Baulkham Hills High School

2026 HIGHER SCHOOL CERTIFICATE TRIAL EXAMINATION

Mathematics Advanced

General Instructions

- Reading time – 10 minutes
- Working time – 3 hours
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided at the back of this paper
- For questions in Section II, show relevant mathematical reasoning and/ or calculations
- Write your Student Number at the top of this page

Total marks: 100

Section I – 10 marks (pages 4-7)

- Attempt Questions 1–10.
- Allow about 15 minutes for this section.

Section II – 90 marks (pages 8-36)

- Attempt Questions 11–32.
- Allow about 2 hours and 45 minutes for this section.

Section I

10 marks
Attempt Questions 1–10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1–10

1 What is the domain of $f(x) = \ln(x-2) + \sqrt{9-x}$?

A. $x > 2$ and $x < 9$

B. $x \geq 2$ and $x \leq 9$

C. $x > 2$ and $x \leq 9$

D. $x \geq 2$ and $x < 9$

2 What is the value of $\sum_{r=1}^{20} -(-1)^r$?

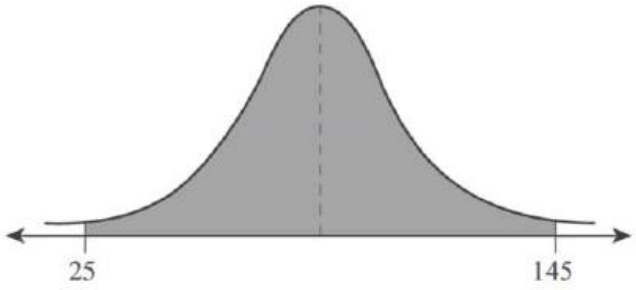
A. -1

B. 0

C. 1

D. No possible solution

3 The graph shows a normal distribution. Approximately 99.7% of the area under the curve is bounded by $x = 25$ and $x = 145$.



What is the mean and variance that best represents this information?

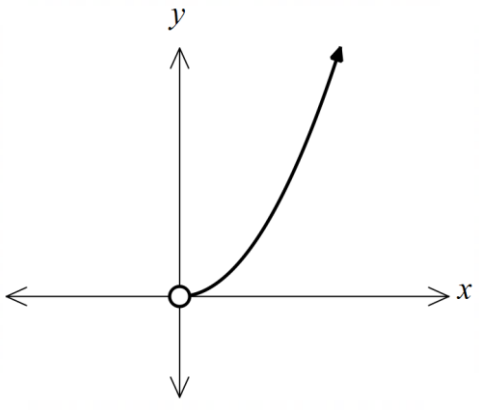
A. mean = 60 and variance = 20

B. mean = 60 and variance = 400

C. mean = 85 and variance = 20

D. mean = 85 and variance = 400

4 Which equation best represents the graph drawn below?



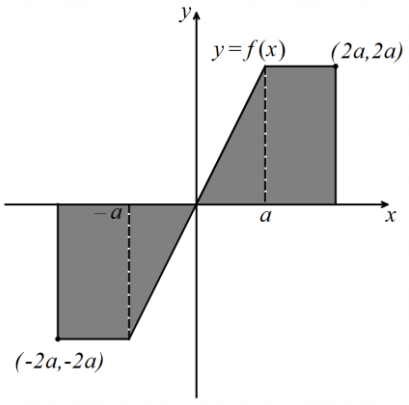
A. $y = 4x^2 + x,\ x > 0$

B. $y = x^2(x + 1),\ x > 0$

C. $y = 4^{x+1},\ x > 0$

D. $y = \frac{4}{x+1},\ x > 0$

5 The diagram below shows a shaded region bounded by $y = f(x), y = 0, x = 2a$ and $x = -2a$.



Which of these expressions does **NOT** find the shaded area?

A. $\int_{-2a}^{2a} f(x)\,dx$

B. $-2\int_{0}^{-2a} f(x)\,dx$

C. $2\int_{0}^{a} f(x)\,dx + 4a^2$

D. $3(a \times 2a)$

6 A table of future value interest factors is shown below.

Period	Interest rate per period					
	1%	2%	3%	4%	6%	8%
2	2.0100	2.0200	2.0300	2.0400	2.0600	2.0800
3	3.0301	3.0604	3.0909	3.1216	3.1836	3.2464
4	4.0604	4.1216	4.1836	4.2465	4.3746	4.5061
5	5.1010	5.2040	5.3091	5.4163	5.6371	5.8666
6	6.1520	6.3081	6.4684	6.6330	6.9753	7.3359
7	7.2135	7.4343	7.6625	7.8983	8.3938	8.9228
8	8.2857	8.5830	8.8923	9.2142	9.8975	10.6366
9	9.3685	9.7546	10.1591	10.5828	11.4913	12.4876
10	10.4622	10.9497	11.4639	12.0061	13.1808	14.4866
11	11.5668	12.1687	12.8078	13.4864	14.9716	16.6455
12	12.6825	13.4121	14.1920	15.0258	16.8699	18.9771

An annuity involves making equal contributions of $2500 into an account every quarter for 2 years at an interest rate of 4% per annum compounded quarterly.
Based on the information provided, what is the future value of this annuity?

A. $5 100.00

B. $20 400.00

C. $20 714.25

D. $23 035.50

7 Olivia and Sarah are participating in a fitness program. The probability that Olivia will complete the program is 0.8, and the probability that Sarah will complete the program is 0.7.

What is the probability that the only one of them will complete the program?

A. 0.14

B. 0.24

C. 0.38

D. 0.56

8 What is the simplified form of $\cot(270 + x)° \cos(-x)°$?

A. $-\sin x$

B. $\sin x$

C. $-\cot x \ \text{cosec}\ x$

D. $\cot x \ \text{cosec}\ x$

9 Consider a dataset with mean 10 and standard deviation 2. Two scores, 10.5 and 9.5 are added to the dataset. Which of the following shows how the mean and the standard deviation are affected by the additional scores?

A. Both mean and standard deviation will change.

B. Both mean and standard deviation will stay the same.

C. Only standard deviation will increase.

D. Only standard deviation will decrease.

10 Let $h(x) = f\big(g(x)\big)$, with $f(4) = 1, f'(x) = 6, g(2) = 4$ and $g'(2) = 4$.
What is the equation of the tangent of the graph of $y = h(x)$ at $x = 2$?

A. $y = x - 6$

B. $y = 24x - 2$

C. $y = 6x - 11$

D. $y = 24x - 47$

Mathematics Advanced
Section II Answer Booklet

90 Marks
Attempt Questions 11-32
Allow about 2 hours and 45 minutes for this section

Instructions

- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response
- Your responses should include relevant mathematical reasoning and/or calculations
- Extra writing space is provided at the back of this booklet. If you use this space, clearly indicate which question you are answering

Question 11 (2 marks)

Find the exact gradient of the tangent to the curve $y = \log_e(\tan x)$ at the point where $x = \frac{\pi}{3}$. **2**

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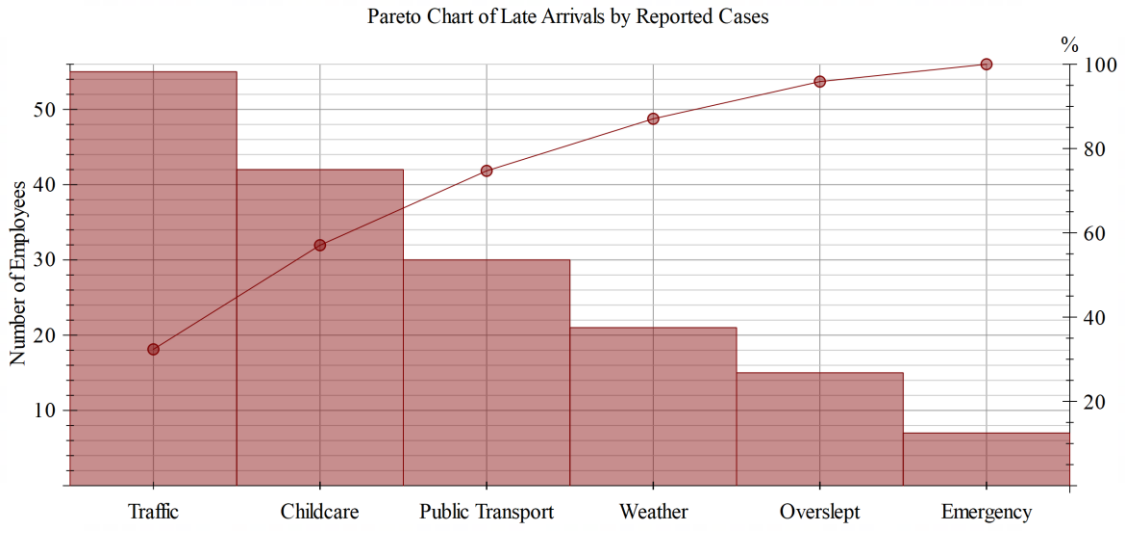
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Question 12 (1 marks)

A company owner collected data related to the reason given by employees for being late to work. **1**

The pareto chart shows the data collected.



Estimate the percentage of employees that were late because of public transport.

