NAME: _____

Blacktown Boys' High School

2026 Year 12

Trial Examination

Mathematics Advanced

General Instructions

- Reading time – 10 minutes
- Working time – 3 hours
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided for the paper
- In Section II, show all relevant mathematical reasoning and/or calculations

Total marks: 100

Section I – 10 marks (page 3 - 7)

- Attempt Questions 1 - 10
- Allow about 15 minutes for this section

Section II – 90 marks (pages 9 - 38)

- Attempt Questions 11 - 34
- Allow about 2 hours 45 minutes for this section

Assessor: Mrs Chhabra

Students are advised that this is a trial examination only and cannot in any way guarantee the content of format of the 2026 Higher School Certificate Examination.

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Section I

10 marks
Attempt Questions 1–10
Allow about 15 minutes for this section

Use the multiple choice answer sheet for Questions 1–10.

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1 The first three terms of an arithmetic sequence are 5,9 and 13.
What is the 18^{th} term of the sequences?

A. 81

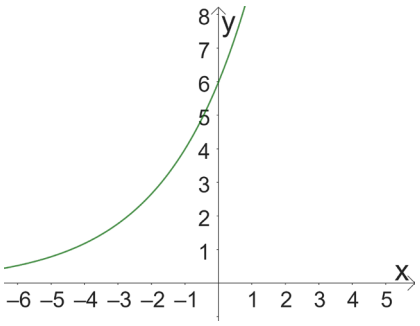
B. 77

C. 73

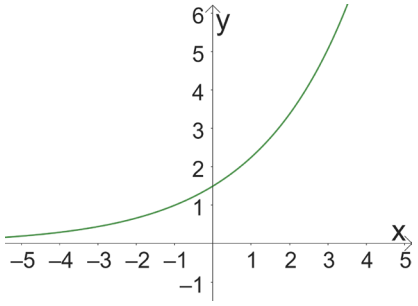
D. 69

2 Which of the following best represents the graph of $y = 6(1.5)^x$?

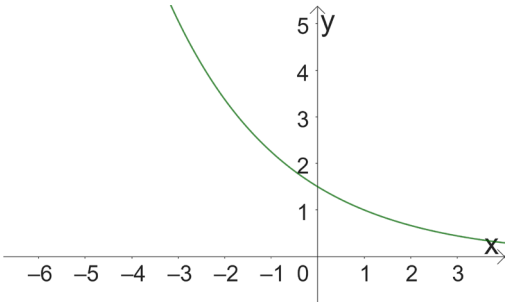
A.



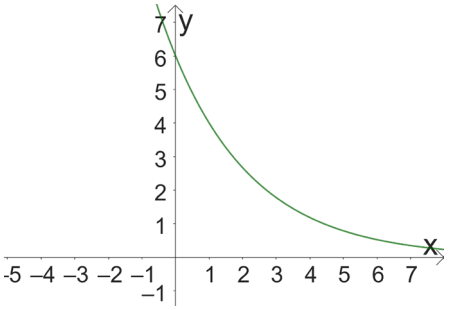
B.



C.



D.



3 Which is equivalent to $\int \frac{1}{(3x-2)^2}\,dx$?

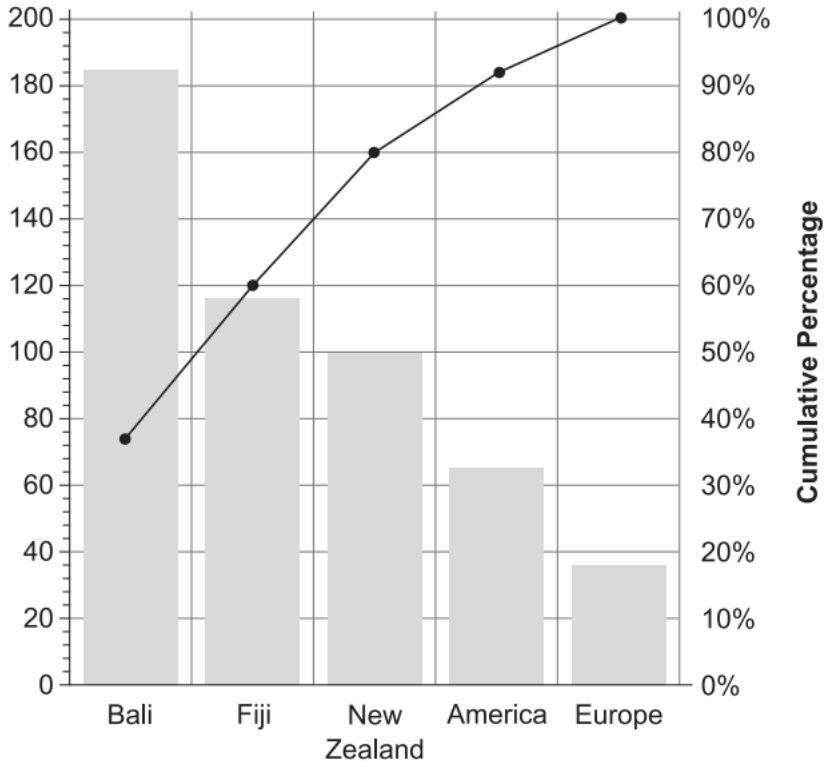
A. $\frac{1}{3(3x-2)} + C$

B. $\frac{-1}{3(3x-2)} + C$

C. $3\ln(3x-2) + C$

D. $\frac{1}{3}\ln(3x-2) + C$

4



The Pareto chart shows the number of holiday bookings made to different destinations. Which of the following is the closest percentage of the total bookings made for America?

A. 93%

B. 33%

C. 20%

D. 13%

5 Let $f(x)$ be any function with domain all real numbers.
Which of the following is an odd function, regardless of the choice of $f(x)$?

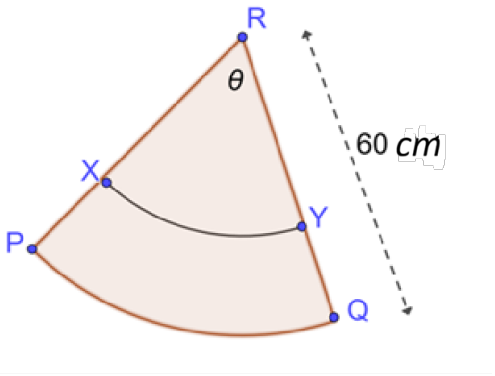
A. $f(x) + f(-x)$

B. $x(f(x) + f(-x))$

C. $\big(f(x)\big)^2 + \big(f(-x)\big)^2$

D. $f(x)f(-x)$

6 The region PQR is a sector of a circle with radius 60 cm, centred at R. The angle of the sector is θ. The arc XY lies on another circle also centred at R, as shown in the diagram. The arc XY divides the sector PQR into two regions of equal area.
Find the exact length of the arc RX?



A. $45\sqrt{2}$

B. 30

C. $30\sqrt{2}$

D. $15\sqrt{2}$

7 If $f(x+2) = x^2 + 4x + 1$, then $f(x)$ is equal to:

A. $x^2 - 3$

B. $x^2 + 1$

C. $x^2 - 4x + 1$

D. $x^2 + 4x + 5$

8 Two functions, f and g, are differentiable for all real values of x. It is known that

$$f(3) = 4, g(3) = -2, f'(3) = 5$$

If the tangent to the curve $y = f(x)g(x)$ at $x = 3$ has a gradient -6, find the value of $g'(3)$.

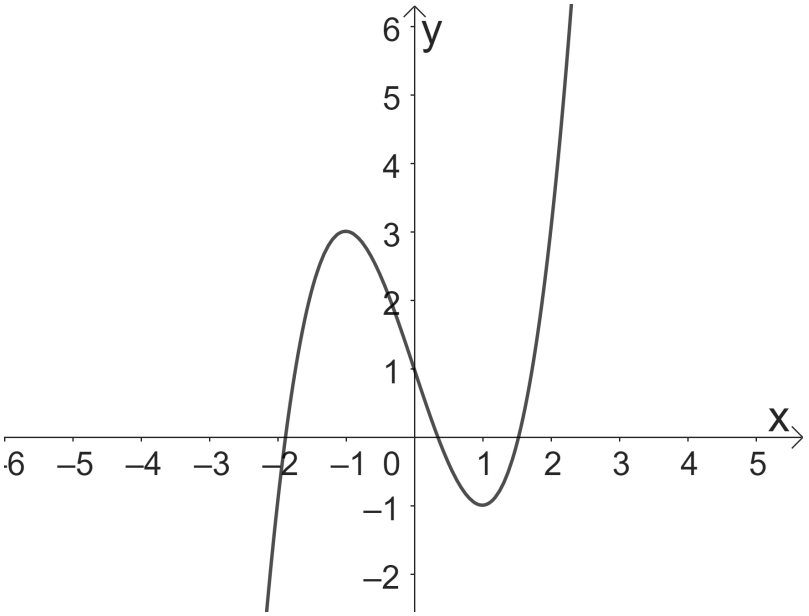
A. 2

B. 0

C. -1

D. 1

9 The diagram shows a part of the graph of $y = f(x)$, which has a local maximum at $x = -1$ and a local minimum at $x = 1$. Which of the following shows the correct relationship between $f''(-1), f'(1)$ and $f(0)$?



A. $f(0) < f'(1) < f''(-1)$

B. $f'(1) < f''(-1) < f(0)$

C. $f''(-1) < f'(1) < f(0)$

D. $f'(1) < f(0)) < f''(-1)$

10 A probability density function $f(x)$ is given by

$$f(x) = \begin{cases} \frac{1}{54}(27x - x^3), & 0 \le x \le 2.086 \\ 0, & \text{otherwise} \end{cases}$$

The 25^{th} percentile p of this distribution satisfies which of the following equations?

