
Student number

Gosford High School

2026 **Year 12 Trial HIGHER SCHOOL CERTIFICATE EXAMINATION**

Mathematics Advanced

General Instructions

- Reading time – 10 minutes
- Working time – 3 hours
- Write using black or blue pen. DO NOT use a pen with erasable ink
- Calculators approved by NESA may be used
- A laminated reference sheet is provided
- For all questions in Section II, show relevant mathematical reasoning and/or calculations
- Write your student number at the top of this page

Total Marks 100

Section I – 10 marks (pages 2 – 6)

- Attempt Questions 1-10
- Allow about 15 minutes for this section

- **Section II – 90 marks** (pages 7 – 24)
- Attempt Questions 11 – 30
- Allow about 2 hours and 45 minutes for this section

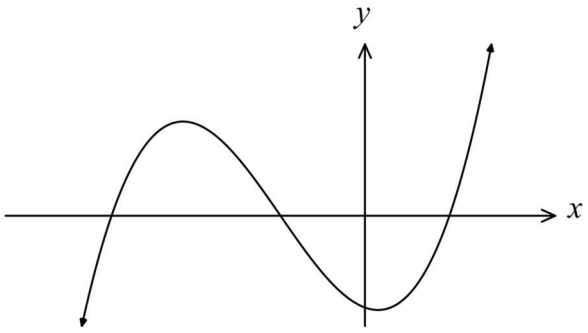
Section I

10 marks
Attempt Questions 1-10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1-10

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1 The graph of $y = f(x)$ is shown.



Which of the following could be the equation of this graph?

A. $y = (x^2 - 1)(x - 3)$

B. $y = (x^2 - 1)(x + 3)$

C. $y = (x - 1)^2(x - 3)$

D. $y = (x - 1)^2(x + 3)$

2 Consider the following dataset.

10 15 15 16 20 24

Which row of the table shows how the interquartile range and standard deviation are affected when a score of 18 is added to the dataset?

	Interquartile range	*Standard deviation*
A.	Changes	Increases
B.	Stays the same	Increases
C.	Changes	Decreases
D.	Stays the same	Decreases

3 What is the domain of $f(x)=\dfrac{1}{\sqrt{4-x^2}}$

A. $x<2$

B. $x\leq 2$

C. $-2<x<2$

D. $-2<x<0 \cup 0<x<2$

4 What is $\int \pi\sin(\pi x)\,dx$?

A. $-\cos(\pi x)+c$

B. $-\pi^2\cos(\pi x)+c$

C. $\pi\cos(\pi x)+c$

D. $\dfrac{\pi\sin^2(\pi x)}{2}+c$

5 Which of the following is the range of $y=-(x-5)^2+3$?

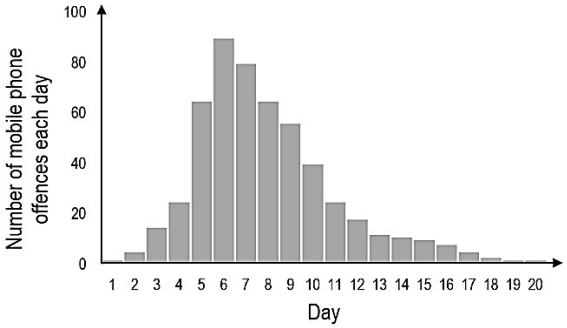
A. $(-\infty,5]$

B. $(-\infty,3]$

C. $[5,\infty)$

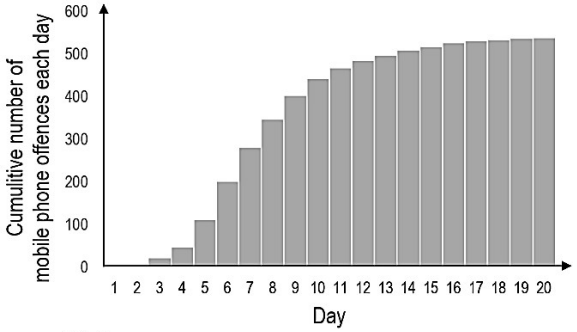
D. $[3,\infty)$

6 A mobile phone detection camera that uses AI and infrared to target illegal phone use by drivers was recently installed in Woodbridge. The number of offences detected each day over a 20 day period is shown in the following graph.

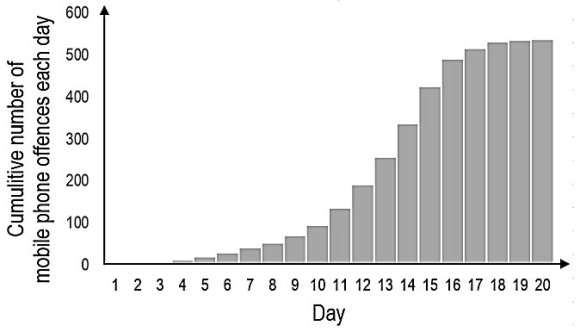


Which of the following graphs best shows the cumulative number of mobile phone offences detected up to and including each day?

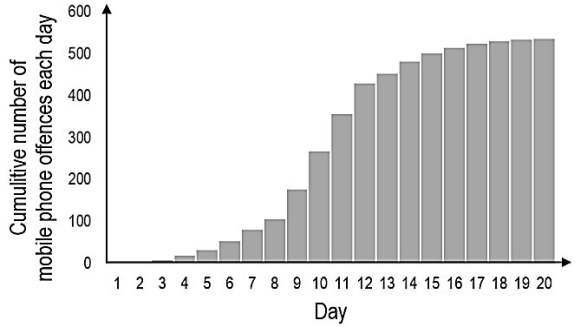
A.



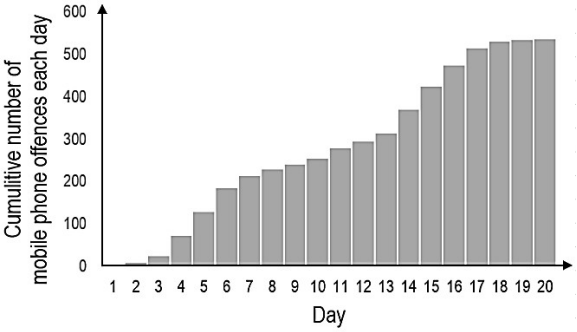
B.



C.



D.



7 How many solutions of the equation $(\cot x + 1)(\cos^2 x - 1) = 0$ lie between 0 and 2π?

A. 2

B. 3

C. 4

D. 5

8 A is known to vary inversely with the square of x. Which equation best represents the relation between A and x?

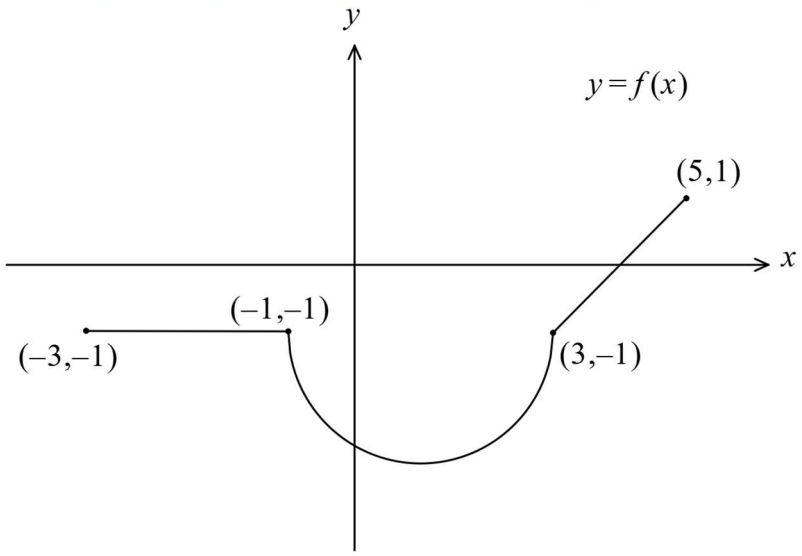
A. $A^2 = \frac{k}{x}$

B. $A^2 = kx$

C. $A = \frac{k}{x^2}$

D. $A = kx^2$

9 $y = f(x)$ is continuous and consists of a semi-circle joined to straight line segments as shown.



What is the value of $\int_{-3}^{5} f(x)\,dx$?

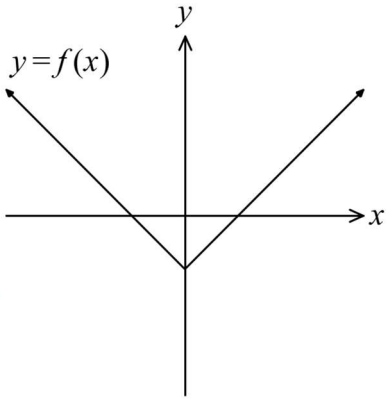
A. $2\pi - 6$

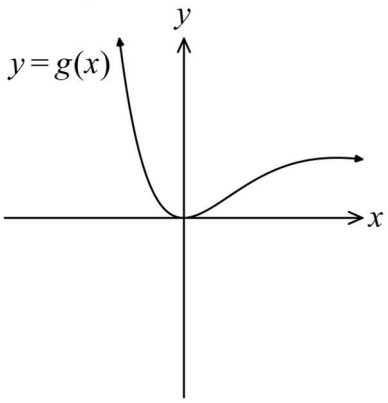
B. $3 + 2\pi$

C. $-3 - 2\pi$

D. $-2(3 + \pi)$

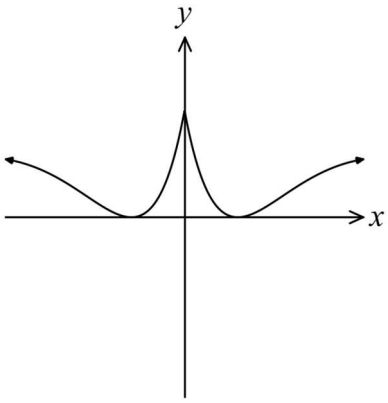
10 The graphs of $y = f(x)$ and $y = g(x)$ are shown.