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Question 13 (3 marks)

An arithmetic sequence from start to end is 51, 44, 37… -47.

(a) Write a simplified expression for the nth term for this sequence. **1**

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(b) How many terms are there in the sequence? **1**

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(c) Find the sum of this series. **1**

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Question 14 (6 marks)

Differentiate the following:

(a) $y = \sqrt[3]{2x - 11}$ **2**

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(b) $y = e^x \tan(5x)$ **2**

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(c) $y = \log_e \left[\frac{x}{(x+1)^4}\right]$ **2**

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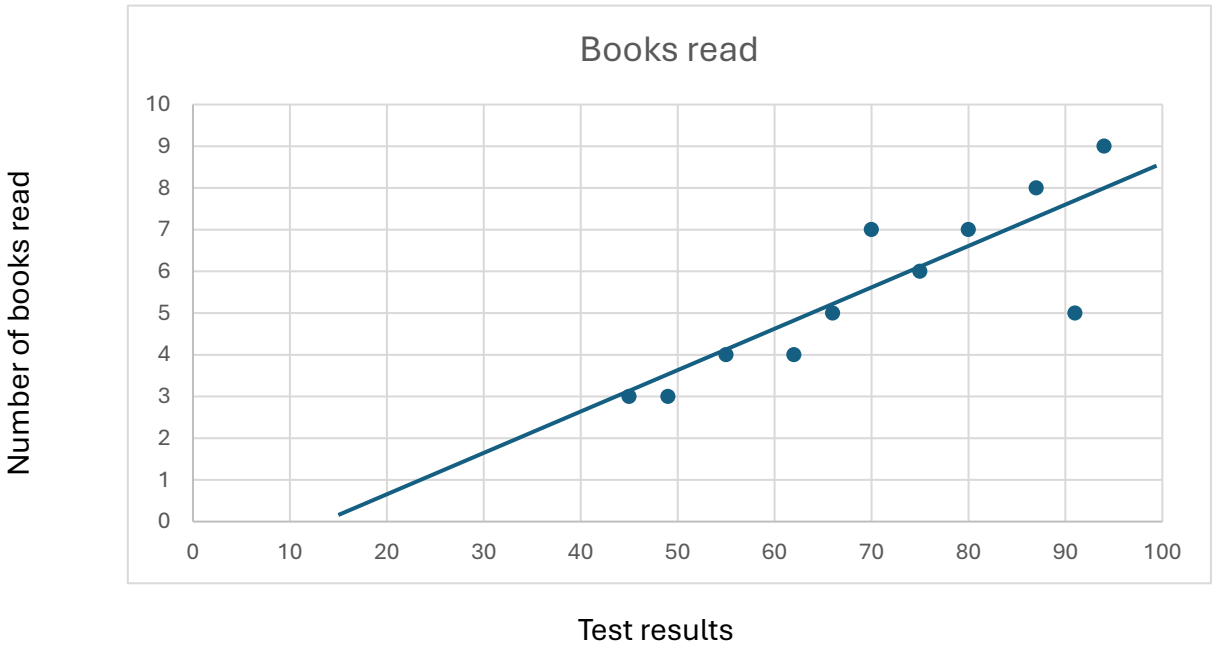
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Question 15 (3 marks)

A group of Year 6 students were surveyed about their reading habits.

The students were then given a reading comprehension test.

The scatter plot below shows the results of the test plotted against the number of books that the student had read in the previous month.



The line of best fit for the data has also been added.

(a) How many students were part of the survey? **1**

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(b) Pearson's correlation coefficient for this scatterplot is 0.843. **1**
Describe the correlation in the context of the data?

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(c) A student who was not part of the survey has read 7 books in the past month. **1**
Predict the mark that this student would get if they sat the reading comprehension test.

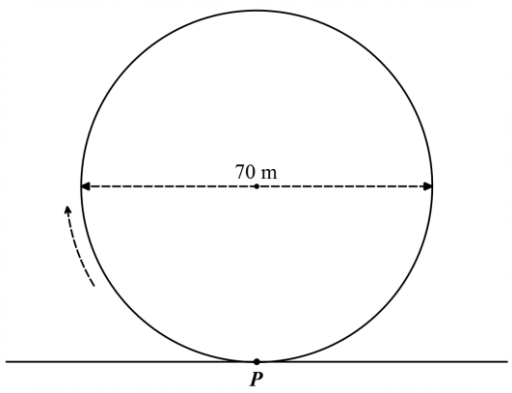
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Question 16 (9 marks)

The diagram below shows a Ferris wheel with a diameter of 70 metres. Passengers board the Ferris wheel when its carriages reaches its lowest point P, at ground level. The wheel rotates at a constant rate in a clockwise direction and completes one revolution every 20 minutes.



Let h be the vertical height of a carriage t minutes after it passes point P.

(a) Find the height of the carriage after 5 minutes. **1**

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(b) It is known that h can be expressed in the form $h(t) = a\cos(bt) + c$, where a, b and c are constants. Find the values of a, b and c. **2**

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Question 16 continues on page 14

(c) In the space provided below, sketch the graph of h for $0 \leq t \leq 40$. **2**



(d) Let R be the rate at which h changes over time. **2**
By first finding R, find the maximum rate of change of R with respect to time.

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(e) When Leo enters the carriage, he is excited to reach a height of 50m above the ground. **2**
Determine how long it will take him to reach this height. Give your answer to the nearest second.

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Question 17 (3 marks)

Find $\int x\sqrt{5x^2+1}\,dx$ **3**

Question 18 (5 marks)

When cooking oil cools down, its temperature, T, in degrees Celsius, after t minutes is given by the formula

$$T = 60 + 150e^{-0.1t}$$

(a) What is the initial temperature of the oil? **1**

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(b) At what temperature is the oil decreasing at a rate of 10°C per minute? Leave your answer to the nearest degree. **3**

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(c) What temperature does the oil eventually cool to? Leave your answer to the nearest degree. **1**

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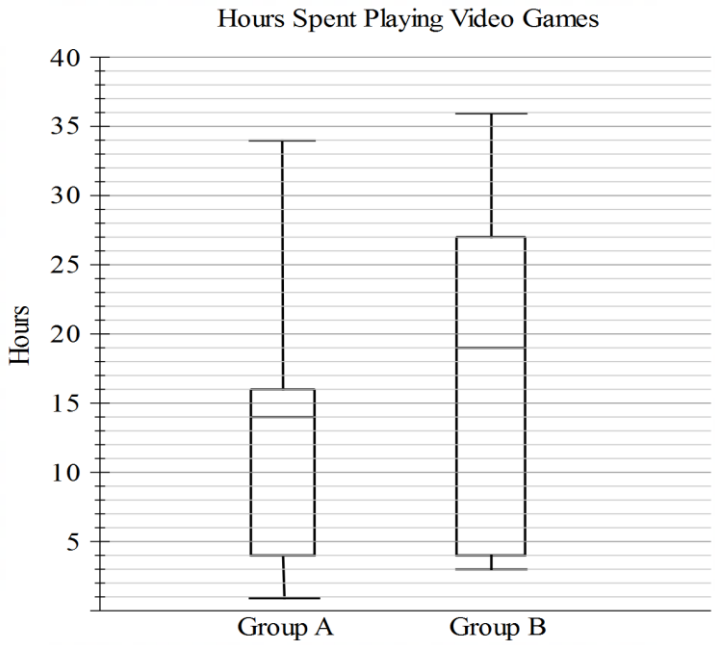
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Question 19 (4 marks)

The parallel boxplots below show the number of hours spent playing video games each week for two groups of students.



(a) What is the median number of hours spent playing video games for group A? **1**

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(b) Determine the interquartile range for group B. **1**

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(c) Which group displays a more consistent number of hours? **1**
Justify your answer with reference to an appropriate data measurement.

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(d) A student that wasn't part of the survey spends 36 hours a week playing video games. **1**
Would this student be considered an outlier in group A?
Justify your decision with appropriate calculations.

