Student Name: ____________________

Teacher: _________________________

Sydney Technical High School

2025

HIGHER SCHOOL CERTIFICATE
TRIAL EXAMINATION

Mathematics Advanced

General Instructions

- Reading time – 10 minutes
- Working time – 3 hours
- Write using black pen
- Approved calculators may be used
- A reference sheet is provided at the back of this paper
- For questions in Section II, show relevant mathematical reasoning and/or calculations

Total marks : 100

Section I – 10 marks (pages 3 – 6)

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section
- **Answer on the Multiple choice answer sheet on page 37**

Section II – 90 marks (pages 8 – 28)

- Attempt Questions 11 – 35
- Allow about 2 hours and 45 minutes for this section

Markers only:

Q 1 - 10	Q 11 - 18	Q 19 - 23	Q 24 - 29	Q 30 - 35	TOTAL
/ 10	/23	/ 20	/25	/22	/100

Section I

10 marks
Attempt Questions 1–10
Answer on the multiple-choice answer sheet on page 37.
Allow about 15 minutes for this section

<u>Use the multiple-choice answer sheet for Questions 1 – 10</u>

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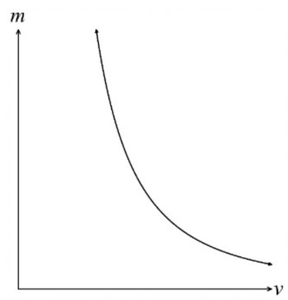
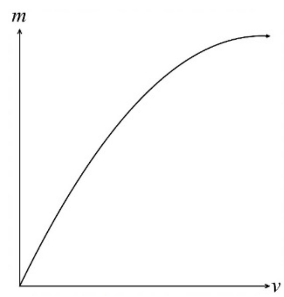
1. The mass of an object is graphed against the velocity of it motion.

Which of the following shows $\frac{dm}{dv} > 0 \; and \; \frac{d^2m}{dv^2} < 0$?

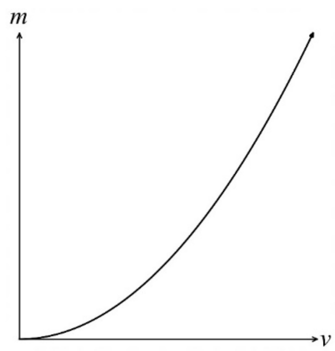
A.



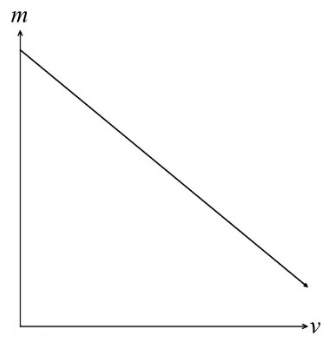
B.



C.



D.



2. Consider the graph of $y = 2x - 5$

   What is the equation of the graph after it has been translated 4 units to the left?

   A. $y = 2x + 1$

   B. $y = 2x - 1$

   C. $y = 2x - 13$

   D. $y = 2x + 3$

3. The solutions to $(2x-1)(x+2)=18$ are:

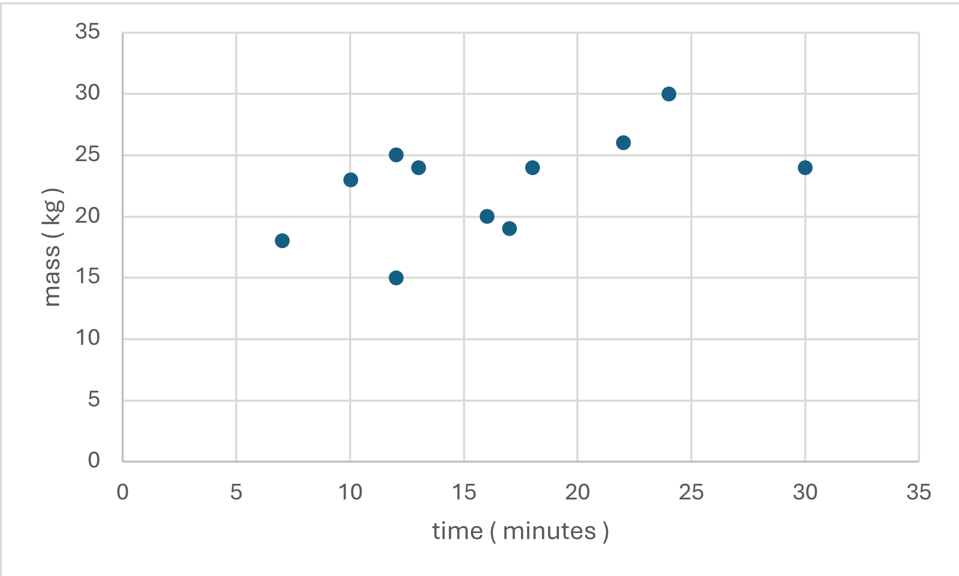
A. $x=\frac{1}{2}, x=-2$

B. $x=-\frac{1}{2}, x=2$

C. $x=\frac{5}{2}, x=-4$

D. $x=-\frac{5}{2}, x=4$

4. The scatter plot below shows the mass in kilograms of kiwifruit being packed compared to the time taken in minutes by a worker on their first 11 days at the job.



Which of the following values if the most appropriate Pearson's correlation coefficient for the data?

A. 0.95

B. 0.55

C. 0.05

D. 0.99

5. The population of a country town is given by the equation $P = 30000e^{-0.06t}$, where t is taken in years from 2025. The time taken for the population to reach a critical 25% of its current population is closest to:

A. 5 years

B. 23 years

C. 233 years

D. 29553 years

6. Twelve students sit in a lecture theatre, with exactly twelve seats. Five in the front row and seven in the second row. Two students are chosen at random from the group to go and present to the rest of the class. What is the probability that they are from different rows?

