\cancel{(1+e)}}{\cancel{1+e}}\right| \\
&= \ln e = 1
\end{aligned}$$

**Alternatively,** manipulate integrand as such:

$$\begin{aligned}
\frac{1}{1+e^{-x}} &= \frac{1+e^{-x}-e^{-x}}{1+e^{-x}} \\
&= \frac{1+e^{-x}}{1+e^{-x}} - \frac{e^{-x}}{1+e^{-x}} \\
&= 1 + \frac{-e^{-x}}{1+e^{-x}}
\end{aligned}$$

and then integrate to obtain $(x - \ln|1+e^{-x}|)$ as the primitive.

**Question 22** (5 marks) (Lam)

(a) (3 marks)

- ✓ [1] for recognising a geometric series and subsequently, $\frac{T_3}{T_2} = \frac{T_2}{T_1}$ in terms of the cosine terms.
- ✓ [2] for finding $A = \frac{\pi}{6}$ (other potential solutions include $A = \frac{5\pi}{6}$ and also when $\cos A = -\frac{\sqrt{3}}{2}$, or equivalent merit.
- ✓ [3] for fully correct solution, with due consideration of the possible quadrant for $\angle A$ and also $T_1 > T_2 > T_3$.

$$\begin{aligned}
\frac{\frac{1}{2}}{1+\cos A} &= \frac{1-\cos A}{\frac{1}{2}} \\
1 - \cos^2 A &= \frac{1}{4} \\
\cos^2 A &= \frac{3}{4} \\
\cos A &= \pm\frac{\sqrt{3}}{2} \\
\therefore A &= \frac{\pi}{6}, \frac{5\pi}{6} \quad (\text{as } A < \pi)
\end{aligned}$$

Since $T_1 > T_2 > T_3$, $\frac{T_2}{T_1} < 1$. If $A = \frac{\pi}{6}$,

$$\frac{\frac{1}{2}}{1+\cos\frac{\pi}{6}} = \frac{1}{2+\sqrt{3}} \approx 0.2679 < 1$$

If $A = \frac{5\pi}{6}$,

$$\begin{aligned}
\frac{\frac{1}{2}}{1+\cos\frac{5\pi}{6}} &= \frac{1}{2-\sqrt{3}} \approx 3.73 > 1 \\
A &= \frac{\pi}{6} \text{ only}
\end{aligned}$$

(b) (2 marks)

- ✓ [1] for utilising the value of $A$ to find a value for the common ratio.
- ✓ [2] for fully showing the reason why the series has a limiting sum.

Since $A = \frac{\pi}{6}$,

$$\begin{aligned}
T_1 = 1 + \frac{\sqrt{3}}{2} &\qquad T_2 = \frac{1}{2} \\
r &= \frac{\frac{1}{2}}{1+\frac{\sqrt{3}}{2}} \\
&= \frac{\frac{1}{2}}{\frac{2+\sqrt{3}}{2}} \\
&= \frac{1}{2+\sqrt{3}} \\
&\approx 0.2679\cdots < 1
\end{aligned}$$

Hence a limiting sum exists.

**Question 23** (3 marks) (Lam)

✓ [1] for expansion of $y^2$ equation, or equivalent merit.

✓ [2] for the values of $T_3 - T_2$ and $T_2 - T_1$ in terms of $y$, $m$ and $n$, or equivalent merit.

✓ [3] for correct solution. Note that students who automatically assumed $T_3 - T_2 = T_2 - T_1$ can only receive 2 out of 3 marks maximum, as they assumed the terms were equal prior to showing the terms form an arithmetic sequence.

$$\begin{aligned} y^2 &= (2m-y)(2n-y) \\ \cancel{y^2} &= 4mn - 2my - 2ny + \cancel{y^2} \\ 2my + 2ny &= 4mn \\ y &= \frac{2mn}{m+n} \\ \frac{1}{y} &= \frac{m+n}{2mn} \end{aligned}$$

Inspecting $T_2 - T_1$:

$$\begin{aligned} T_2 - T_1 &= \frac{1}{y} - \frac{1}{m} \\ &= \frac{m+n}{2mn} - \frac{1}{m}{\scriptstyle\frac{\times 2n}{\times 2n}} \\ &= \frac{m+n-2n}{2n} \\ &= \frac{m-n}{2n} \end{aligned}$$

Inspecting $T_3 - T_2$:

$$\begin{aligned} T_3 - T_2 &= \frac{1}{n} - \frac{1}{y} \\ &= {\scriptstyle\frac{\times 2m}{\times 2m}}\frac{1}{n} - \frac{m+n}{2mn} \\ &= \frac{2m-m-n}{2mn} \\ &= \frac{m-n}{2mn} = T_2 - T_1 \end{aligned}$$

As $T_3 - T_2 = T_2 - T_1$, the three terms form an arithmetic sequence.

