Girraween High School

2026

TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Advanced

General Instructions

- Reading time: 10 minutes
- Working time: 3 Hours
- Write using a black pen
- NESA approved calculators may be used
- Laminated reference sheets are provided
- Answer multiple-choice questions by completely colouring in the appropriate circle on your multiple-choice answer sheet.
- Answer questions on the lines provided and show all relevant mathematical reasoning and/or calculations.

Total Marks: 100

Section 1 (Pages 1 – 5) **10 Marks**

- Attempt Q1 - Q10
- Allow about 15 minutes for this section

Section 2 (Pages 6-26) **90 marks**

- Attempt Q11 – Q32
- Allow about 2 hours and 45 minutes for this section

Section I

10 marks
Attempt Questions 1– 10
Allow about 15 minutes for this section
Use the multiple-choice answer sheet for Questions 1–10

Question 1

The solution to $|x - 3| = 6$ is

A) $x = -3, 9$

B) $x = 3, -9$

C) $x = -3, -9$

D) $x = 3, 9$

Question 2

What is the gradient of the tangent to the graph of $y = 2\sin \pi x$ at $x = \frac{1}{3}$.

A) $\frac{\pi}{2}$

B) π

C) $\sqrt{3}\pi$

D) 2π

Examination continues next page…

Question 3

Which expression is equivalent to $\tan\theta + \cot\theta$?

A) $\operatorname{cosec}\theta + \sec\theta$

B) $\sec\theta \operatorname{cosec}\theta$

C) 2

D) 1

Question 4

The graph of the function $f(x) = \dfrac{3x+2}{5-x}$, has asymptotes at:

A) $x = -5, y = \dfrac{3}{2}$

B) $x = \dfrac{2}{3}, y = -3$

C) $x = 5, y = 3$

D) $x = 5, y = -3$

Question 5

$$\lim_{x \to 0} \frac{\sin\frac{x}{4}}{x} =$$

A) 1

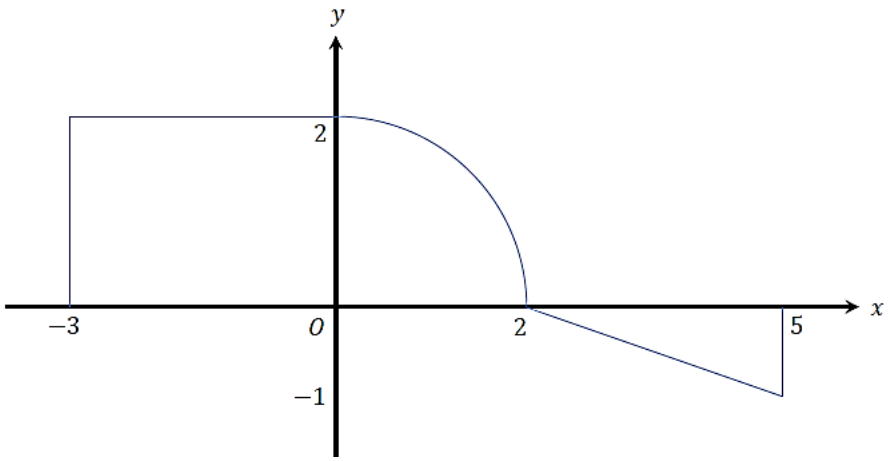
B) 4

C) $\dfrac{1}{4}$

D) 0

Examination continues next page…

Question 6



What is the exact value of $\int_{-3}^{5} f(x)\,dx$.

A. $\pi + \frac{9}{2}$

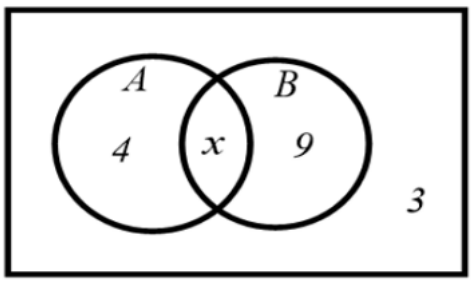
B. $\pi + \frac{15}{2}$

C. $\pi - \frac{15}{2}$

D. $\pi - \frac{9}{2}$

Question 7

In the Venn diagram below, events A and B are independent.



What is the value of x?

A) 6

B) 8

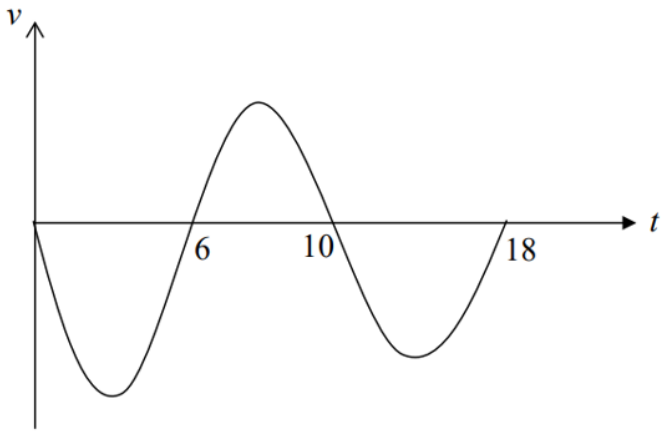
C) 12

D) 15

Examination continues next page…

Question 8

A particle moves along a straight line. Its velocity v at time t is shown in the graph below.



Find what value of t is the displacement of the particle a maximum?

A) 18

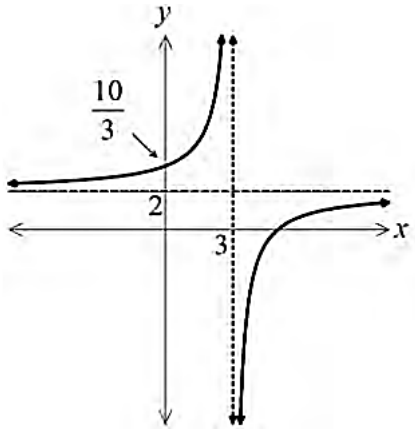
B) 10

C) 8

D) 6

Question 9

Consider the graph of $y = \frac{k}{x+b} + c$ below, with y- intercept at $\left(0, \frac{10}{3}\right)$.



Which of the following is true?

A) $k < b < c$

B) $b < k < c$

C) $k < c < b$

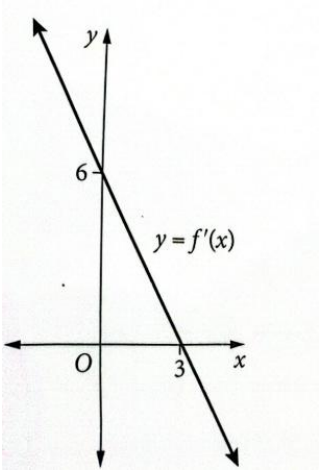
D) $c < k < b$

Examination continues next page…

Question 10

The graph of $y = f'(x)$ is shown.

What is the equation of $y = f(x)$, if it has a maximum value of 16?



A) $y = x^2 - 6x + 16$

B) $y = x^2 - 6x - 11$

C) $y = -x^2 + 6x + 7$

D) $y = -x^2 + 6x + 25$

Examination continues next page…

Section II

90 marks
Attempt Questions 11–32
Allow about 2 hours and 45 minutes for this section

- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of the response.
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided at the end of this booklet. If you use this space, clearly indicate which question you are answering.

