Caringbah High School

Year 12 2026

Mathematics Advanced

HSC Course

Assessment Task 4

Trial HSC Examination

General Instructions

- Reading time – 10 minutes
- Working time – 3 hours
- Write using a blue or black pen
- Calculators approved by NESA may be used
- A reference sheet is provided
- For questions in Section II, show relevant mathematical reasoning and/or calculations
- Marks may not be awarded for partial or incomplete answers

Total marks – 100

Section I **10 marks**

- Attempt Questions 1–10
- Allow about 15 minutes for this section
- Mark your answers on the answer sheet provided.

Section II **90 marks**

- Attempt Questions 11–31
- Allow about 2 hours 45 minutes for this section
- Write your answers for each question in the space provided. Extra writing space is available on page 32.

Student Number: _____ **Class:**_____

Marker's Use Only								
Section I	**Section II**							**Total**
Q1-10	**Q11-15**	**Q16-18**	**Q19-21**	**Q22-23**	**Q24-26**	**Q27-29**	**Q30-31**	
/10	/11	/14	/12	/11	/14	/13	/15	/100

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Section I

10 marks

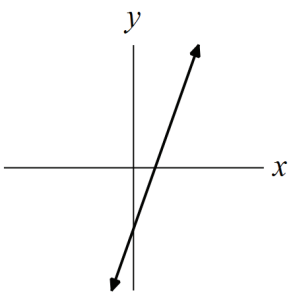
Attempt Questions 1-10

Allow about 15 minutes for this section

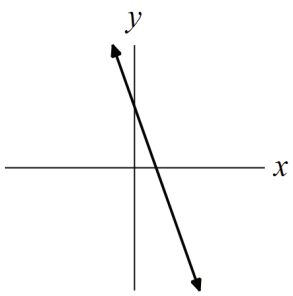
Use the multiple-choice answer sheet for Questions 1–10

1. Which of the following could be the graph of $3x + y = 5$?

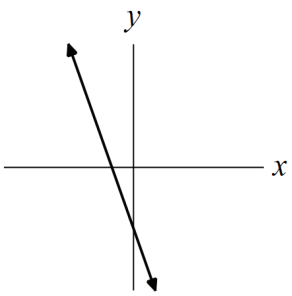
(A)



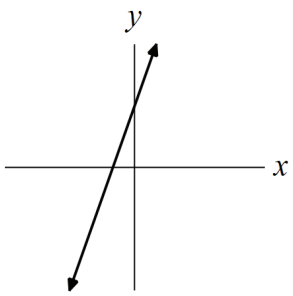
(B)



(C)



(D)



2. In an arithmetic series, the first term is –2 and the third term is 8.

What is the sum of the first 15 terms?

(A) 395

(B) 450

(C) 495

(D) 540

Do NOT write in this area.

3. Consider $x = \log_a(2y)$, where $a > 1$.

Which of the following is the correct expression for the function, with y as the subject?

(A) $y = a^{\frac{x}{2}}$

(B) $y = a^{2x}$

(C) $y = \frac{a^x}{2}$

(D) $y = 2a^x$

4. A group of campers is about to begin a hike. Before the hike begins, the weights of the campers are measured, and the mean, standard deviation and interquartile range (IQR) of the campers' weights are calculated. The campers then put on identical backpacks that all have the same weight.

Which row of the table identifies how the weight of the backpacks would affect the mean, standard deviation and IQR of the campers' weights?

	Mean	Standard deviation	IQR
(A)	Increases	Increases	Increases
(B)	Increases	No change	Increases
(C)	Increases	No change	No change
(D)	Increases	Increases	No change

Do NOT write in this area.

5. Which of the following is equivalent to the given integral?

$$\int \frac{1}{(2x+1)^2}\,dx$$

(A) $\dfrac{-1}{2(2x+1)} + C$

(B) $\dfrac{-1}{4(2x+1)} + C$

(C) $\dfrac{-1}{3(2x+1)^3} + C$

(D) $\dfrac{-1}{6(2x+1)^3} + C$

6. The cubic function $y = (x-4)^3 + 5$ is reflected about the y-axis and then translated 3 units to the left. What is the equation of the resulting cubic?

(A) $y = -(x+1)^3 + 2$

(B) $y = -(x+7)^3 + 5$

(C) $y = (x+1)^3 + 2$

(D) $y = (x+7)^3 + 5$

7. Consider the function $f(x) = (4x-1)^3 + 2x - 3$. The derivative $f'(x)$ has a minimum value of p. What is the value of p?

(A) $p = -1$

(B) $p = 0$

(C) $p = 1$

(D) $p = 2$

Do NOT write in this area.

8. A bag contains eucalyptus lollies and lemon lollies. The probability of randomly selecting a lemon lolly is $\frac{4}{7}$. After six lemon lollies are removed, the probability of randomly selecting a lemon lolly becomes $\frac{5}{11}$.

How many eucalyptus lollies are in the bag?

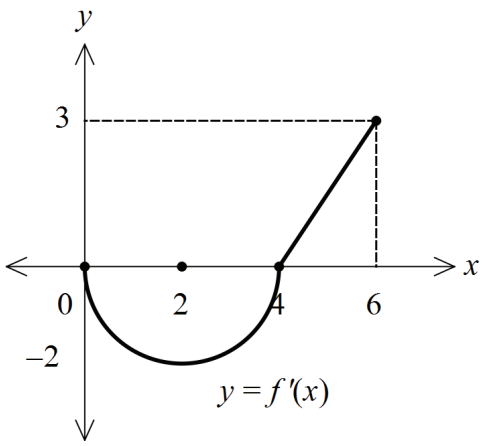
(A) 3

(B) 6

(C) 9

(D) 12

9. The graph of the derivative function $y = f'(x)$, for $0 \leq x \leq 6$, is shown below. The graph consists of a semicircle of radius 2 centred on the x-axis, and a straight line.



Given that $f(0) = 5$, what is the exact value of $f(6)$?

(A) $5 + 2\pi$

(B) $11 - 2\pi$

(C) $8 - 2\pi$

(D) $8 + 2\pi$

Do NOT write in this area.

10. Rachita is training for a marathon and uses an app on her phone to track her physical activity and fitness. The app calculates the number of calories that Rachita burns per minute as modelled by the function

$$f(V) = \frac{1}{100}\left(\frac{V^3}{800} - \frac{V^2}{2} + 57V\right),$$

where V is Rachita's speed in metres per minute.

If Rachita runs for 10 km at a constant speed, what is the minimum number of calories that she burns?

(A) 200 calories

(B) 500 calories

(C) 600 calories

(D) 700 calories

End of Section I

Do NOT write in this area.

Section II

90 marks
Attempt Questions 11–31
Allow about 2 hours and 45 minutes for this section

Your responses should include relevant mathematical reasoning and/or calculations.
Extra writing space is available on pages 31-32.

Question 11 (2 marks)

Solve $|4 - 3x| = 11$ **2**

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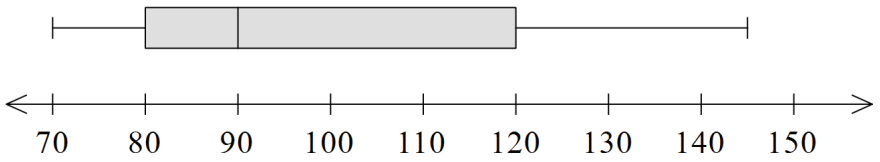
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Question 12 (2 marks)

A group of 680 students did as many star jumps as they could in 5 minutes.

The results are recorded in the box plot.



(a) Describe the shape of the distribution. **1**

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(b) Find the number of students who can do more than 120 star jumps. **1**

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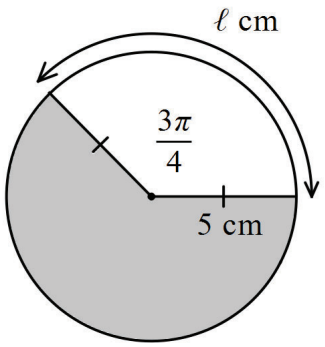
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Question 13 (3 marks)

The diagram shows a circle of radius 5 cm and a minor arc of length ℓ cm.

subtending an angle of $\frac{3\pi}{4}$ radians at the centre. The major sector is also shaded.



