Fort Street High School

NESA Number:

Name:

2026 TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Advanced

Teacher	Finance	Hussein	Ngo	Shi	Stamoulos
	☐	☐	☐	☐	☐

General Instructions

- Reading time – 10 minutes
- Working time – 3 hours
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided
- For questions in Section II, show relevant mathematical reasoning and/ or calculations
- Marks may be deducted for careless or badly arranged work.

Total marks: 92

Section I – 10 marks

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section

Section II – 90 marks

- Attempt Parts A to F
- Allow about 2 hours and 45 minutes for this section
- Write your student number on each answer booklet

Question	Mark
Multiple Choice	**/10**
Part A	**/16**
Part B	**/14**
Part C	**/17**
Part D	**/14**
Part E	**/12**
Part F	**/17**
Total	**/100**

Mathematics Advanced
Task **2026**
Section I

Multiple Choice Questions

NESA NUMBER

Attempt Questions 1–10 (10 marks)

Instructions

- Write your student number at the top of this page.
- Answer the questions on the multiple choice answer sheet.

Please turn over

10 marks
Attempt Questions 1-10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1-10.

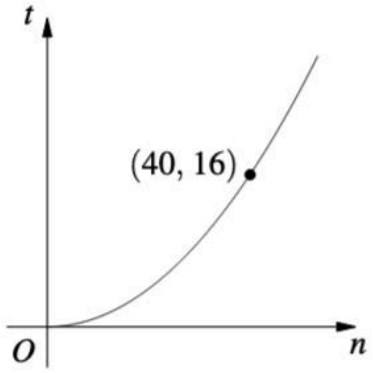
Question 1.

In a warehouse, the time t minutes, taken to process an order varies directly with the number of orders placed, n.

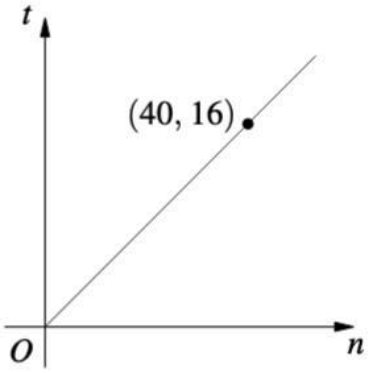
If 40 orders are placed on a certain day, the processing time for each order is 16 minutes.

Which of the following diagrams best represents the relationship between t and n?

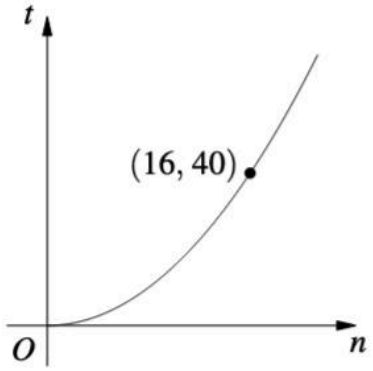
A.



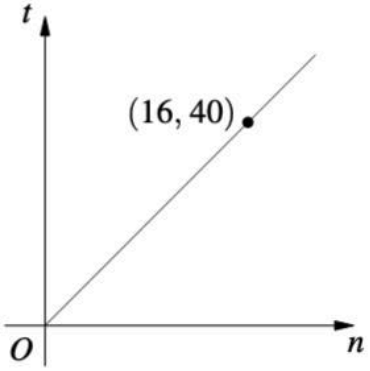
B.



C.



D.



Question 2.

In an arithmetic series, the first term is -2 and the third term is 8. What is the 10^{th} term?

A. 38

B. 41

C. 43

D. 45

Question 3.

In a mathematics examination, 25% of students scored less than 40 marks, 75% of students scored less than 86 marks and the highest mark was 96.

Given that the distribution is symmetrical with no outliers, what is the lowest mark and the median of this data set?

	Lowest mark	Median
A	30	63
B	30	68
C	36	63
D	36	68

Question 4.

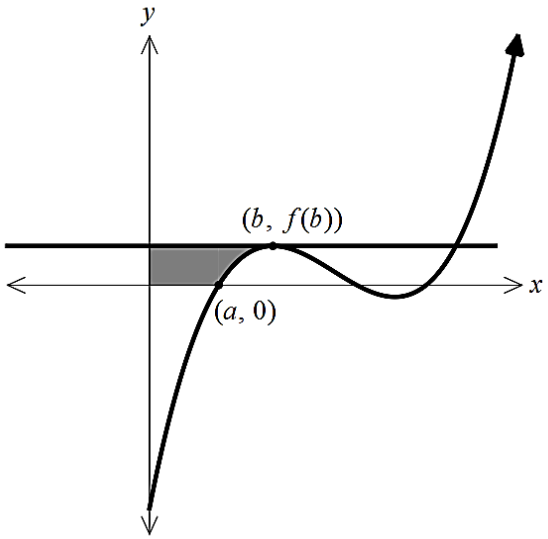
Let $f(x) = \sqrt{x}$ and $g(x) = \frac{1}{x-1}$

What is the domain and range of $f(g(x))$?

	Domain	Range
A	$[-1,1]$	$(-\infty, \infty)$
B	$(0, \infty)$	$(-\infty, 0) \cup (0, \infty)$
C	$[0, \infty)$	$(0, \infty)$
D	$(1, \infty)$	$(0, \infty)$

Question 5.

The graph of $f(x)$ is shown below. The horizontal line is a tangent to the local maximum $(b, f(b))$. The shaded area is bounded by the graph of $f(x)$, the coordinate axes and the horizontal line as shown below.



Which is the correct expression for the area of the shaded region?

A. $a\,f(b) - \int_a^b f(x)\,dx$

B. $a\,f(b) + \int_a^b f(x)\,dx$

C. $b\,f(b) - \int_a^b f(x)\,dx$

D. $-b\,f(b) + \int_0^b f(x)\,dx$

Question 6.

The limiting sum of a geometric series is $\frac{10}{9}$ and the sum of its first three terms is $\frac{26}{25}$. What is the common ratio of this series?

A. 1.39

B. 1.25

C. 0.40

D. 0.25

Question 7.

The normal to the graph of $f(x) = \ln\left(\frac{1}{x+2}\right)$ at point P has a gradient of 4.

What is the x-coordinate of point P?

A. $x = 2$

B. $x = -2$

C. $x = \frac{1}{2}$

D. $x = -\frac{1}{2}$

Question 8.

What is $\int \frac{1}{(2x+1)^2}\,dx$?

A. $\frac{-1}{2(2x+1)} + C$

B. $\frac{-1}{4(2x+1)} + C$

C. $\frac{-1}{3(2x+1)^3} + C$

D. $\frac{-1}{6(2x+1)^3} + C$

Question 9.

In a bag of n coloured discs, exactly 7 are red.

Two discs are selected at random, without replacement.

The probability that both discs are red is $\frac{21}{55}$.

Which quadratic equation could be used to find the value of n ?

A. $n^2 + n + 110 = 0$

B. $n^2 - n - 110 = 0$

C. $n^2 + n - 110 = 0$

D. $n^2 - n + 110 = 0$

Question 10.

Consider the equation $\left(\sin\theta - \frac{\sqrt{2}}{2}\right)\left(\tan 2\theta - \frac{\sqrt{3}}{3}\right) = 0$

How many solutions are there in the interval $0 \leq \theta \leq \pi$?

A. 1

B. 2

C. 3

D. 4

Mathematics Advanced
Trial HSC Examination 2026
Section II

Part A

NESA NUMBER

Attempt Questions 11–16 (16 marks)

Instructions

- Write your student number at the top of this page.
- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
- Your responses should include relevant mathematical reasoning and/or calculations.
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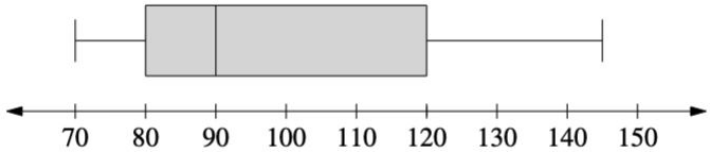
11	12	13	14	15	16	Total

Please turn over

Question 11 (2 marks)

A group of 680 students did as many starjumps as they could in 5 minutes.

The results are recorded in the box-plot below



(a) Describe the shape of the distribution. **1**

(b) Find the number of students who can do more than 120 starjumps. **1**

Question 12 (2 marks)

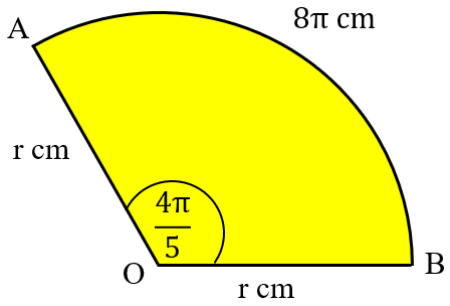
The equation of a circle is given by $x^2 + 2x + y^2 - 4y = 0$.

Find the centre and radius of the circle.

Question 13 (2 marks)

In the diagram, AOB is a sector of a circle with centre at O and radius r cm.