Question 11 (2 marks)

Differentiate: $y = e^{2x}(x + 2)$. 2

Question 12 (2 marks)

Find $\int \sqrt{4x + 1}\,dx$. 2

Examination continues next page…

Question 13 (2 marks)

For the function $f(x) = x^3 - 9x$, state the domain where $f(x)$ is increasing. 2

Question 14 (2 marks)

Find the exact value of $\int_1^2 \frac{3}{4x - 1}\, dx$ 2

Examination continues next page…

Question 15 (3 marks)

i. Differentiate $\frac{14}{7x^2 + 2}$ 1

ii. Hence, find $\int \frac{14x}{(7x^2 + 2)^2}\, dx$. 2

Question 16 (3 marks)

Prove that $\sec^2 x + \sec x \tan x = \frac{1}{1 - \sin x}$. 3

Examination continues next page…

Question 17 (2 marks)

Consider the function $f(x) = e^x$ and $g(x) = \ln(x - 2)$.

i. Find the composite function $f(g(x))$. 1

ii. Find in interval notation the range of $f(g(x))$. 1

Question 18 (2 marks)

The function below is a probability density function for the given interval:

$$f(x) = \begin{cases} ax^2, & 0 \leq x \leq 6 \\ 0, & \text{otherwise.} \end{cases}$$

Find the value of a. 2

Examination continues next page…

Question 19 (3 marks)

Bob plays archery competitively as part of a team. The number of attempts(X) that he needs to hit the target is given by the probability distribution table:

x	1	2	3	4	5
$P(X = x)$	0.12	0.35	0.32	0.17	0.04

i. Find the expected number of attempts Bob needs to hit the target. 1

ii. The team's coach would like to select someone from the team to be captain. Another archer in the team, Karen, has the same expected number of attempts as Bob, and a standard deviation of 1.2. If the coach bases his decision on performance, are they more likely to choose Bob or Karen? Justify your answer with appropriate statistical reasoning. 2

Examination continues next page…

Question 20 (4 marks)

Tom did a survey of a group of people he knew about their age and how much they earn each week. The results are shown in the table below.

Age (years) (x)	18	45	28	15	32	68
Wage (\$/week) (W)	715	2350	1530	438	1690	1320

i. Using your calculator, find the correlation coefficient(r), and explain the strength of the correlation this data gives. 2

ii. Using your calculator, find the equation of the least-squares regression line in the form $W = Bx + A$, where A and B are integers. 1

iii. Could your equation from part(ii) be used to make valid estimate for ages greater than 68 or less than 15 years? Justify your answer with reasons. 1

Examination continues next page…

Question 21 (6 marks)

In Alex's first year of fulltime employment his annual salary is \$75 000. At the beginning of the second year his salary increases to \$79 000. His salary continues to increase by \$4000 at the beginning of each successive year.

i. What will Alex's salary be at the beginning of his 25^{th} year? 1

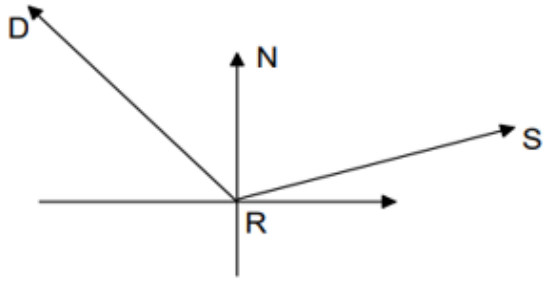
ii. Calculate Alex's total earnings after working for 25 years? 2

iii. During which year of employment will his total earnings first exceed \$2 000 000? 3

Examination continues next page…

Question 22 (5 marks)

David and Sarah are pilots of two light planes which leave Rodeo Pilot Station at the same time. David flies on a bearing of 330°T at a speed of 180 km/h and Sarah flies on a bearing of 080°T at a speed of 240 km/h.



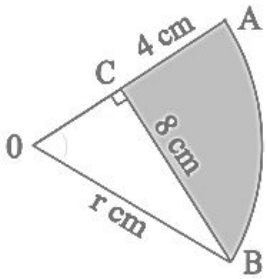
i. How far apart are David and Sarah after 2 hours? 2

ii. What is the bearing of Sarah from David after 2 hours? (to the nearest degree.) 3

Examination continues next page…

Question 23 (4 marks)

The diagram shows the sector OAB with centre O and radius r. The perpendicular from B meets the radius OA at C. $BC = 8$ cm and $AC = 4$ cm.



NOT TO SCALE

i. Show that the radius r of the circle is 10 cm. 2

ii. Find the area of the shaded region in this sector. 2

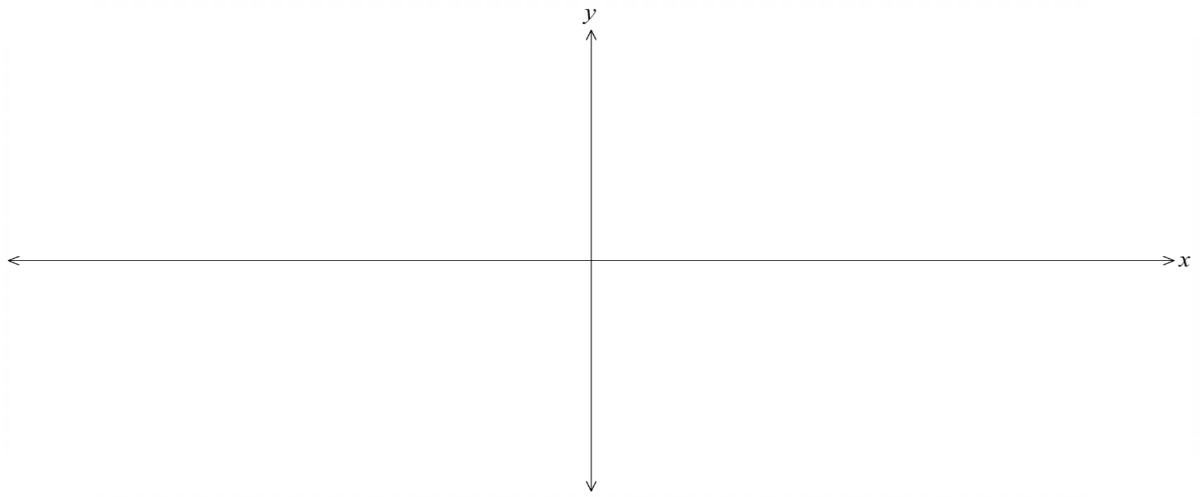
Question 24 (7 marks)

i. Solve the equation $1 - 2\cos x = 0$ for $0 \leq x \leq 2\pi$. 2

Examination continues next page…

Question 24 continued

ii. Sketch the graph of the curve $y = 1 - 2\cos x$ for $0 \leq x \leq 2\pi$, showing clearly the coordinates of the endpoints and the maximum turning point. 2



iii. Find in simplest exact form the area of the region bounded by the curve $y = 1 - 2\cos x$ and the x-axis between $x = 0$ and $x = \pi$. 3

Examination continues next page

Question 25 (6 marks)

In a laboratory, a food product is heated in an oven set at $210°C$ to kill harmful bacteria. The number of bacteria recorded when the food product is first placed in the oven is 1800. After 10 minutes the number of bacteria recorded is 550.

It is found that the number N, of bacteria remaining t minutes after being in the

oven is given by $N = N_0\, e^{-kt}$, where N_0 and $k$are constants.

i. Calculate the value of k correct to 4 decimal places. 3

ii. If an acceptable number of bacteria present is 100 or less, for how long should the food product be in the oven? Express your answer to the nearest minute. 3

Examination continues next page…

Question 26 (7 marks)

The velocity of an object is given by the equation $v = 6t - 5 - t^2$ where t is in seconds and velocity v in m/s. It begins its motion at $x = 5$ metres.

i. Find an equation for the displacement of the object. 2

ii. At what two times is the object stationary? 2

iii. Find the distance travelled by the object in first 3 seconds. 3

Examination continues next page…

Question 27 (4 marks)

Whilst performing some experiments, researchers have found that the rate of change of the distance (in cm/s) travelled by a particle is given by $A(t) = 40 \ln(1 + 5t)$, where t is the number of seconds elapsed since the start of the research.