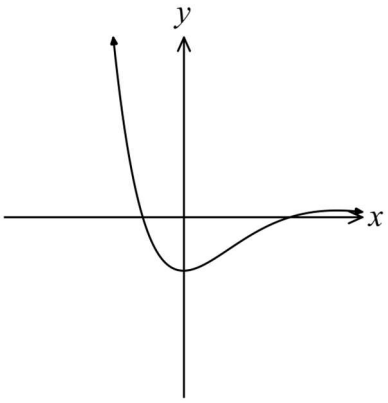


Which graph best represents $g(f(x))$?

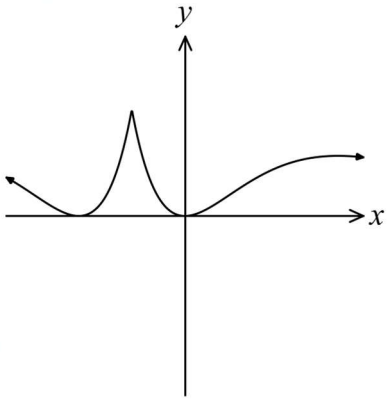
A.



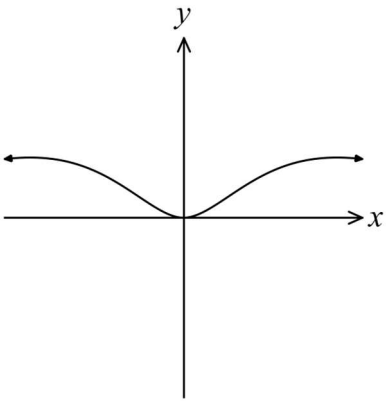
B.



C.



D.



2026 **Year 12 Trial**

HIGHER SCHOOL CERTIFICATE EXAMINATION

Student number

Mathematics Advanced

Section II Answer Booklet

90 marks
Attempt Questions 11 – 30
Allow about 2 hours and 45 minutes for this section

Instructions

- Write your student number at the top of this page
- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response
- Your responses should include relevant mathematical reasoning and/or calculations
- Extra writing space is provided at the back of this booklet. If you use this space, clearly indicate which question you are answering

Please turn over

Question 11 (2 marks)

Solve $(5x-1)^2 = 16$ **2**

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Question 12 (2 marks)

Solve $e^{2\ln x} = x + 12$ **2**

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Question 13 (6 marks)

A box contains 10 chocolates that appear identical. However, 6 of the chocolates have yellow centres and 4 have green. Todd ate three chocolates chosen randomly from the box. Find the probability that:

(a) the first two chocolates that Todd ate had a yellow centre **2**

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(b) at least two of the chocolates that Todd ate had yellow centres **2**

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(c) all of the chocolates that Todd ate had yellow centres given that two were known to have been yellow. **2**

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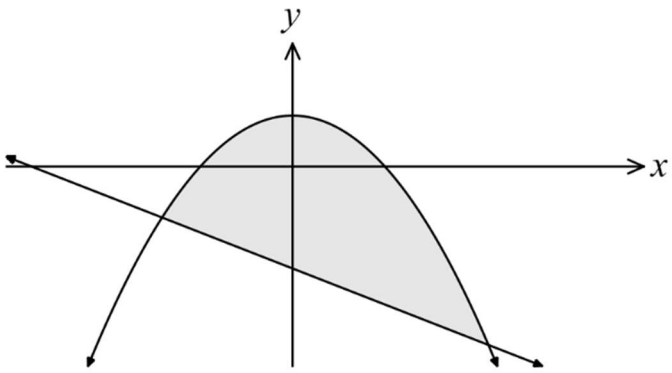
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Question 14 (5 marks)

The curves $y = 2 - x^2$ and $y = -x - 4$ intersect at two points as shown in the diagram.



NOT TO SCALE

(a) Find the x ordinates of the points of intersection of the two curves. **2**

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(b) Find the area enclosed by the two curves. **3**

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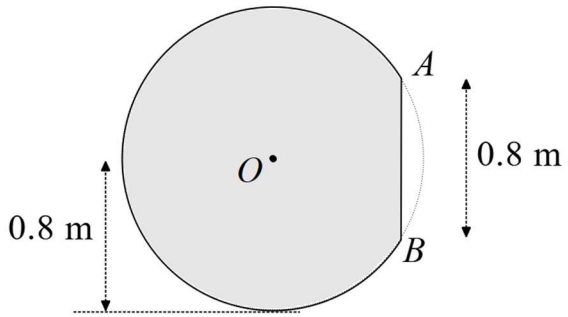
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Question 15 (4 marks)

A small table-top is in the shape of a circle with a small segment removed as shown. The circle has centre O and radius 0.8 metres. The length of the straight edge AB is also 0.8 metres.



NOT TO SCALE

(a) Explain why $\angle AOB = \frac{\pi}{3}$ **1**

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(b) Find the exact area of the table-top. **3**

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Question 16 (10 marks)

A particle is moving along the x axis. Its position at time t is given by $x = \frac{t}{2} + \sin t$.

(a) At what times during the period $0 \le t \le 3\pi$ is the particle stationary? **3**

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(b) At what times during the period $0 \le t \le 3\pi$ is the acceleration equal to 0? **3**

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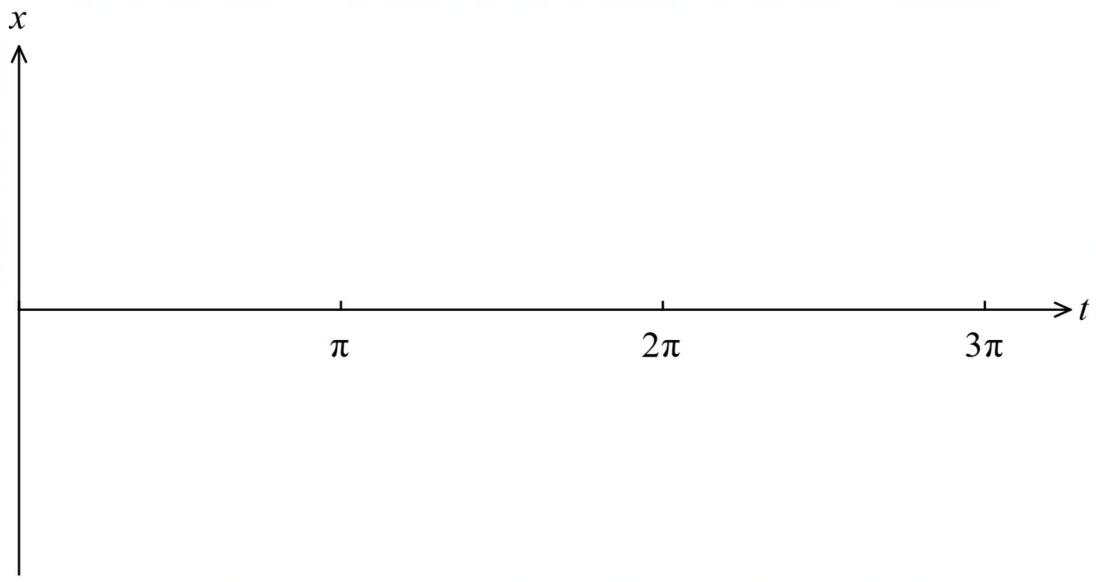
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(c) Carefully sketch the graph of $x = \frac{t}{2} + \sin t$ for $0 \le t \le 3\pi$. **4**

Clearly label any stationary points and any points of inflection.



Question 17 (3 marks)

It is given that $y = f(x)g(x)$, where $f(2) = -3$, $f'(2) = 1$, $g(2) = 8$, and $g'(2) = 4$. **3**

What is the equation of the tangent to $y = f(x)g(x)$ at the point on the curve where $x = 2$?

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Question 18 (3marks)

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Find the exact value of $\int_0^{\frac{1}{3}} (e^{3x} + x)\, dx$.

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Question 19 (3 marks)

For what values of x is $f(x) = x^3 - 2x^2$ increasing? **3**

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Question 20 (7 marks)

The probability distribution for a discrete random variable X is given by

$$P(X = x) = \begin{cases} bx(x-2) & x = 3, 4, 5, 6 \\ 0 & \text{otherwise} \end{cases}$$

(a) Show that the value of the constant b is $\frac{1}{50}$ **3**

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(b) Calculate the value of $E(X)$ **2**

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(c) Find $P(x \leq E(X))$ **2**

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Question 21 (3 marks)

Without use of calculus, sketch the graph of $y = \frac{1}{2-x} + 4$. **3**

Clearly show any asymptotes and intercepts with the coordinate axes.