NOT TO SCALE

(a) Find the exact length of the minor arc, ℓ cm. **1**

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(b) Find the exact area of the major sector. **2**

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Question 14 (2 marks)

Given that $f(x) = x - 2$ and $g(x) = -\sqrt{x+3}$, find the range of $f\big(g(x)\big)$. **2**

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Question 15 (2 marks)

Find, in simplest form **2**

$$\frac{d}{dx}\left(\frac{\sin^2 x}{e^x}\right)$$

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Question 16 (5 marks)

On any given school day, the discrete random variable X represents the number of students who leave early due to illness.

x	0	1	2	3	4	5
$P(X = x)$	p	0.25	0.25	0.13	0.07	0.04

(a) Find the value of p. **1**

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(b) Find the number of students, $E(X)$, who are expected to leave early due to illness on any given school day. **2**

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(c) It is known that at least one student left school early due to illness on a particular day. Find the probability that four students left early on that day. **2**

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Question 17 (4 marks)

A can of soft drink is taken from a cupboard and placed in a cooler filled with ice and water. The temperature of the can in degrees Celsius, T, can be modelled by the equation

$$T = 25e^{-0.1t},$$

where t is the number of minutes since the can was placed in the cooler.

(a) What was the initial temperature of the can? **1**

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(b) What is the temperature of the can after it has been in the cooler for 5 minutes? **1**

Answer correct to the nearest degree Celsius.

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(c) After how long will the temperature of the can fall below 4°C? Answer correct to the nearest second. **2**

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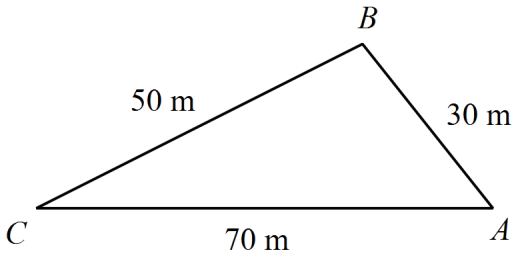
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Question 18 (5 marks)

A triangular plot of land, ABC, is enclosed by fences measuring 30 m, 50 m and 70 m.



NOT TO SCALE

(a) Find the size of $\angle ABC$. **2**

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(b) Find the exact area of the plot of land. **1**

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(c) A goat is tethered to the fence post at point B using a 20 m long rope. **2**

Determine whether the rope is long enough for the goat to reach the fence AC.

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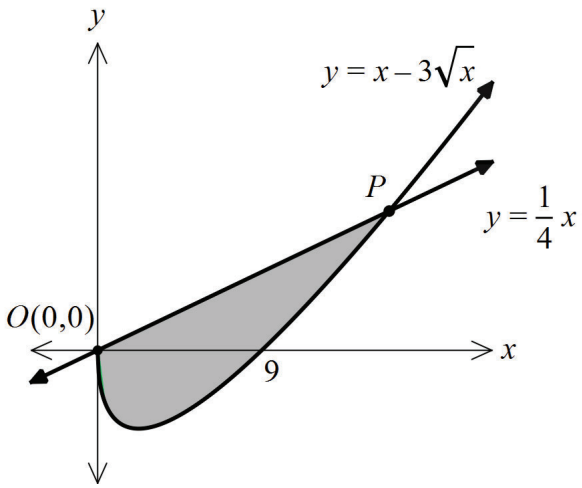
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Question 19 (4 marks)

The diagram shows the graphs of the curve $y = x - 3\sqrt{x}$ and the line $y = \frac{1}{4}x$, which intersect at the points $O(0,0)$ and P.



NOT TO SCALE

(a) Show that the x-coordinate of P is 16. **1**

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(b) Find the exact area of the shaded region enclosed by the curve and the line. **3**

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Question 20 (5 marks)

Consider the circle $x^2 + y^2 - 2y - 15 = 0$.

(a) The circle is reflected in the x-axis. Using an appropriate scale, sketch the reflected circle on the set of axes below, showing the coordinates of the centre and the radius. **3**

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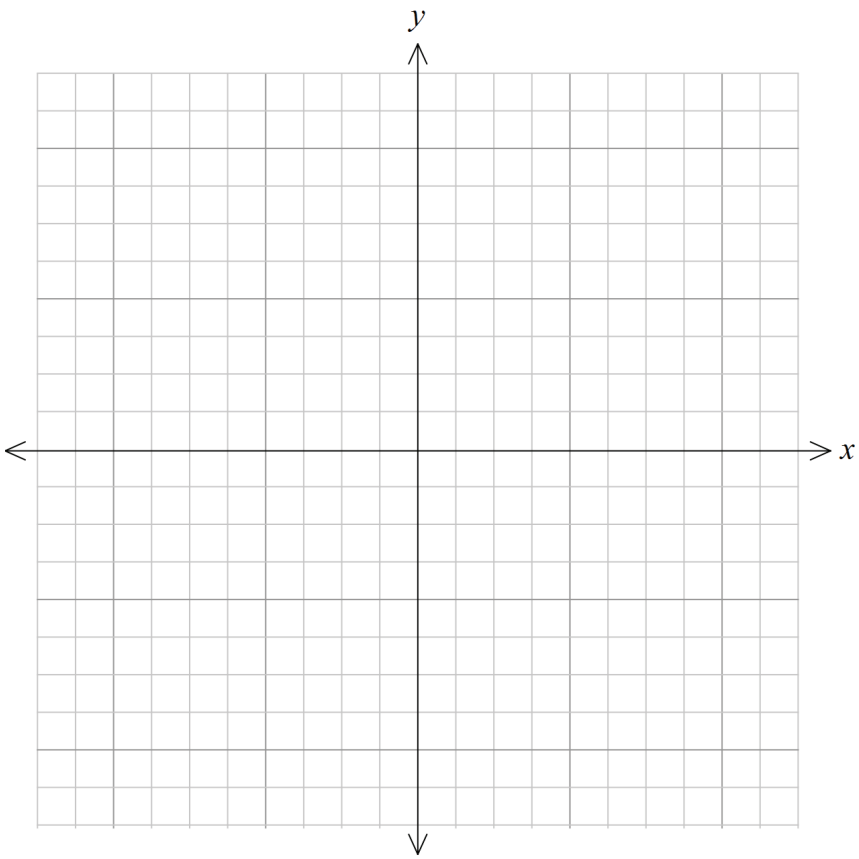
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Question 20 continues on page 15

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(b) The vertex of $y = |x + c|$ lies on the reflected circle. **2**

What are the possible values of c? Leave your answer(s) in exact form.

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Question 21 (3 marks)

The finishing times in minutes of the participants in the Oztown Charity Fun Run were normally distributed with mean μ and standard deviation σ. A time of 28 minutes had a z-score of -2.5 and a time of 64 minutes had a z-score of 1.5. Find the values of μ and σ. **3**

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Question 22 (5 marks)

The table shows the future value interest factors of an annuity of $1.

Future value of an annuity of $1				
Period (years)	Interest rate per period			
	1%	2%	3%	4%
4	4.060	4.122	4.184	4.246
5	5.101	5.204	5.309	5.416
6	6.152	6.308	6.468	6.633

An annuity is opened and annual contributions of $3000 are made at the end of each year for six years. The interest earned by the annuity is calculated just before each contribution is made according to the following rates.

- 3% per annum, compounding annually, for the first four years
- 4% per annum, compounding annually, in the fifth year
- r % per annum, compounding annually, in the sixth year

(a) Using the table, find the total balance of the annuity after the first four years. **1**

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(b) Show that the balance of the annuity immediately after the fifth contribution is $16 054.08. **1**

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Question 22 continues on page 17

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(c) If the annuity's balance immediately after the sixth contribution is \$19 800, find the interest rate, r %, for the sixth year. Give your answer correct to two decimal places. **3**

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Question 23 (6 marks)

The duration, T, in seconds of social media videos uploaded by a content creator is given by the probability density function

$$T(t) = \begin{cases} \frac{1}{20} e^{-\frac{1}{20}(t-20)} & t \geq 20 \\ 0 & t < 20 \end{cases}$$

(a) (i) Calculate $P(T < 34)$, expressing your answer correct to one decimal place. **2**

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(ii) Interpret your answer from part (i) in the given context. **1**

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Question 23 continues on page 19

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(b) It is known that 90% of the videos uploaded by the content creator have a duration of less than m seconds. Find the value of m, correct to the nearest whole number. **3**

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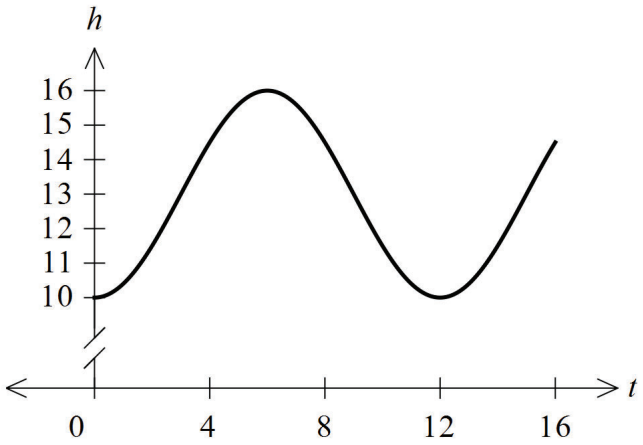
Questions 11-23 are worth 49 marks in total

Question 24 (5 marks)

The water level of a sea channel rises and falls according to changes in the tide. Ships can travel through the channel when the water is at a suitable level. The channel's water level can be modelled by the function $h(t) = A\cos(bt) + 13$, where h is the water level in metres above the sea bed and t is the number of hours after 8 am on a Saturday.