**Question 24** (6 marks) (Lam)

(a) (4 marks)

✓ [1] for the correct value of $AO$.

✓ [2] for applying cosine rule to $\triangle OAB$, or equivalent merit.

✓ [3] for forming a quadratic equation in terms of $OB$.

✓ [4] for correct answer.

In $\triangle AOT$:

$$\begin{aligned} \cos 60^\circ &= \frac{AO}{3\sqrt{3}} \\ AO &= \frac{3\sqrt{3}}{2} \end{aligned}$$

From the bearing provided,

$$\angle AOB = 270^\circ - 240^\circ = 30^\circ$$

Applying the cosine rule in $\triangle AOB$:

$$\begin{aligned}
10^2 &= \left(\frac{3\sqrt{3}}{2}\right)^2 + (OB)^2 - 2\left(\frac{3\sqrt{3}}{2}\right)(OB)\cos 30^\circ \\
\frac{27}{4} &+ (OB)^2 - 3\sqrt{3}\times\frac{\sqrt{3}}{2}(OB) = 100 \\
(OB)^2 &- \frac{9}{2}(OB) - \frac{373}{4} = 0 \\
4(OB)^2 &- 18(OB) - 373 = 0 \\
OB &= \frac{18 \pm \sqrt{18^2 - 4\times 4\times(-373)}}{2\times 4} \\
&= \frac{18 \pm \sqrt{6292}}{8} \\
&= \frac{18 \pm 22\sqrt{13}}{8} \\
&= \frac{9 \pm 11\sqrt{13}}{4}
\end{aligned}$$

Since $OB > 0$,

$$OB = \frac{9 + 11\sqrt{13}}{4} \text{ metres}$$

(b) (2 marks)

- ✓ [1] for the correct value of $OT$.
- ✓ [2] for correct solution.

$$\begin{aligned}
\sin 60^\circ &= \frac{OT}{3\sqrt{3}} \\
OT &= \frac{9}{2} \\
\ell^2 &= OB^2 + OT^2 \\
&= \left(\frac{9 + 11\sqrt{13}}{4}\right)^2 + \left(\frac{9}{2}\right)^2 \\
\ell &= \pm\sqrt{\left(\frac{9 + 11\sqrt{13}}{4}\right)^2 + \left(\frac{9}{2}\right)^2} \\
&= 12.97\cdots \quad (\ell > 0) \\
&= 13\,\text{m}
\end{aligned}$$

**Question 25** (3 marks) (Chaussivert)

- ✓ [1] for finding $m$ in terms of $x$ or equivalent merit
- ✓ [2] for finding $x$ in terms of $k$ or equivalent merit
- ✓ [3] for correct answer

$$\begin{aligned} y &= \ln(kx) \\ \frac{dy}{dx} &= \frac{k}{kx} \\ &= \frac{1}{x} \\ m &= \frac{1}{x} \end{aligned}$$

Since $y = mx$ and $y = \ln(kx)$ intersect:

$$\begin{aligned} mx &= \ln(kx) \\ \text{Sub } m &= \frac{1}{x} \\ \ln(kx) &= \frac{1}{x} \times x \\ \ln(kx) &= 1 \\ kx &= e \\ x &= \frac{e}{k} \\ \therefore m &= \frac{1}{\frac{e}{k}} \\ &= \frac{k}{e} \end{aligned}$$

**Question 26** (6 marks) (Chaussivert)

(a) (3 marks)

- ✓ [1] for factorisation of exponential function or equivalent merit
- ✓ [2] for finding $x$ or equivalent merit
- ✓ [3] for correct answer

$$\begin{aligned} e^x - 1 &= 2e^{-x} \\ e^{2x} - e^x - 2 &= 0 \\ (e^x - 2)(e^x + 1) &= 0 \\ e^x &= -1 \text{ or } 2 \\ e^x &= 2 \quad (e^x > 0) \\ x &= \ln 2 \\ y &= e^{\ln 2} - 1 \\ &= 1 \\ \therefore P&(\ln 2, 1) \end{aligned}$$

(b) (3 marks)