Question 22 (2 marks)

Show that $\sec(\pi - x)\left(1 - \sin^2(\pi - x)\right) = -\cos x$ for all x within the natural domain. **2**

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Question 23 (2 marks)

Find $\int \frac{\tan^2 x + 1}{\tan x + 1}\, dx$ **2**

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Question 24 (3 marks)

Solve $\tan(2\theta - 50)° = -1$ for $0° \leq \theta \leq 180°$ **3**

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Question 25 (4 marks)

A continuous random variable X has probability density function f(x) given by

$$f(x)=\begin{cases}\dfrac{3}{4\sqrt{2}}\sqrt{x-2} & 2\le x\le 4\\ 0 & \text{otherwise}\end{cases}$$

(a) What is the mode of the distribution? **1**

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(b) Find the median of the distribution. **3**

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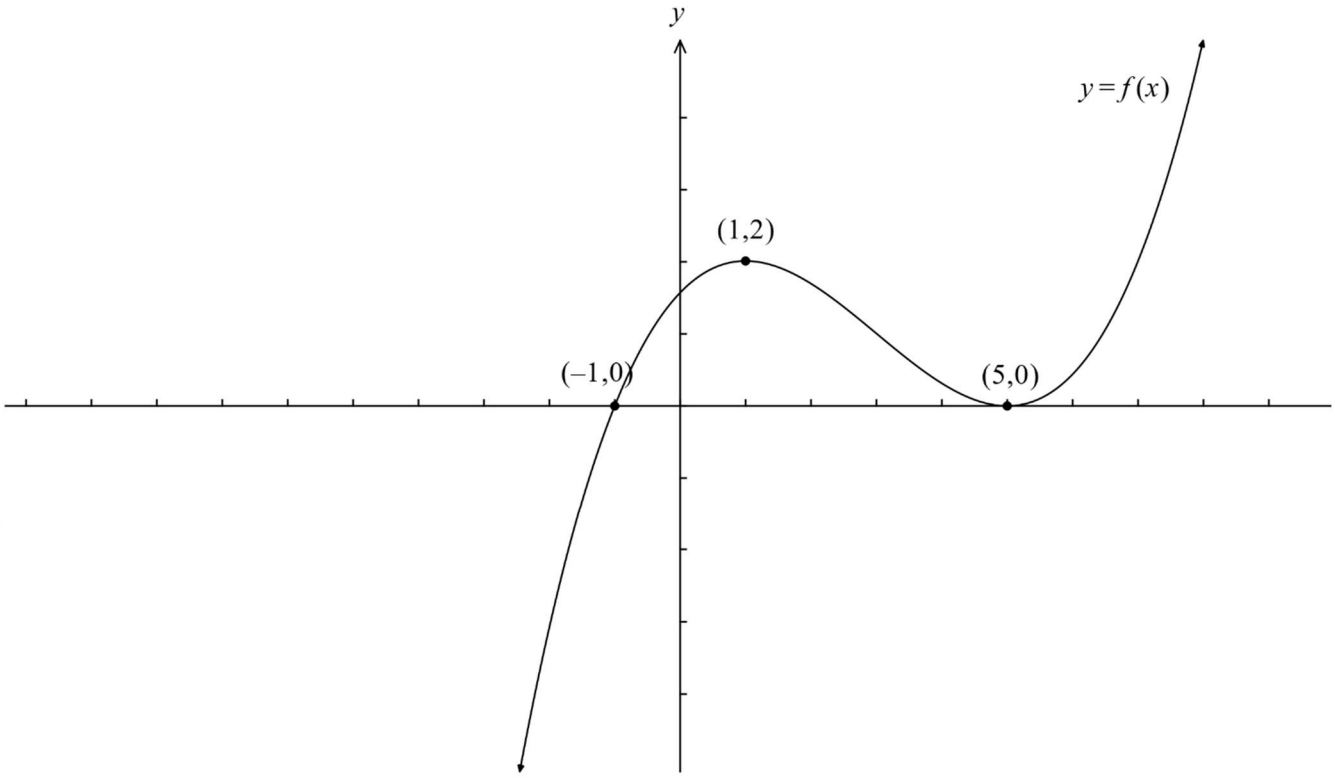
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Question 26 (3 marks)

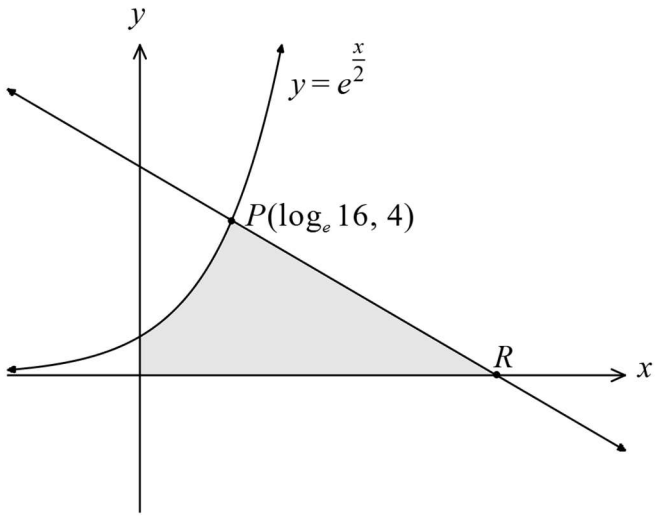
Consider the graph of $y = f(x)$ as shown. **3**



Sketch the graph of $y = f(2 - x)$ and label the coordinates of the turning points and x intercepts.

Question 27 (8 marks)

A normal is drawn to the curve $y = e^{\frac{x}{2}}$ at the point $P(\log_e 16, 4)$. The normal cuts the x axis at R.



NOT TO SCALE

(a) Show that the equation of the normal at P is $y = -\frac{x}{2} + 4 + \log_e 4$ **3**

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(b) Find the coordinates of R. **1**

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(c) Determine the area of the shaded region bound by the curve $y = e^{\frac{x}{2}}$, the normal at P and the coordinate axes. **4**

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Question 28 (9 marks)

A team of environmental scientists studied the relationship between water temperature (in $°C$) and the number of fish in a creek. On each occasion where data was collected, the number of fish, N, and the water temperature, t, were recorded.

The number of fish found on each occasion when data was collected is approximately normally distributed with mean of 42 and standard deviation of 6. The scientists were able to use the least squares regression line to model the data collected with equation $N = -2t + 65$ and Pearson's correlation coefficient for this model is found to be -0.92 .

(a) Explain the meaning of the gradient of the least squares regression line in the context of the data **1**

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(b) Use the model to predict the number of fish on an occasion when the temperature of the water is $17°$. **1**

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(c) What does the Pearson's correlation coefficient indicate about the strength and direction of the relationship between the temperature of the water and the number of fish? **2**

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(d) Find the proportion of samples recorded that contained at least 36 fish. **2**

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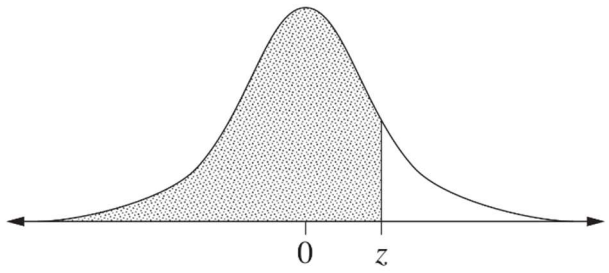
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(e) A random variable is normally distributed with mean 0 and standard deviation 1. The table gives the probability that this random variable is less than z. **3**

z	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4
Probability	0.7257	0.7580	0.7881	0.8159	0.8413	0.8643	0.8849	0.9032	0.9192

The probability values given in the table for different values of z are represented by the shaded area in the following diagram.



The environmental scientists determine that the creek becomes over populated when the water temperature drops below $T°C$. If the creek was over-populated on approximately 11.5% of occasions when the data was collected, find the value of T.

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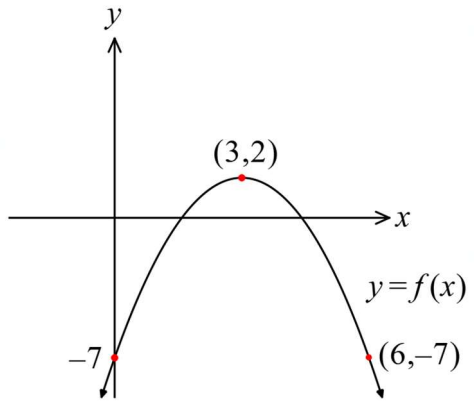
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Question 29 (5 marks)

The graph of $y = f(x)$, where $f(x) = a(x-b)^2 + c$ passes through $(0, -7)$, $(3, 2)$ and $(6, -7)$ as shown in the diagram.



(a) Find the values of a, b and c. **3**

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(b) The graph of $y = g(x)$ is obtained by reflecting $y = f(x)$ through the x – axis then translating it down by k units. Determine the value of k such the solution to $f(x) \geq g(x)$ is $1 \leq x \leq 5$. **2**

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Question 30 (6 marks)

A manufacturer is designing a box in the shape of a rectangular prism with a square base. The volume of the box must be 40 cubic centimetres. The manufacturer determines that the cost, $\$C$, to produce the box is given by $C = 32x + \frac{1}{2}xh - 72\ln x$ where x is the side length of the square and h is the height of the box.

(a) Show that $\frac{dC}{dx} = 32 - \frac{20}{x^2} - \frac{72}{x}$. **3**

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(b) Determine the minimum cost of producing the box. **3**

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End of examination

Additional Working Space

If you use this space, please clearly indicate which question you are answering.

Additional Working Space

Solutions

Gosford High School

Year 12 Trial HIGHER SCHOOL CERTIFICATE EXAMINATION

2026

Mathematics Advanced

General Instructions

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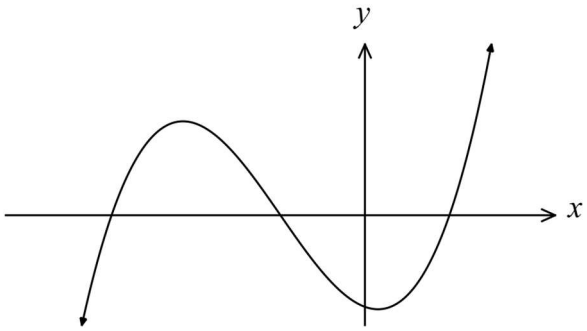
Section I
Multiple Choice answer sheet

2026 Mathematics Advanced – Trial HSC

Write or stick your student number here

1.	A ○	B ●	C ○	D ○
2.	A ○	B ○	C ○	D ●
3.	A ○	B ○	C ●	D ○
4.	A ●	B ○	C ○	D ○
5.	A ○	B ●	C ○	D ○
6.	A ●	B ○	C ○	D ○
7.	A ●	B ○	C ○	D ○
8.	A ○	B ○	C ●	D ○
9.	A ○	B ○	C ○	D ●
10.	A ●	B ○	C ○	D ○

1 The graph of $y = f(x)$ is shown.



Which of the following could be the equation of this graph?