A. $\frac{35}{132}$

B. $\frac{35}{66}$

C. $\frac{35}{144}$

D. $\frac{35}{72}$

7. For the function defined by the rule $f(x) = 2x^3 - 8x + 6$, the **average rate** of change of $f(x)$ with respect to x on the interval [1,3] is

A. 24

B. 18

C. -2

D. 46

8. A function is written as $y = 4xf(x)$. The derivative of its gradient function is;

A. $y' = 4f'(x)$

B. $y' = 4xf'(x) + 4f'(x)$

C. $y'' = 4xf''(x) + 8f'(x)$

D. $y'' = 0$

9. $\int_4^6 f(x)dx = -5$ and $\int_4^{10} f(x)dx = 3$, then $\int_6^{10} [2f(x) + 1\,]dx$ is equal to:

A. 20

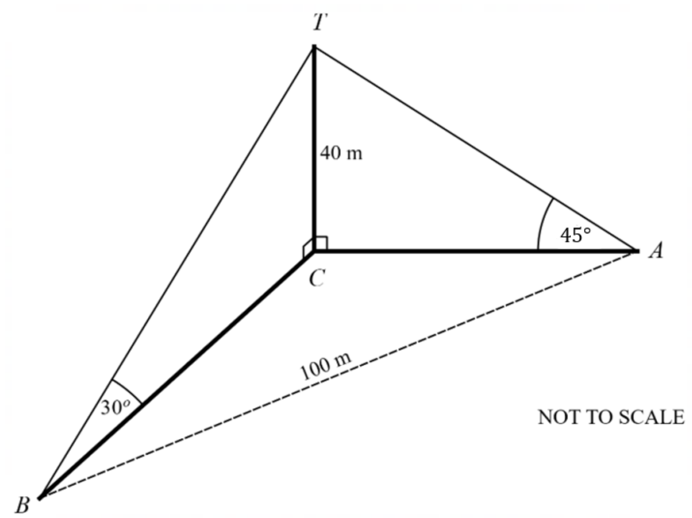
B. 17

C. 8

D. 0

10. A vertical tower TC is 40 metres high. The point A is due east of the base of the tower C, with an angle of elevation to the top of the tower T of $45°$. A second point B is on a different bearing from the point C, as shown in the diagram and has and angle of elevation to the top of the tower of 30°. The points A and B are 100m apart.

The bearing from C to B, correct to the nearest degree is:



A. $131°$

B. $162°$

C. $221°$

D. $311°$

End of section 1

Section II

90 marks

Attempt Questions 11 – 35

Allow about 2 hours and 45 minutes for this section

Instructions

- Answer the question in the spaces provided these spaces provide guidance for the expected length of response.
- Your responses should include relevant mathematical reasoning and or calculations.
- Extra writing space is provided on pages 29 to 34. If you use this space clearly indicate which question you are answering.

Section II begins on page 8

Section II

Question 11 (2 marks)

Solve the equation $12 - 2x = \frac{3x}{5}$

Question 12 (2 marks)

Find the first and second derivative of the function with equation $y = 4\sqrt{x}$

Question 13 (2 marks)

Consider the functions $f(x) = \frac{3}{x+1}$ and $g(x) = 1 - x$.

Let $h(x) = f\big(g(x)\big)$.

Find the domain and range of $y = h(x)$.

Question 14 (3 marks)

Find the equation of the normal to the curve $y = (2x^2 - x + 1)^5$ at the point $(0,1)$.

Question 15 (2 marks)

Solve the equation $|2x - 7| = 9$

Question 16 (4 marks)

Evaluate ,

i. $\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} 2\sin 3x \; dx$ **2**

ii. $\int_{0}^{2} 10^{2x} dx$ **2**

Question 17 (4 marks)

The discrete random variable X has the probability distribution given in the table below.

x	4	5	6	7	8
$P(X = x)$	0.2	A	0.3	B	0.3

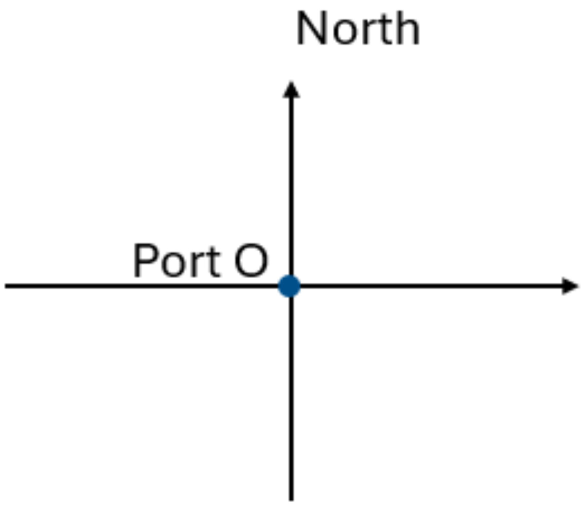
i. Find the values of A and B given the expected value of the distribution is 6.3. **2**

ii. Using the formula $Var(X) = E(x^2) - \mu^2$ to show that that the standard deviation of the distribution is $\sqrt{2.11}$ **2**

Question 18 (4 marks)

A marine ship leaves port O and travels 120 km on a bearing of 057° to a research buoy to collect a sample of water after an oil spill 4 months previously. It then travels a further 150 km to another research buoy on a bearing of 165° to collect another sample, before returning to port.

i. Complete the diagram showing the journey of the marine ship. **1**



ii. Find the distance the ship travels, and, on what bearing, from the second sample site back to port O. **3**

Question 19 (3 marks)

Solve $3\tan(\pi x) = \sqrt{3}$ $\quad 0 \leq x \leq 2$

Question 20 (6 marks)

Differentiate:

i. $\ln\left(\sqrt{2x-1}\right)$ **2**

ii. $e^{lnx^2} + e^{-x} + 3$ **2**

iii. $log_5(3x-5)$ **2**

Question 21 (5 marks)

Find

i. $\int (4x + 7)^5 \, dx$ **1**

ii. $\int \frac{2x}{3x^2+1} \, dx$ **2**

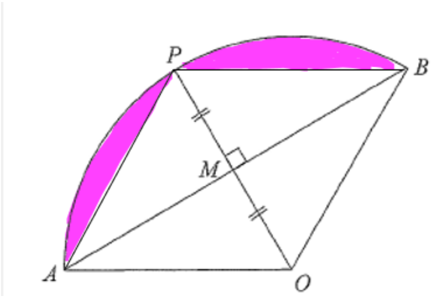
iii. $\int 2 + 2\tan^2 x \, dx$ **2**

Question 22 (3 marks)

Find the equation of the line through the midpoint of the interval from $A(3,2)$ to $B(-7,8)$ perpendicular to the line $\frac{x}{6} + \frac{y}{2} = 1$.