Serena is planning to cross the channel in a ship. She observes the water level from 8 am until midnight on Saturday. At 8 am, during a low tide, the water level is 10 m.

At 2 pm, during a high tide, the water level is 16 m. The graph shows the changes in water level that Serena observed during the 16-hour period.



Serena's ship requires the water level to be at least 11.5 m to sail safely.

(a) Find the values of A and b. **2**

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Question 24 continues on page 21

Do NOT write in this area.

(b) At what times between 8 am and midnight on Saturday can Serena safely sail her ship? **3**

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Question 25 (5 marks)

(a) Show that **2**

$$\frac{\sin x}{1-\sin x} - \frac{\sin x}{1+\sin x} = 2\tan^2 x$$

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(b) Hence, find in simplest exact form **3**

$$\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{\sin x}{1-\sin x} - \frac{\sin x}{1+\sin x}\, dx$$

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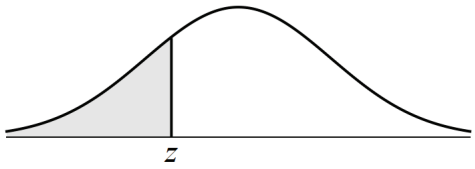
Question 26 (4 marks)

A random variable is normally distributed with a mean of 0 and a standard deviation of 1. **4**

The table gives the probability that this random variable lies below z from some negative values of z.

z	-2.39	-2.38	-2.37	-2.36	-2.35	-2.34	-2.33	-2.32	-2.31	-2.30
$P(Z < z)$	0.0084	0.0087	0.0089	0.0091	0.0094	0.0096	0.0099	0.0102	0.0104	0.0107

The probability values given in the table are represented by the shaded area in the following diagram.



The weight of babies at birth in NSW has a mean of 3.35 kilograms and a standard deviation of 0.52 kilograms. In a group of 1000 babies born in NSW, how many would be expected to weigh more than 2.128 kilograms?

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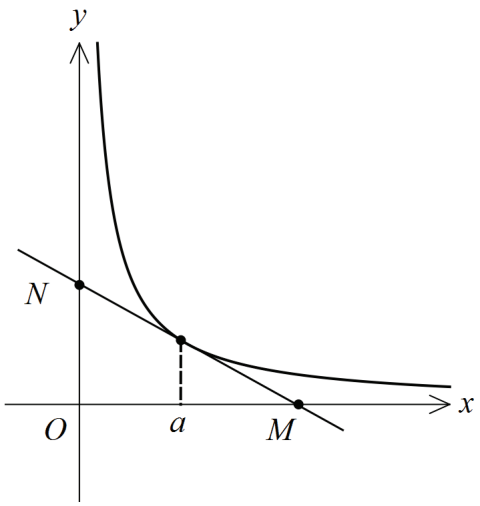
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Question 27 (4 marks)

The diagram shows the curve of $y = \frac{1}{x}$, where $x > 0$, and the tangent to the curve at the point where $x = a$. The tangent meets the x-axis at M and the y-axis at N. **4**



Show that the area of the triangle OMN is independent of a.

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Question 28 (3 marks)

All thirty students in Ozzie's Mathematics class took part in a statistical investigation. Over the course of one year, the number of Mathematics lessons missed, x, and the Yearly Examination mark, y, were recorded for each student. An analysis of the data showed that:

- the least-squares regression line of marks on lessons missed was $y = -0.75x + 88$
- the correlation coefficient, r, between marks and lessons missed satisfied $1 - r^2 = 0.36$

(a) Find the value of r. **2**

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(b) Ozzie missed thirty lessons and had a Yearly Examination mark of 63. Given that he missed two more lessons than the class mean, use the least-squares regression line to calculate the class' mean Yearly Examination mark. **1**

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Question 29 (6 marks)

At the end of 2020, Noel and Noelle bought a farm with a plantation of 2000 Christmas trees. At the start of each year, they plant a number of new trees equal to 5% of the total number of trees in the plantation at the end of the previous year. Towards the end of each year, they cut down and sell 150 trees.

Let A_n denote the number of frees in the plantation at the end of the nth year so that

$$A_0 = 2000 \text{ and } A_n = A_{n-1} \times 1.05 - 150 \text{ for } n = 1, 2, 3, \ldots$$

(a) Show that $A_n = 3000 - 1000 \times 1.05^n$ for $n = 1, 2, 3, \ldots$ **3**

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Question 29 continues on page 27

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(b) Find the first calendar year in which fewer than 50 new trees are planted. **3**

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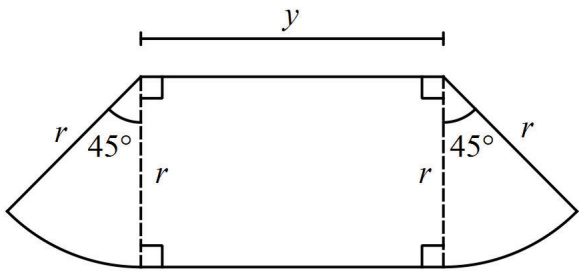
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Question 30 (6 marks)

Alisha has been asked by her local council to design a small community playground. The playground must have a rectangular shape with circular sectors attached to the two shorter sides, as shown in the diagram. The rectangle has a length of r metres and a width of y metres.



The total area of the playground must be 400 m^2. The council wants the fence around the playground to be as short as possible. Let F be the length of the fence in metres.

(a) Show that $y = \frac{400}{r} - \frac{\pi r}{4}$ **1**

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Question 30 continues on page 29

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(b) Show that $F(r) = \frac{800}{r} + 2r$ **1**

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(c) Find the minimum value of F. Justify why your answer corresponds to the minimum of the function. **4**

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Question 31 (9 marks)

Consider the function defined by the geometric series

$$f(x) = e^{-x} + e^{-2x} + e^{-3x} + \cdots \text{ for } x > 0$$

(a) Show that $f'(x) = \dfrac{-e^x}{(e^x-1)^2}$ **2**

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(b) It is known that $f''(x) = \dfrac{e^x(e^x+1)}{(e^x-1)^3}$. Explain why $f''(x) > 0$. **1**

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(c) Use the Trapezoidal rule with three function values to approximate $\displaystyle\int_1^2 f(x)\,dx$, **2**

correct to three decimal places.

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Question 31 continues on page 31

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(d) Find the exact value of $\int_1^2 f(x)\,dx$ **2**

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(e) Using your answers from parts (b), (c) and (d), show that $\ln(e+1) < \frac{4}{3}$ **2**

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End of Examination

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Extra Writing Space

If you use this space, clearly indicate which question you are answering.

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Caringbah High School

Year 12 2026

Mathematics Advanced

HSC Course

Assessment Task 4

Trial HSC Examination

General Instructions

- Reading time – 10 minutes
- Working time – 3 hours
- Write using a blue or black pen
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Section II **90 marks**

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Student Number: Solutions **Class:**_____

Marker's Use Only								
Section I	**Section II**							**Total**
Q1-10	**Q11-15**	**Q16-18**	**Q19-21**	**Q22-23**	**Q24-26**	**Q27-29**	**Q30-31**	
/10	/11	/14	/12	/12	/13	/13	/15	/100

Do NOT write in this area.

Section I

10 marks

Attempt Questions 1-10

Allow about 15 minutes for this section

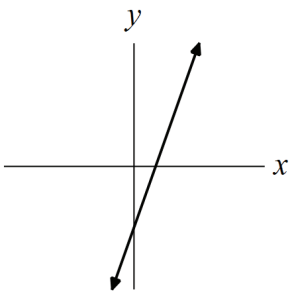
Use the multiple-choice answer sheet for Questions 1–10

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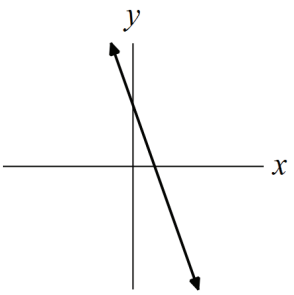
1. Which of the following could be the graph of $3x + y = 5$?