Let D cm be the distance travelled by the particle from $t = 1$ to $t = 5$.

i. Estimate D by using the trapezoidal rule with 4 sub-intervals.
Give answer to the nearest centimetre. 2

ii. Is D an over-estimate or an under-estimate of the exact distance?

Justify your answer. 2

Examination continues next page…

Question 28 (4 marks)

A tennis ball is dropped from a height of 12 metres. After it first hits the ground, it rebounds to a height of 8 metres. It continues to rebound to a height of two thirds its previous rebound height.

i. How high will the ball rise after the 6^{th} time it rebounds from the ground? 2

ii. What is the total vertical distance that the ball will eventually travel? 2

Examination continues next page…

Question 29 (5 marks)

A machine produces metal rods used to reinforce concrete slabs. The diameters of the rods are normally distributed with a mean of 8 mm and a standard deviation of 0.3 mm.

i. Find the diameter of the rod with a z-score of 1.2. 1

ii. If a rod is selected at random, find the probability that the diameter of the rod is less than 7.7 mm. 2

iii. If two rods are selected at random, find the probability that exactly one of the rods will have a diameter that is less than 7.7 mm. 2

Examination continues next page…

Question 30 (6 marks)

a. A table of future value interest factors for an annuity of $1 is shown

Future value of $1						
Period	**1%**	**4%**	**8%**	**12%**	**16%**	**20%**
1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	2.0100	2.0400	2.0800	2.1200	2.1600	2.2000
3	3.0301	3.1216	3.2464	3.3744	3.5056	3.6400
4	4.0604	4.2465	4.5061	4.7793	5.0665	5.3680
5	5.1010	5.4163	5.8666	6.3528	6.8771	7.4416
6	6.1520	6.6330	7.3359	8.1152	8.9775	9.9299

Annie wants to save $ 10 000 over the next 2 years by investing equal payments every 6 months into an account that earns interest 8% p.a. compounded half-yearly.

What should be the value of each payment? 1

b. Joel borrowed $20 000 from a finance company to purchase a car. Interest on the loan is calculated quarterly at the rate of 2.5% per quarter and is charged immediately prior to Joel making his quarterly repayment of $$M$.

Let A_n be the amount in dollars owing on the loan after the n^{th} repayment has been made.

i. Show that $A_3 = 20\,000 \times 1.025^3 - M\,(1 + 1.025 + 1.025^2)$. 1

Examination continues next page…

Question 30b continued.

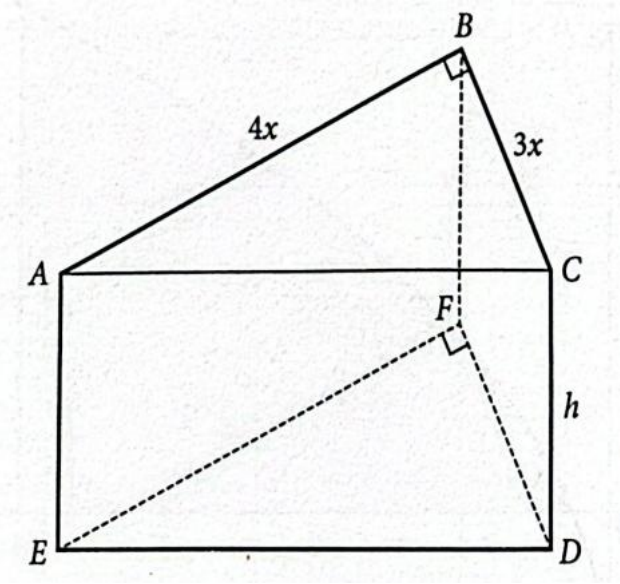
ii. Show that $A_n = 20\,000 \times 1.025^n - 40M(1.025^n - 1)$. 2

iii. If the loan were to be paid out after 7 years what would the value of M be?
Answer correct to the nearest cent. 2

Examination continues next page…

Question 31 (6 marks)

The diagram below shows a right-angled triangular prism, *ABCDEF*, of fixed volume 540cm^3. The sides *AB* and *BC* are in the ratio 4 : 3.



i. Show that the surface area, S, of the prism is given by, $S = 12x^2 + \frac{1080}{x}$. 3

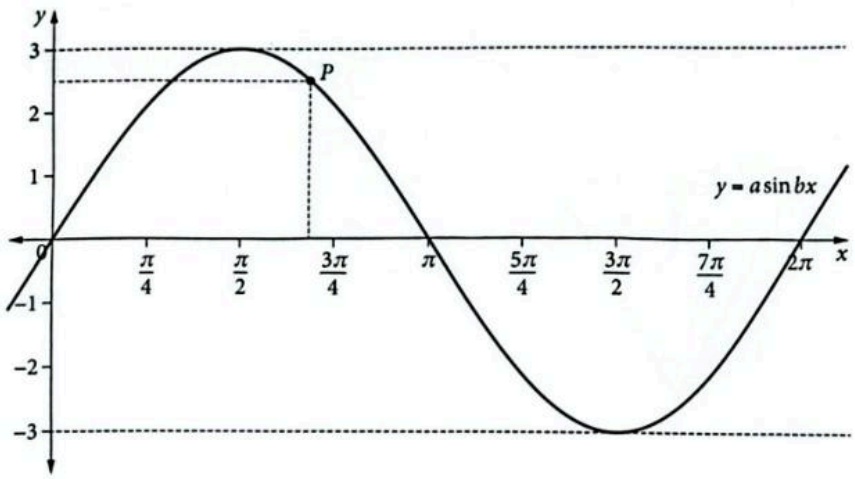
Examination continues next page…

ii. Find the minimum surface area of the prism, to the nearest cm^2. 3

Examination continues next page…

Question 32 (5 marks)

The graph represents the function $y = a \sin bx$.



i. Find the values of a and b. 2

ii. Point $P\left(k, \frac{5}{2}\right)$ lies on the graph, as shown. Find the exact value of $\tan k$. 3

Examination continues next page…

End of examination.

Section II extra writing space

If you use this space, clearly indicate which question you are answering.

Section II extra writing space

If you use this space, clearly indicate which question you are answering.

Section II extra writing space

If you use this space, clearly indicate which question you are answering.

Girraween High School

2026

TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Advanced

General Instructions

- Reading time: 10 minutes
- Working time: 3 Hours
- Write using a black pen
- NESA approved calculators may be used
- Laminated reference sheets are provided
- Answer multiple-choice questions by completely colouring in the appropriate circle on your multiple-choice answer sheet.
- Answer questions on the lines provided and show all relevant mathematical reasoning and/or calculations.

Total Marks: 100

Section 1 (Pages 1 – 5) **10 Marks**

- Attempt Q1 - Q10
- Allow about 15 minutes for this section

Section 2 (Pages 6-26) **90 marks**

- Attempt Q11 – Q32
- Allow about 2 hours and 45 minutes for this section

Section I

10 marks
Attempt Questions 1– 10
Allow about 15 minutes for this section
Use the multiple-choice answer sheet for Questions 1–10

Question 1

The solution to $|x - 3| = 6$ is

A) $x = -3, 9$

B) $x = 3, -9$

C) $x = -3, -9$

D) $x = 3, 9$

$x - 3 = 6$ or $x - 3 = -6$

$x = 9$ or $x = -3$

Question 2

What is the gradient of the tangent to the graph of $y = 2\sin \pi x$ at $x = \frac{1}{3}$.

A) $\frac{\pi}{2}$

B) π

C) $\sqrt{3}\pi$

D) 2π

$y' = 2\pi \cos \pi x$

When $x = \frac{1}{3}$

$y' = 2\pi \cos \frac{\pi}{3}$

$= 2\pi \times \frac{1}{2}$

$= \pi$

Examination continues next page…

Question 3

Which expression is equivalent to $\tan\theta + \cot\theta$?