Question 23 (3 marks)

In the diagram APB is an arc of a circle with centre O, and radius 8 cm, such that the chord AB bisects the line OP at right angles.



i. Show that the angle AOB is $\frac{2\pi}{3}$. **1**

ii. Find the shaded area in simplest exact form. **2**

Question 24 (3 marks)

A and B are two events such that P(A)= 0.25, P(B)=0.3 and $P(A|B) = 0.7$

i. Find $P(A \cup B)$ **2**

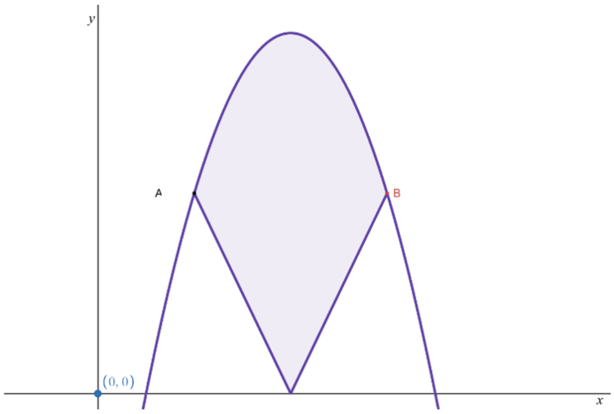
ii. Find in simplest form P(not A and not B) **1**

Question 25 (3 marks)

Find the value of a and b if $\left(2\sqrt{3}+3\right)^2 + \frac{1}{2-\sqrt{3}} = a + b\sqrt{3}$

Question 26 (5 marks)

The curves $y = \frac{5}{4}|x - 8|$ and $y = \frac{1}{12}(3x - 6)(14 - x)$ intersect at two points as shown in the diagram below

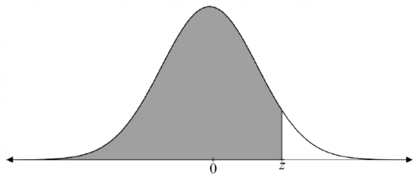


i. Show that the point A has coordinates (4,5) and hence write down the coordinates of point B. **2**

ii. Calculate the shaded area between the two curves. **3**

Question 27 (4 marks)

The population of a town is normally distributed, and the probability for different z scores are represented by the shaded area in the following diagram.



A large country town takes a census as to the age of its population. The mean age of the town is 54 and 2.5% of the population are aged over 66.

i. What is the standard deviation for the age of the population of the town. **1**

ii. What is the probability that a person in the town is aged between 48 and 66? **1**

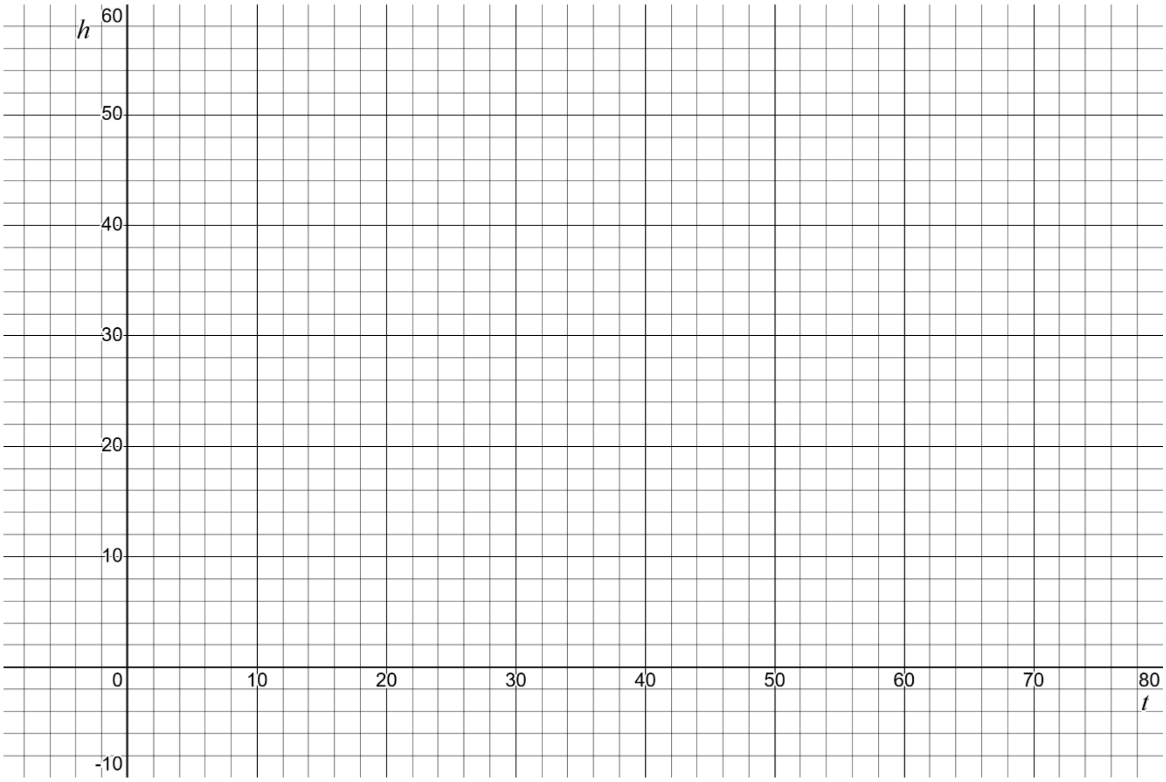
iii. By using the values in the table on page 35, find the probability that a resident of the town is aged between 40 and 45. **2**

Question 28 (4 marks)

An indoor surf pool models the movement of their waves according to the rule

$h = 30 - 20\cos\left(\frac{\pi}{36}t\right)$, where h is the height of the wave above the pool floor, in centimetres and t is the time measured in seconds, from 12 noon.

i. Sketch the motion of the pool for $0 \leq t \leq 72$. Indicate the co-ordinates of any turning points. **2**



ii. A surfer arrives at the pool at noon and wants to enter the pool as the wave is increasing yet wants to have at the height of the wave to be least 30 cm to feel safe.

How many seconds must he wait for the conditions to be ideal? **2**

Question 29 (6 marks)

At an airport, luggage is weighed at check-in to ensure that it meets the airlines strict requirements.

The mass of each piece of luggage, in kilograms, is modelled by a continuous random variable X, whose probability density function is

$$f(x) = \begin{cases} kx^2(30 - x) & 0 \leq x \leq 30 \\ 0 & elsewhere \end{cases}$$

i. Find the value of k. **2**

An airline allows each passenger one piece of luggage free of charge if it weighs 23 kg or less and refuses to carry a bag if the single bag weighs more than 30kg.

They charge $40 excess fee for every kilogram, or part thereof, a bag is over the 23kg.

ii. What is the probability that a passenger will not have to pay for excess luggage? **2**

iii. What is the probability that a passenger will need to pay $200, or more, in excess luggage fees? **2**

Question 30 (6 marks)

i. Sketch the curve $f(x) = 2x^3 - 3x^2 - 12x + 10$ by first finding the coordinates of all the stationary points, checking their nature, and finding the point of inflection. **4**

Question 30 continues on page 23…..

ii. A new curve with equation $g(x) = 10 - f(x+2)$ is to be drawn. **2**

What is the minimum value of this curve in the domain $-5 \leq x \leq 1$?