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Question 20 (6 marks)

Given $f(x) = x - 2$ and $g(x) = (x + 2)(x^2 - 4)$

(a) Find the x-coordinates of all points of intersection between $f(x)$ and $g(x)$. **2**

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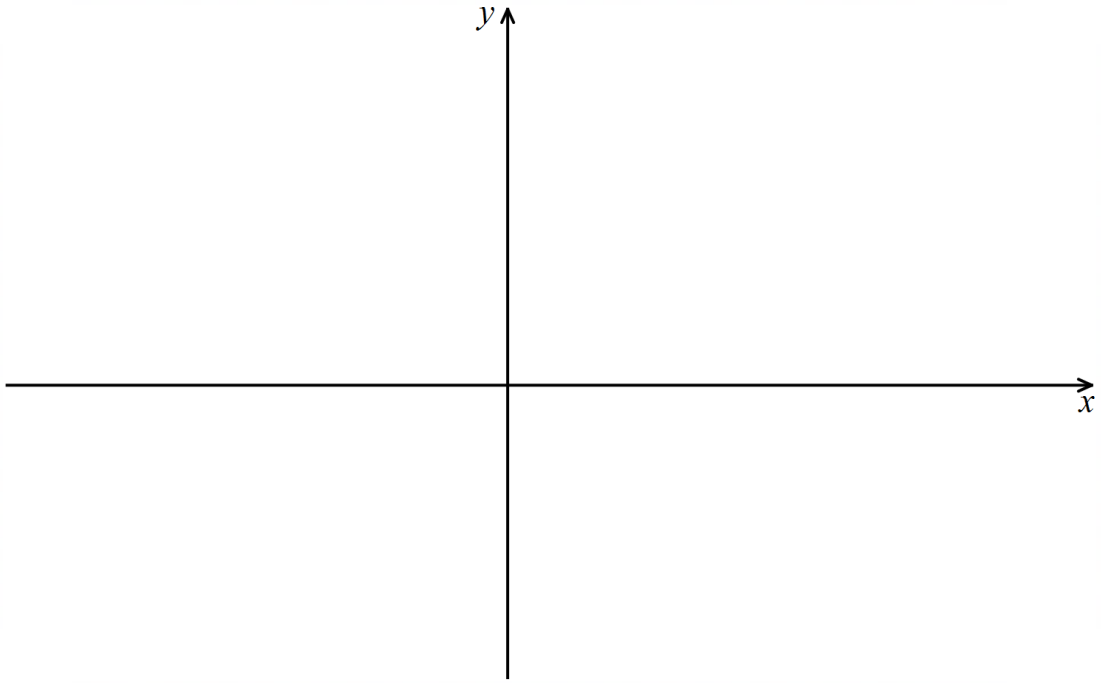
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(b) Sketch the graphs of $f(x)$ and $g(x)$on the same axis, showing all important features, including the points of intersection. (Do not find any stationary points.) **3**



(c) Hence, or otherwise solve the inequality $x - 2 < (x + 2)(x^2 - 4)$ **1**

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Question 21 (4 marks)

The velocity, $\dot{x}$, in m/s of a particle moving in a horizontal straight line is given by:

$$\dot{x} = 3 - \frac{9}{t-2} \text{ for } t > 2,$$

where t is the time in seconds.

(a) In which direction is the particle travelling when $t = 3$? **1**
Justify your answer with calculations.

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(b) Find the time when the particle changes direction. **1**

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Question 21 continues on page 20

(c) Hence, or otherwise, find the distance travelled by the particle between $t = 3$ and $t = 7$. Give your answer correct to 2 decimal places. **2**

Question 22 (3 marks)

Events A and B are such that $P(A) = \frac{3k}{4}, P(B) = k$ and $P(\bar{A} \cap B) = k^2$ where $0 < k < 1$.

(a) Find an expression for $P(A \cap B)$ in terms of k. **1**

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(b) Given that the two events are independent, find the value of k. **2**

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Question 23 (5 marks)

Julio has just retired and invests part of his superannuation into an annuity.

He invests \$400 000 into an account that earns interest at a rate of 3% p.a compounded monthly. and receives regular monthly payments of $\$M$ from the annuity at the end of each month. Let A_n be the amount in the account after n months.

(a) Show that the amount remaining in the annuity after the third monthly payment is given by: **2**

$$A_3 = 400\,000 \times 1.0025^3 - M(1 + 1.0025 + 1.0025^2)$$

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(b) Determine the monthly payment that Julio receives given that the annuity will last for 25 years. **3**

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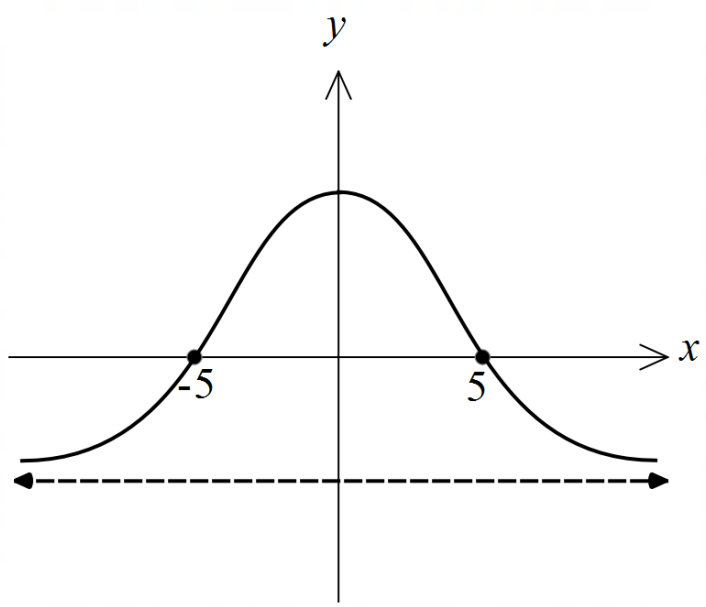
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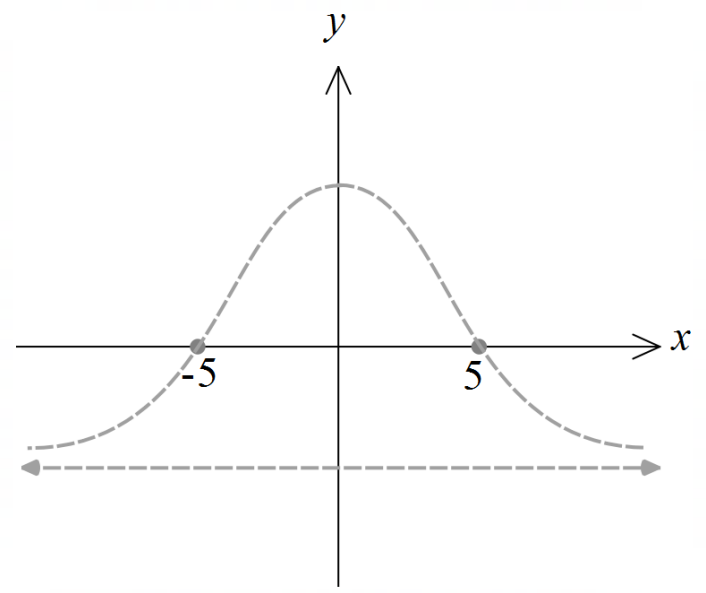
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Question 24 (2 marks)

The graph shows the function $y = f(x)$ with a horizontal asymptote, and points of inflections at (-5,0) and (5,0).



On the axes below, sketch the graph of $y = f'(x)$, clearly showing the behaviour at the intercepts and any asymptotes. **2**



Question 25 (4 marks)

Given a function:

$$f(x) = \begin{cases} \frac{3x}{4}(2-x) & \text{for } 0 \leq x \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

(a) Prove that $f(x)$ represents a probability density function. **2**

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(b) State the mode of the distribution. **1**

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(c) Find $P(X > 1.5)$. **1**

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Question 26 (10 marks)

Consider the function:

$$f(x) = \log_e(x^2 + 1)$$

(a) Find the stationary point(s) of $f(x)$ and determine their nature. **2**

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(b) Find the coordinates of the point(s) of inflection of $f(x)$. **3**

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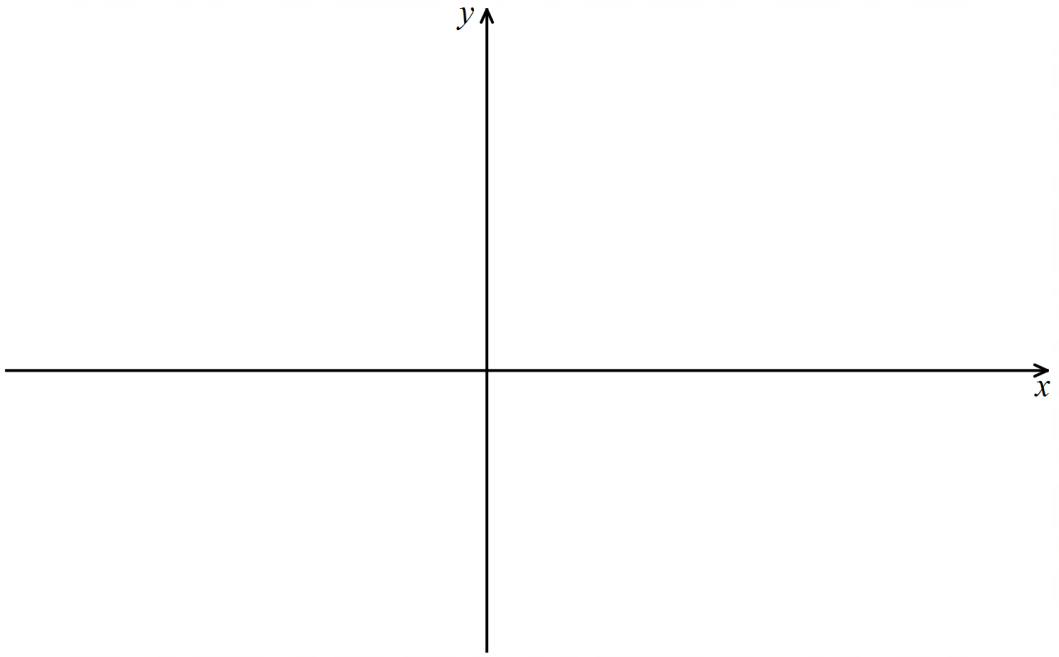
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Question 26 continues on page 26