$y = -3x + 5$

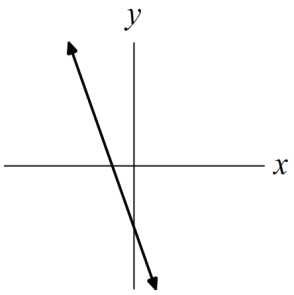
(A)



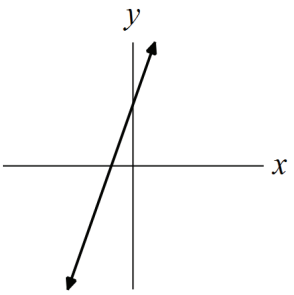
(B)



(C)



(D)



2. In an arithmetic series, the first term is –2 and the third term is 8.

What is the sum of the first 15 terms?

$T_1 = a = -2$

$T_3 = a + 2d = 8$

$d = 5$

$S_{15} = \frac{15}{2}(2(-2) + 14(5))$

$= 495$

(A) 395

(B) 450

(C) 495

(D) 540

Do NOT write in this area.

3. Consider $x = \log_a(2y)$, where $a > 1$.

Which of the following is the correct expression for the function, with y as the subject?

(A) $y = a^{\frac{x}{2}}$

(B) $y = a^{2x}$

(C) $y = \frac{a^x}{2}$

(D) $y = 2a^x$

$x = \log_a(2y)$

$2y = a^x$

$y = \frac{a^x}{2}$

4. A group of campers is about to begin a hike. Before the hike begins, the weights of the campers are measured, and the mean, standard deviation and interquartile range (IQR) of the campers' weights are calculated. The campers then put on identical backpacks that all have the same weight.

Which row of the table identifies how the weight of the backpacks would affect the mean, standard deviation and IQR of the campers' weights?

$x \to x + d$

$\bar{x} \to \bar{x} + d$

	Mean	Standard deviation	IQR
(A)	Increases	Increases	Increases
(B)	Increases	No change	Increases
(C)	Increases	No change	No change
(D)	Increases	Increases	No change

Do NOT write in this area.

5. Which of the following is equivalent to the given integral?

$$\int \frac{1}{(2x+1)^2}\,dx$$

(A) $\frac{-1}{2(2x+1)} + C$

(B) $\frac{-1}{4(2x+1)} + C$

(C) $\frac{-1}{3(2x+1)^3} + C$

(D) $\frac{-1}{6(2x+1)^3} + C$

$= \int (2x+1)^{-2}\,dx$

$= \frac{(2x+1)^{-1}}{-1\times 2} + C$

$= \frac{-1}{2(2x+1)} + C$

6. The cubic function $y = (x-4)^3 + 5$ is reflected about the y-axis and then translated 3 units to the left. What is the equation of the resulting cubic?

(A) $y = -(x+1)^3 + 2$

(B) $y = -(x+7)^3 + 5$

(C) $y = (x+1)^3 + 2$

(D) $y = (x+7)^3 + 5$

$y = (x-4)^3 + 5$

↓ Reflected

$y = (-x-4)^3 + 5$

$y = -(x+4)^3 + 5$

↓ Translated

$y = -(x+3+4)^3 + 5$

$\therefore y = -(x+7)^3 + 5$

7. Consider the function $f(x) = (4x-1)^3 + 2x - 3$. The derivative $f'(x)$ has a minimum value of p. What is the value of p?

(A) $p = -1$

(B) $p = 0$

(C) $p = 1$

(D) $p = 2$

$f'(x) = 12(4x-1)^2 + 2$

$f''(x) = 96(4x-1) = 0$ for max/min $f'(x)$

$x = \frac{1}{4}$

$f'\left(\frac{1}{4}\right) = 12\left(4\left(\frac{1}{4}\right)-1\right)^2 + 2 = p$

$p = 2$

Do NOT write in this area.

8. A bag contains eucalyptus lollies and lemon lollies. The probability of randomly selecting a lemon lolly is $\frac{4}{7}$. After six lemon lollies are removed, the probability of randomly selecting a lemon lolly becomes $\frac{5}{11}$.

How many eucalyptus lollies are in the bag?

(A) 3

(B) 6

(C) 9

(D) 12

① $l:e = 4:3$

② $(l-6):e = 5:6$

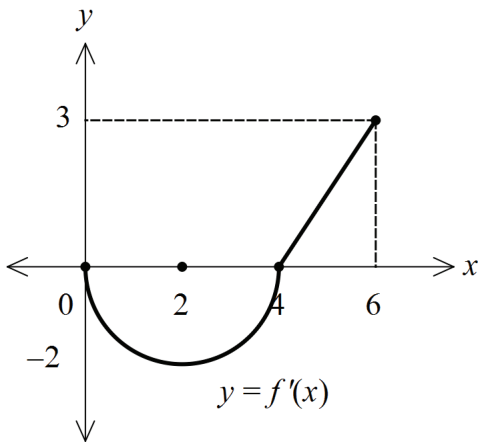
$l:e = 8:6$

$= 16:12$

Check $(l-6):e = 10:12$

$= 5:6$ ✓

9. The graph of the derivative function $y = f'(x)$, for $0 \le x \le 6$, is shown below. The graph consists of a semicircle of radius 2 centred on the x-axis, and a straight line.



Given that $f(0) = 5$, what is the exact value of $f(6)$?

$\int_0^6 f'(x)\,dx = [f(x)]_0^6$

$-\frac{1}{2}\times\pi\times2^2+\frac{1}{2}\times2\times3 = f(6)-f(0)$

$-2\pi+3 = f(6)-5$

$\therefore f(6) = 8-2\pi$

(A) $5+2\pi$

(B) $11-2\pi$

(C) $8-2\pi$

(D) $8+2\pi$

Do NOT write in this area.

10. Rachita is training for a marathon and uses an app on her phone to track her physical activity and fitness. The app calculates the number of calories that Rachita burns per minute as modelled by the function

$$f(V) = \frac{1}{100}\left(\frac{V^3}{800} - \frac{V^2}{2} + 57V\right),$$

where V is Rachita's speed in metres per minute.

If Rachita runs for 10 km at a constant speed, what is the minimum number of calories that she burns?

10 000 m

$S = \frac{D}{T} \rightarrow T = \frac{D}{S} = \frac{10000}{V}$

(A) 200 calories

(B) 500 calories

(C) 600 calories

(D) 700 calories

Let $C(V)$ be total calories burned

$C(V) = f(V) \times T$

$= \frac{1}{100}\left(\frac{V^3}{800} - \frac{V^2}{2} + 57V\right) \times \frac{10000}{V}$

$= 100\left(\frac{V^2}{800} - \frac{V}{2} + 57\right)$

$= \frac{V^2}{8} - 50V + 5700$

$C'(V) = \frac{V}{4} - 50 = 0$ for min C

$V = 200$

$C(200) = \frac{200^2}{8} - 50(200) + 5700$

$= 700$

End of Section I

Do NOT write in this area.

Section II

90 marks
Attempt Questions 11–31
Allow about 2 hours and 45 minutes for this section

Your responses should include relevant mathematical reasoning and/or calculations.
Extra writing space is available on pages 31-32.

Question 11 (2 marks)

Solve $|4 - 3x| = 11$ **2**

$4 - 3x = \pm 11$

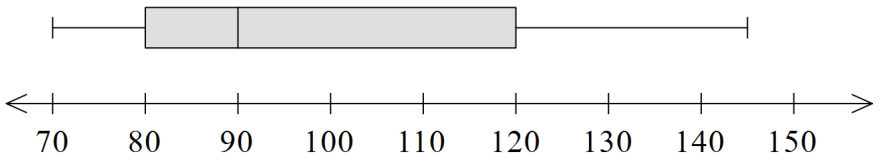
$-3x = 7, -15$

$\therefore x = -\frac{7}{3}, 5$ ✓✓ Both values

Question 12 (2 marks)

A group of 680 students did as many star jumps as they could in 5 minutes.

The results are recorded in the box plot.