A. $p^4 - 54p^2 + 54 = 0$

B. $p^4 - 54p^2 + 27 = 0$

C. $-p^4 + 54p^2 + 27 = 0$

D. $-p^4 + 54p^2 + 54 = 0$

End of Section I

Mathematics Advanced
Section II Answer Booklet 1

NAME: _____

90 marks
Attempt Questions 11 – 33
Allow about 2 hour 45 minutes for this section

Booklet 1 – Attempt Questions 11 – 22 (45 marks)
Booklet 2 – Attempt Questions 23 – 32 (45 marks)

Instructions

- Answer the Questions in the spaces provided. These spaces provide guidance for the expected length of response.
- Your response should include relevant mathematical reasoning and/or calculations
- Extra writing space is provided at the back of this booklet. If you use this space, clearly indicate which question you are answering.

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Question 11 (2 marks)

Evaluate the limiting sum of the series. **2**

$$\frac{5}{6}+\frac{5}{27}+\frac{10}{243}+\cdots$$

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Question 12 (2 marks)

Find $\int(\sin 5x + 3^x + 1)\,dx$ **2**

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Question 13 (3 marks)

(a) Show that $1 - 2\sin x \cos x = (\sin x - \cos x)^2$. **1**

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(b) Hence, solve $1 - 2\sin x \cos x = 0$ for $0 \leq x \leq 2\pi$. **2**

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Question 14 (4 marks)

A discrete random variable X has the following probability distribution.

x	1	3	5	7
$P(X = x)$	p	0.2	q	0.1

(a) Determine p and q, given that $E(X) = 4$. **2**

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(b) Find $P(X \geq 5)$. **1**

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(c) Find $P(X \geq 5 | X \geq 3)$. **1**

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Question 15 (2 marks)

Find the sum of the terms in the arithmetic series. **2**

$$2026 + 2011 + 1996 + \cdots 271.$$

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Question 16 (3 marks)

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Find the equation of the normal to the curve $y = \frac{5e^{2x}}{5 - x^2}$ at $x = 1$.

Question 17 (4 marks)

The table below shows the future value of interest factors for an annuity of \$1.

$Rate\ (r)$ $Period(n)$	0.005	0.01	0.015	0.02	0.03	0.06
7	7.10588	7.21354	7.32300	7.43428	7.66246	8.39384
28	29.97452	32.12910	34.48148	37.05121	42.93092	68.52811
56	64.44140	74.58098	86.79754	101.55826	141.15377	418.82235
84	104.07393	130.67227	166.17264	213.86661	365.88054	2209.41674

Jacob deposits \$900 at the end of each quarter into an account that earns 6% $p.a.$, compounded quarterly, for 7 years.

(a) Find the total amount in Jacob's account after 7 years. **2**

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(b) Taylor invests a single lump sum today in an account that earns 6% $p.a.$, compounded quarterly, for 7 years. Find the amount Taylor must invest now to achieve the same future value as Jacob. **2**

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Question 18 (9 marks)

Let $f(x) = (x+1)(x-2)^2$

(a) Show that $f'(x) = 3x(x-2)$ and find $f''(x)$. **2**

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(b) Find the stationary points of $y = f(x)$ and determine their nature. **2**

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Question 18 continued on next page

Question 18 continued

(c) Determine the point of inflection of $y = f(x)$. **1**

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(d) Sketch the graph of $y = f(x)$, showing x and y intercepts and other key features. **3**

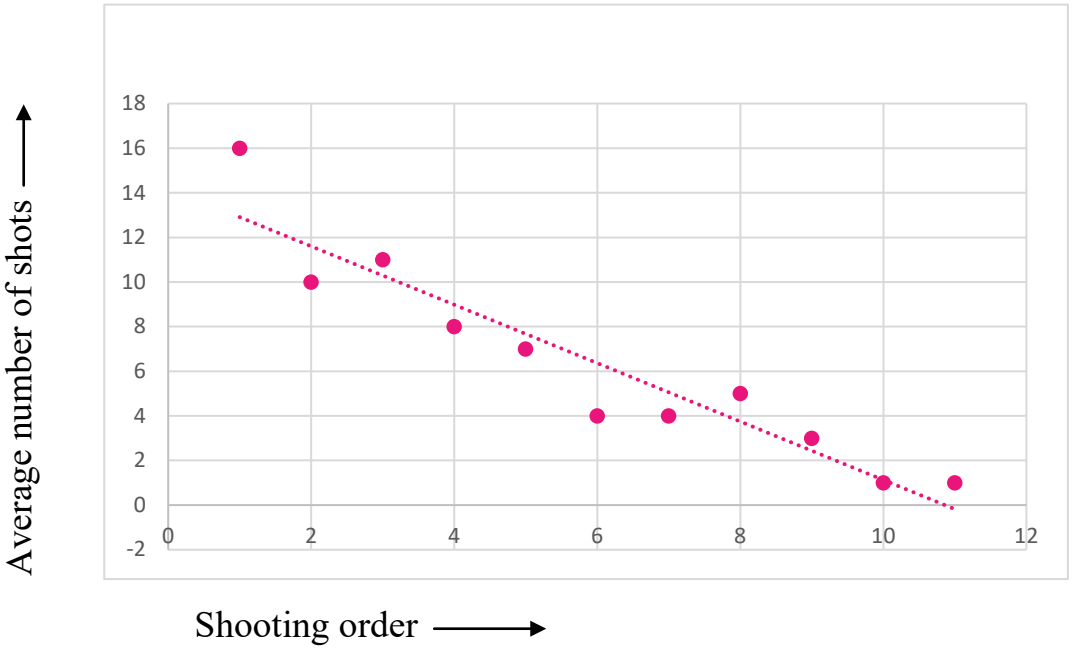
(e) Determine the values of x for which the function $f(x)$ is concaving down. **1**

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Question 19 (6 marks)

The 11 players in a basketball team always shoot five throws in a same order during training. The coach recorded the shooting order and the average number of successful shots made by each player over the season. The data has been displayed as a scatter plot.



(a) Complete the gaps in the table given below. **1**

Shooting Order	Average no of shots
1	16
2	
3	11
4	
5	7
6	4
7	
8	5
9	3
10	1
11	1

(b) State the form and direction of this data set. **2**

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Question 19 continued on next page

Question 19 continued

(c) Determine the equation of the least-squares regression line for this data set, where x represents the shooting order and y represents the average number of shots. Round values correct to 2 decimal places. **1**

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(d) Using the equation of the regression line, determine how many successful shots on average is the 6^{th} player expected to make. **1**

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(e) Explain whether it would be appropriate to use the least-squares regression line to predict the average number of successful shots for a player who is 15^{th} in the shooting order. **1**

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Question 20 (4 marks)

(a) Differentiate $y = \ln \sin(2x)$ **2**

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(b) Hence, find $\int (\cot(2x) + 3\sec^2 x)dx$ **2**

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Question 21 (3 marks)

A music festival is preparing a decorative light display along the main entrance. The lights are arranged in rows. The first row contains 12 lights. Each successive row contains exactly 5 more lights than the previous row. **3**

If there are 792 lights used for this display, how many rows are there?

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Question 22 (3 marks)
Information was collected from a group of employees at a company regarding their modes of transport to work. Each employee indicated whether they travel by train, bus or both. The results showed that 60% of the employees travel by train, 50% of the employees travel by bus, and 25% of the employees travel by both bus and train. It was also found that 36 employees travel by exactly one mode of transport mentioned.

(a) Show that total number of employees in the company is 60. **2**

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(b) Two different employees are selected at random from the group. Find the probability that neither travel by bus. **1**

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Section II extra writing space **NAME: _____**

If you use this space, clearly indicate which question you are answering.

Mathematics Advanced
Section II Answer Booklet 2

NAME: _____

Booklet 2 – Attempt Questions 23 – 32 (45 marks)

Instructions

- Answer the Questions in the spaces provided. These spaces provide guidance for the expected length of response.
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Question 23 (2 marks)

In a geometric sequence, the $10th$ term is 768 and the $13th$ term is 6144. Find the first term, and the common ratio. **2**

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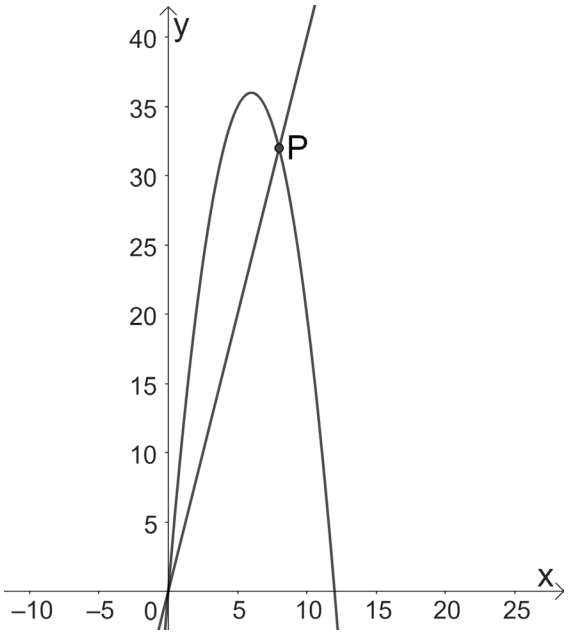
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Question 24 (4 marks)

The graphs of $y = kx$ and $y = 12x - x^2$ intersect at the origin and point P. The coordinates of point P are $(8, 32)$.