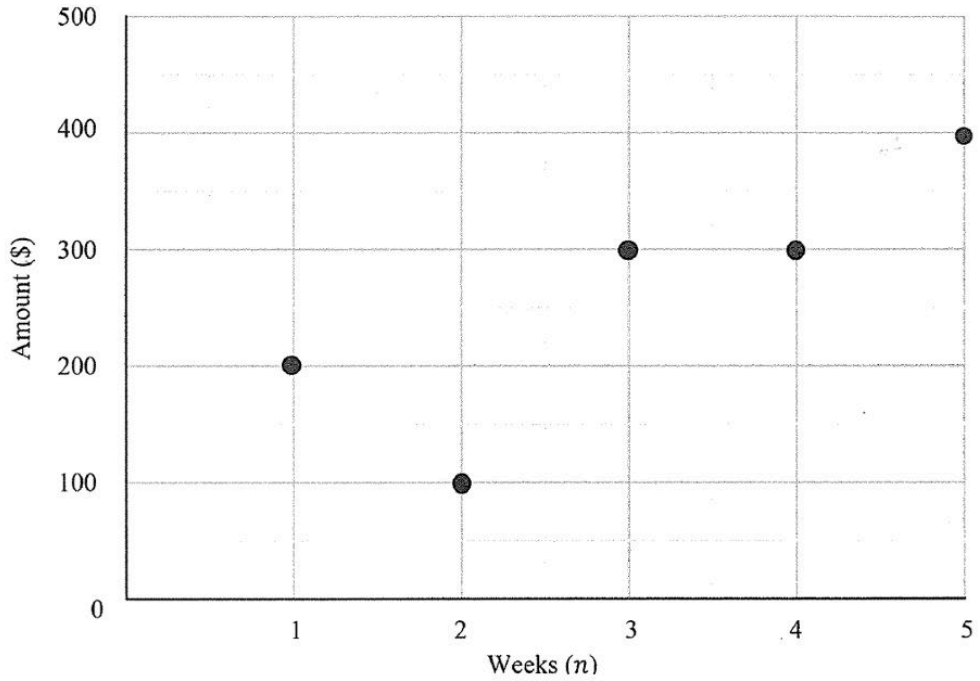
The angle AOB at the centre O is $\frac{4\pi}{5}$ and the length of arc AB is 8π cm.



Find the exact area of the sector.

Question 14 (4 marks)

Ivy has plotted the income that she has received each week for the past five weeks on the graph below.



(a) Over the same five weeks, Ivy's expenses which were originally \$200 per week, increase by \$20 each week. Write the equation for Ivy's expenses. **1**

(b) Write the equation of the least-squares regression line for Ivy's income. **2**

(c) Find the intersection of the lines from Parts (a) and (b). **1**

Question 15 (3 marks)

Let $f(x) = x^2 + kx - 6$. Find the values of k for which the two roots of $f(x)$ have a difference of 7.

Question 16 (3 marks)

Consider the function $f(x) = (x + 1)^2$

The following transformations were applied to the graph of $y = f(x)$ in the order shown below.

1) Horizontal translation to the right by 4 units
2) Horizontal dilation by a factor of $\frac{4}{3}$
3) Vertical dilation by a factor of $\frac{1}{3}$
4) Vertical translation down by 3 units
5) Reflection in the x-axis

Find the equation of the transformed function.

End of Part A.

Mathematics Advanced
Trial HSC Examination 2026
Section II

Part B

NESA NUMBER

Attempt Questions 17–19 (14 marks)

Instructions

- Write your student number at the top of this page.
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- Extra writing space is provided at the ned of this part. If you use this space, clearly indicate which question you are answering.

Office use:

17	18	19	Total

Please turn over

Question 17 (5 marks)

On any given school day, the discrete random variable X represents the number of students who leave early due to illness. The probability distribution is shown below:

x	0	1	2	3	4	5
$P(X = x)$	p	0.25	0.25	0.13	0.07	0.04

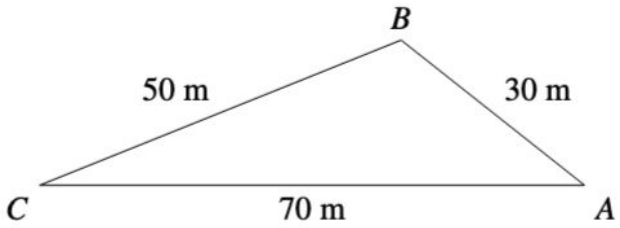
(a) Find the value of p. **1**

(b) Find the number of students, $E(X)$, who are expected to leave early due to illness on any given school day. **2**

(c) It is known that at least one student left school early due to illness on a particular day. Find the probability that four students left early on that day. **2**

Question 18 (6 marks)

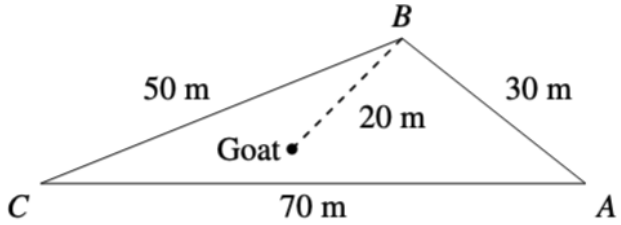
A triangular plot of land, ABC, is enclosed by fences measuring 30 metres, 50 metres and 70 metres as shown.



(a) Find the size of $\angle ABC$, correct to the nearest degree. **2**

(b) Find the area of the plot of land, correct the nearest square metre. **2**

(c) A goat is tethered to the fence post at point B using a 20 metre long rope. **2**



Determine whether the rope is long enough for the goat to reach the fence AC.

Question 17 (3 marks)

Consider the simultaneous equations

$$\begin{cases} \log_{10}(xy) = 2 \\ \log_{10}\left(\dfrac{x}{y}\right) = 1 \end{cases} \qquad \text{where } x, y > 0.$$

Find the values of x and y which satisfy both equations.

End of Part B.

Mathematics Advanced
Trial HSC Examination 2026
Section II

Part C

NESA NUMBER

Attempt Questions 20–22 (17 marks)

Instructions

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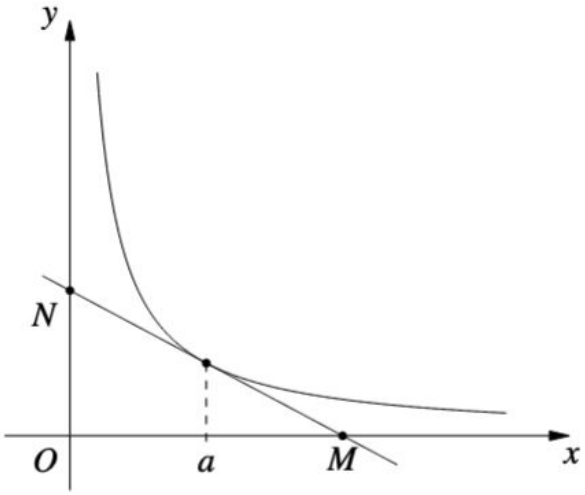
Office use:

20	21	22	Total

Please turn over

Question 20 (3 marks)

The diagram shows the curve of $y = \frac{1}{x}$ where $x > 0$ and the tangent to the curve at the point where $x = a$. The tangent meets the x-axis at M and the y-axis at N.



Show that the area of the triangle OMN is independent of a.

Question 21 (10 marks)

Consider the function $f(x) = -6x^2 + x^4$

(a) Show that $f(x)$ is an even function. **1**

(b) Find the coordinates of the x-intercepts. **1**

(c) Find the stationary points and determine their nature. **3**

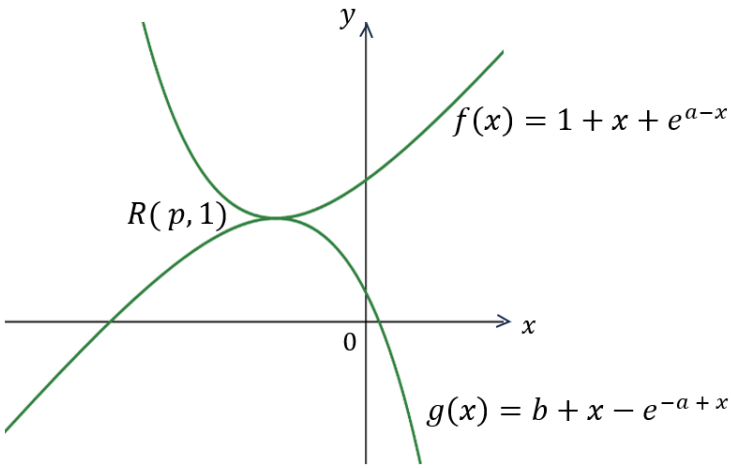
(d) Show that there are two inflection points, and find the gradient at those points. **3**

(e) Sketch the curve, showing stationary points, inflection points, slopes at all those points, and intercepts with the two axes. **2**

Question 22 (4 marks)

The curves of the functions $f(x) = 1 + x + e^{a-x}$ and $g(x) = b + x - e^{-a+x}$ have the same turning point at the point $R(p, 1)$ as shown on the diagram.

Find the coordinates of the point R and the values of a and b.



End of Part C.