(a) Describe the shape of the distribution. **1**

Positively skewed ✓

(b) Find the number of students who can do more than 120 star jumps. **1**

$Q_3 = 120$

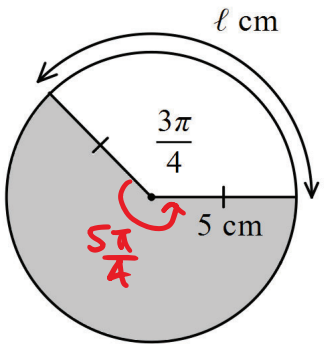
$0.25 \times 680 = 170$ students ✓

Do NOT write in this area.

Question 13 (3 marks)

The diagram shows a circle of radius 5 cm and a minor arc of length ℓ cm.

subtending an angle of $\frac{3\pi}{4}$ radians at the centre. The major sector is also shaded.



NOT TO SCALE

(a) Find the exact length of the minor arc, ℓ cm. **1**

$\ell = r\theta$

$= 5 \times \frac{3\pi}{4}$

$= \frac{15\pi}{4}$ ✓

(b) Find the exact area of the major sector. **2**

$A = \frac{1}{2}r^2\theta$

$= \frac{1}{2} \times 5^2 \times \frac{5\pi}{4}$ ✓

$= \frac{125\pi}{8}$ cm^2 ✓

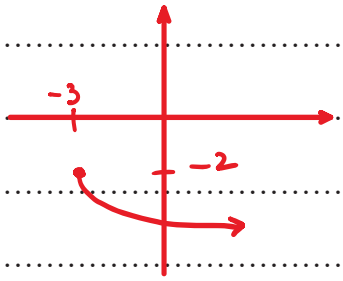
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Question 14 (2 marks)

Given that $f(x) = x - 2$ and $g(x) = -\sqrt{x+3}$, find the range of $f(g(x))$. **2**

$f(g(x)) = f(-\sqrt{x+3})$

$= -\sqrt{x+3} - 2$



Range: $y \leq -2$

Question 15 (2 marks)

Find, in simplest form **2**

$$\frac{d}{dx}\left(\frac{\sin^2 x}{e^x}\right)$$

$\frac{d}{dx}\left(\frac{\sin^2 x}{e^x}\right) = \frac{2\sin x \cos x \cdot e^x - \sin^2 x \cdot e^x}{(e^x)^2}$ ✓ Quotient rule

$= \frac{e^x \sin x (2\cos x - \sin x)}{(e^x)^2}$

$= \frac{\sin x (2\cos x - \sin x)}{e^x}$ ✓ Simplified

Do NOT write in this area.

Question 16 (5 marks)

On any given school day, the discrete random variable X represents the number of students who leave early due to illness.

x	0	1	2	3	4	5	Total
$P(X = x)$	p	0.25	0.25	0.13	0.07	0.04	1
$x.P(X=x)$	0	0.25	0.5	0.39	0.28	0.2	E(X)

(a) Find the value of p. **1**

$p + 0.25 + 0.25 + 0.13 + 0.07 + 0.04 = 1$

$\therefore p = 0.26$ ✓

(b) Find the number of students, $E(X)$, who are expected to leave early due to illness on any given school day. **2**

$E(X) = \sum x\,p(x) = 0 + 0.25 + 0.5 + 0.39 + 0.28 + 0.2$

$= 1.62$ ✓✓ with working (e.g. table)

(c) It is known that at least one student left school early due to illness on a particular day. Find the probability that four students left early on that day. **2**

$P(X = 4 \mid X \geq 1) = \dfrac{P(X=4)}{1 - P(X=0)}$ ✓

$= \dfrac{0.07}{1 - 0.26}$

$= \dfrac{7}{74}$ ✓

Do NOT write in this area.

Question 17 (4 marks)

A can of soft drink is taken from a cupboard and placed in a cooler filled with ice and water. The temperature of the can in degrees Celsius, T, can be modelled by the equation

$$T = 25e^{-0.1t},$$

where t is the number of minutes since the can was placed in the cooler.

(a) What was the initial temperature of the can? **1**

Sub $t=0$, $T = 25e^{0}$

$= 25$ ✓

$\therefore$ initial temperature 25°C

(b) What is the temperature of the can after it has been in the cooler for 5 minutes? **1**

Answer correct to the nearest degree Celsius.

Sub $t=5$, $T = 25e^{-0.5}$

$= 15.163...$

$= 15°C$ (nrst deg) ✓

(c) After how long will the temperature of the can fall below 4°C? Answer correct to the nearest second. **2**

Want $T < 4$

Solve $25e^{-0.1t} = 4$

$e^{-0.1t} = 4/25$ ✓

$-0.1t = \ln(4/25)$

$t = -10\ln(4/25)$

$= 18.325...$

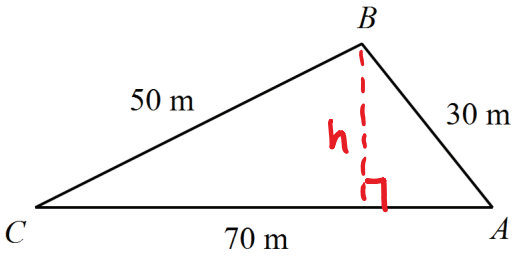
$= 18$ min 20 sec ✓

(nrst sec)

Do NOT write in this area.

Question 18 (5 marks)

A triangular plot of land, ABC, is enclosed by fences measuring 30 m, 50 m and 70 m.



NOT TO SCALE

(a) Find the size of $\angle ABC$. **2**

$\cos\angle ABC = \dfrac{30^2+50^2-70^2}{2(30)(50)}$ ✓

$= -\dfrac{1}{2}$

$\therefore \angle ABC = 120^\circ$ ✓

(b) Find the exact area of the plot of land. **1**

$\text{Area} = \frac{1}{2} \times 30 \times 50 \times \sin 120^\circ$

$= 750 \times \sqrt{3}/2$

$= 375\sqrt{3}\ m^2$ ✓

(c) A goat is tethered to the fence post at point B using a 20 m long rope. **2**

Determine whether the rope is long enough for the goat to reach the fence AC.

$\text{Area} = \frac{1}{2} \times 70 \times h$

$375\sqrt{3} = 35h$ ✓

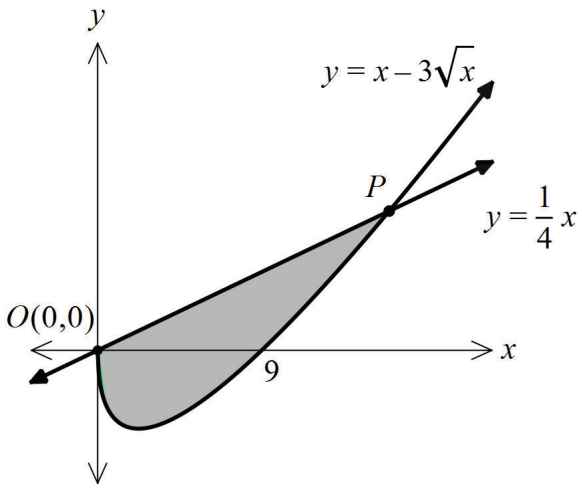
$h = \dfrac{75\sqrt{3}}{7} = 18.557\ldots < 20$

$\therefore$ Yes, the rope is long enough ✓

Do NOT write in this area.

Question 19 (4 marks)

The diagram shows the graphs of the curve $y = x - 3\sqrt{x}$ and the line $y = \frac{1}{4}x$, which intersect at the points $O(0,0)$ and P.



NOT TO SCALE

(a) Show that the x-coordinate of P is 16. **1**

P when $x - 3\sqrt{x} = \frac{1}{4}x$

$\frac{3}{4}x = 3\sqrt{x}$

$\frac{9}{16}x^2 = 9x$

$9x\left(\frac{1}{16}x - 1\right) = 0$

$x = 0, 16$

When $x=0$, origin $(0,0)$

$\therefore$ P when $x=16$ ✓

OR

Sub $x=16$ in both eqns

(b) Find the exact area of the shaded region enclosed by the curve and the line. **3**

Area $= \int_0^{16} \frac{1}{4}x - \left(x - 3\sqrt{x}\right) dx$

$= \int_0^{16} 3x^{\frac{1}{2}} - \frac{3}{4}x \, dx$ ✓

$= \left[\frac{3x^{3/2}}{3/2} - \frac{3}{4}\cdot\frac{x^2}{2}\right]_0^{16}$ ✓

$= 2(16)^{3/2} - \frac{3}{8}(16)^2 - 0$

$= 32$ units^2 ✓

Do NOT write in this area.

Question 20 (5 marks)

Consider the circle $x^2 + y^2 - 2y - 15 = 0$.

