HORNSBY GIRLS HIGH SCHOOL

Mathematics Advanced

Year 12 Higher School Certificate Trial Examination
Term 3 2026

STUDENT NUMBER: _____

General Instructions:

- Reading time – 10 minutes
- Working time – 3 hours
- Write using black pen
- Calculators approved by NESA may be used
- For questions in Section II, show relevant mathematical reasoning and/ or calculations
- A NESA reference sheet is provided

Total Marks:
100

Section I – 10 marks (pages 3–7)

- Attempt Questions 1– 10
- Allow about 15 minutes for this section

Section II – 90 marks (pages 9 - 31)

- Attempt Questions 11 - 33
- Allow about 2 hours and 45 minutes for this section

Q1 - 10	Q11	Q12	Q13	Q14	Q15	Q16	Q17	Q18	Q19	Q20	Q21	
/**10**	/**3**	/**4**	/**2**	/**2**	/**3**	/**3**	/**5**	/**3**	/**3**	/**4**	/**4**	
Q22	Q23	Q24	Q25	Q26	Q27	Q28	Q29	Q30	Q31	Q32	Q33	**Total**
/**2**	/**6**	/**3**	/**5**	/**3**	/**9**	/**2**	/**5**	/**3**	/**5**	/**5**	/**6**	/**100**

Outcomes assessed: MA 12 – 1, 3, 4, 5, 6, 7, 8, 10

This assessment task constitutes 30% of the Higher School Certificate Course School Assessment

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Section I

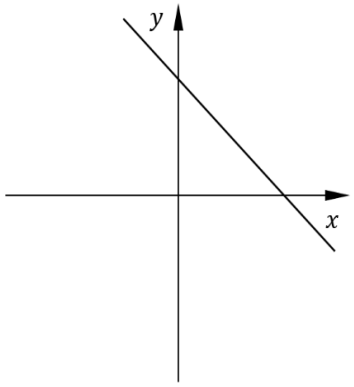
10 marks
Attempt Questions 1 – 10
Allow about 15 minutes for this section

Use the Objective Response answer sheet for Questions 1 – 10

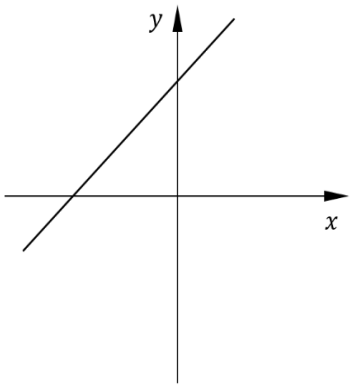
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1 Which of the following could be the graph of $y = x - 1$?

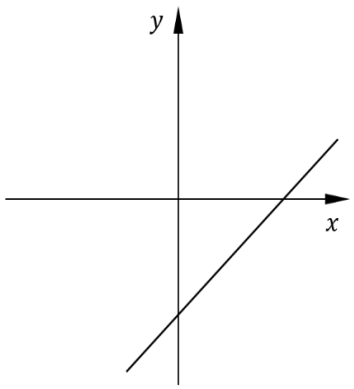
A.



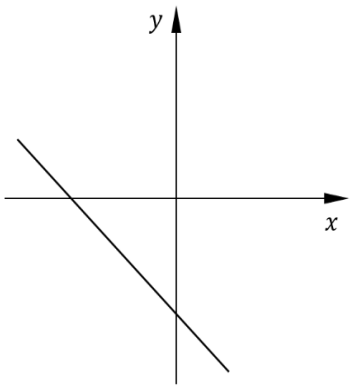
B.



C.



D.



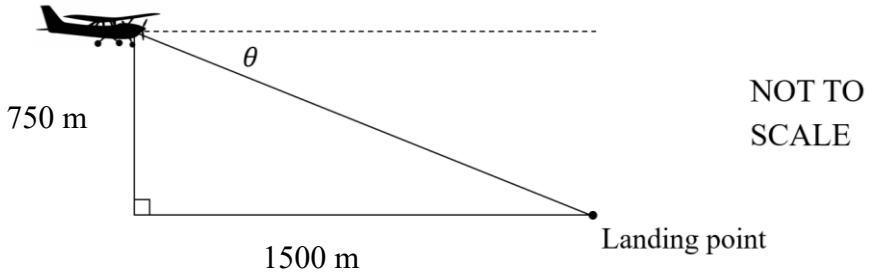
2 Consider the following dataset:

15 18 20 21 22 28

Which row of the table below shows how the mean and standard deviation are affected when each score is increased by 10?