A) $\operatorname{cosec}\theta + \sec\theta$

B) $\sec\theta \operatorname{cosec}\theta$

C) 2

D) 1

$$\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}$$
$$= \frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}$$
$$= \frac{1}{\sin\theta\cos\theta}$$
$$= \operatorname{cosec}\theta \sec\theta$$

Question 4

The graph of the function $f(x) = \dfrac{3x+2}{5-x}$, has asymptotes at:

A) $x = -5, y = \frac{3}{2}$

B) $x = \frac{2}{3}, y = -3$

C) $x = 5, y = 3$

D) $x = 5, y = -3$

Vertical asymptote,
$5 - x = 0$
$x = 5$
Horizontal asymptote
$$\lim_{x\to\infty} \frac{3x+2}{5-x} = \lim_{x\to\infty} \frac{3 + \frac{2}{x}}{\frac{5}{x} - 1}$$
$$y = -3$$

Question 5

$$\lim_{x\to 0} \frac{\sin\frac{x}{4}}{x} =$$

A) 1

B) 4

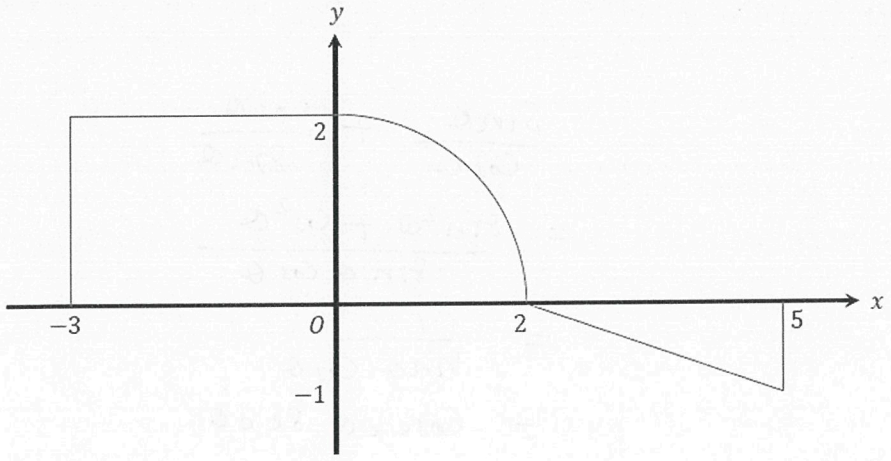
C) $\frac{1}{4}$

D) 0

$$\lim_{x\to 0} \frac{\sin\left(\frac{x}{4}\right)}{4\left(\frac{x}{4}\right)} = \frac{1}{4}\lim_{x\to 0} \frac{\sin\left(\frac{x}{4}\right)}{x/4}$$
$$= \frac{1}{4} \times 1$$
$$= \frac{1}{4}$$

Examination continues next page…

Question 6



What is the exact value of $\int_{-3}^{5} f(x)\,dx$.

$$\int_{-3}^{5} f(x)\,dx = 3\times 2 + \frac{1}{4}\pi(2)^2 - \frac{1}{2}\times 3\times 1$$
$$= 6 + \pi - \frac{3}{2}$$
$$= \pi + \frac{9}{2}$$

A. $\pi + \frac{9}{2}$

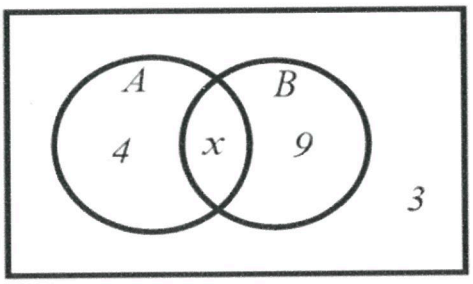
B. $\pi + \frac{15}{2}$

C. $\pi - \frac{15}{2}$

D. $\pi - \frac{9}{2}$

Question 7

In the Venn diagram below, events A and B are independent.



What is the value of x?

Since A and B are independent,

$$P(A/B) = P(A)$$
$$\frac{x}{x+9} = \frac{x+4}{x+16}$$
$$x^2 + 16x = x^2 + 13x + 36$$
$$3x = 36$$
$$x = 12$$

A) 6

B) 8

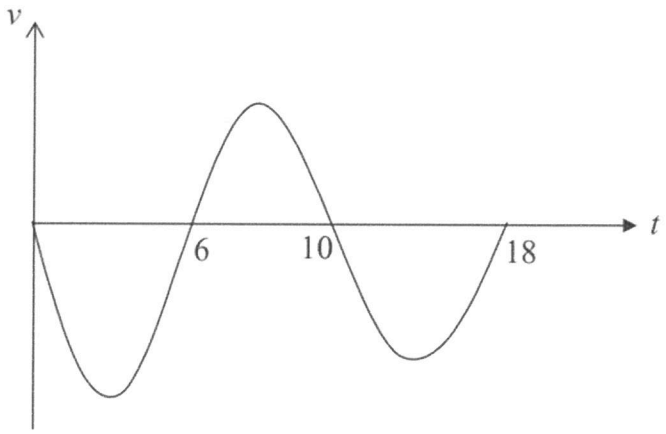
C) 12

D) 15

Examination continues next page…

Question 8

A particle moves along a straight line. Its velocity v at time t is shown in the graph below.



Find what value of t is the displacement of the particle a maximum?

A) 18

B) 10

C) 8

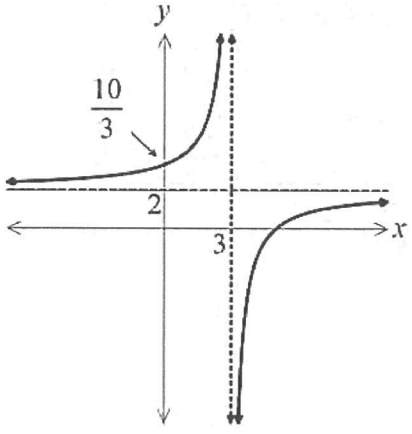
D) 6

Maximum when changing from $v > 0$ to $v < 0$

$\therefore t = 10$

Question 9

Consider the graph of $y = \dfrac{k}{x+b} + c$ below, with y- intercept at $\left(0, \dfrac{10}{3}\right)$.



Which of the following is true?

A) $k < b < c$

B) $b < k < c$

C) $k < c < b$

D) $c < k < b$

$c = 2$

$b = -3$

$y = \dfrac{k}{x-3} + 2$

y intercept $\left(0, \frac{10}{3}\right)$

$\dfrac{10}{3} = \dfrac{k}{-3} + 2$

$10 = -k + 6$

$k = -4$

$-4 < -3 < 2$

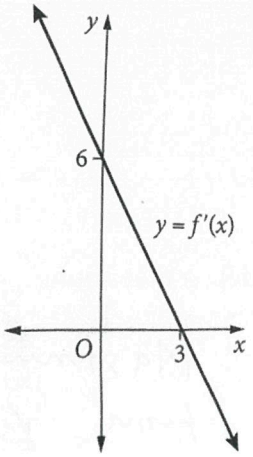
$k < b < c$

Examination continues next page...

Question 10

The graph of $y = f'(x)$ is shown.

What is the equation of $y = f(x)$, if it has a maximum value of 16?



A) $y = x^2 - 6x + 16$

B) $y = x^2 - 6x - 11$

C) $y = -x^2 + 6x + 7$

D) $y = -x^2 + 6x + 25$

$f'(x) = -2x + 6$

$f(x) = -x^2 + 6x + C$

$f'(x) = 0$ when $x = 3$

$-3^2 + 6 \times 3 + C = 16$

$C = 7$

$y = f(x)$

$= -x^2 + 6x + 7$

Examination continues next page…

Section II

90 marks
Attempt Questions 11–32
Allow about 2 hours and 45 minutes for this section

- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of the response.
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided at the end of this booklet. If you use this space, clearly indicate which question you are answering.