(a) The circle is reflected in the x-axis. Sketch the reflected circle on the set of axes below, showing the coordinates of the centre and the radius. **3**

Original: $x^2 + y^2 - 2y + 1 - 16 = 0$

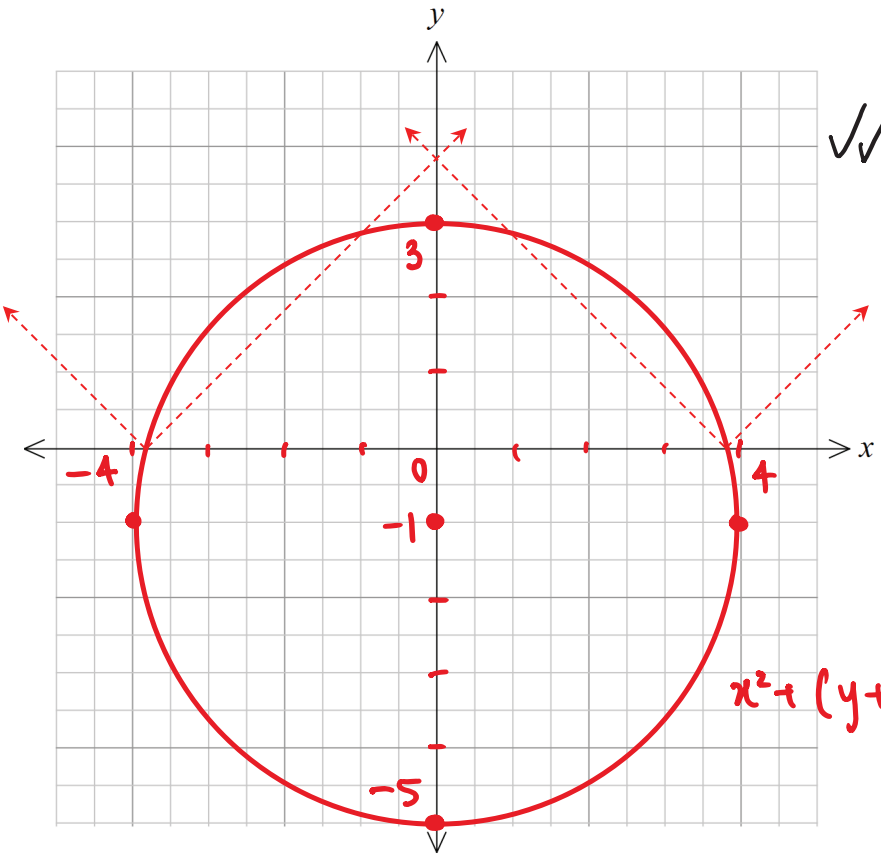
$x^2 + (y-1)^2 = 16$

$C(0,1)$, $r = 4$ units

Reflected in x-axis: $C(0,-1)$, $r = 4$ units

i.e. $x^2 + (y+1)^2 = 16$

✓ Progress towards new circle



✓✓ centre and radius

$x^2 + (y+1)^2 = 16$

Question 20 continues on page 15

Do NOT write in this area.

(b) The vertex of $y = |x + c|$ lies on the reflected circle. **2**

What are the possible values of c? Leave your answer(s) in exact form.

x-int: $y = 0$, $x = c$

$c^2 + (0+1)^2 = 16$

$c^2 = 15$ ✓

$\therefore c = \pm\sqrt{15}$ ✓

Question 21 (3 marks)

The finishing times in minutes of the participants in the Oztown Charity Fun Run were normally distributed with mean μ and standard deviation σ. A time of 28 minutes had a z-score of -2.5 and a time of 64 minutes had a z-score of 1.5. Find the values of μ and σ. **3**

$-2.5 = \frac{28-\mu}{\sigma}$ $\qquad$ $1.5 = \frac{64-\mu}{\sigma}$

$-2.5\sigma = 28 - \mu$ — ① $\qquad$ $1.5\sigma = 64 - \mu$ — ②

✓ two eqns

② − ①: $4\sigma = 36$

$\sigma = 9$ ✓

Sub $\sigma = 9$ in ②: $1.5 \times 9 = 64 - \mu$

$\mu = 50.5$ ✓

$\therefore \mu = 50.5, \sigma = 9$

Do NOT write in this area.

Question 22 (6 marks)

The table shows the future value interest factors of an annuity of $1.

Future value of an annuity of $1				
Period (years)	Interest rate per period			
	1%	2%	3%	4%
4	4.060	4.122	4.184	4.246
5	5.101	5.204	5.309	5.416
6	6.152	6.308	6.468	6.633

An annuity is opened and annual contributions of $3000 are made at the end of each year for six years. The interest earned by the annuity is calculated just before each contribution is made according to the following rates.

- 3% per annum, compounding annually, for the first four years
- 4% per annum, compounding annually, in the fifth year
- r % per annum, compounding annually, in the sixth year

(a) Using the table, find the total balance of the annuity after the first four years. **1**

$3000 × 4.184 = $12552 ✓

(b) Show that the balance of the annuity immediately after the fifth contribution is $16 054.08. **1**

Before 5th contribution = $12552 × 1.04
= $13054.08

After 5th contribution = $13054.08 + $3000
= $16054.08 ✓

Question 22 continues on page 17

Do NOT write in this area.

(c) If the annuity's balance immediately after the sixth contribution is \$19 800, find the interest rate, r %, for the sixth year. Give your answer correct to two decimal places. **3**

$19800 = 16054.08 \times \left(1+\frac{r}{100}\right) + 3000$ ✓

$16800 = 16054.08\left(1+\frac{r}{100}\right)$

$1+\frac{r}{100} = \frac{16800}{16054.08}$

$\frac{r}{100} = \frac{111}{2389}$ ✓

$r = \frac{111}{2389} \times 100$

$= 4.6462...$

$\therefore r = 4.65$ (2dp) ✓

Do NOT write in this area.

Question 23 (6 marks)

The duration, T, in seconds of social media videos uploaded by a content creator is given by the probability density function

$$T(t) = \begin{cases} \frac{1}{20} e^{-\frac{1}{20}(t-20)} & t \geq 20 \\ 0 & t < 20 \end{cases}$$

(a) (i) Calculate $P(T < 34)$, expressing your answer correct to one decimal place. **2**

$P(T<34) = \int_{20}^{34} \frac{1}{20} e^{-\frac{1}{20}(t-20)}\, dt$

$= \frac{1}{20}\left[\frac{e^{-\frac{1}{20}(t-20)}}{-\frac{1}{20}}\right]_{20}^{34}$ ✓

$= -\left(e^{-\frac{1}{20}\times 14} - e^{0}\right)$

$= 0.5034\ldots$

$= 0.5$ (1dp) ✓

(ii) Interpret your answer from part (i) in the given context. **1**

Approx. half of the videos uploaded by the content creator are less than 34 seconds long (or between 20-34 sec) ✓

Question 23 continues on page 19

Do NOT write in this area.

(b) It is known that 90% of the videos uploaded by the content creator have a duration of less than m seconds. Find the value of m, correct to the nearest whole number. **3**

$$\int_{20}^{m} \frac{1}{20} e^{-\frac{1}{20}(t-20)}\, dt = 0.9 \quad \checkmark$$

$$\frac{1}{20}\left[\frac{e^{-\frac{1}{20}(t-20)}}{-1/20}\right]_{20}^{m} = 0.9$$

$$-\left(e^{-\frac{1}{20}(m-20)} - e^{0}\right) = 0.9$$

$$1 - e^{-\frac{1}{20}(m-20)} = 0.9$$

$$0.1 = e^{-\frac{1}{20}(m-20)} \quad \checkmark$$

$$-\frac{1}{20}(m-20) = \ln 0.1$$

$$m - 20 = -20 \ln 0.1$$

$$= 20 \ln 10$$

$$m = 20 + 20 \ln 10$$

$$= 66.051\ldots \quad \checkmark$$

$$\therefore m = 66 \text{ (nrst whole number)}$$

Questions 11-23 are worth 49 marks in total

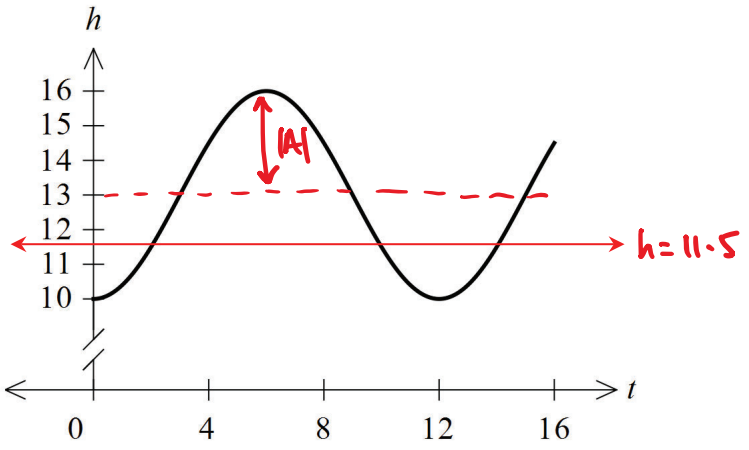
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Question 24 (4 marks)

The water level of a sea channel rises and falls according to changes in the tide. Ships can travel through the channel when the water is at a suitable level. The channel's water level can be modelled by the function $h(t) = A\cos(bt) + 13$, where h is the water level in metres above the sea bed and t is the number of hours after 8 am on a Saturday.