Mathematics Advanced
Trial HSC Examination 2026
Section II

Part D

NESA NUMBER

Attempt Questions 23–26 (14 marks)

Instructions

- Write your student number at the top of this page.
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- Your responses should include relevant mathematical reasoning and/or calculations.
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Office use:

23	24	25	26	Total

Please turn over

Question 23 (3 marks)

Find $\int \sec^2 x\sqrt{\tan(x)+5}\,dx$.

Question 24 (5 marks)

The time a student takes to complete an online assignment is modelled by the continuous random variable T

The cumulative distribution function of T is given by:

$$F(t)=\begin{cases}\dfrac{1}{1728}t^3 & \text{when} \quad 0\le t\le 12\\ 1 & \text{when} \quad t>12\end{cases}, \qquad \text{where } t \text{ is in hours.}$$

(a) Find, to the nearest hour, the time within which 60% of students finish the assignment. **1**

(b) Define the probability density function of T, for all values of t. **1**

(c) It is given that the expected value $E(T)$ of T is 9 and $E(T^2) = 86.4$ **(DO NOT PROVE THIS).** Find the probability, to the nearest percent, that a student takes more than one standard deviation above the mean to complete the assignment. **3**

Question 25 (3 marks)

(a) Differentiate $f(x) = (\ln x)^2$. **1**

(b) Hence, or otherwise, find $\int_1^e \frac{\ln(x^2)}{x}\, dx$. **2**

Question 26 (3 marks)

Prove that $\dfrac{sec^4x - tan^4x}{cosec^2x + 2\,tan^2x - cot^2x} = 1$.

End of Part D.

Mathematics Advanced
Trial HSC Examination 2026
Section II

Part E

NESA NUMBER

Attempt Questions 27–29 (12 marks)

Instructions

- Write your student number at the top of this page.
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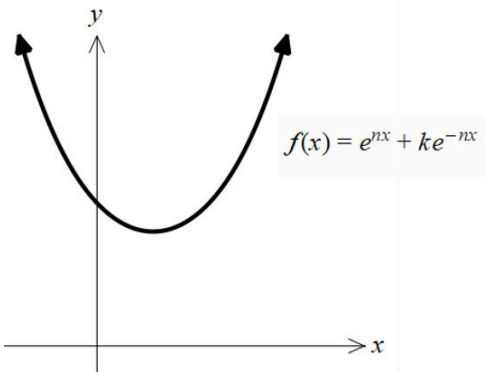
Office use:

27	28	29	Total

Please turn over

Question 27 (4 marks)

The diagram shows the graph of the function $f(x) = e^{nx} + k\,e^{-nx}$ where $k > 1, n > 0$



Find the range of $f(x)$ in simplest form.

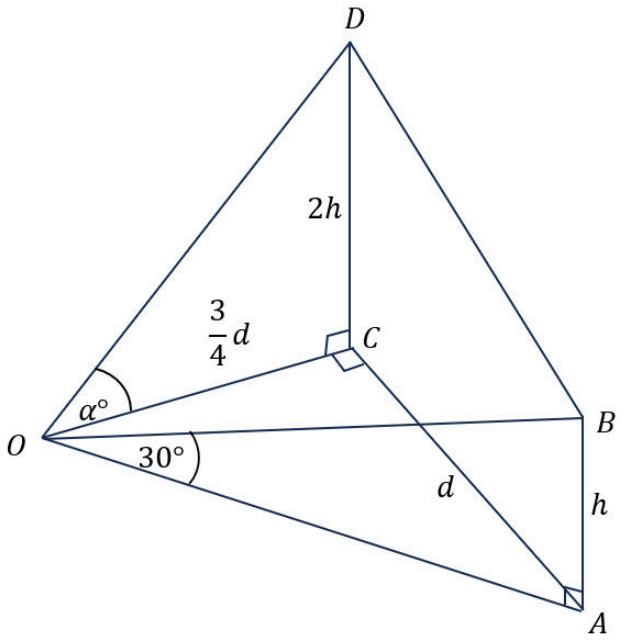
Question 28 (5 marks)

The diagram shows AB and CD which are two towers of height h and $2h$ metres respectively.

AB is south of tower CD, at a distance d metres from it.

O is a point on the ground, west of CD and at the same level as A and C.

The distance OC is $\frac{3}{4}d$ metres. The angle of elevation of B from O is $30°$ and the angle of elevation of D from O is α.



(a) Show that $h = \frac{3}{8}d\tan\alpha$. **1**

(b) By finding another expression for h, find the angle of elevation α to the nearest minute. **4**

Question 29 (3 marks)

A cricket player hits a ball with an angle of elevation $\theta°$, where $\theta°$ is normally distributed with a mean of $30°$ and a standard deviation of $15°$.

The horizontal distance travelled by the ball before hitting the ground is given by $d = 80 \sin 2\theta$, where d is in metres.

Find the probability that the ball travels more than 40 metres horizontally in the air before landing (give your answer to 2 decimal places).

End of Part E.

Mathematics Advanced
Trial HSC Examination 2026
Section II

Part F

NESA NUMBER

Attempt Questions 30–31 (17 marks)

Instructions

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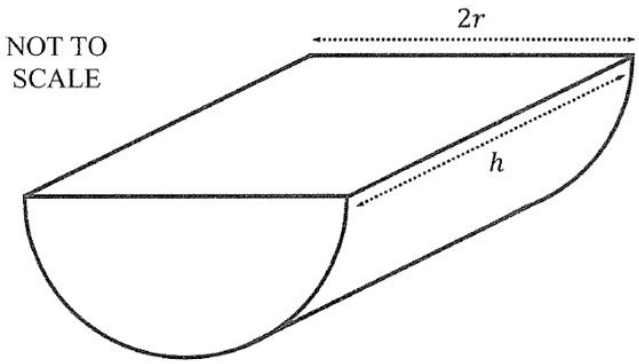
Office use:

30	31	Total

Please turn over

Question 30 (7 marks)

A closed water tank in the shape of a half-cylinder with diameter $2r$ metres and height h metres is to be made from sheet metal as shown below.



The surface area of the tank is A square metres and the volume of the tank is V cubic metres, and is constant.

(a) Given that $V = \frac{1}{2}\pi r^2 h$, show that $A = \pi r^2 + \frac{2V}{\pi}\left(\frac{2+\pi}{r}\right)$. **3**

(b) Show that the amount of sheet metal is minimised when $r = \sqrt[3]{\frac{V(2+\pi)}{\pi^2}}$. **4**

Question 31 (10 marks)

Consider the function defined by the geometric series $f(x) = e^{-x} + e^{-2x} + e^{-3x} + \cdots$ for $x > 0$.

(a) Show that $f(x) = \frac{1}{e^x - 1}$. **2**

(b) Show that $f'(x) = \frac{-e^x}{(e^x - 1)^2}$. **1**

(c) It is known that $f''(x) = \frac{e^x(e^x+1)}{(e^x-1)^3}$. Explain why $f''(x) > 0$. **1**

(d) Use the Trapezoidal rule with three function values to approximate $\int_1^2 f(x)\ dx$, correct to three decimal places. **2**

(e) Find the exact value of $\int_1^2 f(x)\ dx$. **2**

(f) Using your answers from parts (c) and (d), show that $\ln(e+1) < \frac{4}{3}$. **2**

END OF EXAMINATION

10 marks
Attempt Questions 1-10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1-10.

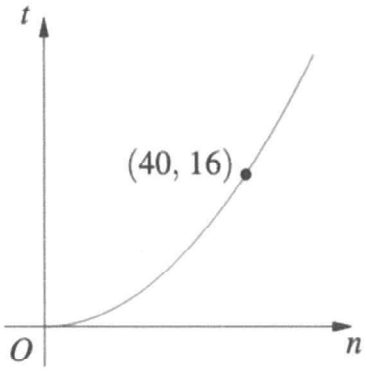
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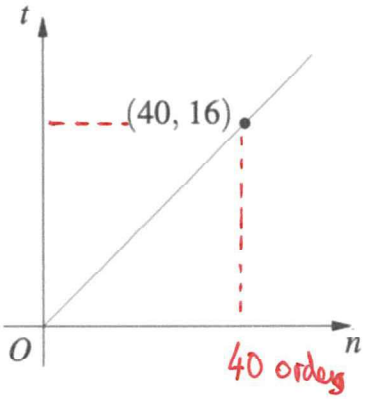
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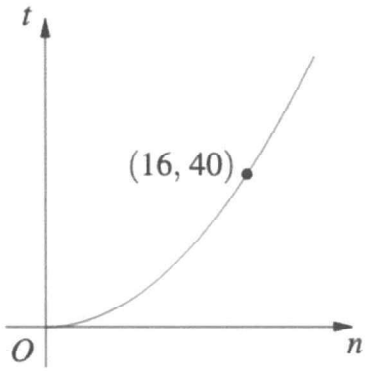
A.



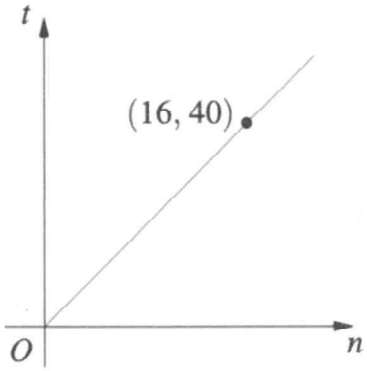
B.