Question 31 (2 marks)

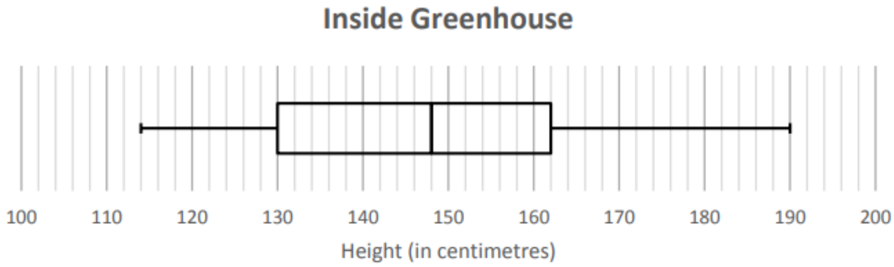
Use the trapezoidal rule with three function values to evaluate the integral $\int_0^1 3e^x \sin x \, dx$. Answer to 3 significant figures.

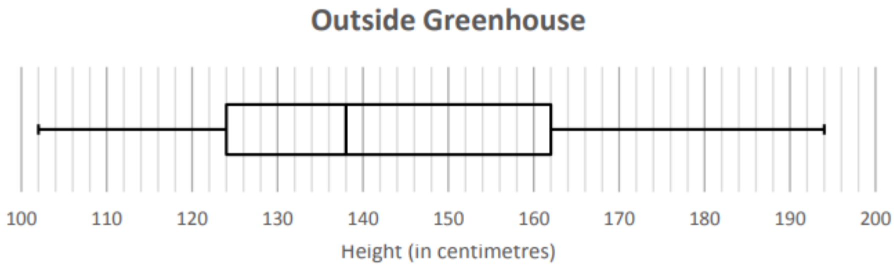
Question 32 (3 marks)

Daniel grows tomatoes.

Some are grown inside a greenhouse while others are grown outside the greenhouse in the garden, with the other vegetables he grows.

The box and whisker plots shown below show the distribution of the heights of the tomato plants inside and outside the greenhouse.





Write down two comparisons, supported with mathematical reasoning, about the heights of the plants grown inside the greenhouse to those grown outside the greenhouse and make a justified recommendation for future plantings.

Question 33 (3 marks)

A particle moves along the x-axis with the acceleration at time t equal to $a = 12(t+3)^2$.

If the particle is initially at rest at the origin find the position of the particle after one second of motion.

Question 34 (3 marks)

A die is loaded such that in a roll of a single die the chance of rolling a 1 is $\frac{x}{4}$, the chance of rolling a 2 is $\frac{1}{4}$ and the chance of rolling a 6 is $\frac{1}{4}(1-x)$. The chance of rolling the other numbers (3,4 or 5) remain one in 6.

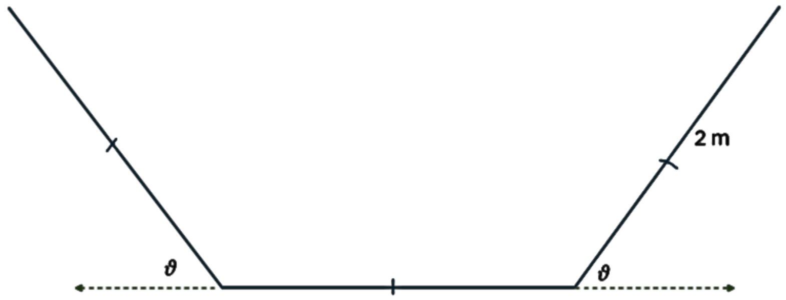
Two such dice are rolled together and the numbers on the uppermost faces are added to obtain a total score for the roll.

The probability of the total score of the two dice being 7 can be expressed as $y = f(x)$

Find the probability of rolling a 1 in a single roll of a die if the probability of rolling a total of 7 with two dice is $\frac{5}{36}$.

Question 35 (5 marks)

A drain is made by folding up a rectangular sheet of metal, with cross- sectional length of 6 m, to form a cross-sectional area in the shape of an isosceles trapezium, as shown in the diagram.



i. Show that the area of the trapezoidal cross section of the drain can be expressed as

$A = \sin\vartheta(4 + 4\cos\vartheta)$ where ϑ is the angle in radians that the metal sheet is bent to form the drain.

2

____

____

____

____

ii. Find the angle ϑ that maximizes the cross-sectional area.

3

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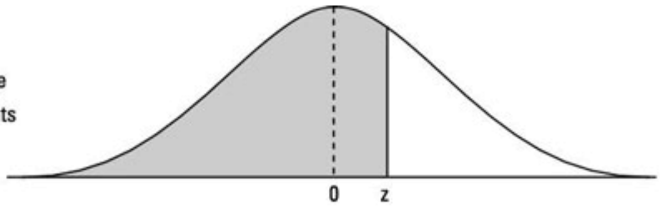
End of assessment task

Appendix 1

Number in the table represents $P(Z \leq z)$



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998
3.5	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998
3.6	.9998	.9998	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999

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STHS Trial Higher School Certificate Examination
Mathematics ADVANCED

Name ______________________ Teacher ____________________

Section I – Multiple Choice Answer Sheet

Allow about 15 minutes for this section

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample: 2 + 4 = (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ⊗ C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word **correct** and drawing an arrow as follows.

A ⊗ B ⊗ (correct) C ○ D ○

	A	B	C	D
1.	○	●	○	○
2.	○	○	○	●
3.	○	○	●	○
4.	○	●	○	○
5.	○	●	○	○
6.	○	●	○	○
7.	○	●	○	○
8.	○	○	●	○
9.	●	○	○	○
10.	○	○	●	○

Section II

Question 11 (2 marks)

Solve the equation $12 - 2x = \frac{3x}{5}$

$60 - 10x = 3x$

$60 = 13x$

$x = \frac{60}{13}$ (2)

Question 12 (2 marks)

Find the first and second derivative of the function with equation $y = 4\sqrt{x}$

$y = 4x^{1/2}$

$y' = 2x^{-1/2}$ (1)

$y'' = -x^{-3/2}$ (1)

$= -1/x\sqrt{x}$

Question 13 (2 marks)

Consider the functions $f(x) = \frac{3}{x+1}$ and $g(x) = 1 - x$.

Let $h(x) = f(g(x))$.

Find the domain and range of $y = h(x)$.

$h(x) = \frac{3}{(1-x)+1}$

D: $x \in R, x \neq 2$ must specify

R: $y \in R, y \neq 0$ which is D (2)

and R.

Question 14 (3 marks)

Find the equation of the normal to the curve $y = (2x^2 - x + 1)^5$ at the point (0,1).