	Mean	*Standard Deviation*
A.	Changes	Changes
B.	Stays the same	Stays the same
C.	Changes	Stays the same
D.	Stays the same	Changes

3 A small plane is approaching an airport to land. It is currently 750 metres above the airport and 1500 metres horizontally from its landing point.



What is the angle of depression, θ, from the plane to its landing point, correct to the nearest degree?

A. 27°

B. 30°

C. 60°

D. 63°

4 Which of the following is the range of the function $f(x) = 2\sqrt{16 - x^2}$?

A. $[-8,8]$

B. $[0,8]$

C. $[0,4]$

D. $[-4,4]$

5 For what values is $y = \dfrac{x^2}{2x^2 + x + 1}$ decreasing?

A. $-2 < x < 0$

B. $x < -2,\ x > 0$

C. $0 < x < 2$

D. $x = 0,\ -2$

6 What is $\int \dfrac{1}{(3x+2)^3}\,dx$

A. $\dfrac{-1}{2(3x+2)^2} + C$

B. $\dfrac{-6}{(3x+2)^2} + C$

C. $\dfrac{-1}{6(3x+2)^2} + C$

D. $\dfrac{-1}{3(3x+2)^2} + C$

7 In a certain Sydney Suburb, the probability of an adult catching the flu is 0.2 . Two adults living in this suburb are chosen at random. The probability of at least one of them catching the flu is:

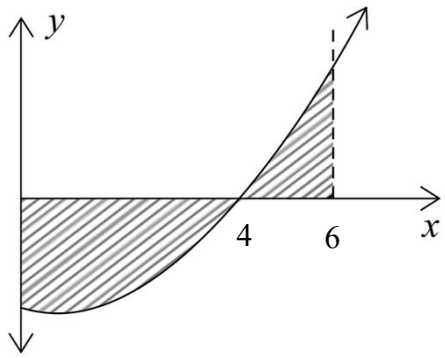
A. 0.04

B. 0.32

C. 0.36

D. 0.64

8 The diagram below is the graph of $y = x^2 + x - 20$.



What is the correct expression for the area bounded by the x-axis and the Graph $y = x^2 + x - 20$ between $0 < x < 6$?

A. $A = \int_0^6 x^2 + x - 20\, dx$

B. $A = \left|\int_0^4 x^2 + x - 20\, dx\right| + \int_4^6 x^2 + x - 20\, dx$

C. $A = \int_0^4 x^2 + x - 20\, dx + \left|\int_4^6 x^2 + x - 20\, dx\right|$

D. $A = \left|\int_0^6 x^2 + x - 20\, dx\right|$

9 A vessel initially contains 100 litres of water.
It is being emptied at the rate of

$$\frac{dv}{dt} = -\left(2 + \frac{20}{t+1}\right)$$ where V is the volume in litres after t minutes.

Which of the following gives the exact volume of water in the vessel after 5 minutes?