Serena is planning to cross the channel in a ship. She observes the water level from 8 am until midnight on Saturday. At 8 am, during a low tide, the water level is 10 m.

At 2 pm, during a high tide, the water level is 16 m. The graph shows the changes in water level that Serena observed during the 16-hour period.



Serena's ship requires the water level to be at least 11.5 m to sail safely.

(a) Find the values of A and b. **2**

$A < 0$ (upside down cosine graph)

$\therefore A = -3$ ✓

Period $= \frac{2\pi}{b}$

$12 = \frac{2\pi}{b}$

$\therefore b = \frac{\pi}{6}$ ✓

Question 24 continues on page 21

Do NOT write in this area.

(b) At what times between 8 am and midnight on Saturday can Serena safely sail her ship? **3**

Need $h(t) \geq 11.5$ for $0 \leq t \leq 16 \rightarrow 0 \leq \frac{\pi}{6}t \leq \frac{8\pi}{3}$

Solve $h(t) = -3\cos\left(\frac{\pi}{6}t\right) + 13 = 11.5$ ✓

$-3\cos\left(\frac{\pi}{6}t\right) = -1.5$

$\cos\left(\frac{\pi}{6}t\right) = 0.5$

$\cos^{-1}(0.5) = \frac{\pi}{3}$ 1st

$\frac{\pi}{6}t = \begin{cases} \pi/3 \\ 2\pi - \pi/3 \end{cases}$

$\frac{\pi}{6}t = \frac{\pi}{3}, \frac{5\pi}{3}, \frac{7\pi}{3}$ (+2π)

$t = 2, 10, 14$ ✓

From graph,

$h(t) \geq 11.5$ when $2 \leq t \leq 10$ and $14 \leq t \leq 16$

∴ Serena can safely sail at 10am–6pm and 10pm–midnight ✓

Do NOT write in this area.

Question 25 (5 marks)

(a) Show that **2**

$$\frac{\sin x}{1-\sin x} - \frac{\sin x}{1+\sin x} = 2\tan^2 x$$

$LHS = \frac{\sin x}{1-\sin x} - \frac{\sin x}{1+\sin x}$

$= \frac{\sin x(1+\sin x) - \sin x(1-\sin x)}{(1-\sin x)(1+\sin x)}$ ✓

$= \frac{\sin x + \sin^2 x - \sin x + \sin^2 x}{1-\sin^2 x}$

$= 2\sin^2 x / \cos^2 x$ ✓

$= 2\tan^2 x$

$= RHS$

(b) Hence, find in simplest exact form **3**

$$\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{\sin x}{1-\sin x} - \frac{\sin x}{1+\sin x}\, dx$$

$\int_{\pi/4}^{\pi/3} \frac{\sin x}{1-\sin x} - \frac{\sin x}{1+\sin x}\, dx = \int_{\pi/4}^{\pi/3} 2\tan^2 x\, dx$

$= 2\int_{\pi/4}^{\pi/3} \sec^2 x - 1\, dx$ ✓

$= 2\left[\tan x - x\right]_{\pi/4}^{\pi/3}$ ✓

$= 2\left(\tan\frac{\pi}{3} - \frac{\pi}{3} - \left(\tan\frac{\pi}{4} - \frac{\pi}{4}\right)\right)$

$= 2\left(\sqrt{3} - \frac{\pi}{3} - 1 + \frac{\pi}{4}\right)$

$= 2\sqrt{3} - 2 - \frac{\pi}{6}$ ✓

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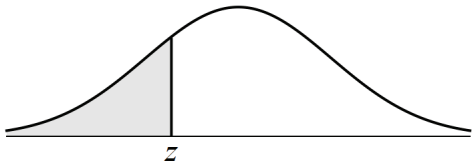
Question 26 (4 marks)

A random variable is normally distributed with a mean of 0 and a standard deviation of 1. **4**

The table gives the probability that this random variable lies below z from some negative values of z.

z	−2.39	−2.38	−2.37	−2.36	−2.35	−2.34	−2.33	−2.32	−2.31	−2.30
$P(Z < z)$	0.0084	0.0087	0.0089	0.0091	0.0094	0.0096	0.0099	0.0102	0.0104	0.0107

The probability values given in the table are represented by the shaded area in the following diagram.



The weight of babies at birth in NSW has a mean of 3.35 kilograms and a standard deviation of 0.52 kilograms. In a group of 1000 babies born in NSW, how many would be expected to weigh more than 2.128 kilograms?

$\mu = 3.35, \quad \sigma = 0.52$

$z = \dfrac{2.128 - 3.35}{0.52}$

$= -2.35$ ✓

$P(X \geq 2.128) = P(z \geq 2.35)$

$= 1 - P(z \leq -2.35)$ ✓

$= 1 - 0.0094$

$= 0.9906$ ✓

Expected number $= 1000 \times 0.9906$

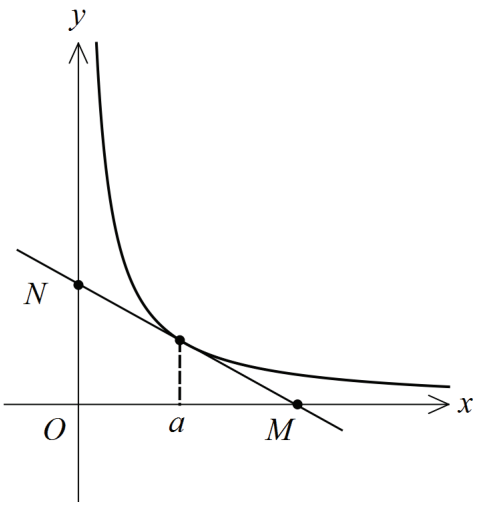
$= 990.6$

$= 991$ babies ✓

Do NOT write in this area.

Question 27 (4 marks)

The diagram shows the curve of $y = \frac{1}{x}$, where $x > 0$, and the tangent to the curve at the point where $x = a$. The tangent meets the x-axis at M and the y-axis at N. **4**



Show that the area of the triangle OMN is independent of a.

$y = \frac{1}{x} = x^{-1}$

$y' = -x^{-2} = \frac{-1}{x^2}$ ✓

At $x = a$, $m_T = -\frac{1}{a^2}$

$y - \frac{1}{a} = \frac{-1}{a^2}(x - a)$

$a^2y - a = -x + a$

$\therefore x + a^2y - 2a = 0$ is eqn of tangent ✓

M when $y = 0$: $x + 0 - 2a = 0$

$x = 2a$ $\therefore M(2a, 0)$

N when $x = 0$: $0 + a^2y - 2a = 0$

$y = 2/a$ $\therefore N(0, 2/a)$ ✓ M and N

Area $\Delta OMN = \frac{1}{2} \times OM \times ON$

$= \frac{1}{2} \times 2a \times 2/a$

$= 2$ units^2 ✓

$\therefore$ Area of ΔOMN is independent of a

Do NOT write in this area.

Question 28 (3 marks)

All thirty students in Ozzie's Mathematics class took part in a statistical investigation. Over the course of one year, the number of Mathematics lessons missed, x, and the Yearly Examination mark, y, were recorded for each student. An analysis of the data showed that:

- the least-squares regression line of marks on lessons missed was $y = -0.75x + 88$
- the correlation coefficient, r, between marks and lessons missed satisfied $1 - r^2 = 0.36$

(a) Find the value of r. **2**

$1-r^2=0.36$

$0.64=r^2$

$r=\pm 0.8$ ✓

But $m=-0.75<0$, negative correlation

$\therefore r=-0.8$ ✓

(b) Ozzie missed thirty lessons and had a Yearly Examination mark of 63. Given that he missed two more lessons than the class mean, use the least-squares regression line to calculate the class' mean Yearly Examination mark. **1**

Sub $\bar{x}=30-2=28$

$\bar{y}=-0.75\times 28+88$

$=67$

$\therefore$ class mean is 67 ✓

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Question 29 (6 marks)

At the end of 2020, Noel and Noelle bought a farm with a plantation of 2000 Christmas trees. At the start of each year, they plant a number of new trees equal to 5% of the total number of trees in the plantation at the end of the previous year. Towards the end of each year, they cut down and sell 150 trees.