(a) Show that $k = 4$. **1**

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(b) Find the area enclosed between $y = kx$ and $y = 12x - x^2$. **3**

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Question 25 (4 marks)

Alex and Ben are playing a game using a two fair six-sided dice. They take turns rolling both dice with Alex going first, followed by Ben, and then continuing in this alternating pattern. The objective of the game is to be the first player to roll a total of exactly 10 on the two dice. Once a player rolls a total of exactly 10, the game ends and that player is declared the winner.

(a) Find the probability that Alex wins the game on his first turn. **1**

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(b) Find the probability that Alex wins the game on either his first turn or second turn. **2**

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(c) Find the probability that Alex eventually wins the game. **2**

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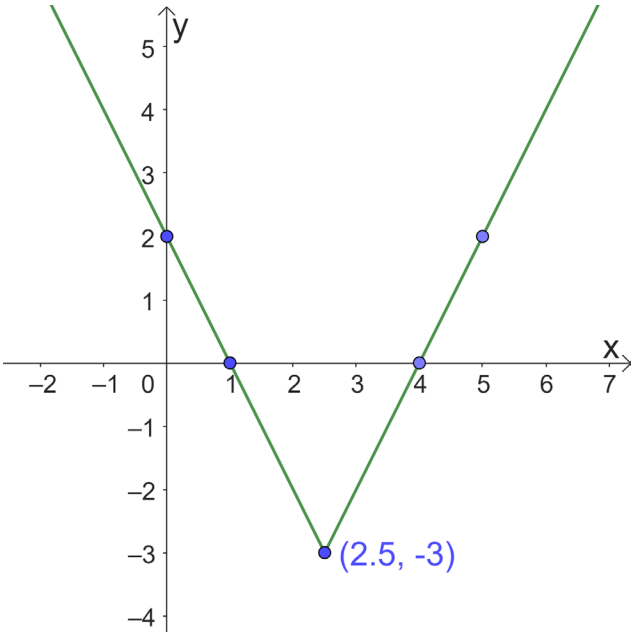
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Question 26 (3 marks)

Consider the graph of $y = f(x)$ as shown.



(a) State, in the correct order, transformations required to obtain the graph of $y = f(0.5(x-2))$ from the graph of $y = f(x)$. **1**

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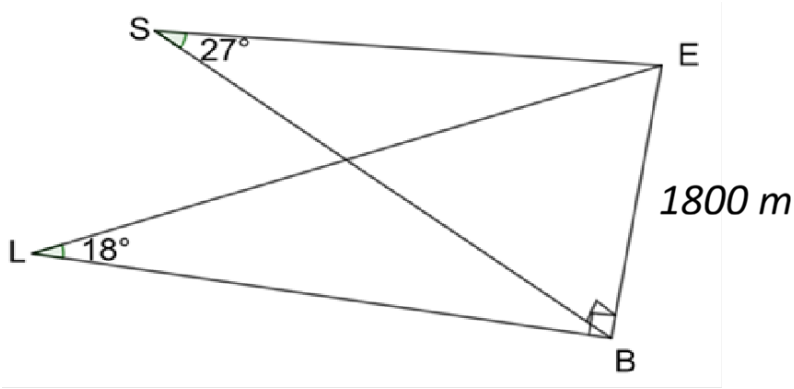
(b) Sketch the graph of $y = f(0.5(x-2))$, showing clearly the new position of the points marked in the original graph. . **2**

Question 27 (4 marks)

A rescue helicopter is searching for two hikers, Liam (L) and Sophia (S), who are stranded on the different sides of a flat levelled valley. Both hikers can see the top of Eagle Peak (E) and are communicating with the rescue team by radio. **4**

Liam reports that Eagle Peak has a bearing of 072° from his position, and the angle of elevation to the top of Eagle Peak is 18°. Sophie reports that Eagle Peak has a bearing of 118° from her position, and the angle of elevation to the top of the peak is 27°. The top of the Eagle Peak is 1800 metres above the valley floor.

Find the distance between Liam and Sophie and determine the bearing of Liam from Sophie's position.



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Question 28 (4 marks)

The amount of time X in minutes that customers spend waiting for a ride at a theme park is modelled by the probability density function.

$$f(x) = \begin{cases} k\cos\frac{\pi x}{10}, & 0 \le x \le 5 \\ 0, & otherwise \end{cases}$$

(a) Show that $k = \frac{\pi}{10}$. **2**

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(b) Find the probability that a customer waits between 2 and 4 minutes, give your answer correct to 4 decimal places. **1**

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(c) What is the probability that a customer waits more than 4 minutes? Give answer correct to 4 decimal places. **1**

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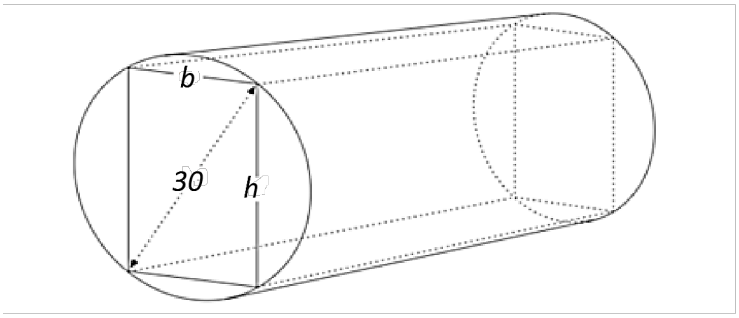
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Question 29 (6 marks)

A rectangular water tank of width b cm and height h cm is designed to fit perfectly inside a cylindrical pipe with diameter of 30 cm as shown, where all vertices of the rectangular prism touch the circumference of the base circles.



The length of the tank is twice its height and the volume (V) of the tank is given by

$$V = bhl$$

(a) Show that $V = 2b(900 - b^2)$. **1**

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(b) Find the dimensions of the rectangular prism that maximises the volume of the tank. Justify your answer. **3**

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Question 29 continued on next page

Question 29 (continued)

(c) A square based tank with a diagonal 30 cm could also be designed to fit inside the pipe. Show that the rectangular water tank is more than 5% greater in volume compared to square based tank. **2**

Question 30 (4 marks)

In a flock of 22400 sheep, the ratio of males to females is 9: 15. The weights of the female sheep are normally distributed with a mean of 78.4 kg and a standard deviation of 5.6 kg. In the flock, 13979 of the female sheep each weigh more than x kg. **4**

Find the value of x.

Question 31 (6 marks)

The temperature, $T°$ C, at a farm over the course of a day is modelled by

$$T(t) = 22 + 8\cos\frac{\pi}{12}(t-4)$$

where t if the time in hours after midnight, $0 \leq t \leq 24$.

(a) Find the temperature at midnight. **1**

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(b) Find $\frac{dT}{dt}$, leave your answer in the simplest form. **1**

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(c) Determine whether the temperature is initially decreasing or increasing. **2**

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(d) Find the maximum temperature and determine the first time at which this occurs. **2**

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Question 32 (7 marks)

A business owner takes an reducing balance loan of $\$500000$ to expand their company. Interest is charged at 0.4% per month, compounded monthly. At the end of each month, interest is added to the loan balance and then a repayment of $\$3200$ is made.

Let A_n be the balance owing on the loan at the end of n months.

(a) Show that $A_2 = 500000(1.004)^2 - 3200(1.004) - 3200$. **2**

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(b) Show that $A_n = 800000 - 300000(1.004)^n$. **3**

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Question 32 continues next page.

Question 32 (continued)

(c) After how many months will the balance owing first be less than \$250000. **2**

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Section II extra writing space **Name:_____**

If you use this space, clearly indicate which question you are answering.

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NAME: Solutions

Blacktown Boys' High School

2026 Year 12

Trial Examination

Mathematics Advanced

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Assessor: Mrs Chhabra

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Section I

10 marks
Attempt Questions 1–10
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What is the 18^{th} term of the sequences?

A. 81

B. 77

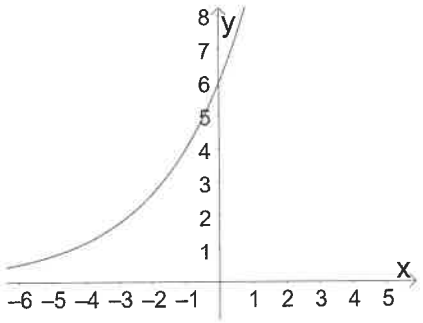
C. 73

D. 69

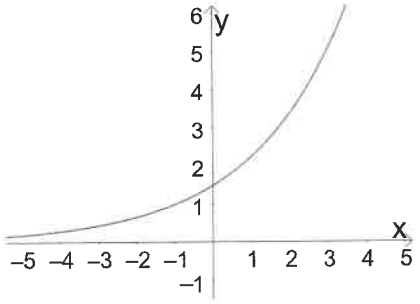
$d=4$
$a_{18} = 5+(18-1)4$
$=73$

2 Which of the following best represents the graph of $y = 6(1.5)^x$?

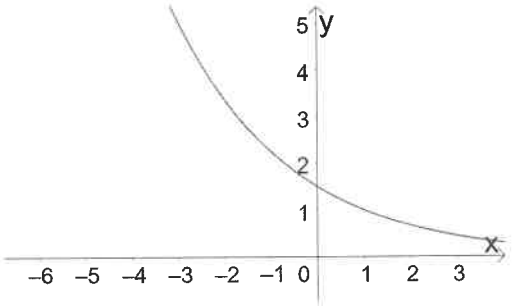
A.