(c) Hence, sketch the curve of $f(x)$, showing all key features. **2**



(d) Use the Trapezoidal Rule with four subintervals to find an approximation to: **2**

$$\int_1^3 \log_e (x^2 + 1)\, dx.$$

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(e) State whether the approximation found in part (b) is greater or less than the exact value of $\int_1^3 \log_e(x^2 + 1)\ dx$. Use your sketch in part (c) to provide a brief justification for your answer. **1**

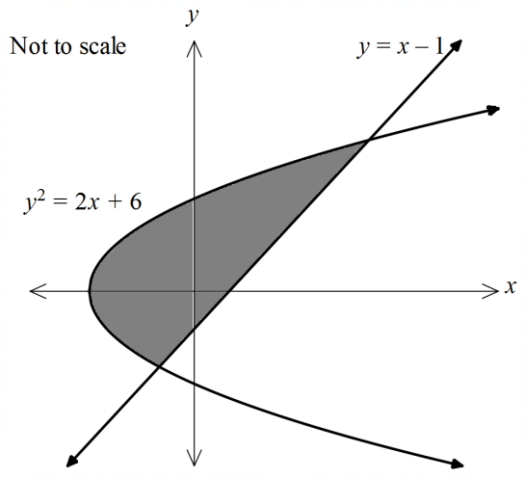
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Question 27 (3 marks)

The graph shows a shaded region bounded by $y^2 = 2x + 6$ and $y = x - 1$ below



Determine the area of the region. **3**

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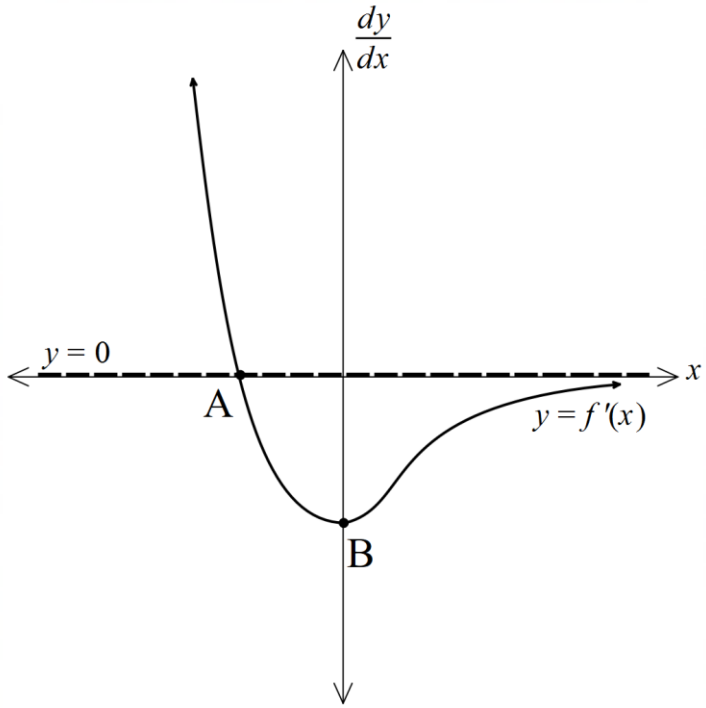
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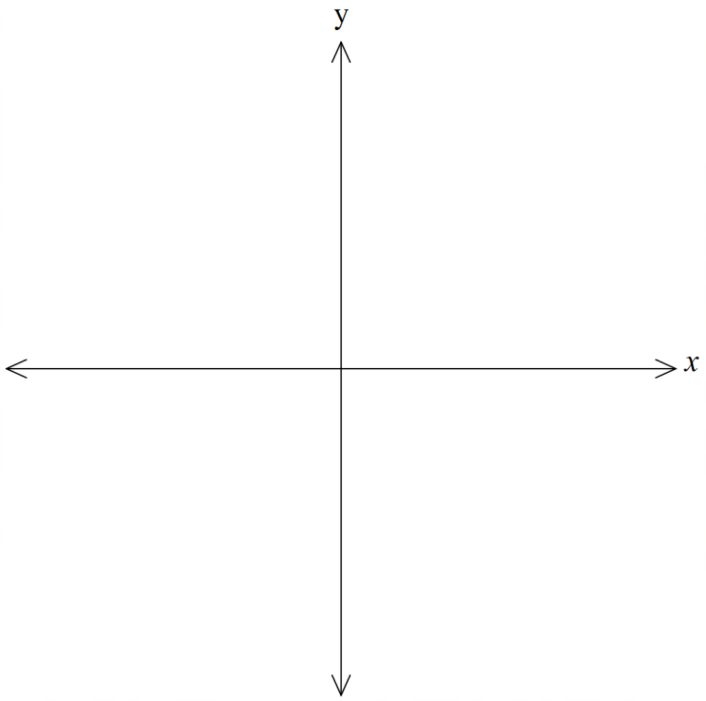
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Question 28 (3 marks)

Below is a graph of $y = f'(x)$, which passes the x-axis at point A, has a minimum turning point at B and asymptote at $y = 0$. **3**



It is known that $y = f(x)$ has an asymptote at $y = 0$. Sketch a possible graph for $y = f(x)$, showing all important key features and plotting points A' and B'.



Question 29 (2 marks)

A random variable is normally distributed with mean 0 and standard deviation 1. The table gives the probability that this random variable lies between 0 and z for different values of z. **2**

	first decimal place									
z	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9
0.	0.5000	0.5398	0.5793	0.6179	0.6554	0.6915	0.7257	0.7580	0.7881	0.8159
1.	0.8413	0.8643	0.8849	0.9032	0.9192	0.9332	0.9452	0.9554	0.9641	0.9713
2.	0.9772	0.9821	0.9861	0.9893	0.9918	0.9938	0.9953	0.9965	0.9974	0.9981
3.	0.9987	0.9990	0.9993	0.9995	0.9997	0.9998	0.9998	0.9999	0.9999	1.0000

Maya is training for long jump for the national age championships. To be able to compete at the national age championships her longest jump must be beaten by less than 0.26% of the general population.

The distance jumped by the general population is normally distributed with a mean of 3.48 metres and standard deviation of 1.08 metres.

Using the table, what is the minimum distance that Maya needs to jump, correct to the nearest centimetre.

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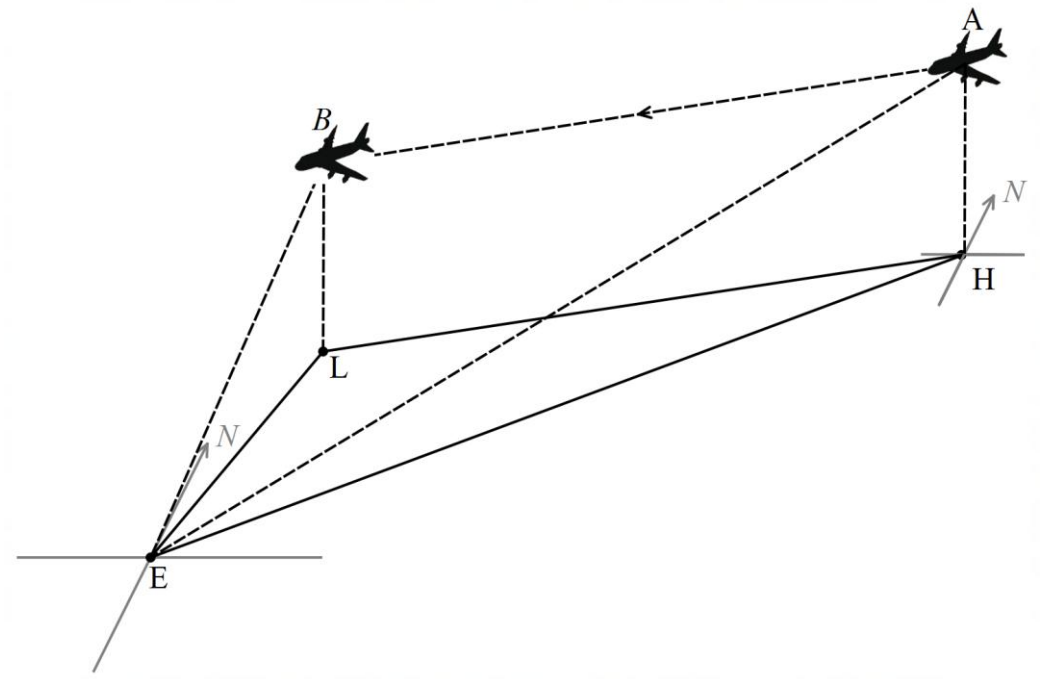
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Question 30 (5 marks)

Emma is plane spotting at a sea-level vantage point (E). From there she can see both her house (H) which is on a bearing of $030°T$, and a plane directly above her house (A) at an angle of elevation of $20°$.