C.



D.



Question 2.

In an arithmetic series, the first term is -2 and the third term is 8. What is the 10^{th} term?

A. 38

B. 41

C. 43

D. 45

$a = -2$

$T_3 = 8 = a + (n-1) \times d = -2 + (3-1) \times d$ so $d = 5$

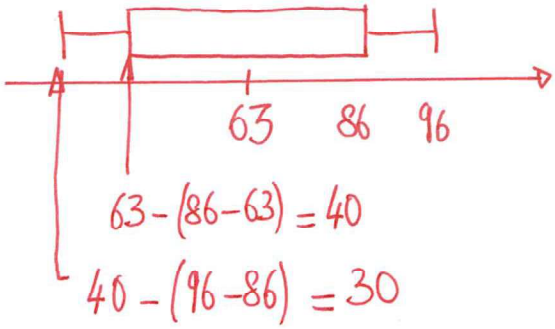
$T_{10} = -2 + (10-1) \times 5 = -2 + 45 = 43$

Question 3.

In a mathematics examination, 25% of students scored less than 40 marks, 75% of students scored less than 86 marks and the highest mark was 96.

Given that the distribution is symmetrical with no outliers, what is the lowest mark and the median of this data set?

	Lowest mark	Median
A	30	63
B	30	68
C	36	63
D	36	68



Question 4.

Let $f(x) = \sqrt{x}$ and $g(x) = \frac{1}{x-1}$

What is the domain and range of $f(g(x))$?

	Domain	Range
A	$[-1,1]$	$(-\infty, \infty)$
B	$(0, \infty)$	$(-\infty, 0) \cup (0, \infty)$
C	$[0, \infty)$	$(0, \infty)$
D	$(1, \infty)$	$(0, \infty)$

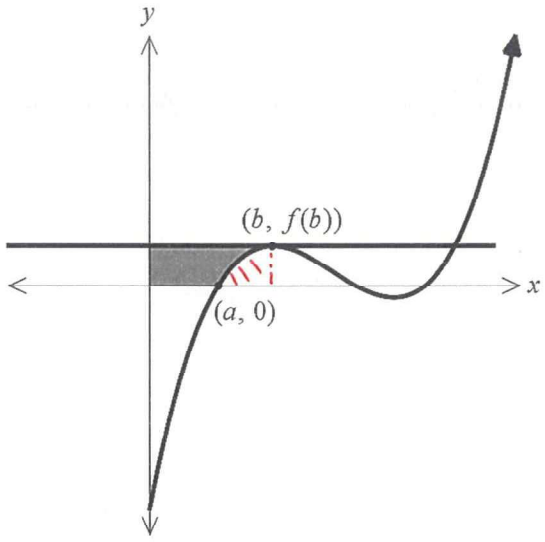
Domain for $g(x)$ is $\mathbb{R} - \{1\}$.

Domain for $f(g(x)) = \frac{1}{\sqrt{x-1}}$ is $x > 1$, i.e. $(1, +\infty)$

For this domain, the range of $f(g(x))$ is indeed $(0, +\infty)$

Question 5.

The graph of $f(x)$ is shown below. The horizontal line is a tangent to the local maximum $(b, f(b))$. The shaded area is bounded by the graph of $f(x)$, the coordinate axes and the horizontal line as shown below.



Which is the correct expression for the area of the shaded region?

A. $a\,f(b) - \int_a^b f(x)\,dx$

B. $a\,f(b) + \int_a^b f(x)\,dx$

C. $b\,f(b) - \int_a^b f(x)\,dx$

D. $-b\,f(b) + \int_0^b f(x)\,dx$

Question 6.

The limiting sum of a geometric series is $\frac{10}{9}$ and the sum of its first three terms is $\frac{26}{25}$. What is the common ratio of this series?

A. 1.39

B. 1.25

C. 0.40

D. 0.25

$S_\infty = \frac{a}{1-r} = \frac{10}{9}$

$S_3 = \frac{26}{25} = \frac{a(1-r^3)}{1-r} = \frac{10}{9} \times (1-r^3)$

So $1-r^3 = \frac{9 \times 26}{10 \times 25} = \frac{117}{125}$

$\therefore r^3 = \frac{8}{125}$ so $r = \frac{2}{5} = 0.4$

Question 7.

The normal to the graph of $f(x) = \ln\left(\frac{1}{x+2}\right)$ at point P has a gradient of 4.

What is the x-coordinate of point P?

A. $x = 2$

B. $x = -2$

C. $x = \frac{1}{2}$

D. $x = -\frac{1}{2}$

$y = \ln\left(\frac{1}{x+2}\right) = \ln 1 - \ln(x+2) = -\ln(x+2)$

Hence $\frac{dy}{dx} = -\frac{1}{x+2}$. therefore the normal would have a gradient of $(x+2)$.

So if the gradient if 4, that means $x = 2$

Question 8.

What is $\int \frac{1}{(2x+1)^2}\,dx$?

A. $\frac{-1}{2(2x+1)} + C$

B. $\frac{-1}{4(2x+1)} + C$

C. $\frac{-1}{3(2x+1)^3} + C$

D. $\frac{-1}{6(2x+1)^3} + C$

$\int \frac{dx}{(2x+1)^2} = \int (2x+1)^{-2}\,dx = \left[\frac{(2x+1)^{-2+1}}{(-2+1)} \times \frac{1}{2}\right] + C$

$= -\frac{1}{2(2x+1)^{-1}} + C$

Question 9.

In a bag of n coloured discs, exactly 7 are red.

Two discs are selected at random, without replacement.

The probability that both discs are red is $\frac{21}{55}$.

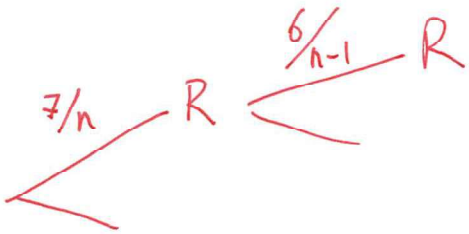
Which quadratic equation could be used to find the value of n ?

A. $n^2 + n + 110 = 0$

B. $n^2 - n - 110 = 0$

C. $n^2 + n - 110 = 0$

D. $n^2 - n + 110 = 0$



$\frac{7}{n} \times \frac{6}{n-1} = \frac{21}{55}$

$\therefore 7 \times 6 \times 55 = 21\,n(n-1)$

$\Leftrightarrow \quad 110 = n(n-1)$

$\Leftrightarrow \quad n^2 - n - 110 = 0$

Question 10.

Consider the equation $\left(\sin\theta - \frac{\sqrt{2}}{2}\right)\left(\tan 2\theta - \frac{\sqrt{3}}{3}\right) = 0$

How many solutions are there in the interval $0 \leq \theta \leq \pi$?

A. 1

B. 2

C. 3

D. 4

either $\sin\theta = \frac{\sqrt{2}}{2}$, i.e. $\theta = \frac{\pi}{4}$ or $\frac{3\pi}{4}$

OR $\tan 2\theta = \frac{\sqrt{3}}{3} = \frac{1}{\sqrt{3}}$ i.e. $2\theta = \frac{\pi}{6}$ or $2\theta = \frac{\pi}{6} + \pi$

$\theta = \frac{\pi}{12}$ or $\theta = \frac{7\pi}{12}$

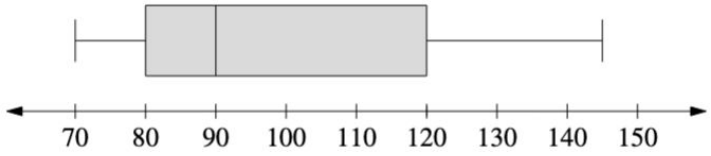
But we note that $\tan 2\theta$ is not defined for $\frac{\pi}{4}$ and for $\frac{3\pi}{4}$, so these two solutions are not possible.

Hence only 2 solutions in $[0, \pi]$, $\frac{\pi}{12}$ and $\frac{7\pi}{12}$.

Question 11 (2 marks)

A group of 680 students did as many starjumps as they could in 5 minutes.

The results are recorded in the box-plot below



(a) Describe the shape of the distribution. **1**

positively skewed.

(b) Find the number of students who can do more than 120 starjumps. **1**

About 25% of students are in the whisker 120 - 145
25% of 680 is 170 students.

Question 12 (2 marks)

The equation of a circle is given by $x^2 + 2x + y^2 - 4y = 0$.

Find the centre and radius of the circle.

$\Leftrightarrow (x+1)^2 - 1 + (y-2)^2 - 4 = 0$

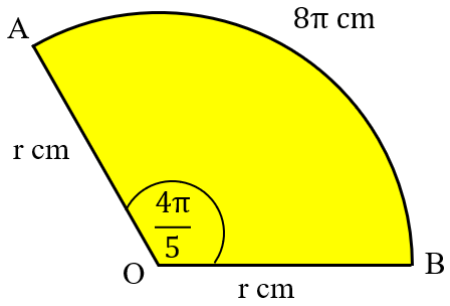
$\Leftrightarrow (x+1)^2 + (y-2)^2 = 5$

So circle of centre $(-1, 2)$, and radius $\sqrt{5}$

Question 13 (2 marks)

In the diagram, AOB is a sector of a circle with centre at O and radius r cm.