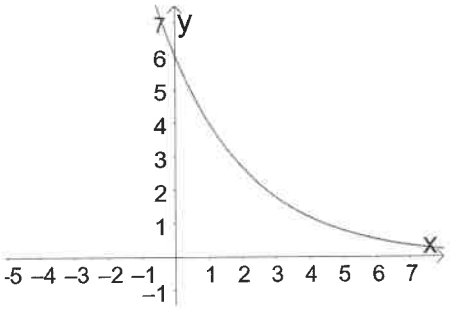
B.



C.



D.



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A. $\frac{1}{3(3x-2)} + C$

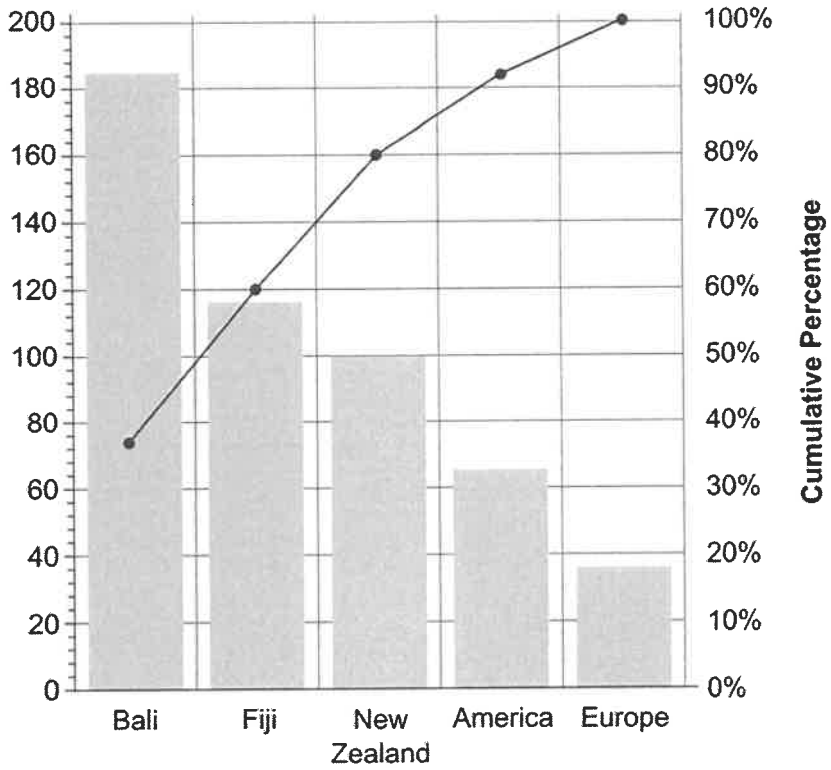
B. $\frac{-1}{3(3x-2)} + C$

C. $3\ln(3x-2) + C$

D. $\frac{1}{3}\ln(3x-2) + C$

$\int (3x-2)^{-2}$
$= \frac{(3x-2)^{-1}}{(-1)3} + C$
$= -\frac{1}{3(3x-2)} + C$

4



The Pareto chart shows the number of holiday bookings made to different destinations. Which of the following is the closest percentage of the total bookings made for America?

A. 93%

B. 33%

C. 20%

D. 13%

90 − 80 = 13%

5 Let $f(x)$ be any function with domain all real numbers.
Which of the following is an odd function, regardless of the choice of $f(x)$?

A. $f(x) + f(-x)$

B. $x(f(x) + f(-x))$

C. $(f(x))^2 + (f(-x))^2$

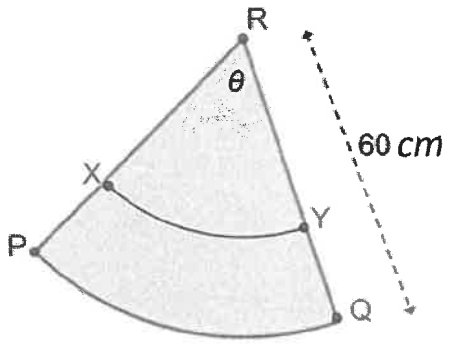
D. $f(x)f(-x)$

$p(x) = x\left(f(x) + f(-x)\right)$

$p(-x) = (-x)\left[f(-x) + f(x)\right]$

$= -p(x)$

6 The region PQR is a sector of a circle with radius 60 cm, centred at R. The angle of the sector is θ. The arc XY lies on another circle also centred at R, as shown in the diagram. The arc XY divides the sector PQR into two regions of equal area.
Find the exact length of the arc RX?



$A_{RPQ} = \frac{1}{2}r^2\theta$
$= \frac{1}{2} \times 60^2 \times \theta$
$= 1800\theta$
$A_{RXY} = \frac{1}{2} A_{RPQ}$
$= \frac{1}{2} \times 1800\theta$
$\frac{1}{2}r^2\theta = \frac{1}{2} \times 1800\theta$
$r^2 = 900 \times 2$
$r = 30\sqrt{2}$

A. $45\sqrt{2}$

B. 30

C. $30\sqrt{2}$

D. $15\sqrt{2}$

7 If $f(x+2) = x^2 + 4x + 1$, then $f(x)$ is equal to:

$f(x+2) = x^2+4x+1$
$y = x+2$
$y-2 = x$
$f(y) = (y-2)^2 + 4(y-2) + 1$
$= y^2 - 3$

A. $x^2 - 3$

B. $x^2 + 1$

C. $x^2 - 4x + 1$

D. $x^2 + 4x + 5$

8 Two functions, $f(x)$ and $g(x)$, are differentiable for all real values of x. It is known that

$$f(3) = 4, g(3) = -2, f'(3) = 5$$

If the tangent to the curve $y = f(x)g(x)$ at $x = 3$ has a gradient -6, find the value of $g'(3)$.

$\frac{dy}{dx} = f(x)g'(x) + g(x)f'(x)$
$-6 = 4g'(3) + (-2)(5)$
$-6 = 4g'(3) - 10$
$-6 = 4g'(3)$
$-\frac{4}{4} = g'(3)$
$-1 = g'(3)$

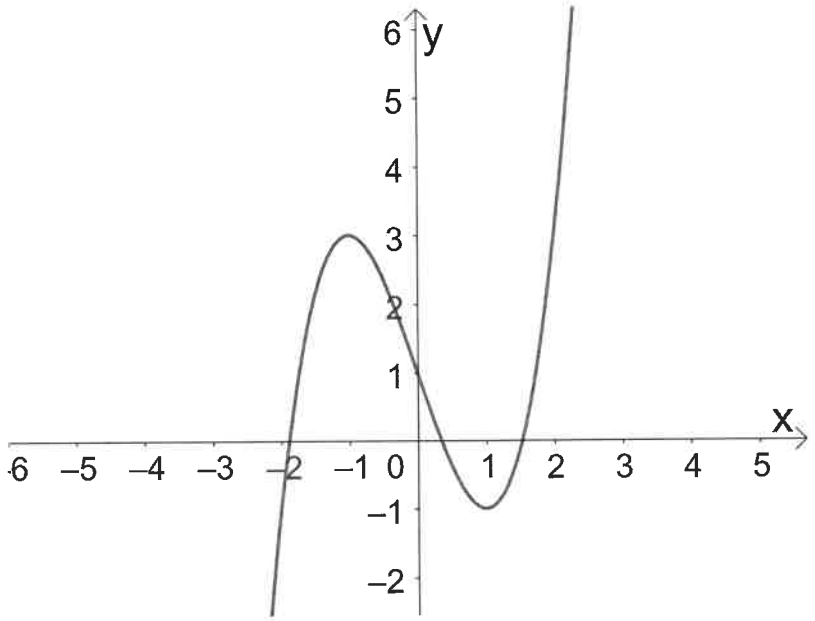
A. 2

B. 0

C. -1

D. 1

9 The diagram shows a part of the graph of $y = f(x)$, which has a local maximum at $x = -1$ and a local minimum at $x = 1$. Which of the following shows the correct relationship between $f''(-1), f'(1)$ and $f(0)$?



A. $f(0) < f'(1) < f''(-1)$

B. $f'(1) < f''(-1) < f(0)$

C. $f''(-1) < f'(1) < f(0)$

D. $f'(1) < f(0)) < f''(-1)$

at $x=1$ local min
$f'(1)=0$
at $x=-1$ local max
$f''(-1)<0$
$f(0)>0$

10 A probability density function $f(x)$ is given by

$$f(x) = \begin{cases} \frac{1}{54}(27x - x^3), & 0 \le x \le 3 \\ 0, & \text{otherwise} \end{cases}$$

The 25^{th} percentile p of this distribution satisfies which of the following equations?

A. $p^4 - 54p^2 + 54 = 0$

B. $p^4 - 54p^2 + 27 = 0$

C. $-p^4 + 54p^2 + 27 = 0$

D. $-p^4 + 54p^2 + 54 = 0$

$$\int_0^p \frac{1}{54}(27x - x^3) = 0.25$$

$$\frac{1}{54}\left[\frac{27x^2}{2} - \frac{x^4}{4}\right]_0^p = 0.25$$

$$\frac{27x^2}{2} - \frac{x^4}{4} = 13.5$$

$$p^4 - 54p^2 + 54 = 0$$

End of Section I

Mathematics Advanced
Section II Answer Booklet 1

NAME: _____

90 marks
Attempt Questions 11 – 32
Allow about 2 hour 45 minutes for this section

Booklet 1 – Attempt Questions 11 – 22 (45 marks)
Booklet 2 – Attempt Questions 23 – 32 (45 marks)

Instructions

- Answer the Questions in the spaces provided. These spaces provide guidance for the expected length of response.
- Your response should include relevant mathematical reasoning and/or calculations
- Extra writing space is provided at the back of this booklet. If you use this space, clearly indicate which question you are answering.