Question 11 (2 marks)

Differentiate: $y = e^{2x}(x+2)$. 2

$y' = e^{2x} \times 1 + (x+2)\,2e^{2x}$ (1)

$= e^{2x}(1+2x+4)$

$= e^{2x}(2x+5)$ (1)

Question 12 (2 marks)

Find $\int \sqrt{4x+1}\,dx$. 2

$= \int (4x+1)^{\frac{1}{2}}\,dx$

$= \dfrac{(4x+1)^{3/2}}{4 \times 3/2} + C$ (1)

$= \dfrac{(4x+1)^{3/2}}{6} + C$ (1)

Examination continues next page…

Question 13 (2 marks)

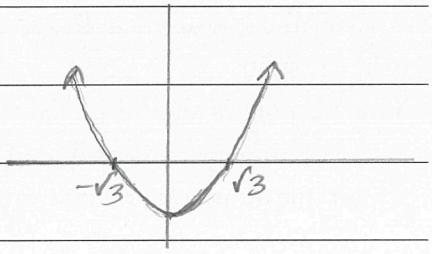
For the function $f(x) = x^3 - 9x$, state the domain where $f(x)$ is increasing. 2

For increasing function, $f'(x) > 0$

$3x^2 - 9 > 0$ (1)

$3(x^2 - 3) > 0$

$x > \sqrt{3}$ or $x < -\sqrt{3}$ (1)



Question 14 (2 marks)

Find the exact value of $\int_1^2 \frac{3}{4x-1}\,dx$ 2

$$\int_1^2 \frac{3}{4x-1}\,dx = 3\left[\frac{\ln|4x-1|}{4}\right]_1^2$$ (1)

$$= \frac{3}{4}\left[\ln|8-1| - \ln|4-1|\right]$$

$$= \frac{3}{4}\left[\ln|7| - \ln|3|\right]$$

$$= \frac{3}{4}\ln\left|\frac{7}{3}\right|$$ (1)

Examination continues next page…

Question 15 (3 marks)

i. Differentiate $\frac{14}{7x^2 + 2}$ 1

$\frac{d}{dx} 14(7x^2+2)^{-1} = -14(7x^2+2)^{-2} \times 14x$

$= \frac{-196x}{(7x^2+2)^2}$

ii. Hence, find $\int \frac{14x}{(7x^2 + 2)^2}\, dx$. 2

$= \frac{-1}{14} \int \frac{-14 \times 14x}{(7x^2+2)^2}\, dx$ (1)

$= -\frac{1}{14} \times \frac{14}{7x^2+2} + C$

$= -\frac{1}{7x^2+2} + C$ (1)

Question 16 (3 marks)

Prove that $\sec^2 x + \sec x \tan x = \frac{1}{1-\sin x}$. 3

LHS $= \sec^2 x + \sec x \tan x$

$= \frac{1}{\cos^2 x} + \frac{1}{\cos x} \times \frac{\sin x}{\cos x}$ (1)

$= \frac{1+\sin x}{\cos^2 x}$

$= \frac{1+\sin x}{1-\sin^2 x}$ (1)

$= \frac{1+\sin x}{(1+\sin x)(1-\sin x)}$

$= \frac{1}{1-\sin x}$ (1)

$=$ RHS

Examination continues next page…

Question 17 (2 marks)

Consider the function $f(x) = e^x$ and $g(x) = \ln(x-2)$.

i. Find the composite function $f(g(x))$. 1

$f(g(x)) = f(\ln(x-2))$

$= e^{\ln(x-2)}$

$= x-2$

ii. Find in interval notation the range of $f(g(x))$. 1

For $\ln(x-2)$, $x > 2$

$\therefore$ Range of $f(g(x))$ is $y > 0$ or $(0, \infty)$

Question 18 (2 marks)

The function below is a probability density function for the given interval:

$$f(x) = \begin{cases} ax^2, & 0 \le x \le 6 \\ 0, & \text{otherwise.} \end{cases}$$

Find the value of a. 2

$\int_0^6 f(x)\,dx = 1$ (as it is a pdf)

$\int_0^6 ax^2\,dx = 1$ (1)

$\left[\frac{ax^3}{3}\right]_0^6 = 1$

$\frac{216a}{3} = 1$

$a = \frac{1}{72}$ (1)

Examination continues next page…

Question 19 (3 marks)

Bob plays archery competitively as part of a team. The number of attempts(X) that he needs to hit the target is given by the probability distribution table:

x	1	2	3	4	5
$P(X = x)$	0.12	0.35	0.32	0.17	0.04

i. Find the expected number of attempts Bob needs to hit the target. 1

$E(X) = 1 \times 0.12 + 2 \times 0.35 + 3 \times 0.32 + 4 \times 0.17 + 5 \times 0.04$

$= 2.66$

ii. The team's coach would like to select someone from the team to be captain. Another archer in the team, Karen, has the same expected number of attempts as Bob, and a standard deviation of 1.2. If the coach bases his decision on performance, are they more likely to choose Bob or Karen? Justify your answer with appropriate statistical reasoning. 2

$Var(X) = 1^2 \times 0.12 + 2^2 \times 0.35 + 3^2 \times 0.32 + 4^2 \times 0.17 \times 5^2 \times 0.04 - (2.66)^2$

$= 1.04$ (2 dp)

$\sigma = 1.02$ (2 dp) (1)

Bob's standard deviation is lower, so more consistent. (1)

∴ Choose Bob

Examination continues next page…

Question 20 (4 marks)

Tom did a survey of a group of people he knew about their age and how much they earn each week. The results are shown in the table below.

Age (years) (x)	18	45	28	15	32	68
Wage (\$/week) (W)	715	2350	1530	438	1690	1320

i. Using your calculator, find the correlation coefficient(r), and explain the strength of the correlation this data gives. 2

$r = 0.5263$ (1)

Moderate correlation (1)

ii. Using your calculator, find the equation of the least-squares regression line in the form $W = Bx + A$, where A and B are integers. 1

From calculator, $A = 706$, $B = 18$

$\therefore W = 18x + 706$

iii. Could your equation from part(ii) be used to make valid estimate for ages greater than 68 or less than 15 years? Justify your answer with reasons. 1

Since there is only moderate correlation, it is not a valid way to estimate for younger or older people.

OR

It is valid for interpolation between 15 and 45 years, and also affected by the outlier (68, 1320)

Examination continues next page…

Question 21 (6 marks)

In Alex's first year of fulltime employment his annual salary is \$75 000. At the beginning of the second year his salary increases to \$79 000. His salary continues to increase by \$4000 at the beginning of each successive year.

i. What will Alex's salary be at the beginning of his 25th year? 1

AP with $a = \$75000$, $d = \$4000$ and $n = 25$

Alex's salary $= \$75000 + 24 \times 4000$

$= \$171000$

ii. Calculate Alex's total earnings after working for 25 years? 2

Total earnings, $S_{25} = \$75000 + \$79000 + \ldots + \$171000$

$= \frac{25}{2}[\$75000 + \$171000]$ (1)

$= \$3075000$ (1)

iii. During which year of employment will his total earnings first exceed \$2 000 000? 3

$\frac{n}{2}[2 \times 75000 + (n-1)4000] = 2000000$ (1)

$150000n + 4000n^2 - 4000n = 4000000$

$4000n^2 + 146000n = 4000000$

$2n^2 + 73n - 2000 = 0$

$n = \frac{-73 \pm \sqrt{73^2 - 4 \times 2 \times 2000}}{4}$ (1)