A. $y = (x^2 - 1)(x - 3)$

B. $y = (x^2 - 1)(x + 3)$

C. $y = (x - 1)^2(x - 3)$

D. $y = (x - 1)^2(x + 3)$

A: Roots at $x = \pm 1, 3$ ➔ Two distinct positive roots. Eliminate A

B: Roots at $x = \pm 1, -3$ ➔ Two distinct negative roots, and correct shape.

C: Double root at $x = 1$, single root at $x = 3$. Eliminate C

D: Double root at $x = 1$, single root at $x = -3$. Eliminate D

2 Consider the following dataset.

10 (15) 15 | 16 (20) 24

Which row of the table shows how the interquartile range and standard deviation are affected when a score of 18 is added to the dataset?

	Interquartile range	*Standard deviation*
A.	Changes	Increases
B.	Stays the same	Increases
C.	Changes	Decreases
D.	Stays the same	Decreases

Existing

$IQR = 20 - 15 = 5$ $\quad \bar{x} = 16\frac{2}{3}$ $\quad \sigma = 4.38$

New

$IQR = 20 - 15 = 5$

18 is closer to the mean than the st.dev, therefore decreasing the st. dev.

3 What is the domain of $f(x)=\dfrac{1}{\sqrt{4-x^2}}$

A. $x<2$

B. $x\leq 2$

C. $-2<x<2$

D. $-2<x<0 \cup 0<x<2$

$4-x^2>0$

$x^2<4$

$\therefore \quad -2<x<2$

4 What is $\int \pi\sin(\pi x)\,dx$?

A. $-\cos(\pi x)+c$

B. $-\pi^2\cos(\pi x)+c$

C. $\pi\cos(\pi x)+c$

D. $\dfrac{\pi\sin^2(\pi x)}{2}+c$

$\int f'(x)\sin f(x)\,dx=-\cos f(x)+c$

5 Which of the following is the range of $y=-(x-5)^2+3$?

A. $(-\infty,5]$

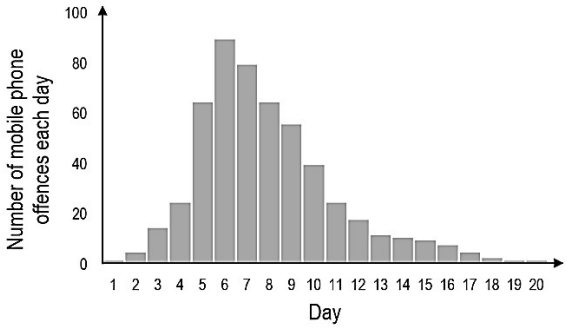
B. $(-\infty,3]$

C. $[5,\infty)$

D. $[3,\infty)$

$y=-(x-5)^2+3$ is a concave down parabola with vertex at (5, 3). The maximum value of the function is 3.

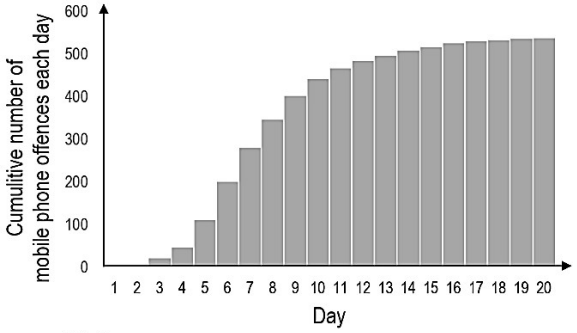
6 A mobile phone detection camera that uses AI and infrared to target illegal phone use by drivers was recently installed in Woodbridge. The number of offences detected each day over a 20 day period is shown in the following graph.



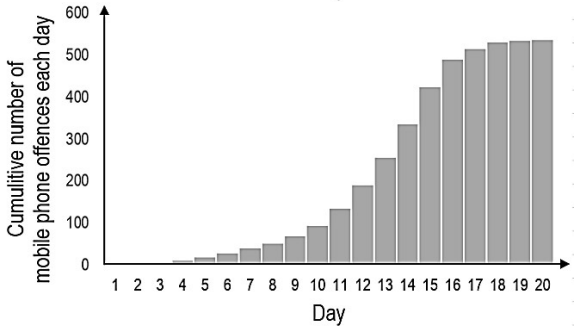
The biggest increase in c.f. occurs on days 6 and 7.

Which of the following graphs best shows the cumulative number of mobile phone offences detected up to and including each day?

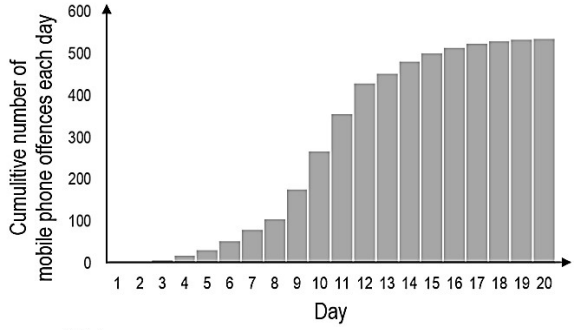
A.



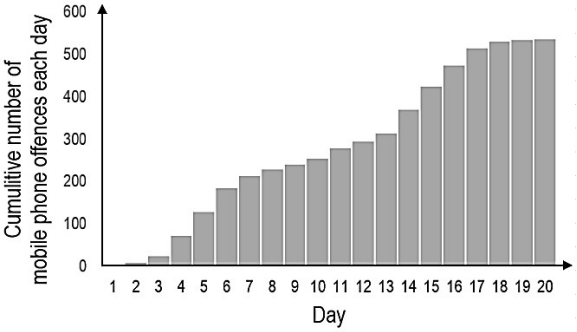
B.



C.



D.



7 How many solutions of the equation $(\cot x + 1)(\cos^2 x - 1) = 0$ lie between 0 and 2π?

A. 2

B. 3

C. 4

D. 5

Note that $\cot x$ is not defined for x = multiples of π. If any of these values of x are found as solutions, they must be disregarded as the original equation will not be defined for those values.

$\cot x = -1$ $\quad\quad$ $\cos^2 x = 1$

$\tan x = -1$ $\quad\quad$ $\cos x = \pm 1$

$x = \frac{3\pi}{4}, \frac{7\pi}{4}$ $\quad\quad$ $x = 0, \pi, 2\pi$

But $\cot x$ is not defined for these values.

Note that the 2014 HSC contained a similar question to this and only 25% of students got this question correct. The 2024 GHS Trial HSC also had a question similar to this. Students clearly have not fully understood or followed up on why some questions have restrictions on the possible answers based on the way the initial problem is defined.

8 A is known to vary inversely with the square of x. Which equation best represents the relation between A and x?

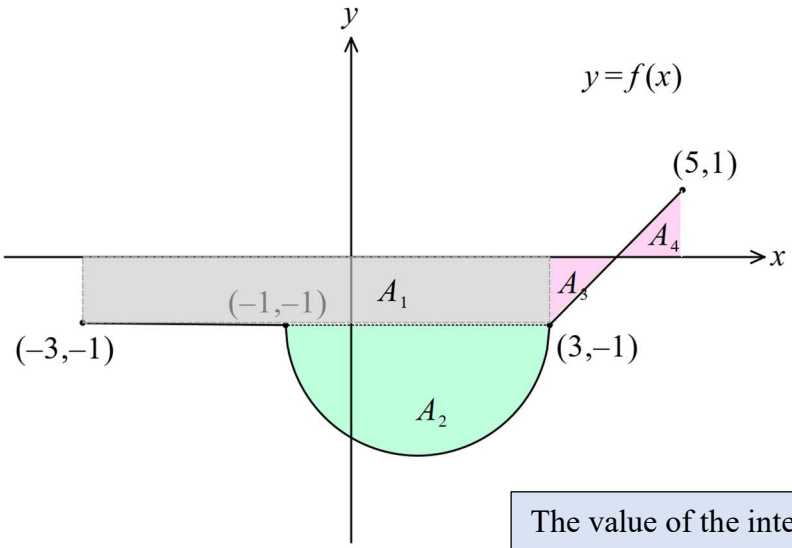
A. $A^2 = \frac{k}{x}$

B. $A^2 = kx$

C. $A = \frac{k}{x^2}$

D. $A = kx^2$

9 $y = f(x)$ is continuous and consists of a semi-circle joined to straight line segments as shown.



What is the value of $\int_{-3}^{5} f(x)\,dx$?

A. $2\pi - 6$

B. $3 + 2\pi$

C. $-3 - 2\pi$

D. $-2(3 + \pi)$

The value of the integral will be the sum of the signed areas.

$$\int_{-3}^{5} f(x)\,dx = -A_1 - A_2 - A_3 + A_4$$

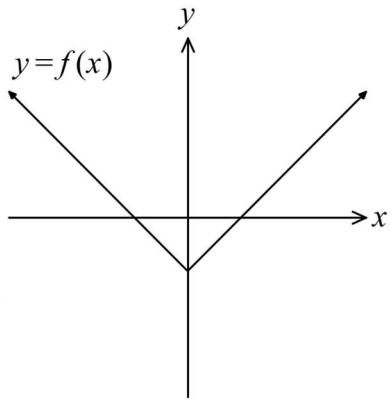
$$= -A_1 - A_2 \text{ since } A_3 = A_4$$

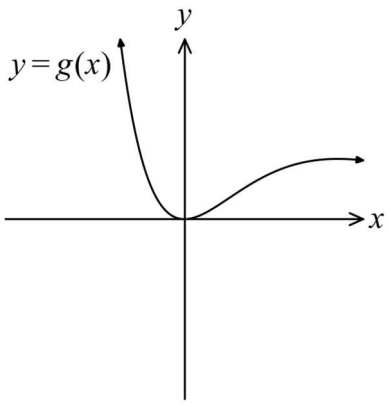
$$= -(1 \times 6) - \left(\frac{1}{2}\pi \times 2^2\right)$$

$$= -6 - 2\pi$$

$$= -2(3 + \pi)$$

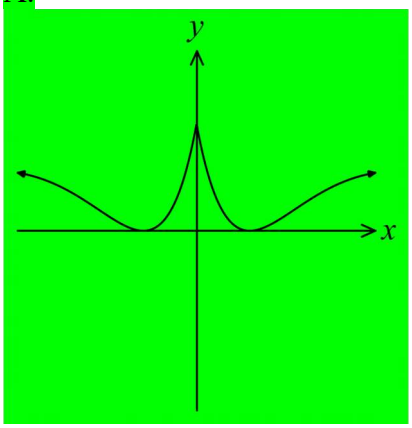
10 The graphs of $y=f(x)$ and $y=g(x)$ are shown.