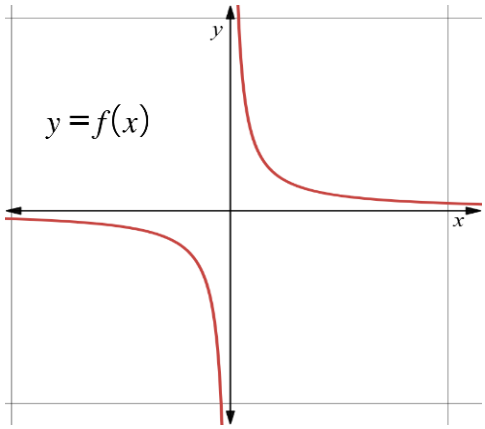
A. $10 + 20\ln 6$

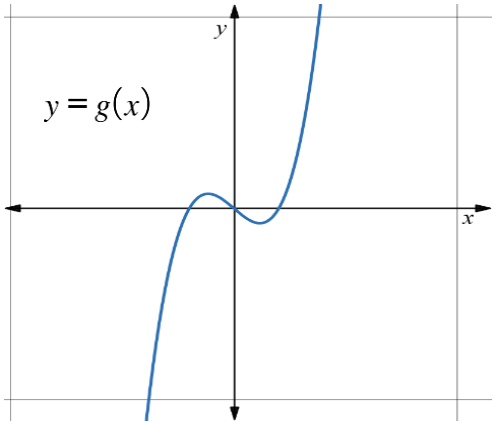
B. $90 - 20\ln 6$

C. $90 + 20\ln 6$

D. $110 + 20\ln 6$

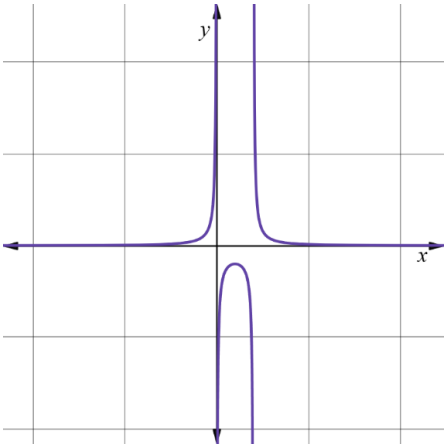
10 The graphs of $y = f(x)$ and $y = g(x)$ are shown.



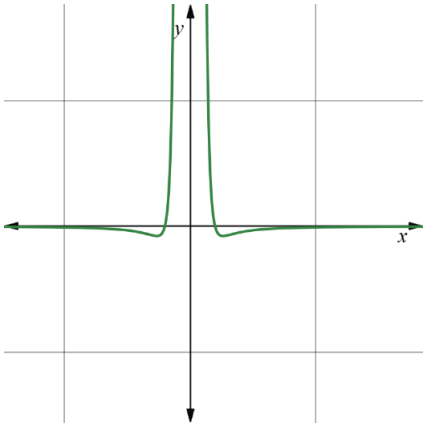


Which graph best represents $y = g(f(x))$?

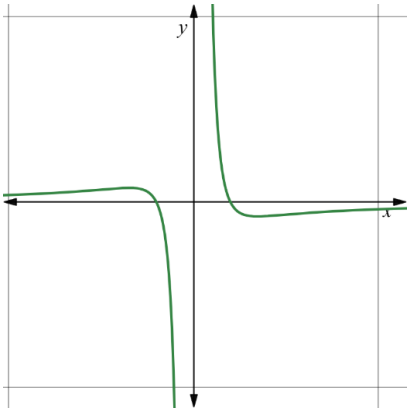
A.



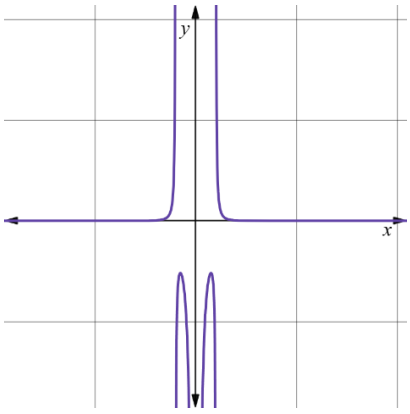
B.



C.



D.



End of Section I

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Section II

90 marks
Attempt questions 11 to 33
Allow about 2 hours and 45 minutes for this section

Answer each question in the spaces provided.
Your responses should include relevant mathematical reasoning and/or calculations.
Extra writing space is provided at the back of the examination paper.

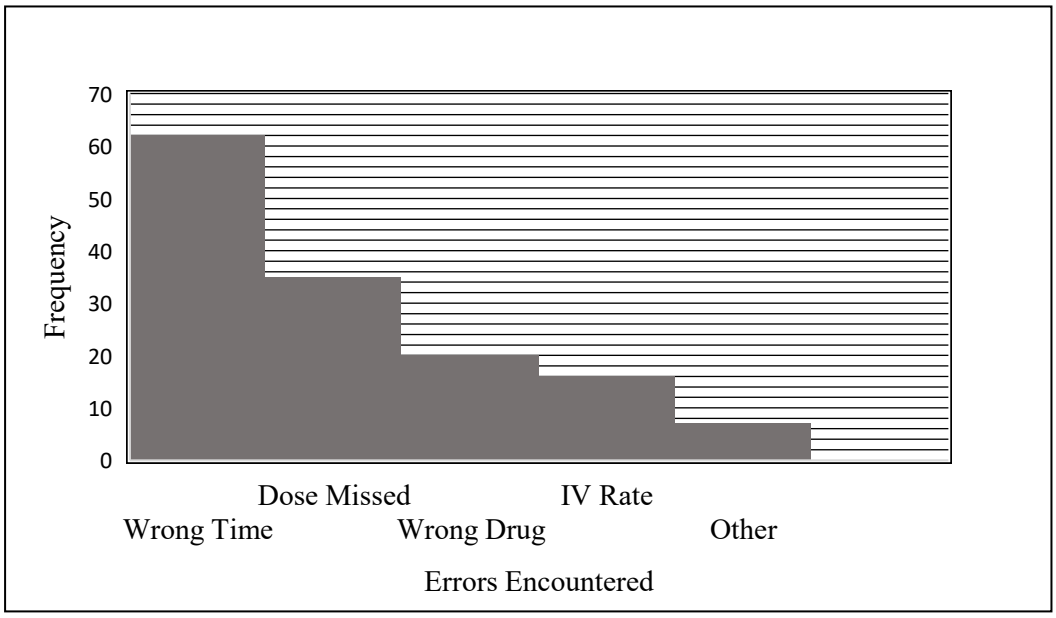
Question 11 (3 marks)