From point (A), the plane travelled at a constant altitude of 900 m on a bearing of $225°T$ to point (B) 1700 m away. Point (B) is directly above a landmark (L)

(a) On the diagram below, label all relevant information listed above. **1**



(b) Determine the new bearing, and angle of elevation, of the aircraft from Emma's observation point (E). **4**

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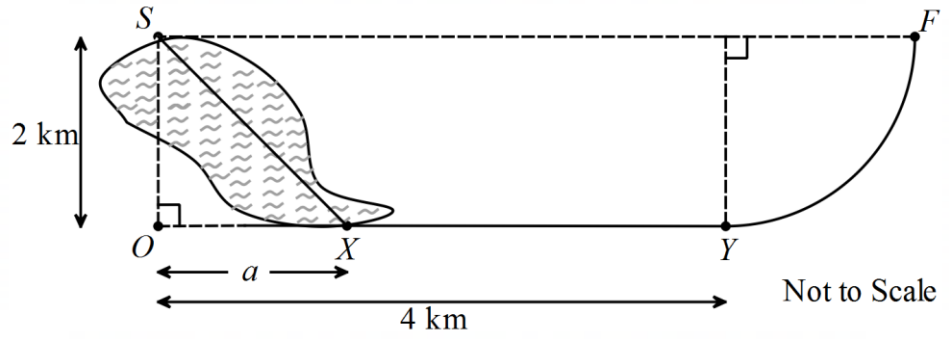
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Question 31 (2 marks)
Given n is a constant, simplify **2**

$$\int x^n(1+x)^n\left(1+\frac{1}{x}\right)^n dx.$$

Question 32 (5 marks)

Helen is training to compete in a mini triathlon. The course she practises on consists of three legs which starts at S and finishes at F. The first leg is a straight line swim from S to a point X. The second leg is a bike ride from X to Y along a straight road OY and the final leg is a jog from Y to F around a circular path. The perpendicular distance from S to O is 2 km while the distance OY is 4 km.



Helen can swim at 4 km/h, bike ride at 16 km/h and jog at 8 km/h.

(a) If the distance $OX = a$ km, show that the time T that it takes Helen to complete the three legs is given by **2**

$$T = \frac{4\sqrt{a^2 + 4} - a + (4 + 2\pi)}{16} \text{ hours.}$$

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Question 32 continues on page 34

(b) Find the value of a that will allow Helen to minimise the time taken to complete the three legs of her practise course. **3**

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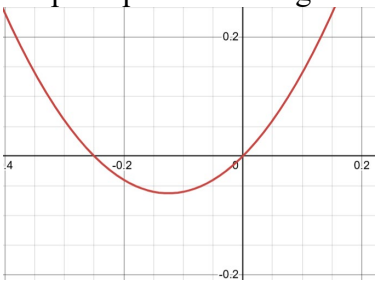
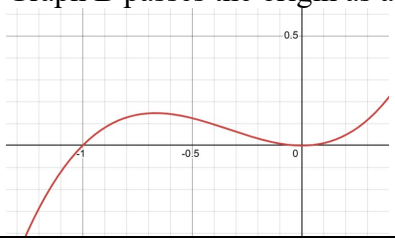
Section II extra writing space

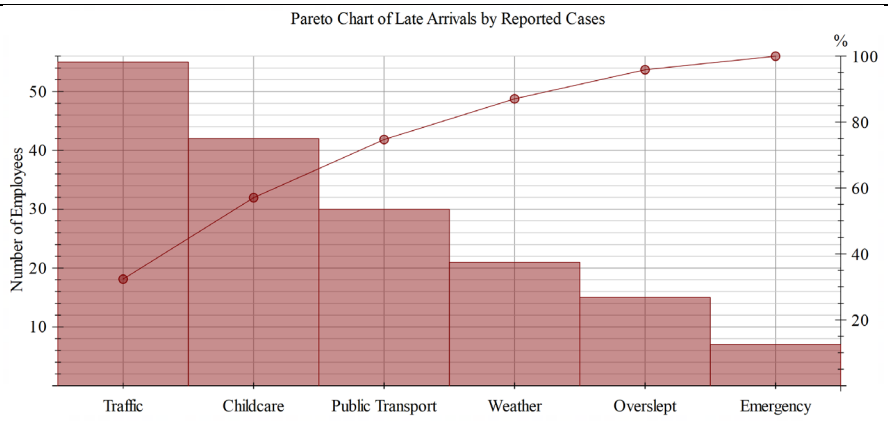
If you use this space, clearly indicate which question you are answering.

Section II extra writing space

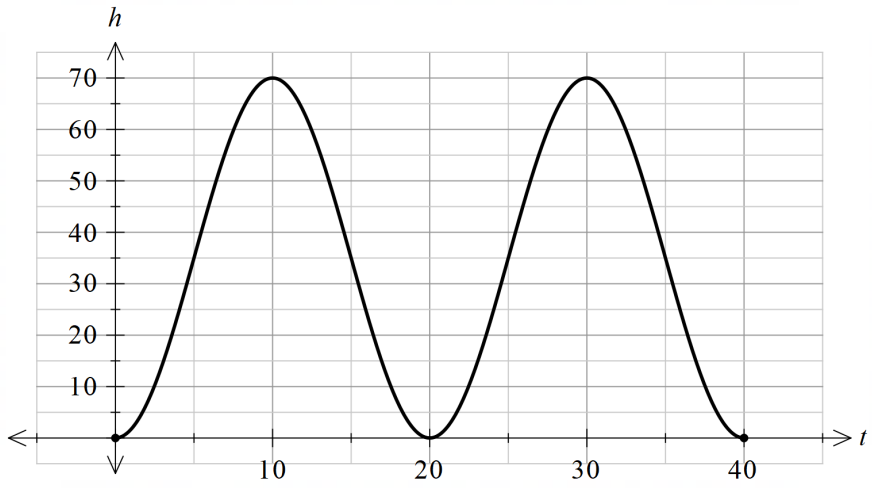
If you use this space, clearly indicate which question you are answering.

Baulkham Hill High School – Trial Advanced Solutions

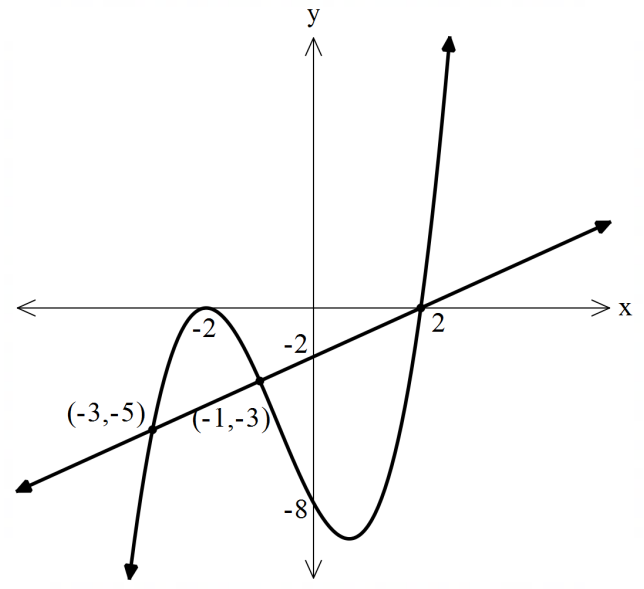
Qn	Worked Solution	Marking Criteria
1.	The argument of the logarithm must be positive: $x-2>0$, $x>2$ | The expression inside the square root must be non-negative: $9-x\geq 0$, $x\leq 9$ $\therefore x>2$ and $x\leq 9$	C
2.	$\sum_{r=1}^{20} -(-1)^r$ | T_1 | T_2 | … | T_{18} | T_{19} | T_{20} | | $-(-1)^1=-1$ | $-(-1)^2=1$ | | $-(-1)^{18}=-1$ | $-(-1)^{19}=1$ | $-(-1)^{20}=-1$ | Pairs will add up to zero so $S_{20}=0$	B
3.	The mean is the midpoint: $\mu=\frac{25+145}{2}$ $\mu=85$ $\mu-3\sigma=25$ $85-3\sigma=25$ $\sigma=20$ varience $=400$	D
4.	Only graphs A and B pass through the origin. Graph A passes the origin as a single root.  Graph B passes the origin as a double root. 	B
5.	A is the only answer that give you 0 as $f(x)$ is odd	A
6.	4% pa = 1% per quarter 2 years = 8 quarterly payments $FV=2500(8.28567)$ $FV=20714.18$	C
7.	$0.8\times 0.3+0.2\times 0.7=0.38$	C

8.	$= -\cot(90 - x) \times \cos x$ $= -\tan x \times \cos x$ $= -\sin x$	A
9.	Average of 2 new scores $= \dfrac{10.5 + 9.5}{2} = 10$ So the average remains the same. But both scores have a distance of 0.5 to the mean, and since the standard deviation is 2, this will reduce the deviation further.	D
10.	$f(4) = 1$ $f'(x) = 6$ $g(2) = 4$ $g'(2) = 4$ $h'(x) = f'\big(g(x)\big) \times g'(x).$ $h'(2) = f'(g(2)) \cdot g'(2)$ $h'(2) = 6 \times 4 = 24.$ So the gradient of the tangent is 24. $h(2) = f\big(g(2)\big)$ $= f(4)$ $= 1$ $\therefore$ tangent passes$(2,1)$ $y - 1 = 24(x - 2)$ $y = 24x - 47$	D
11.	$\dfrac{dy}{dx} = \dfrac{1}{tan\, x} \cdot \dfrac{d}{dx}(\tan x) = \dfrac{\sec^2 x}{\tan x}$ $at\ x = \frac{\pi}{3}$: $\dfrac{dy}{dx} = \dfrac{\sec^2 \frac{\pi}{3}}{\tan \frac{\pi}{3}}$ $= \dfrac{4}{\sqrt{3}} = \dfrac{4\sqrt{3}}{3}$	**2 mark** • Correct answer **1 mark** • Recognises derivative of $\tan x = \sec^2 x$ • Attempts to derive $\ln x$, by having $\tan x$ in the denominator
12.		**1 mark** • Correct answer (Accept 15%-19%)