Question 11 (2 marks)

Evaluate the limiting sum of the series. **2**

$$\frac{5}{6}+\frac{5}{27}+\frac{10}{243}+\cdots$$

$a = \frac{5}{6}$ $\quad r = \frac{\frac{5}{27}}{\frac{5}{6}} = \frac{2}{9}$ — (1)

$S_\infty = \frac{a}{1-r} = \frac{\frac{5}{6}}{1-\frac{2}{9}} = \frac{15}{14}$ — (1)

Question 12 (2 marks)

Find $\int (\sin 5x + 3^x + 1)\, dx$ **2**

$= -\frac{1}{5}\cos 5x + \frac{3^x}{\ln 3} + x + c$

1 mark – correct integration trig | exponential

2 marks – correct solution

Question 13 (3 marks)

(a) Show that $1 - 2\sin x \cos x = (\sin x - \cos x)^2$. **1**

$R.H.S = \sin^2 x - 2\sin x \cos x + \cos^2 x$

$= 1 - 2\sin x \cos x$

$= L.H.S$

(b) Hence, solve $1 - 2\sin x \cos x = 0$ for $0 \le x \le 2\pi$. **2**

$1 - 2\sin x \cos x = 0$

$(\sin x - \cos x)^2 = 0$

$\sin x = \cos x$ 1 mark – one correct x value

$\tan x = 1$

$x = \frac{\pi}{4}, \frac{5\pi}{4}$ 2 marks – correct solution

Question 14 (4 marks)

A discrete random variable X has the following probability distribution.

x	1	3	5	7
$P(X = x)$	p	0.2	q	0.1

(a) Determine p and q, given that $E(X) = 4$. **2**

$p + 0.2 + q + 0.1 = 1$

$p + q = 0.7$ — (1)

$E(X) = 4$

$p + 0.6 + 5q + 0.7 = 4$

$p + 5q = 2.7$ — (2)

(2) − (1)

$p + 5q = 2.7$

$p + q = 0.7$

$4q = 2$

$q = 0.5$

$p = 0.2$

1 mark – both equations correct

2 marks – correct solution

(b) Find $P(X \geq 5)$. **1**

$0.5 + 0.1 = 0.6$

(c) Find $P(X \geq 5 | X \geq 3)$. **1**

$= \dfrac{P(X \geq 5 \cap X \geq 3)}{P(3) + P(5) + P(7)}$

$= \dfrac{P(X = 5) + P(X = 7)}{0.2 + 0.5 + 0.1}$

$= \dfrac{0.5 + 0.1}{0.8} = \dfrac{0.6}{0.8} = \dfrac{3}{4}$

Question 15 (2 marks)

Find the sum of the terms in the arithmetic series. **2**

$$2026 + 2011 + 1996 + \cdots 271.$$

$a = 2026$

$d = -15$

$l = 271$

$T_n = 271$

$271 = 2026 + (n-1)(-15)$

$271 - 2026 = -15n + 15$

$\frac{271 - 2026 - 15}{-15} = n$

$118 = n$ — (1)

$S_{118} = \frac{118}{2}(2026 + 271)$

$= 135523$ — (1)

Question 16 (3 marks)

3

Find the equation of the normal to the curve $y = \dfrac{5e^{2x}}{5 - x^2}$ at $x = 0$.

$y = \dfrac{5e^{2x}}{5-x^2}$

$\dfrac{dy}{dx} = \dfrac{(5-x^2) \times 10e^{2x} - 5e^{2x} \times (-2x)}{(5-x^2)^2}$

$= \dfrac{10e^{2x}(5-x^2+x)}{(5-x^2)^2}$ — (1)

at $x = 0$

$y = \dfrac{5e^{0}}{5-0} = 1$ $(0, 1)$

$m_T = \dfrac{10e^{0}(5-0+0)}{(5-0)^2}$ $m_N = -\dfrac{1}{2}$ — (1)

$= \dfrac{50}{25} = 2$

Equation of normal

$y - 1 = -\frac{1}{2}(x-0)$

$2y - 2 = -x$

$x + 2y - 2 = 0$ — (1)

OR

$y = -\frac{1}{2}x + 1$

Question 17 (4 marks)

The table below shows the future value of interest factors for an annuity of \$1.

Rate (r) / *Period*(n)	0.005	0.01	0.015	0.02	0.03	0.06
7	7.10588	7.21354	7.32300	7.43428	7.66246	8.39384
28	29.97452	32.12910	34.48148	37.05121	42.93092	68.52811
56	64.44140	74.58098	86.79754	101.55826	141.15377	418.82235
84	104.07393	130.67227	166.17264	213.86661	365.88054	2209.41674

Jacob deposits \$900 at the end of each month into an account that earns 6% $p.a.$, compounded quarterly, for 7 years.

(a) Find the total amount in Jacob's account after 7 years. **2**

$r = 0.015$ $n = 7 \times 4 = 28$

$= 900 \times 34.48148$ 1 mark – identifies correct value from the table

$= 31033.332$

2 marks – correct solution

(b) Taylor invests a single lump today in an account that earns 6% $p.a.$, compounded quarterly, for 7 years. Find the amount Taylor must invest now to achieve the same future value as Jacob. **2**

$PV = \frac{FV}{(1+r)^n}$

$= \frac{31033.332}{(1+0.015)^{28}}$ – (1)

$= \$20454.04583$ – (1)

Question 18 (9 marks)

Let $f(x) = (x+1)(x-2)^2$

(a) Show that $f'(x) = 3x(x-2)$ and find $f''(x)$. **2**

$f'(x) = (x-2)^2 + 2(x-2)(x+1)$

$= (x-2)[x-2+2x+2]$

$= 3x(x-2)$ — (1)

$f''(x) = 6x-6$ — (1)

(b) Find the stationary points of $y = f(x)$ and determine their nature. **2**

$f'(x) = 0$

$3x(x-2) = 0$

$x = 0, x = 2$

$y = 4, y = 0$

$(0,4)$ and $(2,0)$

$\therefore$ $(0,4)$ is the max.

$(2,0)$ is the min.

At $x = 0$

$f''(x) = -6 < 0$

concave down

At $x = 2$

$f''(x) = 6 > 0$

concave up

1 mark – both stationary points correct

2 mark – correct solution

Question 18 continued on next page

Question 18 continued

(c) Determine the point of inflection of $y = f(x)$. **1**

$f''(x) = 0$

$6x - 6 = 0$

$x = 1$

$y = (1+1)(1-2)^2$

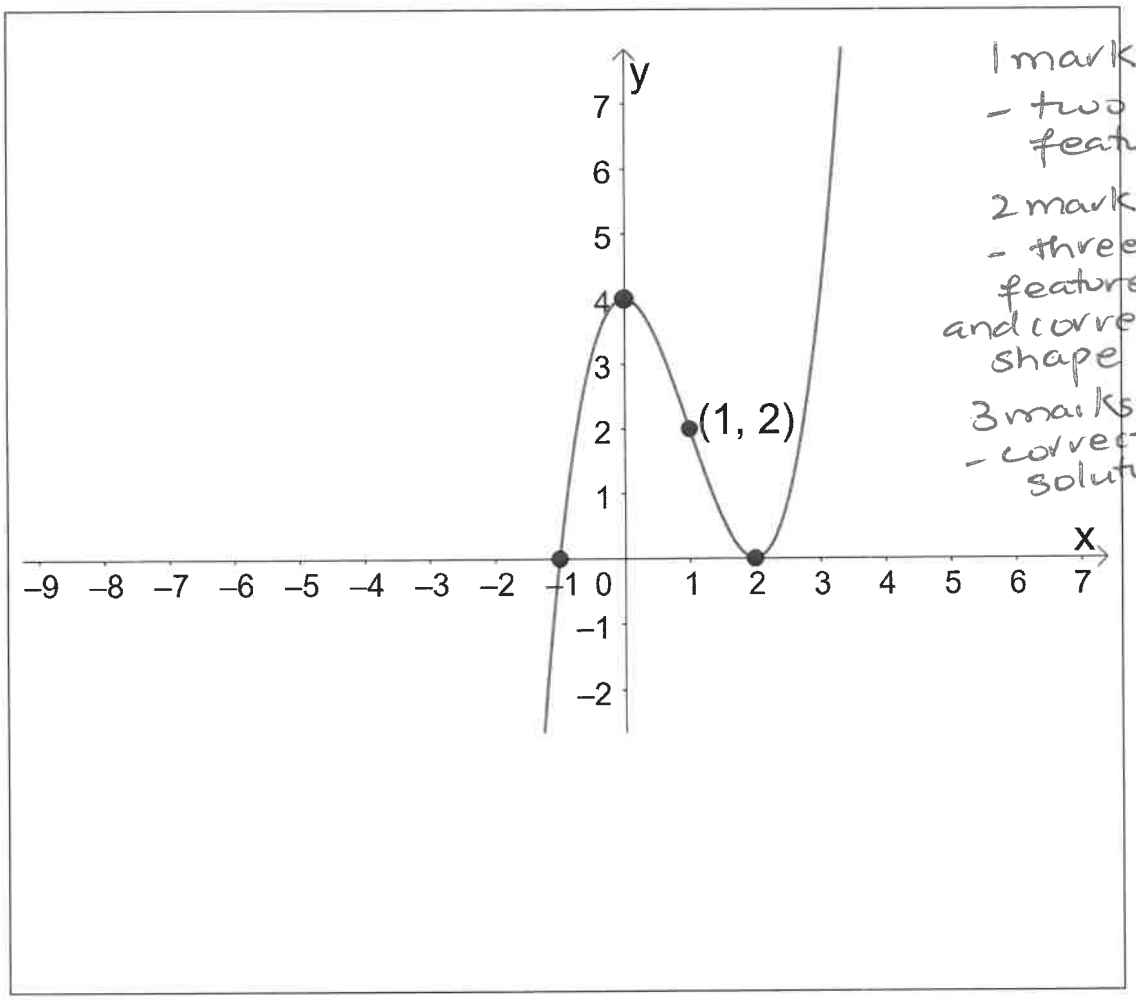
$= 2$

x	0	1	2
$f''(x)$	−6	0	6
	n	·	U

concavity changes

$\therefore (1, 2)$ is the point of inflexion

(d) Sketch the graph of $y = f(x)$, showing x and y intercepts and other key features. **3**