The angle AOB at the centre O is $\frac{4\pi}{5}$ and the length of arc AB is 8π cm.



Find the exact area of the sector.

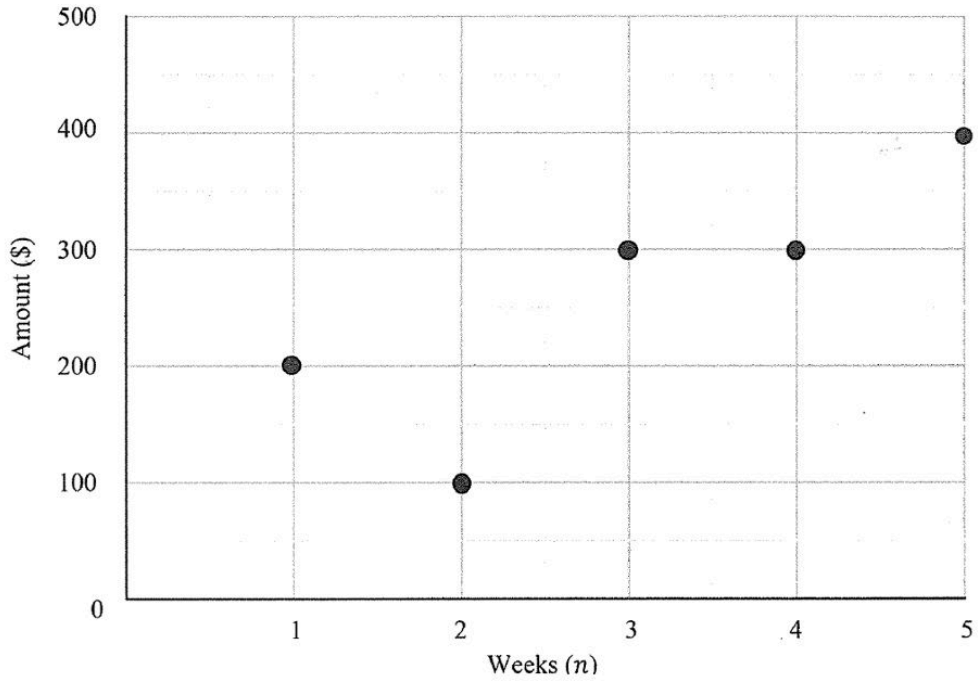
$8\pi = r \times \frac{4\pi}{5}$ (from $l = r\theta$)

So $r = \frac{5 \times 8}{4} = 10$ cm.

$\therefore$ Area $= \frac{1}{2} r^2\theta = \frac{1}{2} \times 10^2 \times \frac{4\pi}{5} = 40\pi$ cm^2

Question 14 (4 marks)

Ivy has plotted the income that she has received each week for the past five weeks on the graph below.



(a) Over the same five weeks, Ivy's expenses which were originally \$200 per week, increase by \$20 each week. Write the equation for Ivy's expenses. **1**

Expenses = 20 w + 200

(b) Write the equation of the least-squares regression line for Ivy's income. **2**

From the calculator, we get Income = 60w + 80

(c) Find the intersection of the lines from Parts (a) and (b). **1**

20w + 200 = 60w + 80

<=> 40w = 120 so w = 3

For w = 3 Expenses = Income = 260

Intersection is (3, 260)

Question 15 (3 marks)

Let $f(x) = x^2 + kx - 6$. Find the values of k for which the two roots of $f(x)$ have a difference of 7.

$\Delta = b^2 - 4ac = k^2 - 4 \times 1 \times (-6) = k^2 + 24$

Two roots are $x_1 = \frac{-b + \sqrt{\Delta}}{2a} = \frac{-k + \sqrt{\Delta}}{2}$

and $x_2 = \frac{-b - \sqrt{\Delta}}{2a} = \frac{-k - \sqrt{\Delta}}{2}$

Hence the difference of two roots is $\frac{\sqrt{\Delta}}{2} \times 2 = \sqrt{\Delta}$

$\therefore \sqrt{k^2+24} = 7$

So $k^2 + 24 = 49$

$k^2 = 25$

$k = \pm 5$

For these values, the difference of roots is 7

Question 16 (3 marks)

Consider the function $f(x) = (x+1)^2$

The following transformations were applied to the graph of $y = f(x)$ in the order shown below.

1) Horizontal translation to the right by 4 units
2) Horizontal dilation by a factor of $\frac{4}{3}$
3) Vertical dilation by a factor of $\frac{1}{3}$
4) Vertical translation down by 3 units
5) Reflection in the x-axis

Find the equation of the transformed function.

1) $f(x) = [(x-4)+1]^2 = [x-3]^2$

2) $f(x) = \left[\frac{3}{4}x - 3\right]^2$

3) $f(x) = \frac{1}{3}\left[\frac{3}{4}x - 3\right]^2$

4) $f(x) = \frac{1}{3}\left[\frac{3}{4}x - 3\right]^2 - 3$

5) $f(x) = -\left[\frac{1}{3}\left(\frac{3}{4}x - 3\right)^2 - 3\right]$

End of Part A.

Turn the page for extra space

Question 17 (5 marks)

On any given school day, the discrete random variable X represents the number of students who leave early due to illness. The probability distribution is shown below:

x	0	1	2	3	4	5
$P(X=x)$	p	0.25	0.25	0.13	0.07	0.04

(a) Find the value of p. **1**

$p + 0.25 + 0.25 + 0.13 + 0.07 + 0.04 = 1$

$\therefore \quad p = 0.26$

(b) Find the number of students, $E(X)$, who are expected to leave early due to illness on any given school day. **2**

$E(X) = 0 \times 0.26 + 1 \times 0.25 + 2 \times 0.25 + 3 \times 0.13 + 4 \times 0.07 + 5 \times 0.04$

$E(X) = 1.62$

1 mark for E(X) formula
1 mark for result

(c) It is known that at least one student left school early due to illness on a particular day. Find the probability that four students left early on that day. **2**

$P(4 \text{ leave early} \mid \text{at least 1 leave early}) = \dfrac{P(4 \text{ leave early AND at least one leave early})}{P(\text{at least one leave early})}$

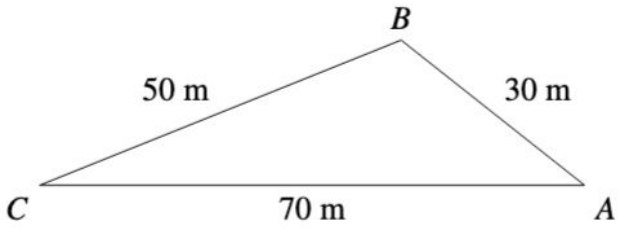
$= \dfrac{P(4 \text{ leave early})}{P(\text{at least one leave early})}$

$= \dfrac{0.07}{0.25 + 0.25 + 0.13 + 0.07 + 0.04} = \dfrac{0.07}{0.74} = \dfrac{7}{74} \approx 0.0946$

1 mark for conditional probability formula
1 mark for substituting and result

Question 18 (6 marks)

A triangular plot of land, ABC, is enclosed by fences measuring 30 metres, 50 metres and 70 metres as shown.



(a) Find the size of $\angle ABC$, correct to the nearest degree. **2**

$\cos \hat{B} = \dfrac{30^2 + 50^2 - 70^2}{2 \times 30 \times 50} = -0.5$

$\therefore \hat{B} = 120^\circ$

1 mark for cosine rule
1 mark for correct result

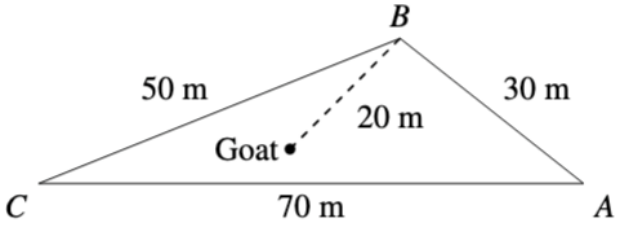
(b) Find the area of the plot of land, correct the nearest square metre. **2**

$\text{Area} = \dfrac{1}{2} ab \sin C = \dfrac{1}{2} \times 50 \times 30 \times \sin 120$

$\text{Area} = 750 \times \dfrac{\sqrt{3}}{2} \approx 650 \text{ m}^2$

1 mark for formula
1 mark for correct result

(c) A goat is tethered to the fence post at point B using a 20 metre long rope. **2**



Determine whether the rope is long enough for the goat to reach the fence AC.

If h=20, the area of the triangle would be $\frac{70 \times 20}{2} = 700\ m^2$.
which is more than the actual area of the triangle (calculated at b) to be $\approx 650\ m^2$)
Hence the goat can reach the fence, as h must be less than 20m

1 mark for calculation of area if h=20
1 mark for conclusion

Question 19 (3 marks)

Consider the simultaneous equations

$$\begin{cases} \log_{10}(xy) = 2 \\ \log_{10}\left(\dfrac{x}{y}\right) = 1 \end{cases} \qquad \text{where } x, y > 0.$$

Find the values of x and y which satisfy both equations.