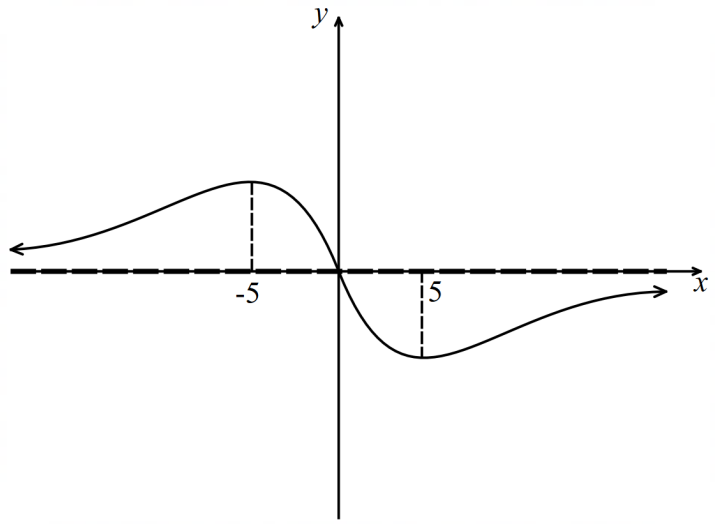
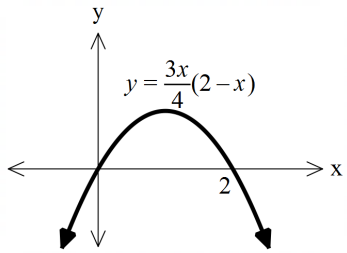
13.(a)	$T_n = 51 + (n-1)(-7)$ $T_n = 58 - 7n$	**1 mark** • Correct answer
13.(b)	$58 + 47 = 7n$ $105 = 7n$ $n = 15$	**1 mark** • Correct answer
13.(c)	$S_{15} = \frac{15}{2}\left(51 + (-47)\right)$ $= 30$	**1 mark** • Correct answer
14.(a)	$y = \sqrt[3]{2x-11} = (2x-11)^{\frac{1}{3}}$ $\frac{dy}{dx} = \frac{1}{3}(2x-11)^{-\frac{2}{3}} \times 2 = \frac{2}{3}(2x-11)^{-\frac{2}{3}}$	**2 mark** • Correct solution **1 mark** • Recognises power is $-\frac{2}{3}$ • Or significant attempt for chain rule.
14.(b)	$\frac{dy}{dx} = e^x \tan(5x) + 5e^x \sec^2(5x)$	**2 mark** • Correct solution **1 mark** • Attempts to use the product rule
14.(c)	$y = \ln x - 4\ln(x+1)$ $\frac{dy}{dx} = \frac{1}{x} - 4\left(\frac{1}{x+1}\right)$ $\frac{dy}{dx} = \frac{1-3x}{x(x+1)}$	**2 mark** • Correct solution **1 mark** • Uses log laws to simplify the question • Correctly uses quotient rule to find $\frac{f'(x)}{f(x)}$
15.(a)	11	**1 mark** • Correct answer
15.(b)	Strong and positive correlation. The more books read, the better they did in exams.	**1 mark** • Correct answer
15.(c)	82%-84%	**1 mark** • Correct answer
16.(a)	$\frac{1}{4}$ revolution $= \frac{1}{2} \times$ diameter $= 35$ m	**1 mark** • Correct answer

16.(b)	Period $= \frac{2\pi}{b} = 20$ $b = \frac{2\pi}{20} = \frac{\pi}{10}$ $c = 35, a = -35$ $h(t) = -35\cos\left(\frac{\pi}{10}t\right) + 35$ $\therefore a = -35, b = \frac{\pi}{10}, c = 35$	**2 marks** • Correct solution **1 mark** • Obtains correct value for 1 variable
16.(c)		**2 mark** • Correct solution **1 mark** • Sine or cosine curve that oscillates between $h = 0$ *and* $h = 70$. Note: Must have a stationary min point at $t = 0$ and $t = 40$
16.(d)	Maximum rate of change: $R = h'(t)$ $R = -35\left(\frac{\pi}{10}\right)\left(-\sin\left(\frac{\pi}{10}t\right)\right)$ $R = \frac{7\pi}{2}\sin\left(\frac{\pi}{10}t\right)$ Differentiate again: $\frac{dR}{dt} = \frac{7\pi}{2} \times \frac{\pi}{10}\cos\left(\frac{\pi}{10}t\right)$ $\frac{dR}{dt} = \frac{7\pi^2}{20}\cos\left(\frac{\pi}{10}t\right)$ $\frac{7\pi^2}{20}$ is maximum (highest point in the graph)	**2 mark** • Correct solution **1 mark** • Correct expression for R
16.(e)	$50 = -35\cos\left(\frac{\pi t}{10}\right) + 35$ $15 = -35\cos\left(\frac{\pi t}{10}\right)$ $-\frac{3}{7} = \cos\left(\frac{\pi t}{10}\right)$ $\cos\left(\frac{\pi t}{10}\right) = -\frac{3}{7}$	**2 mark** • Correct solution (must have 6 min and 25 secs) **1 mark** • Obtains $-\frac{3}{7} = \cos\left(\frac{\pi t}{10}\right)$

	acute angle $= \cos^{-1}\left(-\frac{3}{7}\right)$ $\frac{\pi t}{10} = \cos^{-1}\left(-\frac{3}{7}\right)$ $t = \frac{10}{\pi}\cos^{-1}\left(-\frac{3}{7}\right)$ $t \approx 6.41$ .... $t = 6$ minutes and 25 seconds	
17.	Method 1: $\int x\sqrt{5x^2+1}\,dx = \frac{1}{10}\int 10x\sqrt{5x^2+1}\,dx$ $= \frac{1}{10}(5x^2+1)^{\frac{3}{2}} \times \frac{2}{3} + C$ $= \frac{1}{15}(5x^2+1)^{\frac{3}{2}} + C$ Method 2: If $y = (5x^2+1)^{\frac{3}{2}}$ $\frac{dy}{dx} = 15x(5x^2+1)^{\frac{1}{2}}$ $\int x\sqrt{5x^2+1}\,dx = \frac{1}{15}(5x^2+1)^{\frac{3}{2}} + C$	**3 marks** • Correct solution with +C **2 marks** • Attempts to integrate form the form $k\int f'(x)[f(x)]^n\,dx$ or equivalent merit. **1 mark** • Recognises the integral is of the form $k\int f'(x)[f(x)]^n\,dx$ or equivalent merit.
18.(a)	$T = 60 + 150e^0 = 210$°C	**1 mark** • Correct answer
18.(b)	$\frac{dT}{dt} = -15e^{-0.1t}$ $-10 = -15e^{-0.1t}$ $\frac{2}{3} = e^{-0.1t}$ $-0.1t = \log_e\left(\frac{2}{3}\right)$ $t = -10\log_e\left(\frac{2}{3}\right)$ $t = 4.0547$ sec $T = 60 + 150\left(\frac{2}{3}\right)$ $T = 160°C$	**3 marks** • Correct solution **2 marks** • Finds $t = 4.0547$ sec **1 mark** • Finds $\frac{dT}{dt} = -15e^{-0.1t}$
18.(c)	$60°C$	**1 mark** • Correct answer
19.(a)	14	**1 mark** • Correct answer

19.(b)	$27 - 4 = 23$	**1 mark** • Correct answer
19.(c)	Group A is the IQR is smaller IQR for A = 12 IQR for B = 23	**1 mark** • Correct solution
19.(d)	Group A IQR $= 12$ Upper bound= $16 + 1.5(12) = 34$. Therefore, 36 is an outlier only in Group A.	**1 mark** • Correct solution
20.(a)	$x - 2 = (x + 2)(x^2 - 4)$ $(x - 2) - (x + 2)(x + 2)(x - 2) = 0$ $(x - 2)[1 - (x + 2)^2] = 0$ $(x - 2)[(x + 2)^2 - 1] = 0$ $x - 2 = 0$ | $(x + 2)^2 = 1.$ $x = 2$ | $x = -1\ or - 3$ $\therefore x = -1\ or - 3\ or\ 2$	**2 mark** • Finds all x-values **1 mark** • Obtains $(x - 2)[1 - (x + 2)^2] = 0$ OR finds 2 correct x-values
20.(b)		**3 marks** • Two correct curves with correct intercepts showing points of intersection. **2 marks** • Two correct curves with correct intercepts **1 mark** • One correct curve with correct intercepts
20.(c)	$-3 < x < -1$ or $x > 2$	**1 mark** • Correct answer
21.(a)	$x = 3 - \frac{9}{3 - 2} = -6$ ∴moving left.	**1 mark** • Correct solution, by finding $x = -6$ first.
21.(b)	$3 - \frac{9}{t - 2} = 0$ $t = 5$	**1 mark** • Correct answer