1 mark
- two features

2 mark
- three features and correct shape

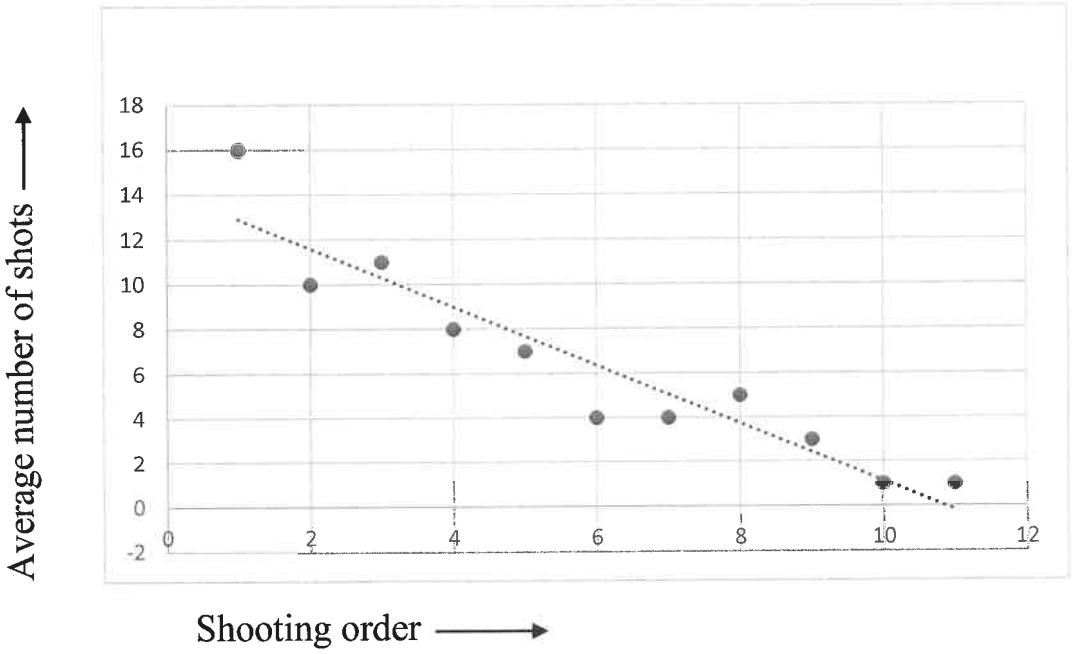
3 marks
- correct solution

(e) Determine the values of x for which the function $f(x)$ is concaving down. **1**

$x < 1$

Question 19 (6 marks)

The 11 players in a basketball team always shoot five throws in a same order during training. The coach recorded the shooting order and the average number of successful shots made by each player over the season. The data has been displayed as a scatter plot.



(a) Complete the gaps in the table given below. **1**

Shooting Order	Average no of shots
1	16
2	10
3	11
4	8
5	7
6	4
7	4
8	5
9	3
10	1
11	1

(b) State the form and direction of this data set. **2**

Form - linear — (1)

Direction - Negative — (1)

Question 19 continued on next page

Question 19 continued

(c) Determine the equation of the least-squares regression line for this data set, where x represents the shooting order and y represents the average number of shots. Round values correct to 2 decimal places. **1**

$y = 14.22 - 1.31x$

(d) Using the equation of the regression line, determine how many successful shots on average is the 6th player expected to make. **1**

$y = 14.22 - 1.31(6)$

$= 6.36$

(e) Explain whether it would be appropriate to use the least-squares regression line to predict the average number of successful shots for a player who is 15th in the shooting order. **1**

Prediction is extrapolation

or

gives negative y value

Question 20 (4 marks)

(a) Differentiate $y = \ln \sin(2x)$ **2**

$\frac{dy}{dx} = \frac{2\cos 2x}{\sin 2x}$ ... 1 mark - correct differentiation

$= 2\cot 2x$ ... 2 marks - correct solution

(b) Hence, find $\int (\cot(2x) + 3\sec^2 x)dx$ **2**

$= \frac{1}{2}\ln|\sin 2x| + 3\tan x + c$

1 mark - correct integral for cot or $\sec^2 x$

2 marks - correct solution

Question 21 (3 marks)

A music festival is preparing a decorative light display along the main entrance. The lights are arranged in rows. The first row contains 12 lights. Each successive row contains exactly 5 more lights than the previous row. **3**

If there are 792 lights used for this display, how many rows are there?

$a = 12 \quad d = 5 \quad S_n = 792$

$S_n = \frac{n}{2}(2a + (n-1)d)$

$792 = \frac{n}{2}(2 \times 12 + (n-1)5)$ — (1) Substitute in the formula

$1584 = 24n + 5n^2 - 5n$

$5n^2 + 19n - 1584 = 0$

$n = \frac{-19 \pm \sqrt{19^2 - 4 \times 5 \times (-1584)}}{10}$ — (1) substitute into quadratic formula

$= \frac{-19 + 179}{10}, \frac{-19 - 179}{10}$

$n > 0$

$\therefore n = 16$ — (1)

Question 22 (3 marks)
Information was collected from a group of employees at a company regarding their modes of transport to work. Each employee indicated whether they travel by train, bus or both. The results showed that 60% of the employees travel by train, 50% of the employees travel by bus, and 25% of the employees travel by both bus and train. It was also found that 36 employees travel by exactly one mode of transport mentioned.

(a) Show that total number of employees in the company is 60. **2**

Total number of employees $= n$

$60\% \times n + 50\% \times n - 25\% \times n - 25\% \times n = 36$

$60\% \times n = 36$

$n = 60$

1 mark – some progress

2 marks – correct solution

(b) Two different employees are selected at random from the group. Find the probability that neither travel by bus. **1**

$P(\text{first employee}) = \frac{30}{60}$

$P(\text{second employee}) = \frac{29}{59}$

$P(\text{neither}) = \frac{1}{2} \times \frac{29}{59} = \frac{29}{118}$

Mathematics Advanced
Section II Answer Booklet 2

NAME: _____

Booklet 2 – Attempt Questions 23 – 32 (45 marks)

Instructions

- Answer the Questions in the spaces provided. These spaces provide guidance for the expected length of response.
- Your response should include relevant mathematical reasoning and/or calculations
- Extra writing space is provided at the back of this booklet. If you use this space, clearly indicate which question you are answering.

Question 23 (2 marks)

In a geometric sequence, the $10th$ term is 768 and the $13th$ term is 6144. Find the first term, and the common ratio. **2**

$T_{10} = 768 \quad T_{13} = 6144$

$ar^9 = 768 \quad -(1)$

$ar^{13} = 6144 \quad -(2)$

$(2) \div (1)$

$$\frac{ar^{12}}{ar^9} = \frac{6144}{768}$$

$r^3 = 8$

$r = 2 \quad -(1)$

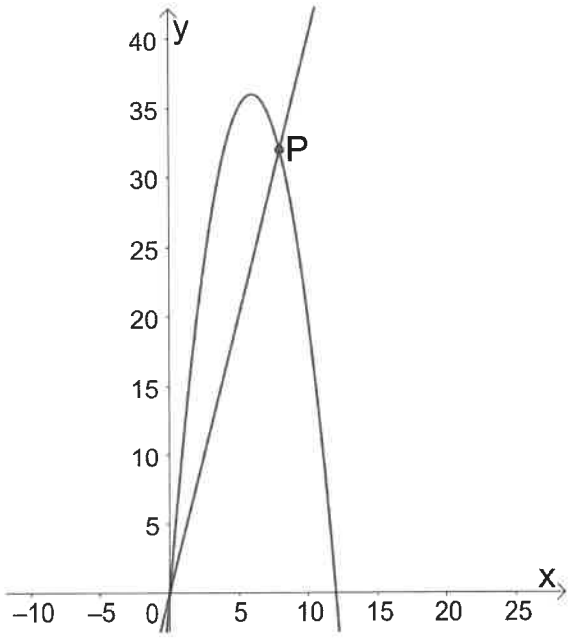
$ar^9 = 768$

$$a = \frac{768}{2^9}$$

$$= \frac{3}{2} \quad -(1)$$

Question 24 (4 marks)

The graphs of $y = kx$ and $y = 12x - x^2$ intersect at the origin and point P. The coordinates of point P are $(8, 32)$.