- ✓ [1] for correct expression of the area or equivalent merit
- ✓ [2] for correct integration or equivalent merit
- ✓ [3] for correct answer

$$\begin{aligned}
A &= \int_0^{\ln 2} 2e^{-x} - e^x + 1 \; dx \\
&= \Big[ -2e^{-x} - e^x + x \Big]_0^{\ln 2} \\
&= -2e^{-\ln 2} - e^{\ln 2} + \ln 2 - \left(-2e^0 - e^0 + 0\right) \\
&= -2\left(\frac{1}{2}\right) - 2 + \ln 2 - (-2 - 1) \\
&= \ln 2 - 3 + 3 \\
&= \ln 2
\end{aligned}$$

**Question 27** (8 marks) (Chaussivert)

(a) (2 marks)

- ✓ [1] for use of trapezoidal rule or equivalent merit
- ✓ [2] for correct answer

| $x$ | 1 | 3 | 5 | 7 |
|---|---|---|---|---|
| $f(x)$ | 0 | $\ln 3$ | $\ln 5$ | $\ln 7$ |

$$\begin{aligned}
h &= \frac{7-1}{3} \\
&= 2 \\
A &= \frac{2}{2}\left[0 + \ln 7 + 2\left(\ln 3 + \ln 5\right)\right] \\
&= 2\ln 3 + 2\ln 5 + \ln 7
\end{aligned}$$

(b) (2 marks)

- ✓ [1] for progress towards answer
- ✓ [2] for correct answer

$$\begin{aligned}
\frac{d}{dx}\left(x \ln x\right) &= \ln x + \frac{1}{x} \times x \\
&= \ln x + 1
\end{aligned}$$

(c) (3 marks)

- ✓ [1] for integrating both sides of equation in part (b) or equivalent merit
- ✓ [2] for correct integration or equivalent merit
- ✓ [3] for correct answer

From part (b):

$$\begin{aligned}
\frac{d}{dx}(x\ln x) &= \ln x + 1 \\
\int_1^7 \ln x + 1 \ dx &= \int_1^7 \frac{d}{dx}(x\ln x) \\
\int_1^7 \ln x \ dx + \int_1^7 dx &= \Big[x\ln x\Big]_1^7 \\
\int_1^7 \ln x \ dx &= \Big[x\ln x\Big]_1^7 - \Big[x\Big]_1^7 \\
&= 7\ln 7 - 0 - (7-1) \\
&= 7\ln 7 - 6
\end{aligned}$$

(d) (1 mark)

- ✓ [1] for correct answer

Equating part (a) and (c):

$$\begin{aligned}
7\ln 7 - 6 &= 2\ln 3 + 2\ln 5 + \ln 7 \\
6\ln 7 &= 2\ln 3 + 2\ln 5 + 6 \\
\ln 7 &\approx \frac{1}{3}(\ln 3 + \ln 5) + 1
\end{aligned}$$

**Question 28** (7 marks) (Lee)

(a) (2 marks)

- ✓ [1] for integration of acceleration or equivalent merit
- ✓ [2] for correct answer

$$\begin{aligned}
\dot{x} &= \int 4\sin 2t \ dt \\
&= -2\cos 2t + C
\end{aligned}$$

When $t = 0$, $\dot{x} = -1$

$$\begin{aligned}
-1 &= -2\cos 0 + C \\
C &= 1 \\
\therefore \dot{x} &= 1 - 2\cos 2t
\end{aligned}$$

(b) (2 marks)

✓ [1] for finding $\cos 2t$ or equivalent merit

✓ [2] for correct answer

When $\dot{x} = 0$, $t = ?$

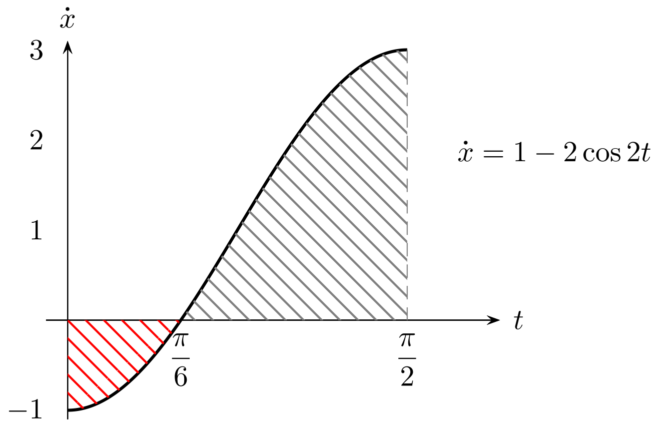
$$\begin{aligned} 1 - 2\cos 2t &= 0 \\ \cos 2t &= \frac{1}{2} \\ 2t &= \frac{\pi}{3} \\ t &= \frac{\pi}{6} \text{ seconds} \end{aligned}$$