$y' = 5(2x^2 - x + 1)^4 \times (4x - 1)$ ← (1)

at $x = 0$

$m_T = 5 \times (1)^4 (-1)$

$= -5$

$\therefore m_N = 1/5$ ← (1)

Now $y - y_1 = m(x - x_1)$

$y - 1 = 1/5(x - 0)$

$5y - 5 = x$

$x - 5y + 5 = 0$

any (1)

Question 15 (2 marks)

Solve the equation $|2x - 7| = 9$

$2x - 7 = 9$ or $-2x + 7 = 9$

$2x = 16$ $\quad$ $-2x = 2$

$x = 8$ $\quad$ $x = -1$

$\therefore x = -1$ or $x = 8$ (2)

Question 16 (4 marks)

Evaluate ,

i. $\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} 2\sin 3x \; dx$ 2

$\left. -\frac{2}{3} \cos 3x \right]_{\pi/3}^{\pi/2}$

$= -\frac{2}{3}\left[\cos\frac{3\pi}{2} - \cos\pi\right]$

1 = $\int$ correctly

1 = value

$= -\frac{2}{3}\left[0 - (-1)\right]$

$= -\frac{2}{3}$

ii. $\int_0^2 10^{2x} dx$ 2

$= \left. \frac{10^{2x}}{2\ln 10} \right]_0^2$ R.S -(1)

$= \frac{1}{2\ln 10}\left[10^4 - 10^0\right]$

$= \frac{9999}{2\ln 10}$

(1) either

Question 17 (4 marks)

The discrete random variable X has the probability distribution given in the table below.

x	4	5	6	7	8
$P(X = x)$	0.2	A	0.3	B	0.3

i. Find the values of A and B given the expected value of the distribution is 6.3. **2**

$E(x) = 0.8 + 5A + 1.8 + 7B + 2.4 = 6.3$ —(1)

$A + B = 0.2$ —(2)

from(1) $5A + 7B = 1.3$

(2) × 7 $7A + 7B = 1.4$ —

$2A = 0.1$

$A = 0.05$

$\therefore B = 0.15$

1 = setup $E(x)$

1 = Both A and B correct

ii. Using the formula $Var(X) = E(x^2) - \mu^2$ to show that that the standard deviation of the distribution is $\sqrt{2.11}$ **2**

$E(x^2) = 4^2 \times 0.2 + 5^2 \times 0.05 + 6^2 \times 0.3 + 7^2 \times 0.15 + 8^2 \times 0.3$ ← (1)

$= 41.8$

Now $E(x^2) - \mu^2 = 41.8 - 6.3^2$

$Var(x) = 2.11$

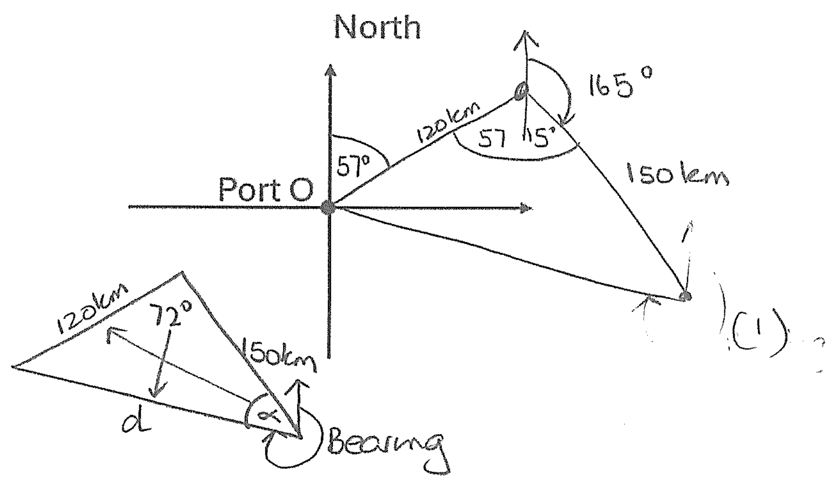
$\sigma = \sqrt{Var(x)}$ } (1)

$= \sqrt{2.11}$

Question 18 (4 marks)

A marine ship leaves port O and travels 120 km on a bearing of 057° to a research buoy to collect a sample of water after an oil spill 4 months previously. It then travels a further 150 km to another research buoy on a bearing of 165° to collect another sample, before returning to port.

i. Complete the diagram showing the journey of the marine ship. **1**



ii. Find the distance the ship travels, and, on what bearing, from the second sample site back to port O. **3**

$d^2 = 120^2 + 150^2 - 2(120)(150)\cos 72°$

$d = 160.547$ km —(1)

and $\dfrac{\sin\alpha}{120} = \dfrac{\sin 72°}{160.547}$

$\alpha = \sin^{-1}(0.71\ldots)$

$= 45.3°$ (1dp) —(1)

Bearing $= 360° - (15 + 45)$

$= 300°T$ —(1)

Question 19 (3 marks)

Solve $3\tan(\pi x) = \sqrt{3}$ $\quad 0 \le x \le 2$

$\tan(\pi x) = \sqrt{3}/3$

$\pi x = \pi/6, 7\pi/6$

$x = \frac{1}{6}, \frac{7}{6}$ — (2)

Question 20 (6 marks)

Differentiate:

i. $\ln(\sqrt{2x-1}) = \frac{1}{2}\ln(2x-1)$ **2**

$y' = \dfrac{\frac{1}{2} \times 2}{2x-1}$

$= \dfrac{1}{2x-1}$ — (2)

ii. $e^{lnx^2} + e^{-x} + 3$ **2**

$y = x^2 + e^{-x} + 3$

$y' = 2x - e^{-x}$

iii. $log_5(3x-5)$ **2**

$f'(x) = 3 \quad f(x) = 3x-5 \quad a = 5$ (R.S)

$y' = \dfrac{3}{(\ln 5)(3x-5)}$

Question 21 (5 marks)

Find

i. $\int (4x+7)^5\, dx$ **1**

$= \frac{1}{24}(4x+7)^6 + C$

ii. $\int \frac{2x}{3x^2+1}\, dx$ **2**

$= \frac{1}{3}\ln|3x^2+1| + C$

iii. $\int 2 + 2\tan^2 x\, dx$ **2**

$= 2\int 1 + \tan^2 x \, dx$

$= 2\int \sec^2 x \, dx$

$= 2\tan x + C$

Question 22 (3 marks)

Find the equation of the line through the midpoint of the interval from $A(3,2)$ to $B(-7,8)$ perpendicular to the line $\frac{x}{6}+\frac{y}{2}=1$.