$\Leftrightarrow \begin{cases} \log_{10} x + \log_{10} y = 2 \\ \log_{10} x - \log_{10} y = 1 \end{cases}$

let $X = \log_{10} x$ and $Y = \log_{10} y$

$\Leftrightarrow \begin{cases} X + Y = 2 \\ X - Y = 1 \end{cases} \Leftrightarrow 2X = 3 \ (\text{i.e. } X = 1.5)$ and $Y = 0.5$

So $\log_{10} x = 1.5$ (i.e. $x = 10^{1.5} = 10\sqrt{10}$)
and $\log_{10} y = 0.5 \Rightarrow y = \sqrt{10}$

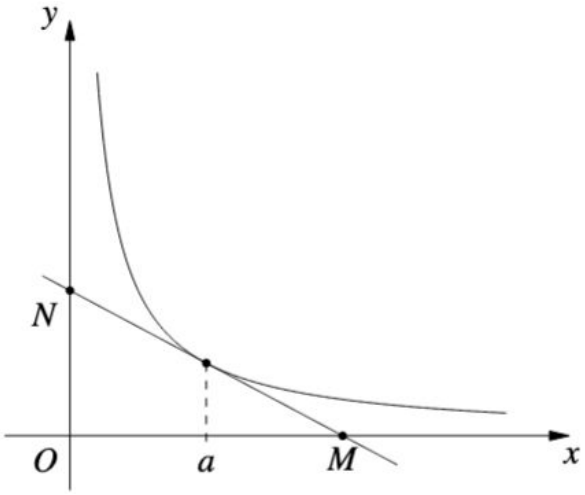
1 mark for applying log laws to modify equations
1 mark for change of variables and solving simultaneous equations
1 mark for finding values of x and y

End of Part B.

Extra writing space

Question 20 (3 marks)

The diagram shows the curve of $y = \frac{1}{x}$ where $x > 0$ and the tangent to the curve at the point where $x = a$. The tangent meets the x-axis at M and the y-axis at N.



Show that the area of the triangle OMN is independent of a.

The slope of the tangent is $\frac{dy}{dx} = -\frac{1}{x^2}$.

At $x = a$, $\frac{dy}{dx} = -\frac{1}{a^2}$, and it goes through $\left(a, \frac{1}{a}\right)$

So the equation of the tangent is $y - \frac{1}{a} = -\frac{1}{a^2}(x - a)$

$\Leftrightarrow y = -\frac{x}{a^2} + \frac{1}{a} + \frac{1}{a}$, i.e. $y = -\frac{x}{a^2} + \frac{2}{a}$

When $x = 0$ $y = 2/a$

When $y = 0$ $\frac{x}{a^2} = \frac{2}{a}$ so $x = 2a$.

The area of the triangle is therefore:

$A = 2a \times \frac{2}{a} \times \frac{1}{2} = 2$ units^2.

1 mark for finding equation of tangent
1 mark for finding axes intercepts
1 mark for area and conclusion

Question 21 (10 marks)

Consider the function $f(x) = -6x^2 + x^4$

(a) Show that $f(x)$ is an even function. **1**

$f(-x) = -6(-x)^2 + (-x)^4 = -6x^2 + x^4 = f(x)$

Hence $f(x)$ is an even function, as for all x $f(-x) = f(x)$.

(b) Find the coordinates of the x-intercepts. **1**

x intercepts are when $y = 0$, i.e. $-6x^2 + x^4 = 0$

$\Leftrightarrow x^2(x^2 - 6) = 0 \Leftrightarrow x^2(x - \sqrt{6})(x + \sqrt{6}) = 0$

Three x-intercepts at $(0,0)$, $(-\sqrt{6}, 0)$ and $(\sqrt{6}, 0)$.

(c) Find the stationary points and determine their nature. **3**

$\frac{dy}{dx} = -12x + 4x^3 = 4x(x^2 - 3) = 4x(x - \sqrt{3})(x + \sqrt{3})$

Stationary points at $x = 0$ and $\pm\sqrt{3}$

For $x = 0$, $f(0) = -6 \times 0^2 + 0^4 = 0$ so $(0,0)$

For $x = \sqrt{3}$, $f(\sqrt{3}) = -6(\sqrt{3})^2 + (\sqrt{3})^4 = -9$

So stationary point at $(\sqrt{3}, -9)$ and, as f is even, also at $(-\sqrt{3}, -9)$

To check whether these points are minimum or maximum we calculate the 2nd derivative.

$\frac{d^2y}{dx^2} = -12 + 12x^2 = 12(x^2 - 1) = 12(x + 1)(x - 1)$

at $\sqrt{3}$, $d^2y/dx^2 > 0$ so $(\sqrt{3}, -9)$ is a minimum, and so is $(-\sqrt{3}, -9)$

at 0, $d^2y/dx^2 < 0$ so $(0,0)$ is a maximum.

- 4 -

1 mark for derivative
1 mark for stationary points
1 mark for discussion on min or max

(a) Show that there are two inflection points, and find the gradient at those points. **3**

$\frac{d^2y}{dx^2} = 12(x+1)(x-1)$ so two inflection points at $x = \pm 1$

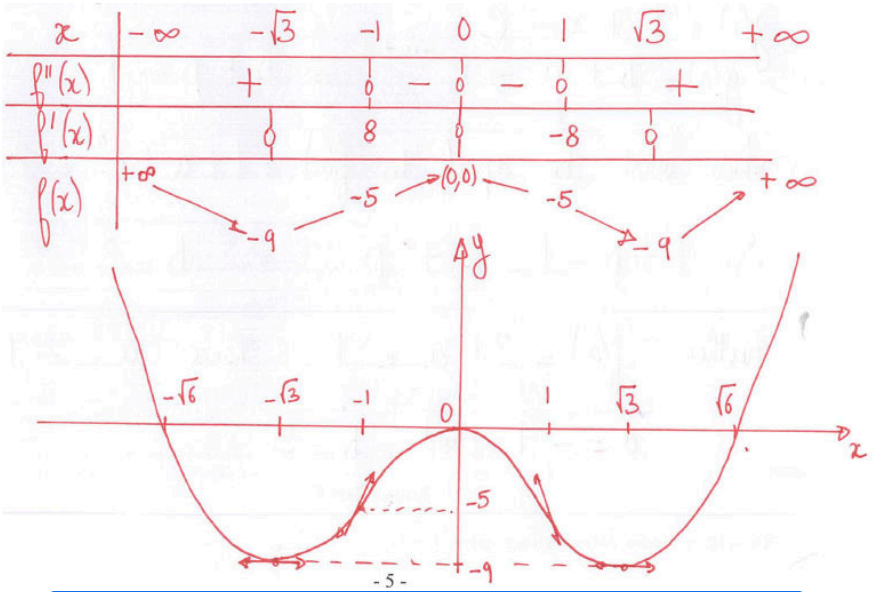
when $x = 1$, $\frac{dy}{dx} = 4 \times 1(1^2 - 3) = -8$ and $f(1) = -6 + 1 = -5$

So inflection point at $(1, -5)$, where the gradient is -8.

And as f is even, inflection point at $(-1, -5)$ where the gradient is 8.

1 mark for 2nd derivative
1 mark for inflection points
1 mark for gradient at inflection points

(b) Sketch the curve, showing stationary points, inflection points, slopes at all those points, and intercepts with the two axes. **2**

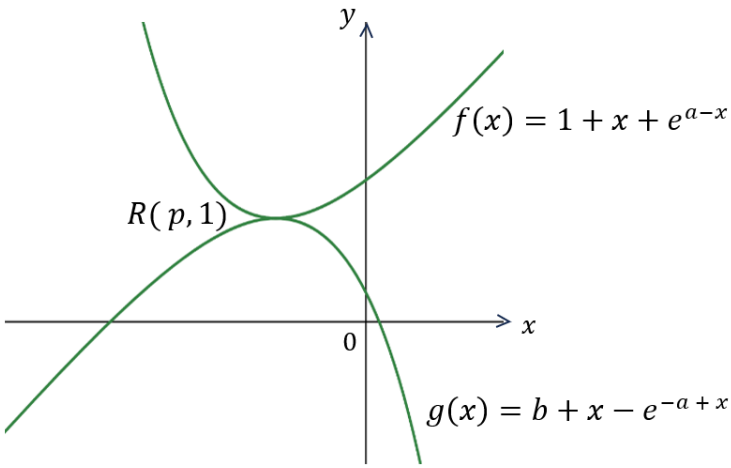


1 mark for correct curve
1 mark for stationary points, intercepts and gradients

Question 22 (4 marks)

The curves of the functions $f(x) = 1 + x + e^{a-x}$ and $g(x) = b + x - e^{-a+x}$ have the same turning point at the point $R(p, 1)$ as shown on the diagram.

Find the coordinates of the point R and the values of a and b.



$f(x) = 1 + x + e^{a-x}$ $\quad f'(x) = 1 - e^{a-x}$

$f'(x) = 0$ when $e^{a-x} = 1$ so $a - x = 0$ $\quad x = a$.