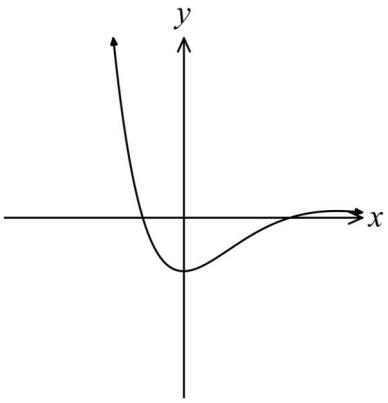


Which graph best represents $g(f(x))$?

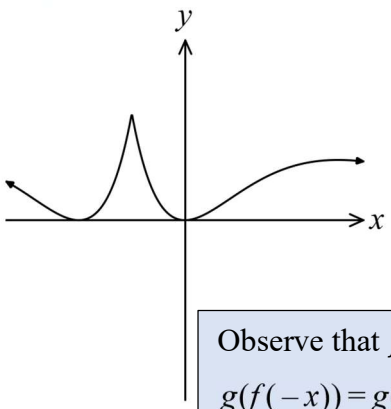
A.



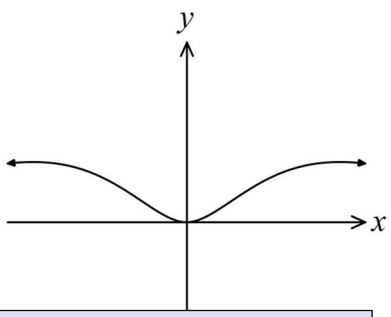
B.



C.



D.



Observe that $f(x)$ is an even function.

$g(f(-x))=g(f(x))$ since $f(x)$ is even.

Hence $g(f(x))$ is also even.

Question 11 (2 marks)

Solve $(5x-1)^2=16$ **2**

$$(5x-1)^2=16$$
$$5x-1=\pm 4$$
$$5x=1\pm 4$$
$$=-3 \text{ or } 5$$
$$\therefore \quad x=-\frac{3}{5} \text{ or } x=1$$

Mark	Criteria
2	Correct solution
1	Finds one correct solution, or, demonstrates knowledge of an appropriate method but contains an error.

Question 12 (2 marks)

Solve $e^{2\ln x}=x+12$ **2**

$$e^{2\ln x}=x+12,\ x>0 \text{ so that } \ln x \text{ is defined.}$$
$$(e^{\ln x})^2=x+12$$
$$x^2=x+12$$
$$x^2-x-12=0$$
$$(x-4)(x+3)=0$$
$$x=4 \text{ or } x=-3$$

But $x>0$ $\therefore x=4$ only.

Mark	Criteria
2	Correct solution
1	Applies appropriate mathematics to reduce the problem to a quadratic equation, or equivalent merit.

Question 13 (6 marks)

A box contains 10 chocolates that appear identical. However, 6 of the chocolates have yellow centres and 4 have green. Todd ate three chocolates chosen randomly from the box. Find the probability that:

(a) the first two chocolates that Todd ate had a yellow centre **2**

$$P(Y,Y)=\frac{6}{10}\times\frac{5}{9}$$
$$=\frac{1}{3}$$

Mark	Criteria
2	Correct solution
1	Applies the product rule, but does not consider that the chocolates are not replaced (as Todd ate them)

(b) at least two of the chocolates that Todd ate had yellow centres **2**

$$P(\geq 2Y)=P(2Y)+P(3Y)$$
$$=3\left(\frac{6}{10}\times\frac{5}{9}\times\frac{4}{8}\right)+\left(\frac{6}{10}\times\frac{5}{9}\times\frac{4}{8}\right)$$
$$=\frac{2}{3}$$

Mark	Criteria
2	Correct solution
1	Finds the probability that either two are yellow, or three are yellow, or equivalent merit.

(c) all of the chocolates that Todd ate had yellow centres given that two were known to have been yellow. **2**

$$P(3Y \mid \geq 2Y) = \frac{P(3Y)}{P(\geq 2Y)} \text{ fdfd}$$

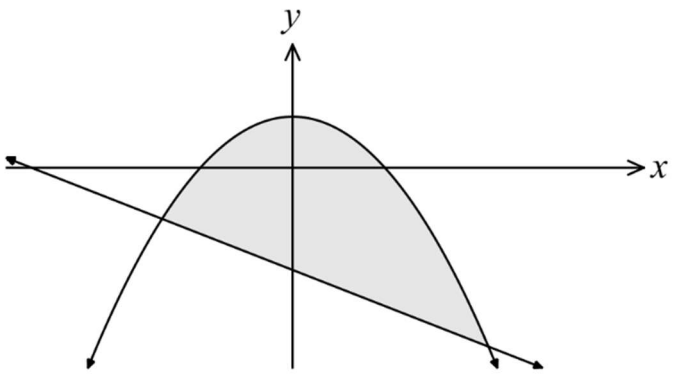
$$= \frac{\frac{1}{6}}{4\left(\frac{1}{6}\right)}$$

$$= \frac{1}{4}$$

Mark	Criteria
2	Correct solution, allowing for errors in (b) to be carried through
1	Attempts to apply the conditional probability formula and incorporates the probability that P(YYY)

Question 14 (5 marks)

The curves $y = 2 - x^2$ and $y = -x - 4$ intersect at two points as shown in the diagram.



NOT TO SCALE

(a) Find the x ordinates of the points of intersection of the two curves. **2**

Solving simultaneously:

$y = 2 - x^2$ ①

$y = -x - 4$ ②

Substitute $y = 2 - x^2$ into ②

$2 - x^2 = -x - 4$

$0 = x^2 - x - 6$

$0 = (x - 3)(x + 2)$

$\therefore \quad x = -2$ or $x = 3$ at the points of intersection.

Mark	Criteria
2	Correct solution
1	Attempts to solve the equations simultaneously

(b) Find the area enclosed by the two curves. **3**

$$\text{Area} = \int_{-2}^{3} (2-x^2)-(-x-4)\,dx$$

$$= \int_{-2}^{3} -x^2+x+6\,dx$$

$$= \left[-\frac{x^3}{3}+\frac{x^2}{2}+6x\right]_{-2}^{3}$$

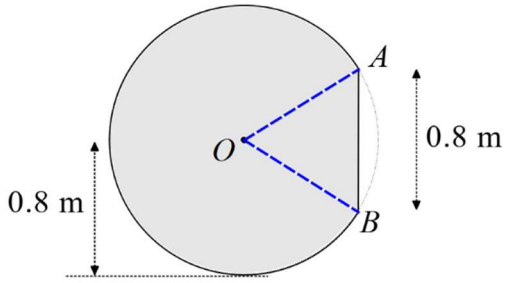
$$= \left(-\frac{(3)^3}{3}+\frac{(3)^2}{2}+6(3)\right)-\left(-\frac{(-2)^3}{3}+\frac{(-2)^2}{2}+6(-2)\right)$$

$$= 20\tfrac{5}{6} \text{ sq. units.}$$

Mark	Criteria
3	Correct solution
	1 mark each for: • Expression that gives required area • Finds primitive function • Substitution of bounds into primitive function

Question 15 (4 marks)

A small table-top is in the shape of a circle with a small segment removed as shown. The circle has centre O and radius 0.8 metres. The length of the straight edge AB is also 0.8 metres.



NOT TO SCALE

(a) Explain why $\angle AOB = \frac{\pi}{3}$ **1**

$OA = OB = 0.8$ m, (radius of circle centred at O)

$AB = 0.8$ m, (Given information)

Hence ΔAOB is equilateral and $\angle AOB = \frac{\pi}{3}$

Mark	Criteria
1	States that ΔAOB is equilateral, or equivalent merit

(b) Find the exact area of the table-top. **3**

$$\begin{aligned}\text{Area} &= \text{Area(Major sector)} + \text{Area}\left(\Delta AOB\right)\\ &= \frac{1}{2}(0.8)^2\left(2\pi - \frac{\pi}{3}\right) + \frac{1}{2}(0.8)^2\sin\left(\frac{\pi}{3}\right)\\ &= \frac{1}{2}\times\left(\frac{4}{5}\right)^2\left(\frac{5\pi}{3}\right) + \frac{1}{2}\times\left(\frac{4}{5}\right)^2\left(\frac{\sqrt{3}}{2}\right)\\ &= \frac{8\pi}{15} + \frac{4\sqrt{3}}{25}\ m^2\\ &= \frac{40\pi + 12\sqrt{3}}{75}\ m^2\end{aligned}$$

Mark	Criteria
3	Correct solution with the answer given in exact form.
	Allow 1 mark each for valid expressions to determine: • Area of a sector (major or minor) • Area of ΔAOB

Question 16 (10 marks)

A particle is moving along the x axis. Its position at time t is given by $x = \frac{t}{2} + \sin t$.