A hospital wants to identify the most significant errors encountered when it comes to administering medications.
A random survey was conducted, and the results are shown in the table below.

Error	**Frequency**
Dose missed	35
Wrong Drug	20
Wrong Time	62
IV rate	16
Other	7

The receptionist was asked to construct a Pareto chart, to illustrate the data collected from the survey.

Below is a partially constructed Pareto chart.



a) Complete the Pareto chart above, by choosing an appropriate scale for the vertical axis on the right-hand side of the Pareto chart and hence drawing a cumulative frequency percentage line graph. **2**

b) What percentage of errors will be solved if there is more focus put on improving the two main errors when administering medication? (Give your answer to 2 decimal places) **1**

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Question 12 (4 marks)

The time (H hours) it takes to travel to a location varies inversely with the speed (S km/hour) travelled. It takes 4 hours driving at an average speed of 60 km/hour to reach a location.

(a) Find the equation relating H and S. **2**

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(b) By first completing the table of values, graph the relationship between H and S. **2**
Express values correct to the nearest hour.

S	100	60	40
H			

Question 13 (2 marks)

Use 2 applications of the trapezoidal rule to find an approximate value of $\int_2^4 \ln x \, dx$. Give your answer to 2 decimal places. **2**

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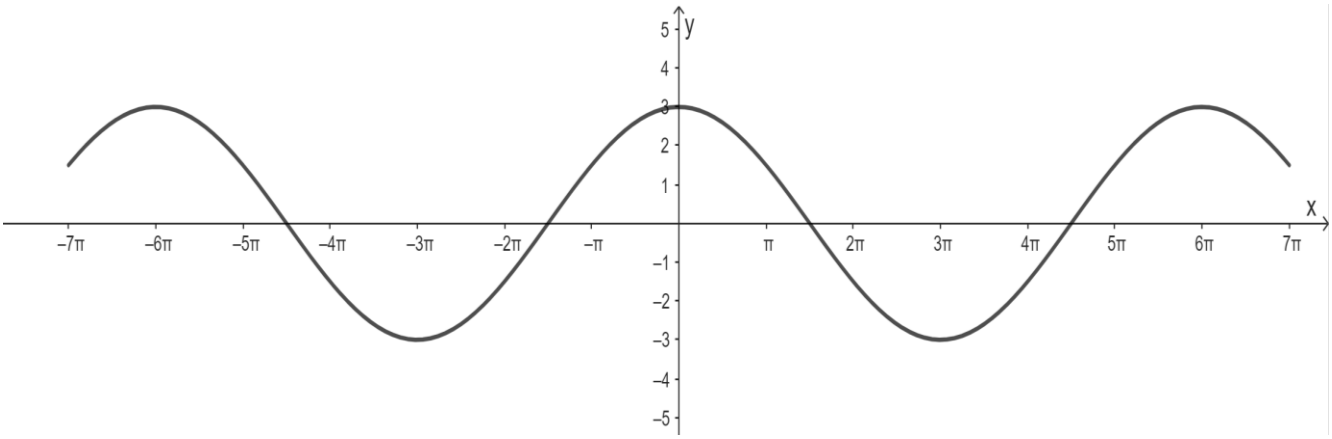
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Question 14 (2 marks)

The graph of $y = k\cos(ax)$ is shown. **2**



What are the values of k and a?

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Question 15 (3 marks)

Remy orders a burger and a shake from the drive-through at her favourite fast-food restaurant.