(a) Show that $k = 4$. **1**

$32 = K \times 8$

$K = 4$

(b) Find the area enclosed between $y = kx$ and $y = 12x - x^2$. **3**

$\int_0^8 (12x - x^2 - 4x)\,dx$ — (1)

$= \int_0^8 (8x - x^2)\,dx$

$= \left[\frac{8x^2}{2} - \frac{x^3}{3}\right]_0^8$

$= \frac{8(8)^2}{2} - \frac{(8)^3}{3} - 0$

$= 4 \times (8)^2 - \frac{(8)^3}{3}$

$= 256 - \frac{512}{3} = \frac{256}{3}$ units^2

2 marks – correct integration

3 marks – correct solution

Question 25 (4 marks)

Alex and Ben are playing a game using a two fair six-sided dice. They take turns rolling both dice with Alex going first, followed by Ben, and then continuing in this alternating pattern. The objective of the game is to be the first player to roll a total of exactly 10 on the two dice. Once a player rolls a total of exactly 10, the game ends and that player is declared the winner.

(a) Find the probability that Alex wins the game on his first turn. **1**

$(5,5)\ (6,4)\ (4,6)$

$\frac{3}{36} = \frac{1}{12}$

(b) Find the probability that Alex wins the game on either his first turn or second turn. **2**

Alex(wins) $= \frac{1}{12}$ Alex(loses) $= \frac{11}{12}$

$P = \frac{1}{12} +$ Alex(Loss) × Ben(loss) × Alex(Win)

$= \frac{1}{12} + \left(\frac{11}{12}\right)\left(\frac{11}{12}\right)\left(\frac{1}{12}\right)$

$= \frac{265}{1728}$

1 mark = some progress in identifying second turn

2 marks = correct solution

(c) Find the probability that Alex eventually wins the game. **2**

$\frac{1}{12} + \left(\frac{11}{12}\right)^2\frac{1}{12} + \left(\frac{11}{12}\right)^4\frac{1}{12} + \ldots$

$= \frac{1}{12}\left(1 + \left(\frac{11}{12}\right)^2 + \left(\frac{11}{12}\right)^4 + \cdots\right)$

Geometric series

$a = 1$

$r = \left(\frac{11}{12}\right)^2$

$S_\infty = \frac{1}{1-\left(\frac{11}{12}\right)^2}$

1 mark = sets G.P. and identifies a and r

2 marks = correct solution

$P = \frac{1}{12} \cdot \frac{1}{1-\left(\frac{11}{12}\right)^2}$

$= \frac{12}{23}$

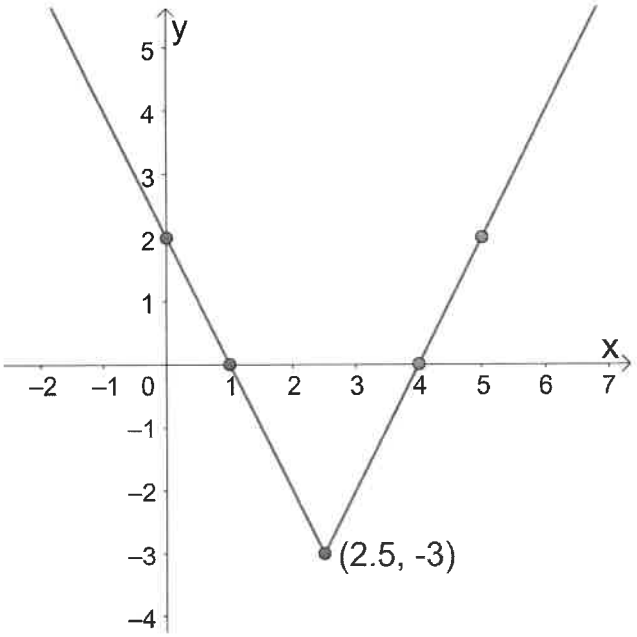
Question 26 (3 marks)

Consider the graph of $y = f(x)$ as shown.

OR

1) Horizontally moved to the right by one unit.

2) Horizontally dilated by a factor of 2



(a) State, in the correct order, transformations required to obtain the graph of $y = f(0.5(x-2))$ from the graph of $y = f(x)$. **1**

① Horizontally dilated by a factor of 2

② Translated 2 units to the right

(b) Sketch the graph of $y = f(0.5(x-2))$, showing clearly the new positions of the points marked in the original graph. . **2**

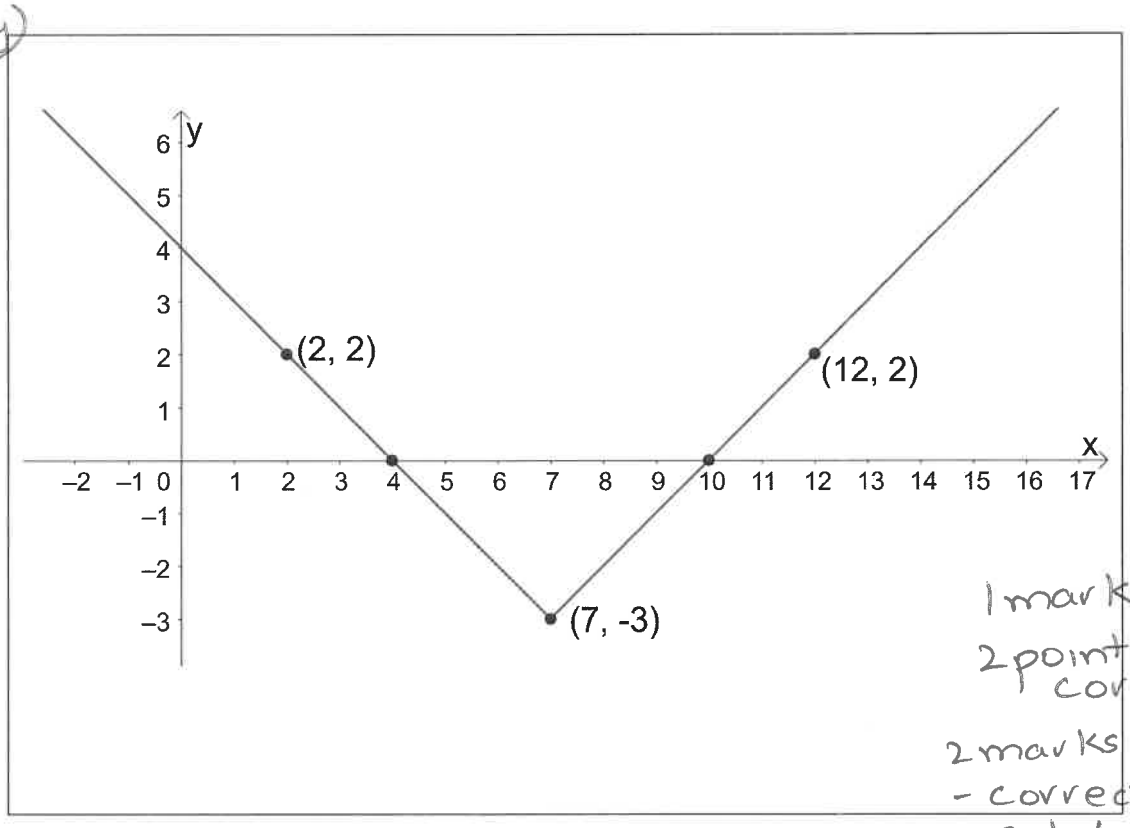
$(x,y) \to (2x+2,y)$

$(0,2) \to (2,2)$

$(1,0) \to (4,0)$

$(4,0) \to (10,0)$

$(5,2) \to (12,2)$



1 mark - 2 points correct

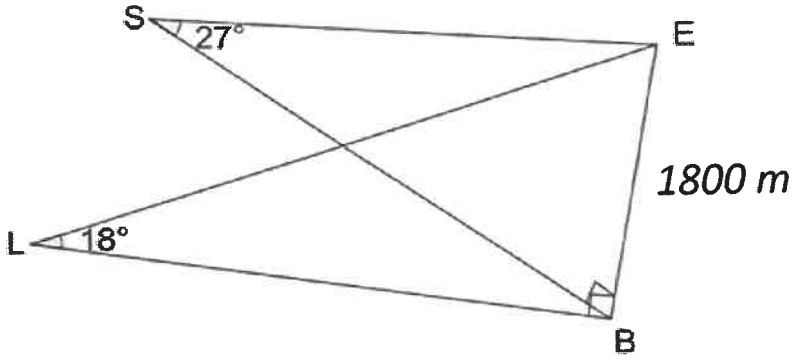
2 marks - correct solution

Question 27 (4 marks)

A rescue helicopter is searching for two hikers, Liam (L) and Sophia (S), who are stranded on the different sides of a flat levelled valley. Both hikers can see the top of Eagle Peak (E) and are communicating with the rescue team by radio. **4**

Liam reports that Eagle Peak has a bearing of 072° from his position, and the angle of elevation to the top of Eagle Peak is 18°. Sophie reports that Eagle Peak has a bearing of 118° from her position, and the angle of elevation to the top of the peak is 27°. The top of the Eagle Peak is 1800 metres above the valley floor.

Find the distance between Liam and Sophie and determine the bearing of Liam from Sophie's position.