Let A_n denote the number of trees in the plantation at the end of the nth year so that

$$A_0 = 2000 \text{ and } A_n = A_{n-1} \times 1.05 - 150 \text{ for } n = 1, 2, 3, \ldots$$

(a) Show that $A_n = 3000 - 1000 \times 1.05^n$ for $n = 1, 2, 3, \ldots$ **3**

$A_0 = 2000$

$A_1 = A_0 \times 1.05 - 150$

$= 2000 \times 1.05 - 150$

$A_2 = A_1 \times 1.05 - 150$

$= (2000 \times 1.05 - 150) \times 1.05 - 150$

$= 2000 \times 1.05^2 - 150 \times 1.05 - 150$

$A_3 = A_2 \times 1.05 - 150$

$= (2000 \times 1.05^2 - 150 \times 1.05 - 150) \times 1.05 - 150$

$= 2000 \times 1.05^3 - 150 \times 1.05^2 - 150 \times 1.05 - 150$ ✓pattern

$\vdots$

$A_n = 2000 \times 1.05^n - 150 \times 1.05^{n-1} - 150 \times 1.05^{n-2} - \ldots - 150$

$= 2000 \times 1.05^n - 150 \times (1 + 1.05 + 1.05^2 + \ldots + 1.05^{n-1})$

$= 2000 \times 1.05^n - 150 \times \frac{1(1.05^n - 1)}{1.05 - 1}$ ✓

$= 2000 \times 1.05^n - 3000(1.05^n - 1)$

$= 2000 \times 1.05^n - 3000 \times 1.05^n + 3000$

$= 3000 - 1000 \times 1.05^n$, as required ✓

Question 29 continues on page 27

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(b) Find the first calendar year in which fewer than 50 new trees are planted. **3**

Want $0.05 \times A_n < 50$

i.e. $A_n < 1000$ ✓

Solve $A_{n-1} = 3000 - 1000 \times 1.05^n = 1000$

$2000 = 1000 \times 1.05^n$

$1.05^n = 2$

$n = \dfrac{\ln 2}{\ln 1.05}$

$= 14.206\ldots$ ✓

Check when $n = 14$, end of 2034, $A_n = 1020.06\ldots$

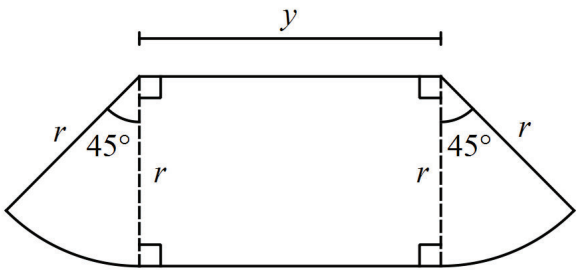
when $n = 15$, end of 2035, $A_n = 921.07\ldots$

∴ In 2036, fewer than 50 trees are planted ✓ calendar year

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Question 30 (6 marks)

Alisha has been asked by her local council to design a small community playground. The playground must have a rectangular shape with circular sectors attached to the two shorter sides, as shown in the diagram. The rectangle has a length of r metres and a width of y metres.



The total area of the playground must be 400 m^2. The council wants the fence around the playground to be as short as possible. Let F be the length of the fence in metres.

(a) Show that $y = \frac{400}{r} - \frac{\pi r}{4}$ **1**

Area = $yr + 2 \times \frac{1}{8}\pi r^2$

$400 = yr + \frac{\pi r^2}{4}$

$yr = 400 - \frac{\pi r^2}{4}$

$\therefore y = \frac{400}{r} - \frac{\pi r}{4}$ ✓ Show

Question 30 continues on page 29

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(b) Show that $F(r) = \frac{800}{r} + 2r$ **1**

$F(r) = 2y + 2r + 2 \times r \times \frac{\pi}{4}$

$= 2\left(\frac{400}{r} - \frac{\pi r}{4}\right) + 2r + \frac{\pi r}{2}$

$= \frac{800}{r} - \frac{\pi r}{2} + 2r + \frac{\pi r}{2}$

$= \frac{800}{r} + 2r$ ✓ Show

(c) Find the minimum value of F. Justify why your answer corresponds to the minimum of the function. **4**

$F(r) = 800r^{-1} + 2r$

$F'(r) = -\frac{800}{r^2} + 2 = 0$ for max/min $F(r)$ ✓

$-800 + 2r^2 = 0$

$2r^2 = 800$

$r^2 = 400$

$r = 20 \quad (r > 0)$ ✓

$F''(r) = \frac{1600}{r^3}$

$F''(20) = \frac{1600}{20^3} = \frac{1}{5} > 0$ ↺ ∴ min ✓

$F(20) = \frac{800}{20} + 2(20)$

$= 80$ ✓

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Question 31 (9 marks)

Consider the function defined by the geometric series

$$f(x) = e^{-x} + e^{-2x} + e^{-3x} + \cdots \quad \text{for } x > 0$$

GP: $a = e^{-x}$, $r = e^{-x}$, $S = \frac{a}{1-r}$ since $|r| < 1$

(a) Show that $f'(x) = \dfrac{-e^x}{(e^x-1)^2}$ **2**

$f(x) = \dfrac{e^{-x}}{1-e^{-x}} = \dfrac{1/e^x}{1-1/e^x} = \dfrac{1}{e^x-1}$ ✓ $f(x)$

$f(x) = (e^x-1)^{-1}$

$f'(x) = -(e^x-1)^{-2} \times e^x$

$= \dfrac{-e^x}{(e^x-1)^2}$ ✓ Show

(b) It is known that $f''(x) = \dfrac{e^x(e^x+1)}{(e^x-1)^3}$. Explain why $f''(x) > 0$. **1**

Since $x > 0$, $e^x > 1$

So $e^x(e^x+1) > 0$ and $(e^x-1)^3 > 0$

$\therefore f''(x) = \dfrac{e^x(e^x+1)}{(e^x-1)^3} > 0$ ✓ Explain

(c) Use the Trapezoidal rule with three function values to approximate $\displaystyle\int_1^2 f(x)\,dx$, **2**

correct to three decimal places.

x	1	3/2	2
$f(x)$	$\frac{1}{e-1}$	$\frac{1}{e^{1.5}-1}$	$\frac{1}{e^2-1}$

use $f(x) = \dfrac{1}{e^x-1}$

$\displaystyle\int_1^2 f(x)\,dx \approx \frac{1/2}{2}\left[\frac{1}{e-1} + 2\left(\frac{1}{e^{1.5}-1}\right) + \frac{1}{e^2-1}\right]$ ✓

$= 0.328$ (3dp) ✓

Question 31 continues on page 31

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(d) Find the exact value of $\int_1^2 f(x)\,dx$ **2**

$\int_1^2 f(x)\,dx = \int_1^2 \frac{e^{-x}}{1-e^{-x}}\,dx$

$= \left[\ln\left(1-e^{-x}\right)\right]_1^2$ ✓

$= \ln\left(1-e^{-2}\right) - \ln\left(1-e^{-1}\right)$

$= \ln\left(\frac{1-e^{-2}}{1-e^{-1}}\right)$

$= \ln\left(\frac{(1+e^{-1})(1-e^{-1})}{1-e^{-1}}\right)$

$= \ln\left(1+e^{-1}\right)$

$= \ln\left(\frac{e+1}{e}\right)$

$= \ln(e+1) - \ln e$

$= \ln(e+1) - 1$ ✓

or up to here

(e) Using your answers from parts (b), (c) and (d), show that $\ln(e+1) < \frac{4}{3}$ **2**

From (b), $f''(x) > 0$ so $f(x)$ is concave up ↑↑ ✓

i.e. Exact area < Trapezoidal rule approx.

$\ln(e+1) - 1 < 0.328$

$\ln(e+1) < 1.328 < \frac{4}{3}$

$\therefore \ln(e+1) < \frac{4}{3}$ ✓

End of Examination

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