$= -54.76$ or 18.26

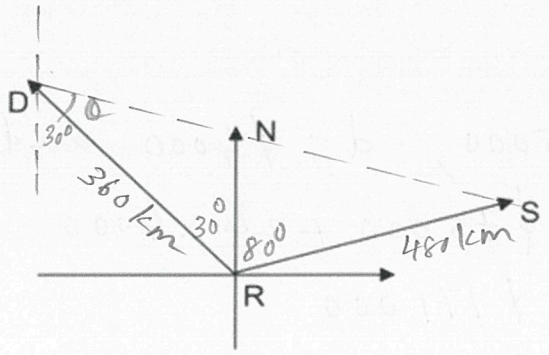
Since $n > 0$, $n \approx 18.26$

∴ During the 19th year first exceed \$2000000. (1)

Examination continues next page…

Question 22 (5 marks)

David and Sarah are pilots of two light planes which leave Rodeo Pilot Station at the same time. David flies on a bearing of 330°T at a speed of 180 km/h and Sarah flies on a bearing of 080°T at a speed of 240 km/h.



i. How far apart are David and Sarah after 2 hours? 2

$\angle DRS = 30^\circ + 80^\circ = 110^\circ$

$DS^2 = 360^2 + 480^2 - 2 \times 360 \times 480 \times \cos 110^\circ$ (1)

$= 478202.16$

$DS = 691.52$ km (1)

ii. What is the bearing of Sarah from David after 2 hours? (to the nearest degree.) 3

$\frac{\sin\theta}{480} = \frac{\sin 110^\circ}{691.52}$ (1)

$\sin\theta = \frac{480 \times \sin 110^\circ}{691.52} = 0.65226..$

$\theta = 40.71$

$\approx 41^\circ$ (1)

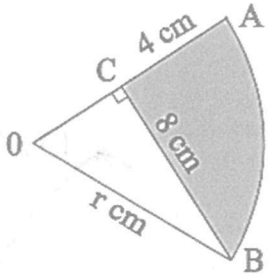
$\therefore$ Bearing $= 180 - 41 - 30$

$= 109^\circ T$ (1)

Examination continues next page…

Question 23 (4 marks)

The diagram shows the sector OAB with centre O and radius r. The perpendicular from B meets the radius OA at C. $BC = 8$ cm and $AC = 4$ cm.



NOT TO SCALE

i. Show that the radius r of the circle is 10 cm. 2

$(r-4)^2 + 8^2 = r^2$ (1)

$r^2 - 8r + 16 + 64 = r^2$

$-8r = -80$ (1)

$r = 10$

ii. Find the area of the shaded region in this sector. 2

$\sin\theta = \frac{8}{10}$, $\theta = 0.9273$ (in radians).

$\therefore$ Shaded area = Area of sector AOB − Area of $\triangle OCB$

$= \frac{1}{2} \times 10^2 \times 0.9273 - \frac{1}{2} \times 8 \times 6$ (1)

$= 22.37 \text{ cm}^2$ (2 dp) (1)

Question 24 (7 marks)

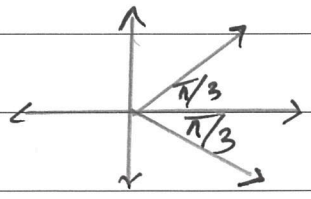
i. Solve the equation $1 - 2\cos x = 0$ for $0 \leq x \leq 2\pi$. 2

$2\cos x = 1$, $0 \leq x \leq 2\pi$

$\cos x = \frac{1}{2}$ (1)

Related angle is $\pi/3$

$\therefore x = \frac{\pi}{3}$ or $\frac{5\pi}{3}$ (1)

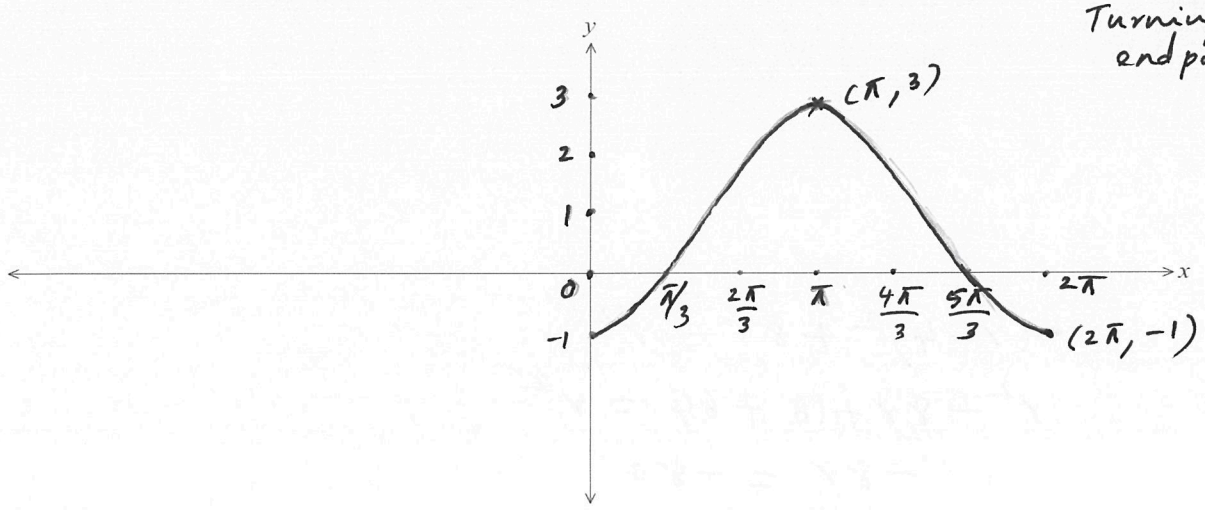


Examination continues next page…

Question 24 continued

ii. Sketch the graph of the curve $y = 1 - 2\cos x$ for $0 \leq x \leq 2\pi$, showing clearly the coordinates of the endpoints and the maximum turning point. 2

intercepts → 1
Turning point & end points → 1



iii. Find in simplest exact form the area of the region bounded by the curve $y = 1 - 2\cos x$ and the x-axis between $x = 0$ and $x = \pi$. 3

$$\text{Area} = \left| \int_0^{\pi/3} (1 - 2\cos x)\,dx \right| + \int_{\pi/3}^{\pi} (1 - 2\cos x)\,dx \quad (1)$$

$$= \left| \left[x - 2\sin x \right]_0^{\pi/3} \right| + \left[x - 2\sin x \right]_{\pi/3}^{\pi} \quad (1)$$

$$= \left| \frac{\pi}{3} - 2\sin\frac{\pi}{3} - 0 \right| + \left[\pi - 2\sin\pi - \left(\frac{\pi}{3} - 2\sin\frac{\pi}{3} \right) \right]$$

$$= \left| \frac{\pi}{3} - 2 \times \frac{\sqrt{3}}{2} \right| + \left[\pi - 0 - \frac{\pi}{3} + 2 \times \frac{\sqrt{3}}{2} \right]$$

$$= \left| \frac{\pi}{3} - \sqrt{3} \right| + \frac{2\pi}{3} + \sqrt{3}$$

$$= \sqrt{3} - \frac{\pi}{3} + \frac{2\pi}{3} + \sqrt{3} \quad \left(\text{as } \sqrt{3} > \frac{\pi}{3} \right)$$

$$= 2\sqrt{3} + \frac{\pi}{3} \quad (1)$$

Examination continues next page

Question 25 (6 marks)

In a laboratory, a food product is heated in an oven set at $210°C$ to kill harmful bacteria. The number of bacteria recorded when the food product is first placed in the oven is 1800. After 10 minutes the number of bacteria recorded is 550.