(c) (3 marks)

✓ [1] for expression of distance or equivalent merit

✓ [2] for correct integration or equivalent merit

✓ [3] for correct answer




$$\begin{aligned} d &= \left| \int_0^{\frac{\pi}{6}} 1 - 2\cos 2t \;\, dt \right| + \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} 1 - 2\cos 2t \;\, dt \\ &= \left| t - \sin 2t \Big|_0^{\frac{\pi}{6}} \right| + \Big[ t - \sin 2t \Big]_{\frac{\pi}{6}}^{\frac{\pi}{2}} \\ &= \left| \frac{\pi}{6} - \sin\frac{\pi}{3} - 0 \right| + \frac{\pi}{2} - \sin\pi - \left( \frac{\pi}{6} - \sin\frac{\pi}{3} \right) \\ &= \left| \frac{\pi}{6} - \frac{\sqrt{3}}{2} \right| + \frac{\pi}{2} - \left( \frac{\pi}{6} - \frac{\sqrt{3}}{2} \right) \\ &= \frac{\sqrt{3}}{2} - \frac{\pi}{6} + \frac{\pi}{2} - \frac{\pi}{6} + \frac{\sqrt{3}}{2} \\ &= \sqrt{3} + \frac{\pi}{6} \text{ metres} \end{aligned}$$

**Question 29** (7 marks) (Lee)

(a) (3 marks)

- ✓ [1] for finding $QR$ or equivalent merit
- ✓ [2] for finding $y$ or equivalent merit
- ✓ [3] for correct answer

In $\triangle QOR$:

$$\begin{aligned} \tan\theta &= \frac{QR}{x} \\ QR &= 2x \end{aligned}$$

In $\triangle POS$:

$$\begin{aligned} (x+y)^2 + PS^2 &= OP^2 \\ (x+y)^2 + (2x)^2 &= 5^2 \\ (x+y)^2 &= 25 - 4x^2 \end{aligned}$$

Since $x + y > 0$

$$\begin{aligned} x + y &= \sqrt{25 - 4x^2} \\ y &= -x + \sqrt{25 - 4x^2} \\ P &= 2\,(2x) + 2\left(-x + \sqrt{25 - 4x^2}\right) \\ &= 2x + 2\sqrt{25 - 4x^2} \end{aligned}$$

(b) (4 marks)

- ✓ [1] for finding $\dfrac{dP}{dx}$ or equivalent merit
- ✓ [2] for finding $x$ or equivalent merit
- ✓ [3] for showing maximum at $x$ or equivalent merit
- ✓ [4] for correct answer

$$\begin{aligned}
P &= 2x + 2\left(25 - 4x^2\right)^{\frac{1}{2}} \\
\frac{dP}{dx} &= 2 + 2 \times \frac{1}{2}\left(25 - 4x^2\right)^{-\frac{1}{2}} \times -8x \\
&= 2 - \frac{8x}{\sqrt{25 - 4x^2}}
\end{aligned}$$

Let $\dfrac{dP}{dx} = 0$

$$\begin{aligned}
2 - \frac{8x}{\sqrt{25 - 4x^2}} &= 0 \\
\frac{8x}{\sqrt{25 - 4x^2}} &= 2 \\
4x &= \sqrt{25 - 4x^2} \\
16x^2 &= 25 - 4x^2 \\
20x^2 &= 25 \\
x^2 &= \frac{5}{4} \\
x &= \frac{\sqrt{5}}{2} \quad (x > 0)
\end{aligned}$$

| $x$ | $1$ | $\dfrac{\sqrt{5}}{2}$ | $2$ |
|---|---|---|---|
| $\dfrac{dP}{dx}$ | $0.25$ | $0$ | $-\dfrac{10}{3}$ |
| $P$ | $\nearrow$ | $-$ | $\searrow$ |

$\therefore$ maximum at $x = \dfrac{\sqrt{5}}{2}$

$$\begin{aligned}
P &= 2 \times \frac{\sqrt{5}}{2} + 2\sqrt{25 - 4\left(\frac{\sqrt{5}}{2}\right)^2} \\
&= \sqrt{5} + 2\sqrt{20} \\
&= 5\sqrt{5} \text{ metres}
\end{aligned}$$