$\text{midpoint} = \left(\frac{3+-7}{2}, \frac{2+8}{2}\right)$

$= (-2,5)$

$x+3y=6$

$3y=6-x \qquad m=-1/3$

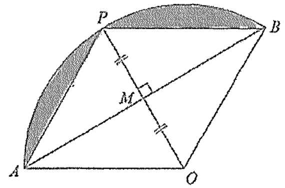
$M\perp=3 \qquad (M_1 \times M_2=-1)$

$\therefore \quad y-5=3(x+2)$

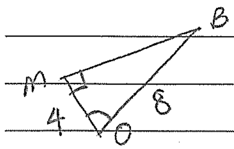
$y=3x+11.$

Question 23 (3 marks)

In the diagram APB is an arc of a circle with centre O, and radius 8 cm, such that the chord AB bisects the line OP at right angles.



i. Show that the angle AOB is $\frac{2\pi}{3}$. **1**



$\cos\theta = 4/8$

$\theta = \pi/3$

$\angle AOM = \angle MOB$

$\angle AOB = 2\times\pi/3$

$= 2\pi/3$

ii. Find the shaded area in simplest exact form. **2**

$\text{Segment} = [\text{Sector} - \triangle \times 2]$

$= \frac{1}{2}r^2\theta - \frac{1}{2}ab\sin C \times 2$

$= \frac{1}{\cancel{2}}\times 8^2 \times \frac{\cancel{2}\pi}{3} - \frac{1}{2}\times 8\times 8\times \sin\frac{2\pi}{3}\times 2$

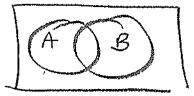
$= \frac{64\pi}{3} - 32\times\frac{\sqrt{3}}{2}\times 2$

$= \frac{64\pi}{3} - 32\sqrt{3} \quad u^2$

Question 24 (3 marks)

A and B are two events such that P(A)= 0.25, P(B)=0.3 and $P(A|B) = 0.7$

i. Find $P(A \cup B)$ **2**



$P(A|B) = \dfrac{P(A \cap B)}{P(B)}$

$0.7 = \dfrac{P(A \cap B)}{0.3}$

$P(A \cap B) = 0.21$

$P(A \cup B) = 0.04 + 0.21 + 0.09$

$P(A \cup B) = 0.34$

ii. Find in simplest form P(not A and not B) **1**

$1 - P(P \cup B)$

$= 1 - 0.34$

$= 0.66$

Question 25 (3 marks)

Find the value of a and b if $(2\sqrt{3}+3)^2 + \frac{1}{2-\sqrt{3}} = a + b\sqrt{3}$

LHS $= (2\sqrt{3}+3)^2 + \frac{1}{2-\sqrt{3}} \times \frac{2+\sqrt{3}}{2+\sqrt{3}}$ ← 1

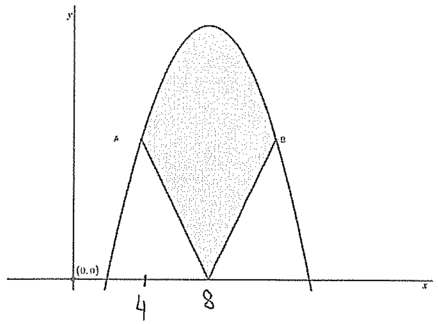
$= 12 + 12\sqrt{3} + 9 + 2 + \sqrt{3}$ ←

$= 23 + 13\sqrt{3}$

$a = 23 \quad b = 13$ ← 2

Question 26 (5 marks)

The curves $y = \frac{5}{4}|x-8|$ and $y = \frac{1}{12}(3x-6)(14-x)$ intersect at two points as shown in the diagram below



i. Show that the point A has coordinates (4,5) and hence write down the coordinates of point B. **2**

when $x = 4$ $y = \frac{5}{4}|4-8|$ & $y = \frac{1}{12}(3\times4-6)(14-4)$

$= 5$ $\qquad = \frac{1}{12}(6)(10)$

$= 5$ ✓

∴ meet @ A (4,5) with symmetry

B = (12,5) ✓

ii. Calculate the shaded area between the two curves. **3**

$A = \int_4^{12} \frac{1}{12}(3x-6)(14-x)\,dx - 2\times\frac{1}{2}\times5\times4$

$= \frac{1}{12}\int_4^{12} 42x - 3x^2 - 84 + 6x \; dx \quad - 20$

$= \frac{1}{12}\int_4^{12} 48x - 3x^2 - 84\,dx \quad - 20$

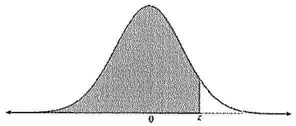
$= \frac{1}{12}\left[24x^2 - x^3 - 84x\right]_4^{12} - 20$

$= \frac{1}{12}\left[24\times12^2 - 12^3 - 84\times12 - (24\times16 - 4^3 - 84\times4)\right] - 20$

$= 41\frac{1}{3}\ u^2$

Question 27 (4 marks)

The population of a town is normally distributed, and the probability for different z scores are represented by the shaded area in the following diagram.



A large country town takes a census as to the age of its population. The mean age of the town is 54 and 2.5% of the population are aged over 66.

i. What is the standard deviation for the age of the population of the town. **1**

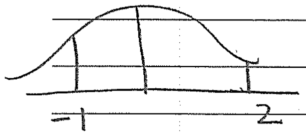
$54 + 2\sigma = 66$

$2\sigma = 12$

$\sigma = 6$

ii. What is the probability that a person in the town is aged between 48 and 66? **1**

$X = 66 \quad Z = 2 \qquad X = 48 \quad Z = -1$



$P(48 \le X \le 66) = P(-1 \le Z \le 2)$

$= 34 + (50 - 2.5)\%$

$= 81.5\%$

iii. By using the values in the table on page 35, find the probability that a resident of the town is aged between 40 and 45. **2**

$X = 40 \quad Z_1 = -2.33 \qquad X = 45 \quad Z_2 = -1.5$

$P(Z \le -2.33) = 1 - 0.9901$

$= 0.0099$

$P(Z \le -1.5) = 1 - 0.9332$

$= 0.0668$

$\therefore P(40 \le X \le 45) = 0.0668 - 0.0099$

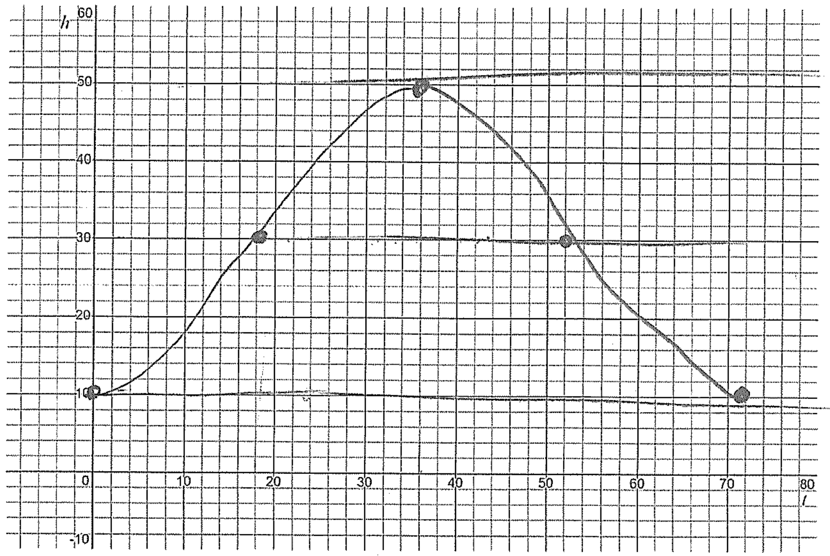
$= 0.0569$

Question 28 (4 marks)