(a) At what times during the period $0 \le t \le 3\pi$ is the particle stationary? **3**

$$\begin{aligned}&\text{Stationary} \Rightarrow \dot{x} = 0\\ &\dot{x} = \frac{1}{2} + \cos t\\ &0 = \frac{1}{2} + \cos t\\ &\cos t = -\frac{1}{2},\ 0 \le t \le 3\pi\\ &t = \pi - \frac{\pi}{3},\ \pi + \frac{\pi}{3},\ 3\pi - \frac{\pi}{3}\\ &\therefore\quad t = \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{8\pi}{3}\end{aligned}$$

Mark	Criteria
3	Correct solution.
	Allow 1 mark each for: • Differentiates to find the velocity function • Finds a solution by solving v=0 in the given domain

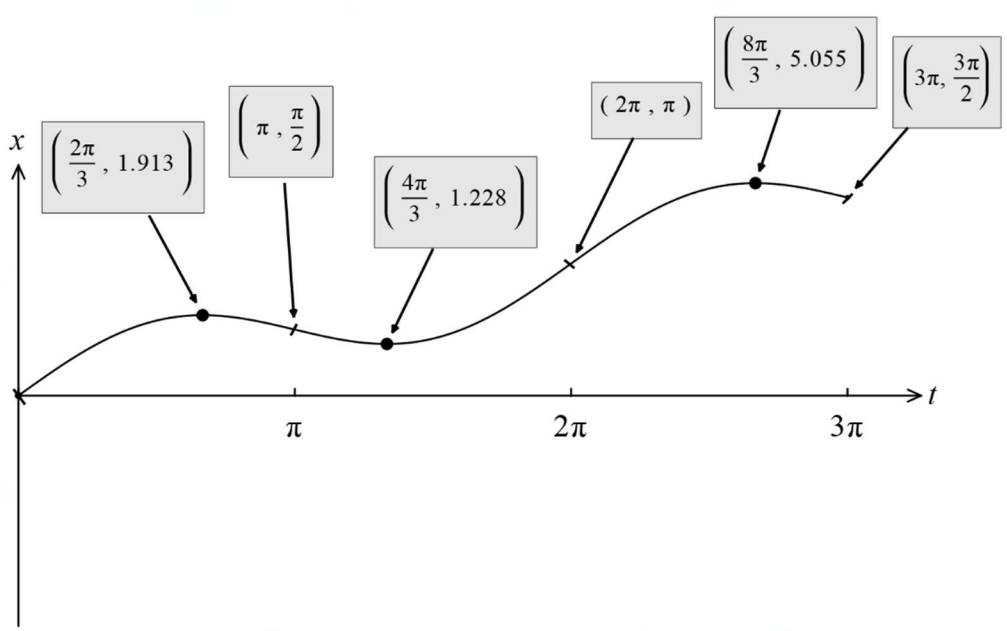
(b) At what times during the period $0 \le t \le 3\pi$ is the acceleration equal to 0? **3**

$$\begin{aligned}&\text{Acceleration } 0 \Rightarrow \ddot{x} = 0\\ &\ddot{x} = -\sin t\\ &0 = \sin t,\ 0 \le t \le 3\pi\\ &t = 0,\ \pi,\ 2\pi,\ 3\pi\end{aligned}$$

Mark	Criteria
3	Correct solution.
	Allow 1 mark each for: • Differentiates to find the acceleration function • Finds a solution by solving a=0 in the given domain

(c) Carefully sketch the graph of $x = \frac{t}{2} + \sin t$ for $0 \leq t \leq 3\pi$. **4**

Clearly label any stationary points and any points of inflection.



Mark	Criteria
4	Correct graph within required domain, with stationary points, points of inflection and endpoints clearly shown.
3	Correct graph drawn with most features labelled, or, the location of all features found but the graph is poorly drawn or does not reflect the location of the key features.
2	Part of the graph is correctly drawn with some of the key features located.
1	General shape of graph matching the required information OR some y ordinates have been determined for the key points but no progress

Question 17 (3 marks)

It is given that $y = f(x)g(x)$, where $f(2) = -3, f'(2) = 1, g(2) = 8$, and $g'(2) = 4$. **3**

What is the equation of the tangent to $y = f(x)g(x)$ at the point on the curve where $x = 2$?

When $x = 2$, $y = f(2)g(2)$

$= (-3)(8)$

$= -24$

Gradient of tangent: Evaluate y' at $x = 2$:

$y' = f(2)g'(2) + g(2)f'(2)$

$= (-3)(4) + (8)(1)$

$= -4$

Equation of tangent $y - y_1 = m(x - x_1)$:

$y - f(2)g(2) = -4(x - 2)$

$y - (-24) = -4(x - 2)$

$\therefore \quad y = -4x - 16$

Mark	Criteria
3	Correct solution
	Allow 1 mark each for • Determining the *y* value where $x = 2$ **and** demonstrates further valid engagement with the question • Applying the product rule • Using the point gradient formula with appropriate values, or equivalent merit.

Question 18 (3marks)

Find the exact value of $\int_0^{\frac{1}{3}} (e^{3x} + x)\, dx$. **3**

$$\int_0^{\frac{1}{3}} (e^{3x} + x)\, dx = \left[\frac{e^{3x}}{3} + \frac{x^2}{2}\right]_0^{\frac{1}{3}}$$

$$= \left(\frac{e^{3\left(\frac{1}{3}\right)}}{3} + \frac{\left(\frac{1}{3}\right)^2}{2}\right) - \left(\frac{e^{3(0)}}{3} + \frac{(0)^2}{2}\right)$$

$$= \frac{e}{3} + \frac{1}{18} - \frac{1}{3} - 0$$

$$= \frac{e}{3} - \frac{5}{18}$$

$$= \frac{6e - 5}{18}$$

Mark	Criteria
3	Correct solution
	Allow 1 mark each for: • Finds the primitive of $e^{3x} + x$ • Substitutes bounds into the primitive function • Simplifies to an appropriate form (i.e. applies index laws where appropriate.)

Question 19 (3 marks)

For what values of x is $f(x)=x^3-2x^2$ increasing? **3**

$$f'(x)=3x^2-4x$$

Increasing $\Rightarrow f'(x)>0$

$$3x^2-4x>0$$

$$x(3x-4)>0$$

$$\therefore \quad x<0 \text{ or } x>\frac{4}{3}$$

Mark	Criteria
3	Correct solution supported by appropriate mathematical reasoning
2	Finds the derivative and identifies the bounds of the inequality
1	Allow 1 mark each for • Finds the derivative • Identifies the correct critical values for their quadratic inequality • Correct direction of inequality signs for their quadratic inequality

Question 20 (7 marks)

The probability distribution for a discrete random variable X is given by

$$P(X=x)=\begin{cases} bx(x-2) & x=3,\ 4,\ 5,\ 6 \\ 0 & \text{otherwise} \end{cases}$$

(a) Show that the value of the constant b is $\frac{1}{50}$ **3**

x	3	4	5	6
$P(X=x)$	$3b$	$8b$	$15b$	$24b$

$$3b+8b+15b+24b=1 \quad (\Sigma p(x)=1)$$

$$50b=1$$

$$\therefore \quad b=\frac{1}{50}$$

Mark	Criteria
3	Shows the required result by applying appropriate properties of discrete random variables.
	Allow 1 mark each for • Demonstrates knowledge that $\Sigma\, P(X=x)=1$ • Finds valid expressions for each $P(X=x)$ • Uses the method for a continuous probability distribution to find $b=\frac{1}{36}$

(b) Calculate the value of $E(X)$ **2**

$$E(X)=\Sigma(x\times p(x))$$

$$=3(3b)+4(8b)+5(15b)+6(24b)$$

$$=260b$$

$$=260\times\frac{1}{50}$$

$$=5.2$$

Mark	Criteria
2	Correct solution
1	Determines $x \times P(x)$ values, or equivalent merit.

(c) Find $P(x \leq E(X))$ **2**

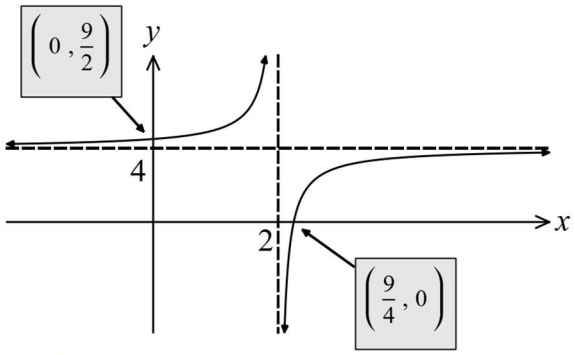
$$\begin{aligned} P(x \leq E(X)) &= P(X=3)+P(X=4)+P(X=5) \\ &= 1-P(X=6) \\ &= 1-\frac{24}{50} \\ &= \frac{26}{50} \\ &= \frac{13}{25} \end{aligned}$$

Mark	Criteria
2	Correct solution
1	Valid progress towards the correct solution.

Question 21 (3 marks)

Without use of calculus, sketch the graph of $y = \dfrac{1}{2-x} + 4$. **3**

Clearly show any asymptotes and intercepts with the coordinate axes.



Mark	Criteria
3	Correct graph with the location of the asymptotes and intercepts clearly marked.
2	Appropriate graph but missing a required detail, or all features identified but the graph does not approach the asymptotes
1	Some features identified, but graph not drawn/correct, or an appropriate graph drawn with some features identified.

Question 22 (2 marks)

Show that $\sec(\pi - x)\left(1 - \sin^2(\pi - x)\right) = -\cos x$ for all x within the natural domain. **2**

$$\begin{aligned} LHS &= \sec(\pi-x)\left(1-\sin^2(\pi-x)\right) \\ &= -\sec(x)\left(\cos^2(\pi-x)\right) \\ &= -\frac{1}{\cos(x)}(-\cos(x))^2 \\ &= -\frac{\cos^2 x}{\cos x} \\ &= -\cos(x) \\ &= RHS \text{ as required} \end{aligned}$$

Mark	Criteria
2	Valid, clearly communicated solution.
1	Demonstrates knowledge of the relationship between $sec(\pi - x)$ and $-\cos x$, or applies the Pythagorean identity, or equivalent merit.