$\tan 18^\circ = \frac{1800}{LB}$ $\tan 27^\circ = \frac{1800}{SB}$

$LB = 5539.8\text{ m}$ $SB = 3532.7\text{ m}$

$\angle LBS = 118^\circ - 72^\circ = 46^\circ$

$LS = \sqrt{(5539.8)^2 + (3532.7)^2 - 2(5539.8)(3532.7)\cos 46^\circ}$

$= 3997.4738$

1 mark - either LB/SB correct

$\frac{\sin S}{LB} = \frac{\sin 46^\circ}{LS}$

2 mark - correct LS

$\sin S = \frac{LB \sin 46^\circ}{LS}$

3 mark - correct angle S

$S = \sin^{-1}(0.9964)$

$\geq 85^\circ$

4 marks - correct solution

Bearing $= 118 + 85^\circ = 203$

Question 28 (4 marks)

The amount of time X in minutes that customers spend waiting for a ride at a theme park is modelled by the probability density function.

$$f(x) = \begin{cases} k\cos\frac{\pi x}{10}, & 0 \le x \le 5 \\ 0, & otherwise \end{cases}$$

(a) Show that $k = \frac{\pi}{10}$. **2**

$\int_0^5 K\cos\frac{\pi x}{10} = 1$

1 mark – correct integration

$\left[\frac{10K}{\pi}\sin\frac{\pi x}{10}\right]_0^5 = 1$

2 marks – correct solution

$\frac{10K}{\pi}\left(\sin\frac{5\pi}{10} - 0\right) = 1$

$\frac{10K}{\pi} = 1 \quad \therefore K = \frac{\pi}{10}$

(b) Find the probability that a customer waits between 2 and 4 minutes, give your answer correct to 4 decimal places. **1**

$\int_2^4 \frac{\pi}{10}\cos\frac{\pi x}{10}$

$= \left[\sin\frac{\pi x}{10}\right]_2^4$

$= \sin\frac{4\pi}{10} - \sin\frac{2\pi}{10}$

$= 0.363271264 \approx 0.3633$

(c) What is the probability that a customer waits more than 4 minutes? Give answer correct to 4 decimal places. **1**

$P(X>4) = \int_4^5 \frac{\pi}{10}\cos\frac{\pi x}{10}$

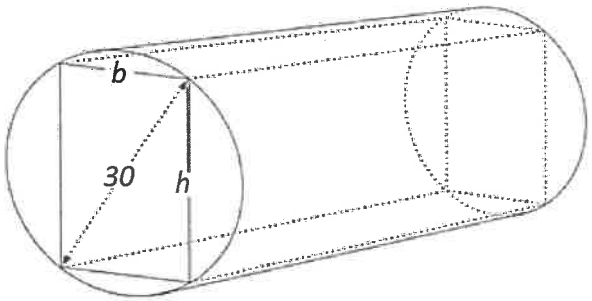
$= \left[\sin\frac{\pi x}{10}\right]_4^5$

$= \sin\frac{\pi}{2} - \sin\frac{4\pi}{10}$

$= 0.048943 \approx 0.0489$

Question 29 (6 marks)

A rectangular water tank of width b cm and height h cm is designed to fit perfectly inside a cylindrical pipe with diameter of 30 cm as shown, where all vertices of the rectangular prism touch the circumference of the base circles.



The length of the tank l, is twice its height h, and the volume (V) of the tank is given by

$$V = bhl$$

(a) Show that $V = 2b(900 - b^2)$. **1**

$b^2 + h^2 = 30^2$

$h = \sqrt{900 - b^2}$

$l = 2h$

$V = bhl$

$= b \times \sqrt{900-b^2} \times 2\sqrt{900-b^2}$

$= 2b(900 - b^2)$

(b) Find the dimensions of the rectangular prism that maximises the volume of the tank. Justify your answer. **3**

$V = 1800b - 2b^3$

$\frac{dV}{db} = 1800 - 6b^2$

$1800 - 6b^2 = 0$

$b = \sqrt{300} \quad (b > 0)$

$b = 10\sqrt{3}$ — (1)

$\frac{d^2V}{db^2} = -12b$

when $b = 10\sqrt{3}$

$\frac{d^2y}{dx^2} = -120\sqrt{3} < 0$

$\therefore$ maximum — (1)

$h = \sqrt{900 - 300}$

$h = \sqrt{600}$

$h = 10\sqrt{6}$ — (1 mark both h and l correct)

$l = 20\sqrt{6}$

Dimensions

$10\sqrt{3} \times 10\sqrt{6} \times 20\sqrt{6}$

Question 29 continued on next page

Question 29 (continued)

© A square based tank with a diagonal 30 cm could also be designed to fit inside the pipe. Show that the rectangular water tank is more than 5% greater in volume compared to square based tank. **2**

$V_{rectangular\ tank} = 10\sqrt{3} \times 10\sqrt{6} \times 20\sqrt{6}$

$= 12000\sqrt{3}$

Using Pythagoras

$b^2 + b^2 = 30^2$

$b = \sqrt{\frac{900}{2}}$

$= \sqrt{450} = 15\sqrt{2}$

$V_{square\ tank} = b^2 l \qquad l = 2b$

$= (15\sqrt{2})^2 \times 2(15\sqrt{2})$

$= 13500\sqrt{2}$

$\frac{V_{rectangular}}{V_{square}} = \frac{12000\sqrt{3}}{13500\sqrt{2}} = 1.088662$

$\frac{12000\sqrt{3} - 13500\sqrt{2}}{13500\sqrt{2}} = 0.0887$

$= 8.87\%$

$8.87\% > 5\%$

1 mark – both volume correct

2 marks – correct solution

Question 30 (4 marks)

In a flock of 22400 sheep, the ratio of males to females is 9: 15. The weights of the female sheep are normally distributed with a mean of 78.4 kg and a standard deviation of 5.6 kg. In the flock, 13979 of the female sheep each weigh more than x kg. **4**

Find the value of x.

No. of females $= \frac{15}{24} \times 22400$

$= 14000$ — (1)

$P(X > x) = \frac{13979}{14000} = 0.9985$ — (1)

$P(X < x) = 1 - P(X > x)$

$= 1 - 0.9985$

$= 0.0015$ — (1)

$z = \frac{x - \bar{x}}{\mu}$

$-3 = \frac{x - 78.4}{5.6}$

$x = 78.4 + (-3)(5.6)$

$= 61.6$ — (1)

Question 31 (6 marks)

The temperature, $T°$ C, at a farm over the course of a day is modelled by

$$T(t) = 22 + 8\left(\cos\frac{\pi}{12}(t-4)\right)$$

where t if the time in hours after midnight, $0 \leq t \leq 24$.

(a) Find the temperature at midnight. **1**

At $t=0$

$T(0) = 22+8\left(-\cos\frac{4\pi}{12}\right)$

$= 22+8\left(\frac{1}{2}\right) = 26°C$

(b) Find $\frac{dT}{dt}$, leave your answer in the simplest form. **1**

$\frac{dT}{dt} = -\frac{8\pi}{12}\sin\left(\frac{\pi}{12}(t-4)\right)$

$= -\frac{2\pi}{3}\sin\left(\frac{\pi}{12}(t-4)\right)$

(c) Determine whether the temperature is initially decreasing or increasing. **2**

at $t=0$, $\frac{dT}{dt} = -\frac{2\pi}{3}\sin\left(-\frac{4\pi}{12}\right)$

$= -\frac{2\pi}{3}\left(-\frac{\sqrt{3}}{2}\right)$ — (1)

$= \frac{\pi\sqrt{3}}{3} > 0$ — (1)

∴ Temperature is initially increasing

(d) Find the maximum temperature and determine the first time at which this occurs. **2**

Max temp $= 22+8(1) = 30$ — (1)

$\cos\left(\frac{\pi}{12}(t-4)\right) = 1$

$\frac{\pi}{12}(t-4) = 0$

$t-4 = 0$

$t = 4$ — (1)

Question 32 (7 marks)

A business owner takes an reducing balance loan of \$500000 to expand their company. Interest is charged at 0.4% per month, compounded monthly. At the end of each month, interest is added to the loan balance and then a repayment of \$3200 is made.

Let A_n be the balance owing on the loan at the end of n months.

(a) Show that $A_2 = 500000(1.004)^2 - 3200(1.004) - 3200$. **2**

$A_1 = 500000 \times (1 + 0.004) - 3200$ ..... (1)

$A_2 = A_1 \times 1.004 - 3200$

$= 500000 \times 1.004^2 - 3200 \times 1.004 - 3200$ — (1)

(b) Show that $A_n = 800000 - 300000(1.004)^n$. **3**

$A_3 = A_2 \times 1.004 - 3200$

$= 500000 \times 1.004^3 - 3200 \times (1.004)^2 - 3200 \times 1.004 - 3200$

..... (1)

$A_n = 500000 \times 1.004^n - 3200 \times 1.004^{n-1} - \ldots - 3200 \times 1.004 - 3200$

$= 500000 \times 1.004^n - 3200 \left(1 + 1.004 + \ldots + 1.004^{n-1}\right)$

G.P

$a = 1 \quad r = 1.004$

$S_n = \frac{1.004^n - 1}{1.004 - 1}$

$= 250(1.004^n - 1)$ — (1)

$A_n = 500000 \times 1.004^n - 3200 \times 250(1.004^n - 1)$

$= 500000 \times 1.004^n - 800000(1.004^n - 1)$

$= 800000 - (800000 - 500000)(1.004^n)$

$= 800000 - 300000(1.004)^n$ — (1)

Question 32 continues next page.

s

Question 32 (continued)

(c) After how many months will the balance owing first be less than \$250000. **2**

$800000 - 300000(1.004)^n < 250000$

$-300000(1.004)^n < 250000 - 800000$

$1.004^n > \frac{550000}{300000}$ — (1)

$1.004^n > \frac{11}{6}$

$\therefore \ln 1.004^n > \ln \frac{11}{6}$

$n > \frac{\ln \frac{11}{6}}{\ln 1.004}$

$n > 151.84$

$\therefore$ Balance becomes less than \$250000 after 152 months — (1)