An indoor surf pool models the movement of their waves according to the rule

$h = 30 - 20\cos\left(\frac{\pi}{36}t\right)$, where h is the height of the wave above the pool floor, in centimetres and t is the time measured in seconds, from 12 noon.

i. Sketch the motion of the pool for $0 \le t \le 72$. Indicate the co-ordinates of any turning points. **2**



ii. A surfer arrives at the pool at noon and wants to enter the pool as the wave is increasing yet wants to have at the height of the wave to be least 30 cm to feel safe.

How many seconds must he wait for the conditions to be ideal? **2**

$30 = 30 - 20\cos\left(\frac{\pi}{36}t\right)$

$0 = \cos\alpha \qquad \alpha = \pi/36\, t$

$\alpha = \pi/2,\ 3\pi/2$

$\frac{\pi}{36}t = \frac{\pi}{2}$ Needed to increase wave from graph

$\therefore t = 18\text{ s}.$

Question 29 (6 marks)

At an airport, luggage is weighed at check-in to ensure that it meets the airlines strict requirements.

The mass of each piece of luggage, in kilograms, is modelled by a continuous random variable X, whose probability density function is

$$f(x) = \begin{cases} kx^2(30-x) & 0 \le x \le 30 \\ 0 & elsewhere \end{cases}$$

i. Find the value of k. 2

$$k\int_0^{30} 30x^2 - x^3 \, dx = 1$$

$$k\left[10x^3 - \tfrac{1}{4}x^4\right]_0^{30} = 1$$

$$k\left[10(30)^3 - \tfrac{1}{4}(30^4) - 0\right] = 1$$

$$k(67500) = 1$$

$$k = \tfrac{1}{67500}$$

An airline allows each passenger one piece of luggage free of charge if it weighs 23 kg or less and refuses to carry a bag if the single bag weighs more than 30kg.

They charge $40 excess fee for every kilogram, or part thereof, a bag is over the 23kg.

ii. What is the probability that a passenger will not have to pay for excess luggage? 2

$$\frac{1}{67500}\int_0^{23} 30x^2 - x^3 \, dx$$

$$= \frac{1}{67500}\left[10x^3 - \tfrac{1}{4}x^4\right]_0^{23}$$

$$= \tfrac{1}{67500}\left[10(23)^3 - \tfrac{1}{4}(23)^4 - 0\right]$$

$$= 0.766 \quad (3dp)$$

iii. What is the probability that a passenger will need to pay $200, or more, in excess luggage fees? 2

$$\int_{27}^{30} \frac{1}{67500}(30x^2 - x^3)dx$$

$$\tfrac{1}{67500}\left[10(30)^3 - \frac{30^4}{4} - \left[10(27)^3 - \tfrac{1}{4}(27)^4\right]\right]$$

$$= 0.0523 \ldots \quad (4 \text{ sig fig})$$

Question 30 (6 marks)

i. Sketch the curve $f(x) = 2x^3 - 3x^2 - 12x + 10$ by first finding the coordinates of all the stationary points, checking their nature, and finding the point of inflection. 4

$f'(x) = 6x^2 - 6x - 12 = 0$ for stat pts

$6(x-2)(x+1) = 0$

$x = 2 \quad x = -1$

$y = -10 \quad y = 17$

nature of stat pts (1st deriv test shown)

x	-2	-1	0
$f'(x)$ y'	$6\times -4 \times 7$	0	$6(-2)\times 1$ -12
	/	—	\

x	0	2	3
$f'(x)$ y'		0	$6\times 1\times 4$
	\	—	/

$(2, -10)$ relative min TP $\quad (-1, 17)$ relative Max T.P.

Inflexion $f''(x) = 12x - 6 = 0$

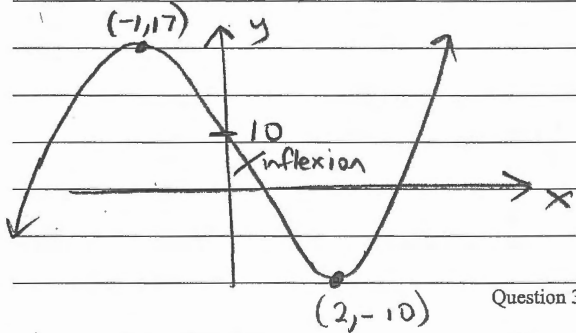
$x = \tfrac{1}{2}$

$y = \tfrac{7}{2}$

Check $f''(0) = -6$ ∩

$f''(\tfrac{1}{2}) = 0$

$f''(1) = 6$ ∪ $\quad$ ∴ concavity change at $(\tfrac{1}{2}, \tfrac{7}{2})$



4

Question 30 continues on page 23…..

marking Perfect = 4

no test inflex = 3

errors in points or missing steps for Inflex & 2 TP's = 2

only d/dx = 1

ii. A new curve with equation $g(x) = 10 - f(x+2)$ is to be drawn. **2**

What is the minimum value of this curve in the domain $-5 \le x \le 1$?

$g(x) = -f(x+2) + 10$

∴ as a reflection is occuring a min will come from the max T.P.

$(-1, 17) \longrightarrow (-1, -17)$ after reflection

Translate 2 units left

$\longrightarrow (-3, -17)$

up 10 $(-3, -7)$ ∴ MIN value $= -7$ ← 1

1

Question 31 (2 marks)

Use the trapezoidal rule with three function values to evaluate the integral $\int_0^1 3e^x \sin x \, dx$. Answer to 3 significant figures.

x	0	0.5	1
$3e^x \sin x$	0	$3e^{1/2}\sin 1/2$	$3e\sin 1$

↑ Radians used

$a = 0$

$b = 1$

$n = 2$

$\frac{b-a}{2n}\left[f(a) + f(b) + 2 \times f(\ )\right]$ R.S

$\frac{1-0}{2(2)}\left[0 + 3e\sin 1 + 2 \times 3e^{1/2}\sin 1/2\right]$

(Full marks here)

$= 2.901\ldots$

$= 2.90$

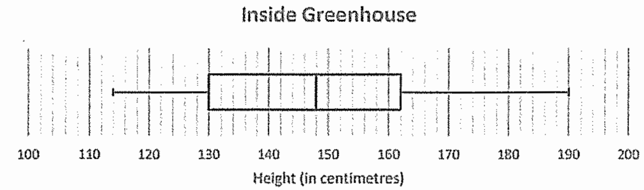
No mark deduction if rounding not done

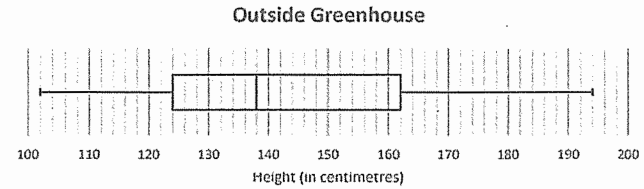
Question 32 (3 marks)

Daniel grows tomatoes.