Question 23 (2 marks)

Find $\int \frac{\tan^2 x + 1}{\tan x + 1}\, dx$ **2**

$$\int \frac{\tan^2 x + 1}{\tan x + 1}\, dx = \int \frac{\sec^2 x}{\tan x + 1}\, dx \text{ which is of the form } \int \frac{f'(x)}{f(x)} dx$$
$$= \ln|\tan x + 1| + c$$

Mark	Criteria
2	Correct solution including the use of the absolute value signs and constant term.
1	Applies the relevant Pythagorean identity, or equivalent merit.

Question 24 (3 marks)

Solve $\tan(2\theta - 50)° = -1$ for $0° \le \theta \le 180°$ **3**

$$\tan(2\theta - 50)° = -1,\ -50° \le 2\theta - 50° \le 310°$$
$$2\theta - 50° = -45°,\ 135°$$
$$2\theta = 5°, 185°$$
$$\therefore \quad \theta = 2.5°,\ 92.5°$$

Mark	Criteria
3	Correct solution
2	Generates the two correct values for $(2\theta - 50)°$ but does not go on to correctly find θ, or, Generates one correct value for $(2\theta - 50)°$ and correctly solves to find θ
1	Identifies the size of the reference angle and the relevant quadrants in which $2\theta - 50°$ will lie.

Question 25 (4 marks)

A continuous random variable X has probability density function f(x) given by

$$f(x) = \begin{cases} \frac{3}{4\sqrt{2}}\sqrt{x-2} & 2 \le x \le 4 \\ 0 & \text{otherwise} \end{cases}$$

(a) What is the mode of the distribution? **1**

$y = \frac{3}{4\sqrt{2}} \times \sqrt{x-2}$ is a monotonically increasing function. Hence the mode is

$x = 4$

Mark	Criteria
1	Correct answer

(b) Find the median of the distribution. **3**

Let the median be M.

$$\frac{1}{2}=\int_2^M \frac{3}{4\sqrt{2}}\sqrt{x-2}\,dx$$

$$\frac{1}{2}\times\frac{4\sqrt{2}}{3}=\int_2^M \sqrt{x-2}\,dx$$

$$\frac{2\sqrt{2}}{3}=\int_2^M (x-2)^{\frac{1}{2}}\,dx$$

$$=\frac{2}{3}\left[\left((x-2)^{\frac{3}{2}}\right)\right]_2^M$$

$$\sqrt{2}=\left[\left(\sqrt{(x-2)^3}\right]_2^M\right.$$

$$=\sqrt{(M-2)^3}-\sqrt{(2-2)^3}$$

$$=\sqrt{(M-2)^3}$$

$$2=(M-2)^3$$

$$M-2=\sqrt[3]{2}$$

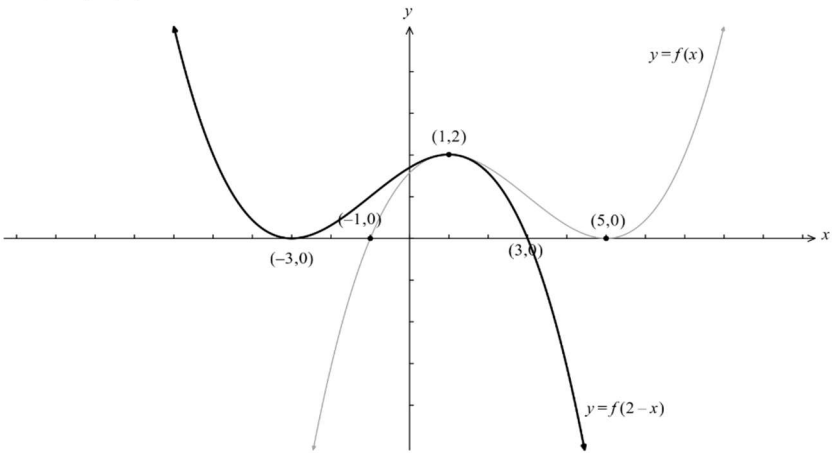
$$\therefore\quad M=2+\sqrt[3]{2}$$

$$\approx 3.2599$$

Mark	Criteria
3	Applies appropriate techniques to proceed to a valid solution
	Allow 1 mark for each of the following: • Find the correct primitive function of $\sqrt{x-2}$ • States that the integral from 2 to M must be equal to 0.5, or equivalent merit.

Question 26 (3 marks)

Consider the graph of $y=f(x)$ as shown. **3**

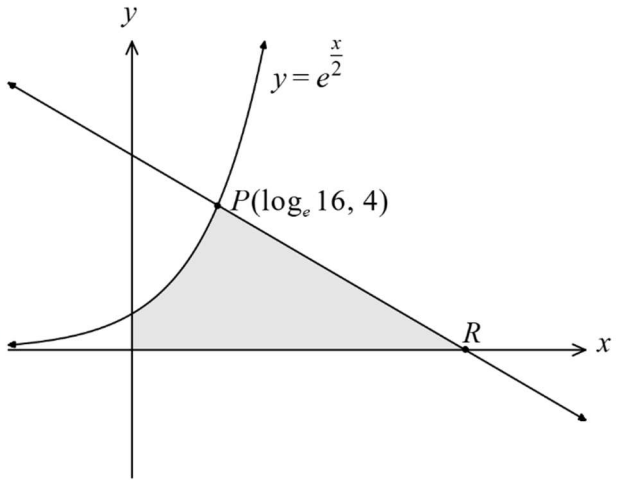


Sketch the graph of $y=f(2-x)$ and label the coordinates of the turning points and x intercepts.

Mark	Criteria
3	Applies transformations in an appropriate order to achieve the correct solution and labels the coordinates of the *x* intercept and turning points.
	Allow 1 mark for each of the following: • Draws a reflection in the y axis • Draw a translation 2 units (either left or right)

Question 27 (8 marks)

A normal is drawn to the curve $y = e^{\frac{x}{2}}$ at the point $P(\log_e 16, 4)$. The normal cuts the x axis at R.



NOT TO SCALE

(a) Show that the equation of the normal at P is $y = -\frac{x}{2} + 4 + \log_e 4$ **3**

$y = e^{\frac{x}{2}}$

$y' = \frac{1}{2} e^{\frac{x}{2}}$

At $x = \ln(16)$

$m_T = \frac{1}{2} e^{\frac{1}{2}\ln(16)}$

$= \frac{1}{2} (e^{\ln(16)})^{\frac{1}{2}}$

$= \frac{1}{2}(16)^{\frac{1}{2}}$

$= \frac{1}{2} \times 4$

$= 2$

$\therefore m_N = -\frac{1}{2}$

Equation of the normal:

$y - y_1 = m(x - x_1)$

$y - 4 = -\frac{1}{2}(x - \ln(16))$

$y = -\frac{x}{2} + \ln\left(16^{\frac{1}{2}}\right) + 4$

$\therefore \quad y = -\frac{x}{2} + 4 + \ln(4)$ as required

Mark	Criteria
3	Clearly communicates the solution using appropriate mathematical language or notation, supported by appropriate justification.
	Allow 1 mark for each of the following • Finding the gradient of the tangent at P • Applying the relationship between the gradients of tangents and normals

(b) Find the coordinates of R. **1**

At R, $y = 0$ for the equation of the normal through P.

$$0 = -\frac{x}{2} + 4 + \ln(4)$$

$$\frac{x}{2} = 4 + \ln(4)$$

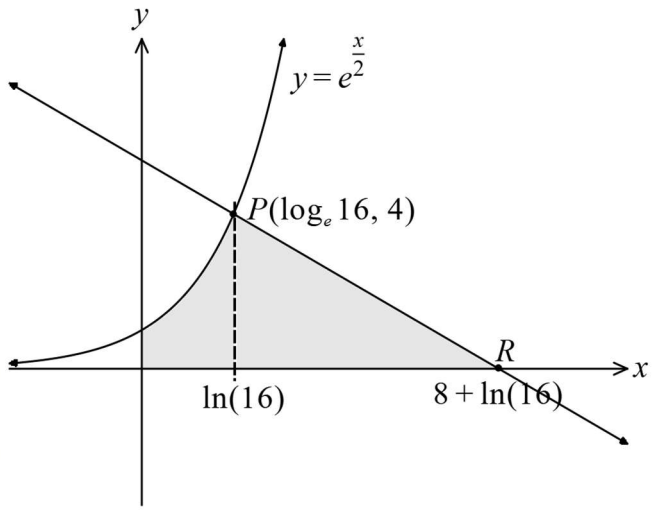
$$x = 8 + 2\ln(4)$$

$$x = 8 + \ln(16)$$

$$\therefore R\left((8 + \ln(16)),\ 0\right)$$

Mark	Criteria
1	Correct solution

(c) Determine the area of the shaded region bound by the curve $y = e^{\frac{x}{2}}$, the normal at P and the coordinate axes. **4**



$$\text{Area} = \int_0^{\ln(16)} e^{\frac{x}{2}}\,dx + \frac{1}{2} \times 8 \times 4 \quad \text{sq.units}$$

$$= 2\int_0^{\ln(16)} \frac{1}{2} e^{\frac{x}{2}}\,dx + 16 \quad \text{sq.units}$$

$$= 2\left[e^{\frac{x}{2}}\right]_0^{\ln(16)} + 16 \quad \text{sq.units}$$

$$= 2\left(e^{\frac{1}{2}\ln(16)} - e^0\right) + 16 \quad \text{sq.units}$$

$$= 22 \text{ sq. units.}$$

Mark	Criteria
4	Correct solution
	Allow 1 mark each for: • Writing a valid expression to determine the area • Finding a primitive of $e^{\frac{x}{2}}$ • Evaluating a definite integral • Finding the area of the triangle

Question 28 (9 marks)

A team of environmental scientists studied the relationship between water temperature (in $°C$) and the number of fish in a creek. On each occasion where data was collected, the number of fish, N, and the water temperature, t, were recorded.