21.(c)	$$\text{Distance} = -\int_3^5 \left(3 - \frac{9}{t-2}\right) dt + \int_5^7 \left(3 - \frac{9}{t-2}\right) dt.$$ $$= [3t - 9\ln(t-2)\,]_3^5 + [3t - 9\ln(t-2)\,]_5^7$$ $$= [(3\times 5 - 9\ln(5-2) - (3\times 3 - 9\ln(3-2)]$$ $$+[(3\times 7 - 9\ln(7-2) - (3\times 5 - 9\ln(5-2)]$$ $$= [9\ln 3 - 6] + \left[6 + 9\ln\left(\frac{3}{5}\right)\right]$$ $$= 5.29 \text{ (2 decimal places)}$$	**2 mark** • Correct solution **1 mark** • Finds the distance of one of the directions.
22.(a)	$$P(A \cap B) = P(B) - P(\bar{A} \cap B)$$ $$= k - k^2$$	**1 mark** • Correct answer
22.(b)	If the events are independent, $$P(A \cap B) = P(A)P(B).$$ $$k - k^2 = \frac{3k}{4} \times k$$ $$3k^2 = 4k - 4k^2$$ $$7k^2 - 4k = 0$$ $$k(7k-4) = 0.$$ $$k = 0 \text{ or } k = \frac{4}{7}.$$ $$\therefore\ k = \frac{4}{7} \text{ (since } 0 < k < 1)$$	**2 mark** • Correct solution **1 mark** • Write a correct equation in terms of k.
23.(a)	$$A_1 = 400\,000(1.0025) - M$$ $$A_2 = 400\,000(1.0025)^2 - M(1.0025) - M$$ $$A_3 = 400\,000(1.0025)^3 - M(1.0025) - M(1.0025)^2 - M$$ $$A_3 = 400\,000(1.0025)^3 - M[1 + 1.0025 + (1.0025)^2]$$	**2 mark** • Correct solution **1 mark** • Finds A_1
23.(b)	$$A_n = 400000(1.0025)^n - M(1 + 1.0025 + \cdots + (1.0025)^{n-1})$$ $$A_n = 400000(1.0025)^n - M\left(\frac{(1.0025)^n - 1}{0.0025}\right)$$ $$n = 25\times 12 = 300, A_n = 0$$ $$0 = 400000(1.0025)^{300} - M\left(\frac{(1.0025)^{300} - 1}{0.0025}\right)$$ $$M\left(\frac{(1.0025)^{300} - 1}{0.0025}\right) = 400000(1.0025)^{300}$$ $$M = 400000(1.0025)^{300} \div \left(\frac{(1.0025)^{300} - 1}{0.0025}\right)$$ $$M = \$1896.85 \text{ per month}$$	**3 marks** • Correct solution **2 marks** • Uses a GP to simplify the expression and lets $A_n = 0$ **1 mark** • Correct expression for A_n

24.		**2 marks** • Shows all special features below **1 mark** • Shows one of the special features below Special features: - Point of inflection at (0,0) - Max point at $x = -5$ min point at $x = 5$
25.(a)	The values of $y = \frac{3x}{4}(2 - x)$ is always positive between $0 \leq x \leq 2$, shown in the graph below. Also, $\frac{3}{4}\int_0^2 (2x - x^2)\, dx = \frac{3}{4}\left[x^2 - \frac{x^3}{3}\right]_0^2$ $= \frac{3}{4}\left(2^2 - \frac{2^3}{3}\right) - \frac{3}{4}\left(0^2 - \frac{0^3}{3}\right) = 1$  $f(x) \geq 0$ between $0 \leq x \leq 2$ Since $f(x) \geq 0$ for all x and $\int_{-\infty}^{\infty} f(x)\, dx = 1$ $f(x)$ is a probability density function	**2 mark** • Correct solution **1 mark** • Shows $\int_{-\infty}^{\infty} f(x)\, dx = 1$ • Explains $f(x) \geq 0$
25.(b)	Mode = 1	**1 mark** • Correct answer
25.(c)	$\frac{3}{4}\int_{1.5}^2 (2x - x^2)\, dx = \frac{3}{4}\left[x^2 - \frac{x^3}{3}\right]_{1.5}^2$ $= \frac{3}{4}\left(2^2 - \frac{2^3}{3}\right) - \frac{3}{4}\left(1.5^2 - \frac{1.5^3}{3}\right)$ $= \frac{5}{32}$	**1 mark** • Correct answer

26.(a)

$$f'(x) = \frac{1}{x^2+1} \times 2x = \frac{2x}{x^2+1}$$

Stationary points occur where $f'(x) = 0$.

$$\frac{2x}{x^2+1} = 0$$
$$2x = 0$$
$$x = 0$$
$$f''(x) = \frac{2(x^2+1) - 2x(2x)}{(x^2+1)^2}$$
$$f''(x) = \frac{2x^2 + 2 - 4x^2}{(x^2+1)^2}$$
$$f''(x) = \frac{2(1-x^2)}{(x^2+1)^2}$$
$$f''(0) = \frac{2(1-0^2)}{(0^2+1)^2} > 0$$
$$\therefore concave\ up, \therefore \min point$$

$$f(0) = \ln(0^2+1) = 0.$$

So the minimum point is (0,0)

2 marks
- Shows $(0,0)$ is a min point

1 mark
- Finds $\frac{2x}{x^2+1} = 0$

26.(b)

Inflection when $f''(x) = 0$

$$2(1-x^2) = 0$$
$$x^2 = 1$$
$$x = \pm 1$$
$$f(\pm 1) = \ln((\pm 1)^2 + 1) = \ln(2)$$

x	-2	-1	0	1	2
$f''(x)$	$-\frac{6}{25}$	0	2	0	$-\frac{6}{25}$
	$\cap$	·	$\cup$	·	$\cap$

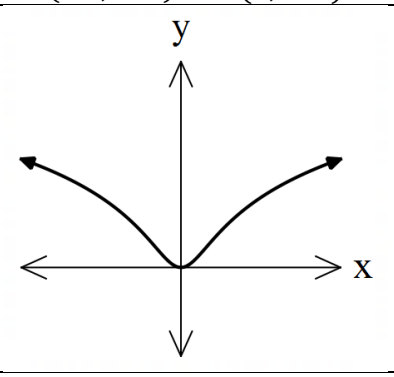
$(-1, \ln 2)$ and $(1, \ln 2)$

3 marks
- Shows the points are points of infections

2 marks
- Finds $(-1, \ln 2)$ and $(1, \ln 2)$

1 mark
- Finds $f''(x) = 0$

26.(c)



2 mark
- Correct solution

1 mark
- Explains that curve is concave down between $-2 < x < 2$

26.(d)

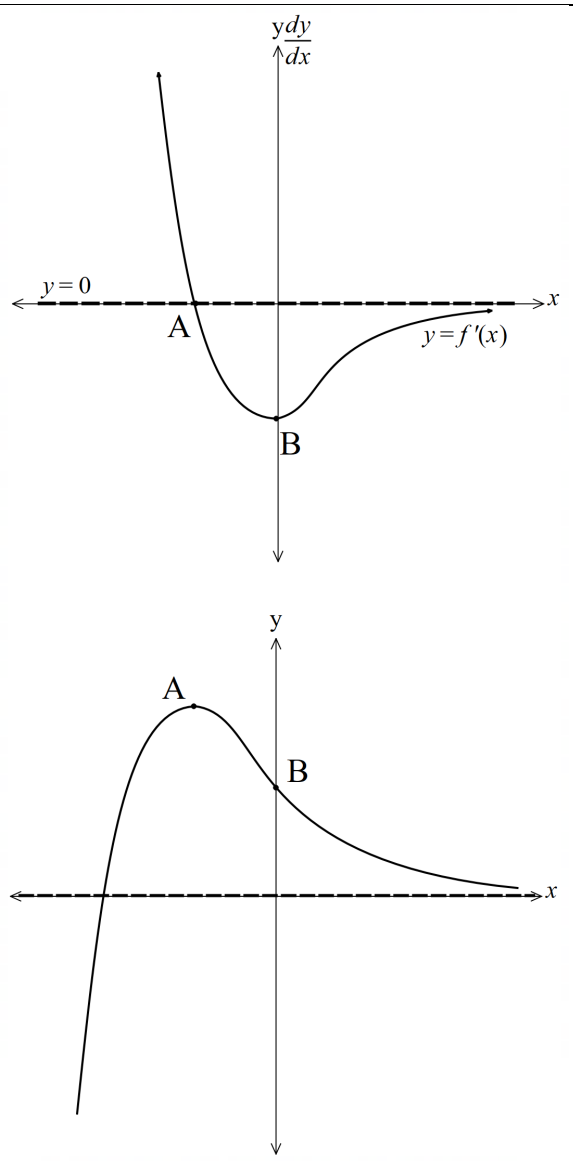
x	1	1.5	2	2.5	3
$f(x)$	$\ln 2$	$\ln 3.25$	$\ln 5$	$\ln 7.25$	$\ln 10$

$$T = \frac{0.5}{2}\left[\ln 2 + 2\left(\ln\frac{13}{4} + \ln 5 + \ln\frac{29}{4}\right) + \ln 10\right]$$
$$= 3.1335$$