From previous experiences, she knows that there is a 90% chance that she will receive the burger she ordered and there is a 95% chance she will get the shake she ordered.

(a) Using the above information, construct a probability tree. **2**

(b) Hence, or otherwise, find the probability that exactly one of the items she receives does not match the items she ordered. **1**

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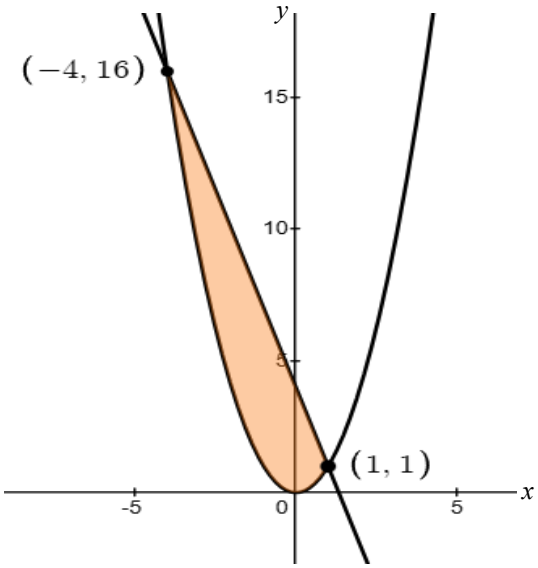
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Question 16 (3 marks)

The parabola $y = x^2$ meets the line $y = 4 - 3x$ at the points $(-4,16)$ and $(1,1)$, as shown in the diagram below. **3**



Find the area enclosed by the parabola and the line.

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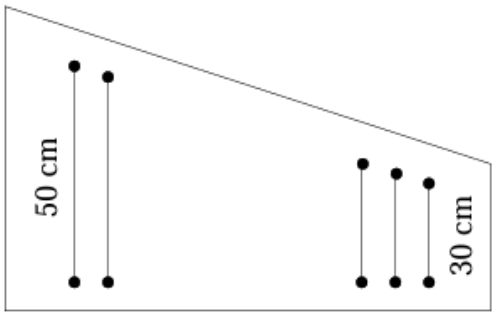
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Question 17 (5 marks)

A musical instrument has many strings attached, as shown in the diagram below. The difference between the lengths of adjacent strings is a constant so that the lengths of the strings are the terms of an arithmetic sequence. The shortest string is 30cm long and the longest string is 50cm. The sum of the lengths of all the strings is 1 240 cm.



(a) Find the number of strings. **2**

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(b) Find the difference in length between adjacent strings. **2**

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(c) What is the length of the tenth string? **1**

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Question 18 (3 marks)

(a) Differentiate $\left(\ln x\right)^2$. **1**

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(b) Hence evaluate $\int_1^4 \frac{\ln x}{x}\,dx$, keeping answer in exact form. **2**

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Question 19 (3 marks)

The graph of the function $f(x) = x^3$ is translated *m* units to the left, dilated vertically by a scale factor of *k*, and then translated 4 units up. The equation of the transformed function is $g(x) = 2x^3 + 18x^2 + 54x + 58$. Find the values of *m* and *k*. **3**

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Question 20 (4 marks**)**

A zoologist is studying the growth of a meerkat colony. The zoologist models the number of meerkats, $N(t)$, by the equation

$$N(t) = 46e^{0.082t},$$

where t is the number of months after starting the experiment.

(a) What is the initial number of meerkats in the experiment? **1**

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(b) What is the number of meerkats one year after starting the experiment? **1**

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(c) What is the rate of increase in the number of meerkats one year after starting the experiment? **2**

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Question 21 (4 marks)

From the top of a $40\,\text{m}$ high building, a ball is projected vertically upwards with an initial velocity of $25\,ms^{-1}$. On its return, it just misses the edge of the building and continues to the ground. If the acceleration due to gravity is $10\,ms^{-2}$, find

(a) The velocity and displacement functions of the ball. **2**

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(b) The greatest height that the ball reaches above the ground. **2**

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Question 22 (2 marks)

The probability that a student studies Physics given they study Mathematics is 0.6. Using the following table, find the probability that a student chosen at random studies both Physics and Maths. **2**

	PHYSICS	NOT PHYSICS
MATHS	A	32
NOT MATHS	28	16

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Question 23 (6 marks)