The number of fish found on each occasion when data was collected is approximately normally distributed with mean of 42 and standard deviation of 6. The scientists were able to use the least squares regression line to model the data collected with equation $N = -2t + 65$ and Pearson's correlation coefficient for this model is found to be -0.92 .

(a) Explain the meaning of the gradient of the least squares regression line in the context of the data **1**

The number of fish decreases by 2 for each 1℃ increase in temperature

Mark	Criteria
1	Valid description incorporating the context of the problem

(b) Use the model to predict the number of fish on an occasion when the temperature of the water is $17°$. **1**

$$N = -2(17) + 65$$
$$= 31$$

Mark	Criteria
1	Correct solution

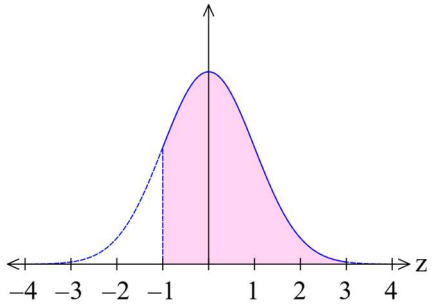
(c) What does the Pearson's correlation coefficient indicate about the strength and direction of the relationship between the temperature of the water and the number of fish? **2**

Strong, negative.

Mark	Criteria
2	Correct solution
1	States the correct strength or direction.

(d) Find the proportion of samples recorded that contained at least 36 fish. **2**

Normally distributed: $\bar{x} = 42, \sigma = 6.\ N = 36 \Longrightarrow z = -1$



$$P(N \geq 36) = P(z \geq -1)$$
$$= \left(\frac{68}{2}\right)\% + 50\%$$
$$= 84\%$$

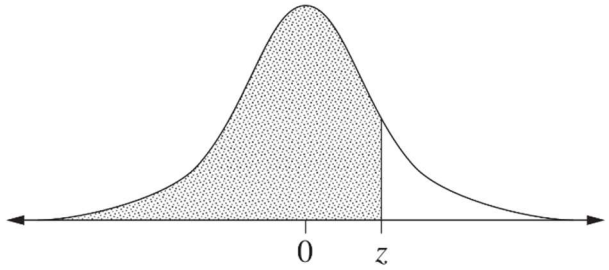
$\therefore$ 84% of samples contained at least 36 fish.

Mark	Criteria
2	Correct solution
1	Determines the z-score, or, calculates a correct probability following on from an incorrect z-score.

(e) A random variable is normally distributed with mean 0 and standard deviation 1. The table gives the probability that this random variable is less than z. **3**

z	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4
Probability	0.7257	0.7580	0.7881	0.8159	0.8413	0.8643	0.8849	0.9032	0.9192

The probability values given in the table for different values of z are represented by the shaded area in the following diagram.



The environmental scientists determine that the creek becomes over populated when the water temperature drops below $T°C$. If the creek was over-populated on approximately 11.5% of occasions when the data was collected, find the value of T.

Let $c =$ the z-score corresponding to the overpopulation limit.

$$P(z > c) = 1 - P(z < c)$$

$$0.115 = 1 - P(z < c)$$

$$\therefore P(z < c) = 0.885$$

The closest value in the table is 0.8849 hence $c = 1.2$

$$z = 1.2 \Rightarrow N = 42 + 1.2 \times 6$$

test to minimise $N = 49.2$

$$N = -2T + 65$$

$$49.2 = -2T + 65$$

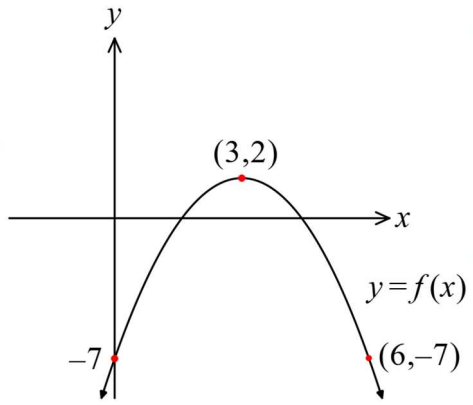
$$T = \frac{65 - 49.2}{2}$$

$$\therefore \quad T = 7.9°C$$

Mark	Criteria
3	Correct solution
	Allow 1 mark for each of the following: • Identifying 0.8849 or z=1.2 from the table • Using a z-score to find a value for N • Using a value of N to determine a value for T.

Question 29 (5 marks)

The graph of $y = f(x)$, where $f(x) = a(x-b)^2 + c$ passes through $(0, -7)$, $(3, 2)$ and $(6, -7)$ as shown in the diagram.



(a) Find the values of a, b and c. **3**

$b = 3$, $c = 2$ from the vertext form of a parabola.

$f(x) = a(x-3)^2 + 2$

$f(0) = -7$

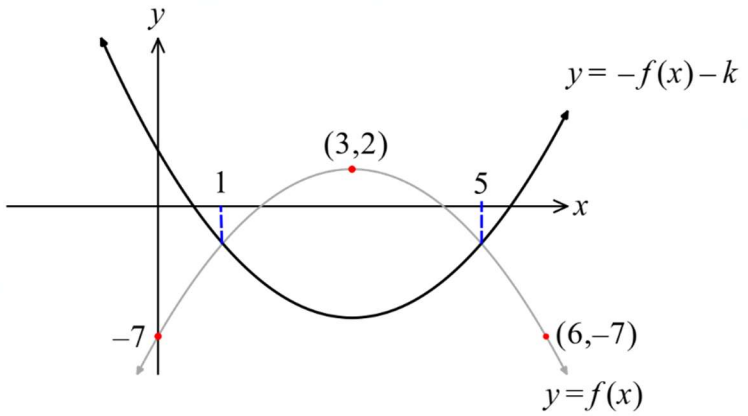
$-7 = a(-3)^2 + 2$

$-9 = 9a$

$\therefore \quad a = -1$

Mark	Criteria
3	Correct answer supported by appropriate mathematical reasoning.
2	Identifies two correct values, or, finds all three values but the answer is not supported by appropriate mathematical reasoning.
1	Identifies correct value, or, uses substitution of given points to generate appropriate simultaneous equations, or equivalent merit.

(b) The graph of $y=g(x)$ is obtained by reflecting $y=f(x)$ through the x – axis then translating it down by k units. Determine the value of k such the solution to $f(x)\geq g(x)$ is $1\leq x\leq 5$. **2**



$$g(x) = -f(x)-k$$
$$= -\left[-(x-3)^2+2\right]-k$$
$$= (x-3)^2-2-k$$

Points of intersection: where $x=1$ or $x=5$, $f(x)=g(x)$.

i.e. $-(x-3)^2+2=(x-3)^2-2-k$

$k=2(x-3)^2-4$

When $x=1$ or $x=5$, $(x-3)^2=4$

$\therefore \quad k=2(4)-4$

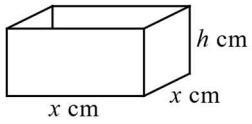
$k=4$

Mark	Criteria
2	Correct solution supported by mathematical reasoning.
1	Draws, or algebraically represents g(x)

Question 30 (6 marks)

A manufacturer is designing a box in the shape of a rectangular prism with a square base. The volume of the box must be 40 cubic centimetres. The manufacturer determines that the cost, $\$C$, to produce the box is given by $C = 32x + \frac{1}{2}xh - 72\ln x$ where x is the side length of the square and h is the height of the box.

(a) Show that $\frac{dC}{dx} = 32 - \frac{20}{x^2} - \frac{72}{x}$. **3**



$$V = x^2h$$
$$40 = x^2h$$
$$\therefore\ h = \frac{40}{x^2}$$

$$C = 32x + \frac{1}{2}xh - 72\ln x$$
$$= 32x + \frac{1}{2}x\left(\frac{40}{x^2}\right) - 72\ln x$$
$$= 32x + 20x^{-1} - 72\ln x$$
$$\frac{dC}{dx} = 32 - 20x^{-2} - 72 \times \frac{1}{x}$$
$$= 32 - \frac{20}{x^2} - \frac{72}{x} \text{ as required.}$$

Mark	Criteria
3	Correct solution supported by clear and appropriate mathematical reasoning.
	Allow 1 mark each for; • Expressing h in terms of x • Finding dC/dx

(b) Determine the minimum cost of producing the box. **3**

Minimum cost occurs when $\frac{dC}{dx} = 0$.

$$0 = 32 - \frac{20}{x^2} - \frac{72}{x}$$
$$= 32x^2 - 72x - 20$$
$$= 4(8x^2 - 18x - 5)$$
$$\therefore\ 0 = (4x + 1)(2x - 5)$$
$$x = -\frac{1}{4} \text{ or } x = \frac{5}{2}$$

But x represents a length hence $x > 0$.

Furthermore, confirm nature: $C'(2.4) = -1.5$, $C'(2.6) = 1.3$ $\therefore$ concave up at $x = 2.5$

$\therefore$ min cost occurs when $x = 2.5$ cm

$$C = 32(2.5) + \frac{20}{2.5} - 72\ln(2.5)$$
$$= 22.027\ldots.$$

$\therefore$ min cost is \$22.03 (nearest cent)

Mark	Criteria
3	Correct solution supported by appropriate justification
2	Finds the minimum cost, or, verifies that the minimum occurs at x=2.5 by using either the first or second derivative.
1	Solves dC/dx=0