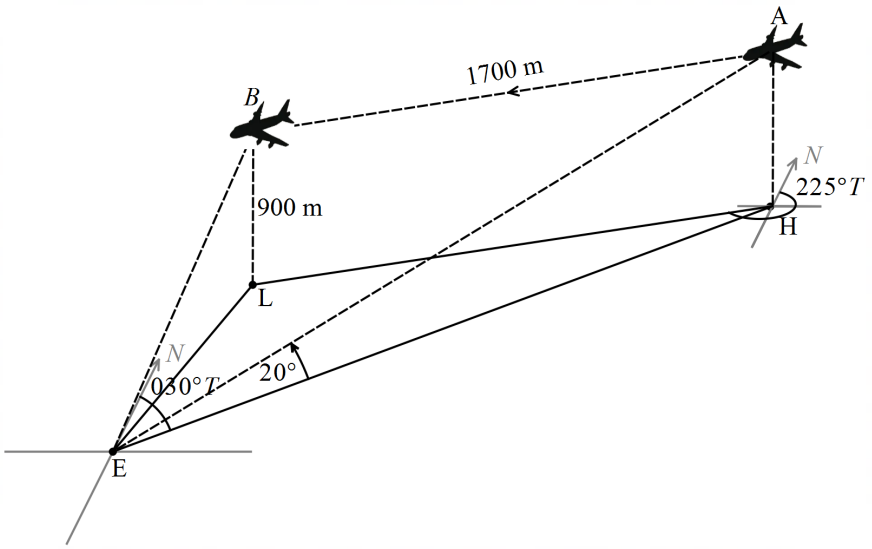
2 marks
- Finds y values

1 mark
- Finds the correct y values using 5 function values

26.(e)	Between $-2 < x < 2$, the curve is a concave down curve. The straight-line chords used in the Trapezoidal Rule lie below the curve. Therefore, the trapezoidal approximation gives an area that is less than the true area.	**1 mark** • Explains that curve is concave down between $-2 < x < 2$, and a line will be above.
27.	$y^2 = 2x + 6 \ldots\ldots (1)$ $y = x - 1$ $x = y + 1. \ldots\ldots\ldots (2)$ Sub (2) into (1) $\frac{y^2 - 6}{2} = y + 1$ $y^2 - 6 = 2y + 2$ $y^2 - 2y - 8 = 0$ $(y - 4)(y + 2) = 0.$ $y = 4$ and $y = -2$ $\therefore$ points of intersection are $(-1, -2)$ and $(5,4)$ $A = \int_{-2}^{4} \left[(y + 1) - \frac{y^2 - 6}{2}\right] dy$ $A = \int_{-2}^{4} \left(-\frac{y^2}{2} + y + 4\right) dy$ $A = \left[-\frac{y^3}{6} + \frac{y^2}{2} + 4y\right]_{-2}^{4}$ $A = \left[-\frac{4^3}{6} + \frac{4^2}{2} + 4(4)\right] - \left[-\frac{(-2)^3}{6} + \frac{(-2)^2}{2} + 4(-2)\right]$ $A = \frac{40}{3} - \frac{14}{3}$ $A = 18 \text{ units}^2$	**3 marks** • Correct solution **2 marks** • Finds the correct integral to find the shaded area. **1 mark** • Finds points of intersection

28.		**3 marks** • Correct solution **2 marks** • Recognises two of the key features below. **1 mark** • Recognises one of the key features below. Key features: - Point A is a maximum point - Point B is a point of inflection. (Must not be a horizontal point of inflection) - graph approaches $y = 0$ as $x \to \infty$
29.	$1 - 0.26 = 0.9974$ $P(Z < z) = 0.9974$ From the standard normal distribution table, $z \approx 2.8$ z-score formula: $z = \frac{x-\mu}{\sigma}$. $2.8 = \frac{x - 3.48}{1.08}$ $x - 3.48 = 3.024$ $x = 6.50\text{m}$ (nearest cm)	**2 mark** • Correct solution **1 mark** • Uses the normal distribution table to find $z \approx 2.8$ • Finds x using wrong z

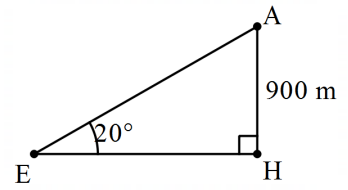
30.(a)



1 mark
- Correctly labels:
  - 1700m
  - 900m
  - $225°T$
  - $030°T$
  - $20°$

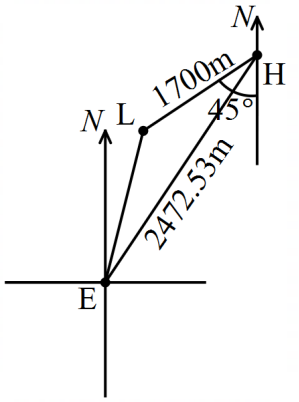
30.(b)

$In\ \Delta EHA$



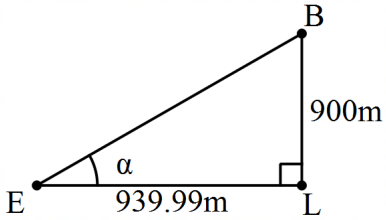
$$\tan 20° = \frac{900}{EH}$$
$$EH = \frac{900}{\tan 20°} \approx 2472.73 \text{ m.}$$

In ΔEHL:



$$\angle EHL = 45° - 30° = 15°$$
$$EL^2 = 1700^2 + 2472.73^2 - 2(1700)(2472.73)\cos 15°$$
$$EL = 939.99 \text{ m}$$

In ΔEBL



Angle of elevation:

$$\tan \alpha = \frac{900}{939.29}$$
$$\alpha = 44°$$

Bearing:

In ΔELH

$$\frac{\sin \beta}{1700} = \frac{\sin 15°}{939.99}$$

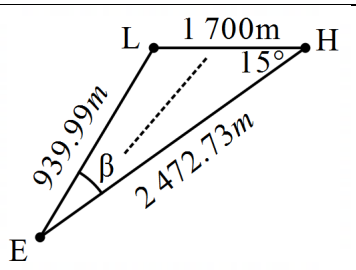
$\beta = 27.90$ (smallest angle opposite the smallest side)

4 marks
- Correct solution

3 marks
- Find either the angle of elevation or bearing (True bearing must be 3 digits)

2 mark
- Calculates EL

1 mark
- Calculates EH

	 ∴ Bearing is $30 - 27.90 = 002°T$ and angle of elevation is 44°	
31.	$\int x^n(1+x)^n\left(1+\frac{1}{x}\right)^n dx = \int (1+x)^n \times \left[x\left(1+\frac{1}{x}\right)\right]^n dx$ $= \int (1+x)^n(1+x)^n dx$ $= \int (1+x)^{2n} dx$ $= \frac{(1+x)^{2n+1}}{2n+1} + C$	**2 marks** • Correct solution **1 mark** - Obtains $(x^2+2x+1)^n$ - Integrates to the power of $2n$
32.(a)	$SX = \sqrt{a^2+4}$ $\therefore\ T = \frac{D}{S} = \frac{\sqrt{a^2+4}}{4}$ hours $XY = 4 - a$ $\therefore\ T = \frac{4-a}{16}$ hours $YF = \frac{2\pi(2)}{4} = \pi$ $\therefore\ T = \frac{\pi}{8}$ hours Combining the three times: $\therefore\ T = \frac{\sqrt{a^2+4}}{4} + \frac{4-a}{16} + \frac{\pi}{8}$ $T = \frac{4\sqrt{a^2+4} - a + (4+2\pi)}{16}$ hours	**2 marks** • Links all three expressions to give the correct answer **1 mark** • Correctly finds the distances SX or XY or YF

32.(b)

$$\frac{dT}{da} = \frac{1}{16}\left[4 \times \frac{1}{2}(a^2+4)^{-\frac{1}{2}} \times (2a) - 1\right]$$

$$\frac{dT}{da} = \frac{1}{16}\left[\frac{4a}{\sqrt{a^2+4}} - 1\right]$$

Stationary point when $\frac{dT}{da} = 0$

$$0 = \frac{4a}{\sqrt{a^2+4}} - 1$$

$$4a = \sqrt{a^2+4}$$

$$16a^2 = a^2 + 4$$

$$15a^2 = 4$$

$$a^2 = \frac{4}{15}$$

$$a = \frac{2}{\sqrt{15}} \approx 0.51 \text{ km} \quad (a > 0)$$

a	0.2	0.3	0.4
$\frac{dT}{da}$	-0.0128	0	0.0356
	\	_	/

$\therefore$ minimum occurs at $a = \frac{2}{\sqrt{15}} \approx 0.51$ km

3 marks
- Correctly shows the value of a is a minimum

2 marks
- Finds a

1 mark
- Correctly finds $\frac{dT}{da}$.

End of Exam