So there's a turning point at $x = a$, hence $p = a$

$g(x) = b + x - e^{-a+x}$

$g(a) = b + a - e^{-a+a} = b + p - 1$

which must be equal to $f(a) = 1 + a + e^{a-a} = 2 + a$.

So $b + p - 1 = 2 + p$. $\quad \therefore b = 3$

Further $f(a) = 2 + a = 1$ $\quad$ hence $a = -1$

$a = -1$ and $b = 3$

1 mark for derivative of f and turning point
1 mark for minimum of g
1 mark for finding b
1 mark for finding a

End of Part C.

Turn the page for extra writing space

Question 23 (3 marks)

Find $\int \sec^2 x\sqrt{\tan(x)+5}\,dx$.

if $f(x) = \tan x + 5$ then $f'(x) = \sec^2 x$.

So $\int \sec^2 x \sqrt{\tan x + 5}\,dx = \int f'(x) \times [f(x)]^{1/2}\,dx$

Using the reverse chain rule results in:

$$\frac{[f(x)]^{\frac{1}{2}+1}}{\frac{1}{2}+1} + C = \frac{(\tan x + 5)^{3/2}}{\frac{3}{2}} + C = \frac{2}{3}(\tan x + 5)^{3/2} + C$$

1 mark for identify f and f'
1 mark for stating reverse chain rule
1 mark for correct result

Question 24 (5 marks)

The time a student takes to complete an online assignment is modelled by the continuous random variable T

The cumulative distribution function of T is given by:

$$F(t) = \begin{cases} \frac{1}{1728}t^3 & \text{when} \quad 0 \le t \le 12 \\ 1 & \text{when} \quad t > 12 \end{cases}$$, where t is in hours.

(a) Find, to the nearest hour, the time within which 60% of students finish the assignment. **1**

$F(t) = 0.6 = \frac{t^3}{1728}$

so $t^3 = 0.6 \times 1728 = 1036.8$

$t = \sqrt[3]{1036.8}$ which is approx 10 hours

(b) Define the probability density function of T, for all values of t. **1**

$F(t) = \int_0^t f(t)\,dt \quad \therefore \quad f(t) = \frac{dF(t)}{dt} = \frac{d}{dt}\left(\frac{t^3}{1728}\right)$

$\therefore f(t) = \frac{3t^2}{1728} = \frac{t^2}{576}$

(c) It is given that the expected value $E(T)$ of T is 9 and $E(T^2) = 86.4$ **(DO NOT PROVE THIS).**

Find the probability, to the nearest percent, that a student takes more than one standard deviation above the mean to complete the assignment. **3**

$Var(T) = E(T^2) - [E(t)]^2$

$Var(T) = 86.4 - 9^2 = 5.4$

$\therefore \sigma(t) = \sqrt{Var(T)} \simeq 2.32379$

$P(T > 9 + 2.32379) = P(T > 11.32379)$

$= 1 - F(11.32379)$

$= 1 - \frac{1}{1728} \times [11.32379]^3$

$\simeq 0.1597 \quad \therefore 16\%$ approx

1 mark for formula
1 mark for writing P(T>...)
1 mark for correct result

Question 25 (3 marks)

(a) Differentiate $f(x) = (\ln x)^2$. **1**

$f'(x) = 2 \times \ln x \times \frac{1}{x} = \frac{2 \ln x}{x} = \frac{\ln x^2}{x}$

(b) Hence, or otherwise, find $\int_1^e \frac{\ln(x^2)}{x}\, dx$. **2**

$\int_1^e \frac{\ln x^2}{x}\, dx = \left[(\ln x)^2\right]_1^e$

$= \left[(\ln e)^2 - (\ln 1)^2\right]$

$= 1^2 - 0^2 = 1$

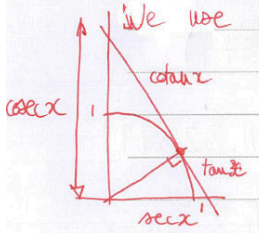
1 mark for using result from a)
1 mark for correct result

Question 26 (3 marks)

Prove that $\frac{sec^4x - tan^4x}{cosec^2x + 2\,tan^2x - cot^2x} = 1$.

$LHS = \frac{(sec^2x - tan^2x)(sec^2x + tan^2x)}{cosec^2x + 2\,tan^2x - cotan^2x}$

We use the Pythagorean relationships of the unit circle



$sec^2x = 1 + tan^2x$

$cosec^2x = 1 + cotan^2x$

$\therefore LHS = \frac{1 \times (1 + 2\,tan^2x)}{(1 + cotan^2x) + 2\,tan^2x - cotan^2x}$

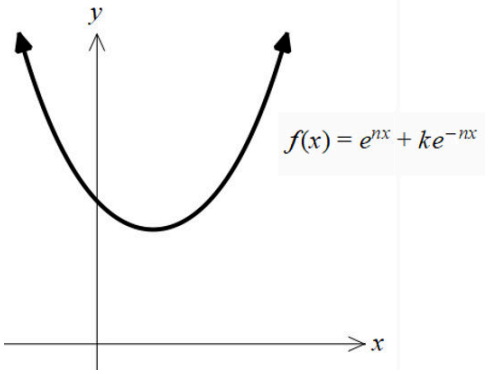
$LHS = \frac{1 + 2\,tan^2x}{1 + 2\,tan^2x} = 1$

1 mark for using Pythagorean relationships
2 marks for simplifying and result

End of Part D.

Question 27 (4 marks)

The diagram shows the graph of the function $f(x) = e^{nx} + k\,e^{-nx}$ where $k > 1, n > 0$



Find the range of $f(x)$ in simplest form.

To find the range, we need to find the minimum of $f(x)$

$f(x)$ is minimum when $f'(x) = 0$

$f(x) = e^{nx} + k\,e^{-nx}$ so $f'(x) = ne^{nx} - kne^{-nx}$

$f'(x) = 0$ when $ne^{nx} = kne^{-nx}$

i.e. $e^{nx} = \frac{k}{e^{nx}}$ or $e^{2nx} = k$

So $2nx = \ln k$ $\quad x = \frac{1}{2n}\ln k$.

For $x = \frac{\ln k}{2n}$, $f\left(\frac{\ln k}{2n}\right) = e^{n\left(\frac{\ln k}{2n}\right)} + k\,e^{-n\times\left(\frac{\ln k}{2n}\right)}$

$= e^{\frac{1}{2}\ln k} + \frac{k}{e^{\frac{1}{2}\ln k}}$

$= e^{\ln\sqrt{k}} + \frac{k}{e^{\ln\sqrt{k}}}$

$= \sqrt{k} + \frac{k}{\sqrt{k}} = 2\sqrt{k}$.

So the range is $[2\sqrt{k}, +\infty)$

1 mark for derivative
1 mark for finding minimum
1 mark for substituting
1 mark for conclusion

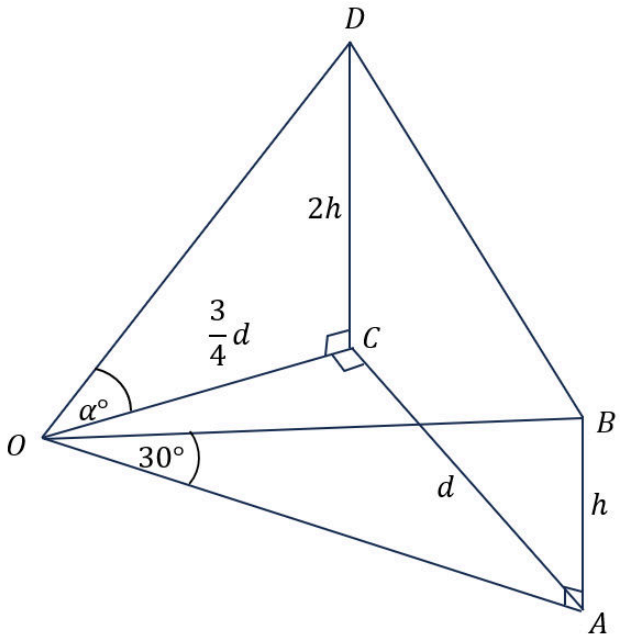
Question 28 (5 marks)

The diagram shows AB and CD which are two towers of height h and $2h$ metres respectively.

AB is south of tower CD, at a distance d metres from it.

O is a point on the ground, west of CD and at the same level as A and C.

The distance OC is $\frac{3}{4}d$ metres. The angle of elevation of B from O is $30°$ and the angle of elevation of D from O is α.



(a) Show that $h = \frac{3}{8}d\tan\alpha$. **1**

$\tan\alpha = \frac{\text{Opposite side}}{\text{Adjacent side}} = \frac{2h}{\frac{3}{4}d} = \frac{8h}{3d}$

$\therefore \quad h = \frac{3d\tan\alpha}{8}$

(b) By finding another expression for h, find the angle of elevation α to the nearest minute. **4**

From the other triangle ($\triangle OAB$), we get

$\tan 30 = \frac{h}{OA}$ $\therefore h = OA \tan 30 = OA \times \frac{1}{\sqrt{3}}$

Using Pythagoras' theorem, we obtain:

$OA^2 = d^2 + \left(\frac{3d}{4}\right)^2 = d^2\left[1 + \frac{9}{16}\right] = \frac{25}{16} d^2$

$\therefore OA = \frac{5}{4} d$.