It is found that the number N, of bacteria remaining t minutes after being in the oven is given by $N = N_0\, e^{-kt}$, where N_0 and k are constants.

i. Calculate the value of k correct to 4 decimal places. 3

$N = N_0 e^{-kt}$, $N_0 = 1800$, $N = 550$, when $t = 10$

$550 = 1800\, e^{-k(10)}$ (1)

$e^{-10k} = \frac{550}{1800}$

$-10k = \ln\left(\frac{550}{1800}\right)$ (1)

$k = 0.1186$ (1)

ii. If an acceptable number of bacteria present is 100 or less, for how long should the food product be in the oven? Express your answer to the nearest minute. 3

$N \leq 100$

$1800\, e^{-0.1186t} \leq 100$ (1)

$e^{-0.1186t} \leq \frac{1}{18}$

$-0.1186t \leq \ln\left(\frac{1}{18}\right)$

$t \geq \frac{\ln\left(\frac{1}{18}\right)}{-0.1186}$ (1)

$t \geq 24.37$

∴ The food should be kept for 25 mins in the oven. (1)

Examination continues next page…

Question 26 (7 marks)

The velocity of an object is given by the equation $v = 6t - 5 - t^2$ where t is in seconds and velocity v in m/s. It begins its motion at $x = 5$ metres.

i. Find an equation for the displacement of the object. 2

$x = \int (6t - 5 - t^2)\,dt$

$x = 3t^2 - 5t - \frac{t^3}{3} + C$ (1)

At $t = 0$, $x = 5$.

$\therefore C = 5$

$x = 3t^2 - 5t - \frac{t^3}{3} + 5$ (1)

ii. At what two times is the object stationary? 2

Object is stationary when $v = 0$

$6t - 5 - t^2 = 0$ or $t^2 - 6t + 5 = 0$ (1)

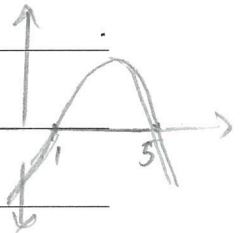
$(t - 5)(t - 1) = 0$

$t = 1, 5$ (1)

iii. Find the distance travelled by the object in first 3 seconds. 3

$\text{Distance} = \left| \int_0^1 (6t - 5 - t^2)\,dt \right| + \int_1^3 (6t - 5 - t^2)\,dt$ (1)

$= \left| \left[3t^2 - 5t - \frac{t^3}{3} \right]_0^1 \right| + \left[3t^2 - 5t - \frac{t^3}{3} \right]_1^3$ (1)



$= \left| \left(3 - 5 - \frac{1}{3}\right) \right| + \left[27 - 15 - \frac{27}{3} - \left(3 - 5 - \frac{1}{3}\right) \right]$

$= \left| -\frac{7}{3} \right| + \frac{16}{3} = \frac{23}{3}$ m (1)

Examination continues next page…

Question 27 (4 marks)

Whilst performing some experiments, researchers have found that the rate of change of the distance (in cm/s) travelled by a particle is given by $A(t) = 40\ln(1+5t)$, where t is the number of seconds elapsed since the start of the research.

Let D cm be the distance travelled by the particle from $t = 1$ to $t = 5$.

i. Estimate D by using the trapezoidal rule with 4 sub-intervals.
Give answer to the nearest centimetre. 2

$D = \int_1^5 A(t)\,dt \qquad h = \frac{5-1}{4} = 1$

t	1	2	3	4	5
$A(t)$	$40\ln(6)$	$40\ln(11)$	$40\ln(16)$	$40\ln(21)$	$40\ln(26)$

$D \approx \frac{h}{2}\left[A(1) + A(5) + 2\{A(2)+A(3)+A(4)\}\right]$

$= \frac{1}{2}\left[40\ln(6) + 40\ln(26) + 2\{40\ln(11)+40\ln(16)+40\ln(21)\}\right]$ (1)

$= 429.597$.

$D \approx 430$ cm (1)

ii. Is D an over-estimate or an under-estimate of the exact distance?

Justify your answer. 2

$A'(t) = \frac{200}{1+5t}$

$A''(t) = 200 \times -1(1+5t)^{-2} \times 5$

$A''(t) = \frac{-1000}{(1+5t)^2} < 0$ for all t in $[1, 5]$ (1)

$\therefore A(t)$ is concave down

$\therefore D$ is an underestimate of the exact distance. (1)

Examination continues next page...

Question 28 (4 marks)

A tennis ball is dropped from a height of 12 metres. After it first hits the ground, it rebounds to a height of 8 metres. It continues to rebound to a height of two thirds its previous rebound height.

i. How high will the ball rise after the 6th time it rebounds from the ground? 2

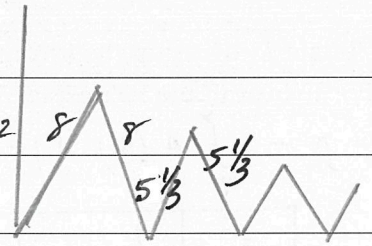
$a = 8, \quad r = \frac{2}{3}, \quad n = 6$

$T_n = ar^{n-1}$

$= 8\left(\frac{2}{3}\right)^5$ (1)

$= 1.05\text{m}$ (2 dp)

$\therefore$ 6th bounce is 1.05 m high. (1)



ii. What is the total vertical distance that the ball will eventually travel? 2

Total vertical distance $= 12 + 8 + 8 + 5\frac{1}{3} + 5\frac{1}{3} + \cdots$

$= 12 + 2\left(8 + 5\frac{1}{3} + \cdots\right)$

$= 12 + 2\left(\frac{8}{1 - 2/3}\right)$ (1) as $\left|\frac{2}{3}\right| < 1$

$= 12 + 2 \times 24$

$= 60\text{m}$ (1)

Examination continues next page…

Question 29 (5 marks)

A machine produces metal rods used to reinforce concrete slabs. The diameters of the rods are normally distributed with a mean of 8 mm and a standard deviation of 0.3 mm.

i. Find the diameter of the rod with a z-score of 1.2. 1

$$Z = \frac{x - M}{\sigma}$$

$$x = z\sigma + M$$

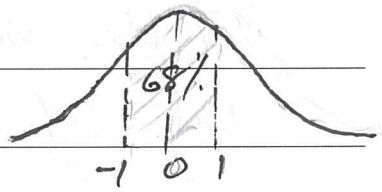
$$= 1.2 \times 0.3 + 8$$

$$= 8.36 \text{ mm}$$

ii. If a rod is selected at random, find the probability that the diameter of the rod is less than 7.7 mm. 2

$$Z = \frac{x - M}{\sigma} = \frac{7.7 - 8}{0.3} = 1$$

$P(-1 < z < 1) = 0.68$ (Using empirical rule)



$100\% - 68\% = 32\%$

$\therefore P(\text{diameter} < 7.7\text{mm}) = P(z < -1) = 0.16$ (using symmetry)

iii. If two rods are selected at random, find the probability that exactly one of the rods will have a diameter that is less than 7.7 mm. 2

$P(x > 7.7\text{mm}) = 1 - 0.16 = 0.84$

$P(\text{exactly one of the rod's diameter} < 7.7\text{mm})$

$= 0.16 \times 0.84 + 0.84 \times 0.16$

$= 0.2688$

Examination continues next page…

Question 30 (6 marks)

a. A table of future value interest factors for an annuity of $1 is shown

Future value of $1						
Period	**1%**	**4%**	**8%**	**12%**	**16%**	**20%**
1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	2.0100	2.0400	2.0800	2.1200	2.1600	2.2000
3	3.0301	3.1216	3.2464	3.3744	3.5056	3.6400
4	4.0604	4.2465	4.5061	4.7793	5.0665	5.3680
5	5.1010	5.4163	5.8666	6.3528	6.8771	7.4416
6	6.1520	6.6330	7.3359	8.1152	8.9775	9.9299

Annie wants to save $ 10 000 over the next 2 years by investing equal payments every 6 months into an account that earns interest 8% p.a. compounded half-yearly.