Between 6 am and 6 pm on 7th April 2026, the height, h, of the tide in a harbour was given by

$$h = 2 + 0.7\sin\left(\frac{\pi}{6}t\right), \text{ for } 0 \le t \le 12, \text{ where } h \text{ is in metres and } t \text{ is in hours.}$$

Taking $t = 0$ at 6 am,

(a) What is the period of the function h? **1**

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(b) What was the value of h at low tide and at what time did low tide occur? **2**

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(c) A ship is able to enter the harbour only if the height of the tide is at least 2.35 m. Find all times between 6 am and 6 pm on 7th April 2026 during which the ship was able to enter the harbour. **3**

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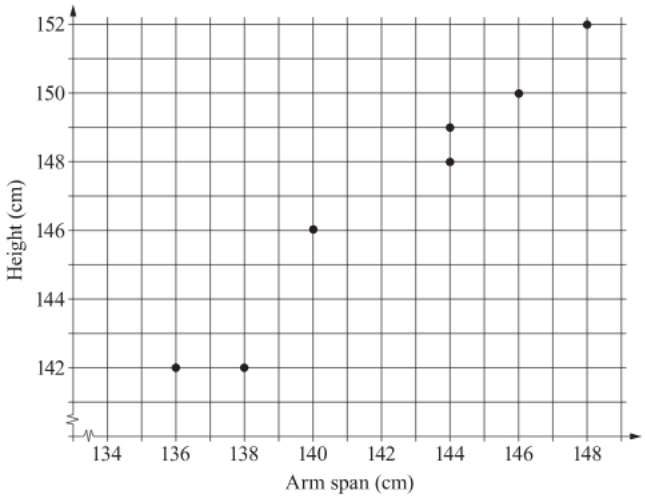
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Question 24 (3 marks)

The height and arm span of seven children is measured. The information is displayed in the scatterplot below.



(a) Calculate Pearson's Correlation Coefficient, correct to 2 decimal places. **1**

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(b) Describe the correlation between height and arm span, in terms of form and direction. **1**

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(c) Using technology, determine the least squares regression line that allows height to be predicted from arm span. **1**

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Question 25 (5 marks)

Water is flowing into an empty tank which has a hole in its base. The rate of change of the level of water in the tank is given by $\frac{dh}{dt} = 3^t - 5$, where h is the depth of the water in the tank in cm and t is the time in minutes after the water starts flowing in.

(a) Show that the depth of the water in the tank is **2**

$$h = \frac{3^t}{\ln 3} - 5t - \frac{1}{\ln 3}$$

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(b) Calculate the depth of the water in the tank after 2 minutes? **3**

Correct answer to two decimal places.

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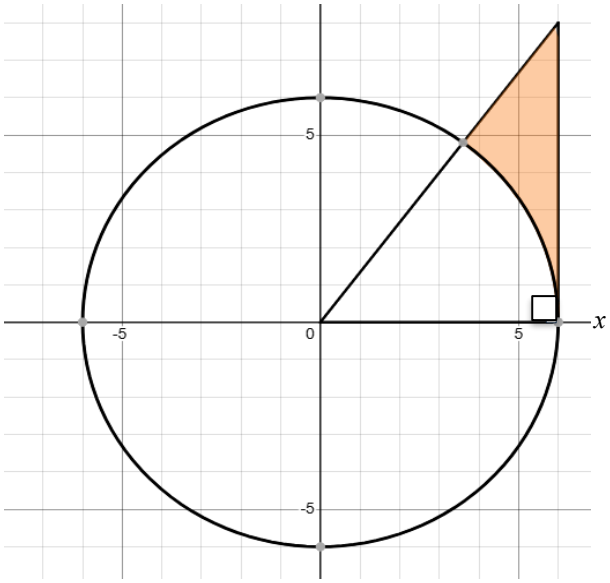
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Question 26 (3 marks)

The diagram below shows a circle of radius 6cm and a right triangle with a perpendicular height of length 8cm. **3**



NOT TO SCALE

Calculate the area of the shaded region, correct to 2 decimal places.

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Question 27 (9 marks)

Consider the function $f(x) = x^4 - 8x^2$.