Some are grown inside a greenhouse while others are grown outside the greenhouse in the garden, with the other vegetables he grows.

The box and whisker plots shown below show the distribution of the heights of the tomato plants inside and outside the greenhouse.





Write down two comparisons, supported with mathematical reasoning, about the heights of the plants grown inside the greenhouse to those grown outside the greenhouse and make a justified recommendation for future plantings.

Inside median height = 148cm

range in height = 190−114 = 76cm

Outside median height = 138cm

range in height = 194−102 = 92cm

∴ There is more variation in heights outside & taller plants inside making the recommendation for future plantings to be inside

Question 33 (3 marks)

A particle moves along the x-axis with the acceleration at time t equal to $a = 12(t + 3)^2$.

If the particle is initially at rest at the origin find the position of the particle after one second of motion.

$V = \int 12(t+3)^2\,dt$

$V = 4(t+3)^3 + C$ $\qquad V = 0$

$0 = 4(3)^3 + C$ $\qquad t = 0$

$C = -108$

$\therefore V = 4(t+3)^3 - 108$ — 1

$x = \int 4(t+3)^3 - 108\,dt$

$x = (t+3)^4 - 108t + C_2$ $\qquad x = 0$

$0 = 81 - 0 + C_2$ $\qquad t = 0$

$C_2 = -81$

$\therefore x = (t+3)^4 - 108t - 81$ — 1

Now $t = 1$

$x = 67$ — 1

Question 34 (3 marks)

A die is loaded such that in a roll of a single die the chance of rolling a 1 is $\frac{x}{4}$, the chance of rolling a 2 is $\frac{1}{4}$ and the chance of rolling a 6 is $\frac{1}{4}(1 - x)$. The chance of rolling the other numbers (3,4 or 5) remain one in 6.

Two such dice are rolled together and the numbers on the uppermost faces are added to obtain a total score for the roll.

The probability of the total score of the two dice being 7 can be expressed as $y = f(x)$

Find the probability of rolling a 1 in a single roll of a die if the probability of rolling a total of 7 with two dice is $\frac{5}{36}$.

$(1,6) + (6,1) + (2,5) + (5,2) + (3,4) + (4,3)$

$f(x) = 2 \times \frac{x}{4} \times \frac{1}{4}(1-x) + \frac{1}{4} \times \frac{1}{6} \times 2 + \frac{1}{6} \times \frac{1}{6} \times 2$

$f(x) = \frac{x}{8}(1-x) + 1/12 + 1/18$ $\leftarrow$ 1

and $f(x) = 5/36$

$\frac{5}{36} = \frac{x}{8}(1-x) + \frac{5}{36}$

$\frac{x}{8}(1-x) = 0$

$x = 0$ or $x = 1$ $\qquad x \neq 0$ as $P(1)$ exists

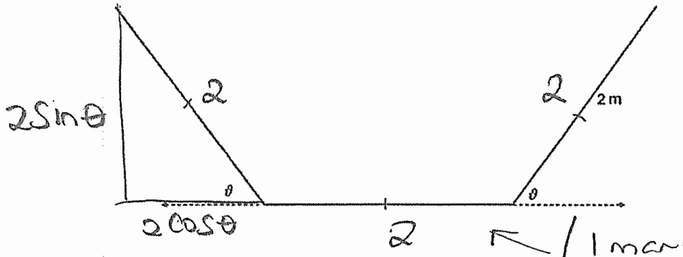
$\therefore x = 1$

$\therefore P(1) = 1/4$ $\leftarrow$ Need for mark

1 for includes explanation of $x \neq 0$ for mark

Question 35 (5 marks)

A drain is made by folding up a rectangular sheet of metal, with cross- sectional length of 6 m, to form a cross-sectional area in the shape of an isosceles trapezium, as shown in the diagram.



(1 mark if on diagram)

i. Show that the area of the trapezoidal cross section of the drain can be expressed as

$A = \sin\vartheta(4 + 4\cos\vartheta)$ where ϑ is the angle in radians that the metal sheet is bent to form the drain. 2

$A = \frac{1}{2}h(a+b)$ or rect $+ 2\times\Delta$

$= \frac{1}{2}(2\sin\theta)(2 + 2 + 4\cos\theta)$

$= \sin\theta(4 + 4\cos\theta)$

must Be explicitly shown

ii. Find the angle ϑ that maximizes the cross-sectional area. 3

$A = 4\sin\theta + 4\sin\theta\cos\theta$

$dA/d\theta = 4\cos\theta + 4\sin\theta(-\sin\theta) + 4\cos\theta\cos\theta$

$= 4\cos\theta - 4(1-\cos^2\theta) + 4\cos^2\theta$

$= 8\cos^2\theta + 4\cos\theta - 4$

$4(2\cos^2\theta + \cos\theta - 1) = 0$ for max/min

$(2\cos\theta - 1)(\cos\theta + 1) = 0$

$\cos\theta = 1/2$ or $\cos\theta = -1$

$0 < \theta < \pi/2$ as a right angle triangle

$\therefore \cos\theta = 1/2$ only

$\theta = \frac{\pi}{3}$

test $\frac{d^2A}{d\theta^2} = 16\cos\theta(-\sin\theta) - 4\sin\theta$

at $\theta = \pi/3$

$= 16\cos\frac{\pi}{3} \times -\sin\frac{\pi}{3} - 4\sin\pi/3$

$= 16 \times 1/2 \times -\sqrt{3}/2 - 4 \times -\sqrt{3}/2$

$= -4\sqrt{3} + 2\sqrt{3}$

$= -2\sqrt{3} < 0$ $\therefore$ Max when angle is $\pi/3$

$\therefore$ angle is $\frac{\pi}{3}$ or $60°$ which maximises the area

End of assessment task

Note As it is a Trig Fnc θ is in radians this should be reflected in the boys solutions!