What should be the value of each payment? 1

interest $= \frac{8\%}{2} = 4\%$

Periods $= 2 \times 2 = 4$

Table value is 4.2465

$\therefore$ Value of the payment $= \frac{10000}{4.2465}$

$= \$2354.88$

b. Joel borrowed $20 000 from a finance company to purchase a car. Interest on the loan is calculated quarterly at the rate of 2.5% per quarter and is charged immediately prior to Joel making his quarterly repayment of $\$M$.

Let A_n be the amount in dollars owing on the loan after the n^{th} repayment has been made.

i. Show that $A_3 = 20\,000 \times 1.025^3 - M\,(1 + 1.025 + 1.025^2)$. 1

Rate = 2.5% per quarter

$A_1 = 20000(1.025) - M$

$A_2 = A_1(1.025) - M$

$= [20000(1.025) - M]1.025 - M$

$= 20000(1.025)^2 - M(1 + 1.025)$

$A_3 = A_2 \times 1.025 - M$

Examination continues next page…

Question 30b continued.

$A_3 = \left[20000(1.025)^2 - M(1+1.025)\right]1.025 - M$

$A_3 = 20000(1.025)^3 - M(1+1.025+1.025^2)$

ii. Show that $A_n = 20\,000 \times 1.025^n - 40M(1.025^n - 1)$. 2

$A_n = 20000 \times 1.025^n - M(1+1.025+\cdots+1.025^{n-1})$ (1)

$= 20000 \times 1.025^n - \dfrac{M(1.025^n - 1)}{1.025 - 1}$ (1) GP with $a=1$, $r=1.025$

$= 20000 \times 1.025^n - 40M(1.025^n - 1)$ as $\dfrac{1}{0.025} = 40$

iii. If the loan were to be paid out after 7 years what would the value of M be?
Answer correct to the nearest cent. 2

Find M when $A_{28} = 0$

$20000(1.025)^{28} - 40M(1.025^{28} - 1) = 0$ (1)

$40M(1.025^{28} - 1) = 20000(1.025)^{28}$

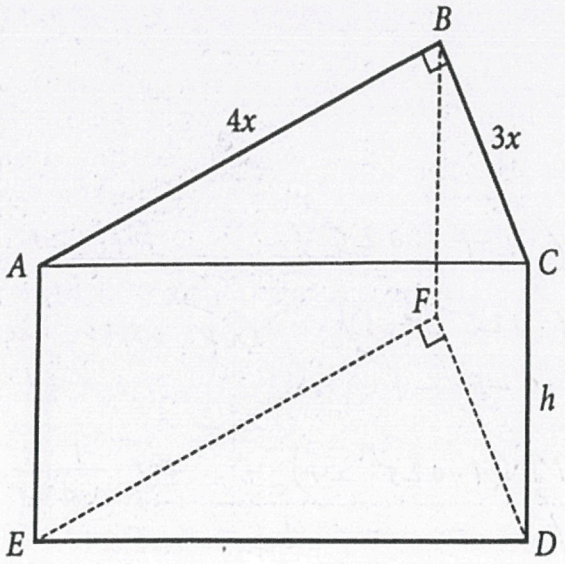
$M = \dfrac{20000(1.025)^{28}}{40(1.025^{28} - 1)}$

$= \$1001.76$ (nearest cent) (1)

Examination continues next page…

Question 31 (6 marks)

The diagram below shows a right-angled triangular prism, *ABCDEF*, of fixed volume 540cm^3. The sides *AB* and *BC* are in the ratio 4 : 3.



i. Show that the surface area, S, of the prism is given by, $S = 12x^2 + \frac{1080}{x}$. 3

Volume $= \frac{1}{2} \times 3x \times 4x \times h = 540$

$6x^2h = 540$

(1) $h = \frac{90}{x^2}$ — (A)

In ΔABC, $AC^2 = (3x)^2 + (4x)^2$ (Pythagoras' theorem).

$AC = 5x$ (1)

$\therefore$ Surface area, $S = 2 \times \frac{1}{2} \times 3x \times 4x + 3xh + 4xh + 5xh$

$= 12x^2 + 12xh$

$= 12x^2 + 12x\left(\frac{90}{x^2}\right)$ } (1) using (A)

$= 12x^2 + \frac{1080}{x}$

Examination continues next page…

ii. Find the minimum surface area of the prism, to the nearest cm^2. 3

$$S = 12x^2 + \frac{1080}{x}$$

$$\frac{dS}{dx} = 24x - \frac{1080}{x^2}$$

For stationary point, $\frac{dS}{dx} = 0$

$$24x - \frac{1080}{x^2} = 0$$

$$24x^3 - 1080 = 0$$

$$x^3 = 45 \quad (1)$$

$$x = \sqrt[3]{45}$$

When $x = \sqrt[3]{45}$, $S = 12\left(\sqrt[3]{45}\right)^2 + \frac{1080}{\sqrt[3]{45}}$

$$= 455$$

$$\frac{d^2S}{dx^2} = 24 + \frac{2160}{x^3} \quad (1)$$

When $x = \sqrt[3]{45}$, $\frac{d^2S}{dx^2} = 24 + \frac{2160}{45} = 72 > 0$

$\frac{d^2S}{dx^2} > 0$, $\therefore$ concave up.

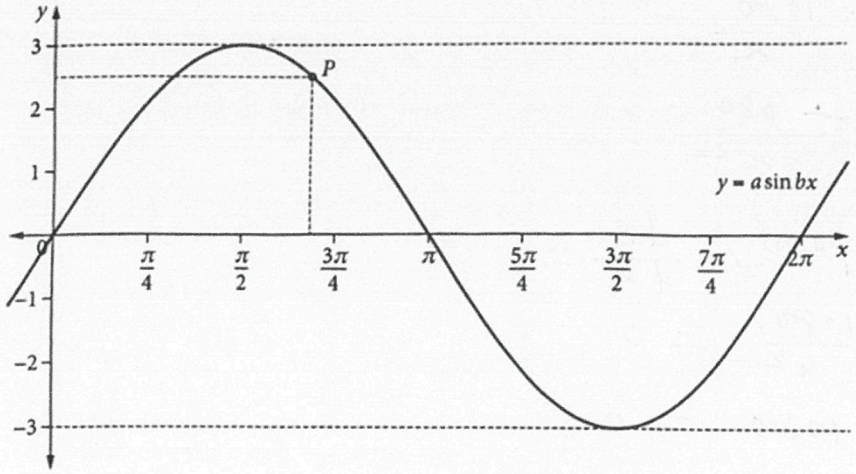
$(\sqrt[3]{45}, 455)$ is a minimum turning point.

$\therefore$ Minimum surface area $= 455\ cm^2$ (1)

Examination continues next page…

Question 32 (5 marks)

The graph represents the function $y = a \sin bx$.



i. Find the values of a and b. 2

$a = 3$ (since the amplitude is 3) (1)

$b = 1$ (since the period is 2π) (1)

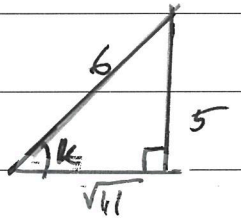
$y = 3 \sin x$.

ii. Point $P\left(k, \frac{5}{2}\right)$ lies on the graph, as shown. Find the exact value of $\tan k$. 3

$y = 3 \sin x$

Substitute $\left(k, \frac{5}{2}\right) \Rightarrow \frac{5}{2} = 3 \sin k$

$\sin k = \frac{5}{6}$ (1)



Base of the triangle, $b = \sqrt{6^2 - 5^2}$

$= \sqrt{11}$

$\therefore \tan k = \frac{5}{\sqrt{11}}$ (1)

Since 'k' is in the second quadrant, $\tan k < 0$

$\therefore \tan k = \frac{-5}{\sqrt{11}}$ (1)

Examination continues next page…