Therefore $h = \frac{5}{4} d \times \frac{1}{\sqrt{3}} = \frac{5d}{4\sqrt{3}}$

So $\frac{5d}{4\sqrt{3}} = \frac{3d}{8} \tan\alpha$

Therefore $\tan\alpha = \frac{5 \times 8}{12\sqrt{3}} = \frac{10}{3\sqrt{3}}$

Hence $\alpha = \tan^{-1}\left(\frac{10}{3\sqrt{3}}\right) \approx 62°33'$

1 mark for finding other expression for h
1 mark for using Pythagoras
1 mark for progress in solving equation
1 mark for result

Question 29 (3 marks)

A cricket player hits a ball with an angle of elevation $\theta°$, where $\theta°$ is normally distributed with a mean of 30° and a standard deviation of 15°.

The horizontal distance travelled by the ball before hitting the ground is given by $d = 80 \sin 2\theta$, where d is in metres.

Find the probability that the ball travels more than 40 metres horizontally in the air before landing (give your answer to 2 decimal places).

$P(d>40) = P(80 \sin 2\theta > 40)$

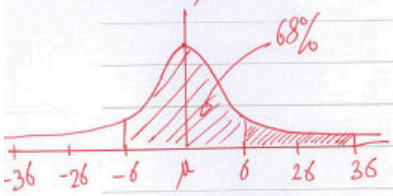
$= P(\sin 2\theta > 1/2)$

$= P(30 < 2\theta < 150)$

$= P(15 < \theta < 75)$

$= P(\mu - \sigma < \theta < \mu + 3\sigma)$

(as $\mu = 30°$ and $\sigma = 15°$).



$P(d>40) = P(\mu - \sigma < \theta < \mu + \sigma) + P(\mu + \sigma < \theta < \mu + 3\sigma)$

$P(d>40) = 0.68 + \left[\frac{0.997 - 0.68}{2}\right]$

$P(d>40) = 83.85\%$

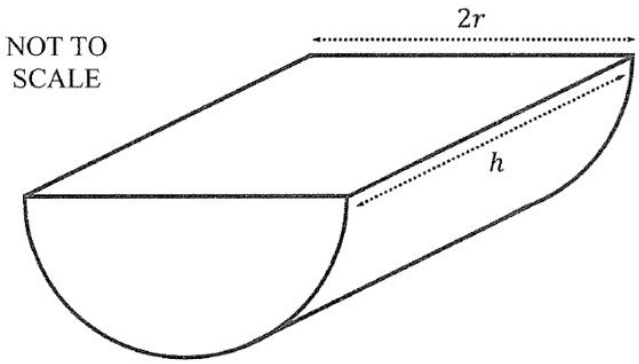
1 mark for P(15<theta<75)
1 mark for P(mu-sigma<theta<mu+3 sigma)
1 mark for solving

End of Part E.

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Question 30 (7 marks)

A closed water tank in the shape of a half-cylinder with diameter $2r$ metres and height h metres is to be made from sheet metal as shown below.



The surface area of the tank is A square metres and the volume of the tank is V cubic metres, and is constant.

(a) Given that $V = \frac{1}{2}\pi r^2 h$, show that $A = \pi r^2 + \frac{2V}{\pi}\left(\frac{2+\pi}{r}\right)$. **3**

$A = \underbrace{2 \times \frac{1}{2}\pi r^2}_{\text{semi-circles at both ends}} + \underbrace{2r \times h}_{\text{rectangle on top}} + \underbrace{h \times \frac{1}{2} \times 2\pi r}_{\text{curved cylindric part}}$

$A = \pi r^2 + 2rh + \pi r h$

$A = \pi r^2 + rh(2+\pi)$

But $V = \frac{1}{2}\pi r^2 h$ so $rh = \frac{2V}{\pi r}$

Hence $A = \pi r^2 + \frac{2V}{\pi r}(2+\pi)$

$A = \pi r^2 + \frac{2V}{\pi}\left(\frac{2+\pi}{r}\right)$

1 mark for showing 3 parts of surface area
1 mark for rearranging and then substituting,
1 mark for simplifying and result

(b) Show that the amount of sheet metal is minimised when $r = \sqrt[3]{\frac{V(2+\pi)}{\pi^2}}$. **4**

A is minimal or maximal when $\frac{dA}{dr} = 0$

$$\frac{dA}{dr} = 2\pi r + \frac{2V}{\pi}(2+\pi)\left(-\frac{1}{r^2}\right)$$

$$\frac{dA}{dr} = 2\pi r - \frac{2V(2+\pi)}{\pi r^2}$$

$$\frac{dA}{dr} = 0 \text{ when } 2\pi r = \frac{2V(2+\pi)}{\pi r^2}$$

$$\Leftrightarrow r^3 = \frac{V(2+\pi)}{\pi^2}$$

For this value of r, A is minimal or maximal.

But $\frac{d^2A}{dr^2} = 2\pi + \frac{2V(2+\pi)}{\pi} \times 2r^3$

which is always positive, so the curve A(r) is concave up, hence has a minimum.

A is minimum when $r = \sqrt[3]{\frac{V(2+\pi)}{\pi^2}}$

1 mark for derivative
1 mark for solving equation
2 marks for argumented discussion on minimum/maximum

Question 31 (10 marks)

Consider the function defined by the geometric series $f(x) = e^{-x} + e^{-2x} + e^{-3x} + \cdots$ for $x > 0$.

(a) Show that $f(x) = \frac{1}{e^x - 1}$. **2**

$f(x) = e^{-x}(1 + e^{-x} + e^{-2x} + \dots)$

as $x > 0$, $e^{-x} < 1$ hence the series converges.

$S_\infty = \frac{a}{1-r} = \frac{1}{1-e^{-x}} = \frac{1}{1-\frac{1}{e^x}} = \frac{e^x}{e^x - 1}$

So $f(x) = e^{-x} \times \frac{e^x}{e^x - 1} = \frac{1}{e^x - 1}$

(b) Show that $f'(x) = \frac{-e^x}{(e^x-1)^2}$. **1**

$f(x) = \frac{1}{e^x - 1} = (e^x - 1)^{-1}$

So $f'(x) = -\frac{1}{(e^x-1)^2} \times e^x = -\frac{e^x}{(e^x-1)^2}$

Note that f' is negative, hence f decreases.

(c) It is known that $f''(x) = \frac{e^x(e^x+1)}{(e^x-1)^3}$. Explain why $f''(x) > 0$. **1**

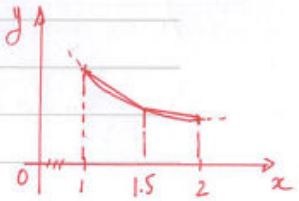
as $e^x > 1$ then $e^x - 1 > 0$ so $(e^x-1)^3 > 0$ (denominator positive)
and also $(e^x+1) > 2$ so the numerator is also positive, hence $f''(x) > 0$. f is concave up.

(d) Use the Trapezoidal rule with three function values to approximate $\int_1^2 f(x)\,dx$, correct to three decimal places. **2**

$$\int_1^2 f(x)\,dx \simeq \frac{2-1}{2\times 2}\left[f(1) + f(2) + 2 \times f(1.5)\right] \quad \text{(Trapezoidal rule)}$$

$$\int_1^2 f(x)\,dx \simeq \frac{1}{4}\left[\frac{1}{e^1-1} + \frac{1}{e^2-1} + 2 \times \frac{1}{e^{1.5}-1}\right]$$

$$\int_1^2 f(x)\,dx \simeq 0.328 \text{ approx}$$



As f decreases and f'' is concave up, this value is an overestimate.

1 mark for formula
1 mark for substituting and result

(e) Find the exact value of $\int_1^2 f(x)\,dx$. **2**

$$\int_1^2 f(x)\,dx = \int_1^2 \frac{1}{e^x-1}\,dx = \int_1^2 \frac{e^{-x}}{1-e^{-x}}\,dx$$

$$= \left[\ln\left(1-e^{-x}\right)\right]_1^2 = \ln\left(1-e^{-2}\right) - \ln\left(1-e^{-1}\right)$$

$$= \ln\left(\frac{1-e^{-2}}{1-e^{-1}}\right) = \ln\left(\frac{e^2-1}{e^2-e}\right) = \ln\left[\frac{(e-1)(e+1)}{e(e-1)}\right]$$

$$= \ln\left(\frac{e+1}{e}\right) = \ln(e+1) - \ln e = \ln(e+1) - 1$$

1 mark for progress
1 mark for result

(f) Using your answers from parts (c) and (d), show that $\ln(e+1) < \frac{4}{3}$. **2**

$\int_1^2 f(x)\,dx = \ln(e+1) - 1$

Further, we have shown that $\int_1^2 f(x)\,dx \approx 0.328$

which we have shown to be an overestimate.

Hence, as $\frac{1}{3} = 0.\dot{3}$, we can state that

$\int_1^2 f(x)\,dx < \frac{1}{3}$

So $\ln(e+1) - 1 < \frac{1}{3}$

$\therefore \ln(e+1) < \frac{4}{3}$

END OF EXAMINATION

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