(a) Find the coordinates of the stationary points of the curve $y = f(x)$ and determine their nature. **3**

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(b) Sketch the curve showing where it meets the axes and any stationary points. **2**

Question 27 continues on page 23

Question 27 (continued)

(c) Find and test for the point(s) of inflection for the function $f(x)$. Mark these on your sketch. **3**

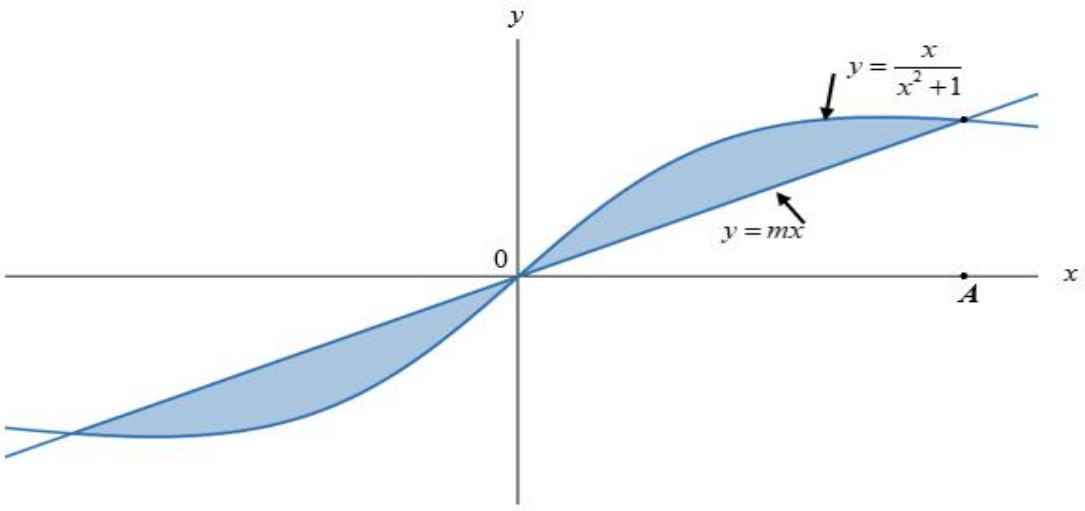
(d) For which values of x is the curve $y = f(x)$ concave up? **1**

Question 28 (2 marks)

The distance between the curves $y = \log_2(x-3)$ and $y = \log_2 x$ at $x = k$ is 2 units. **2**
Find the value of *k*.

Question 29 (5 marks)

The diagram below shows the areas between two odd functions: $y = \dfrac{x}{x^2+1}$ and $y = mx$.



(a) Show that the x-coordinate, A, of the point of intersection of the functions in the first quadrant is given by $\sqrt{\dfrac{1}{m}-1}$. **2**

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(b) Calculate the area of the shaded region in terms of m. **3**

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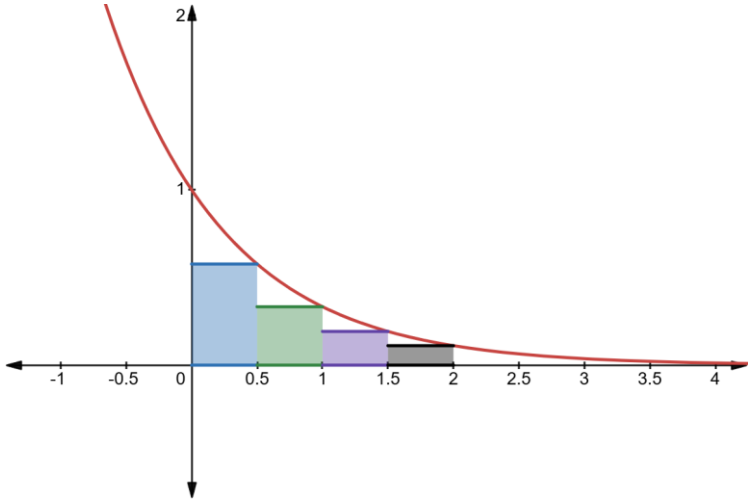
Question 30 (3 marks)

Solve $\tan\left(2x-\frac{\pi}{4}\right)=\frac{1}{\sqrt{3}}$ for $x\in[-\pi,\pi]$

Question 31 (5 marks)

The diagram shows the graph of $y = 3^{-x}$. Also shown on the diagram are the first 4 of an infinite number of rectangular strips of width $\frac{1}{2}$ unit and height $y = 3^{-x}$, for non-negative integer values of x.

NOT TO SCALE



(a) The sum of the areas of the rectangles forms a geometric series. **2**

Show that the limiting sum of this series is $\frac{1}{2\sqrt{3}-2}$.

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Question 30 continues on page 27

Question 31 (continued)

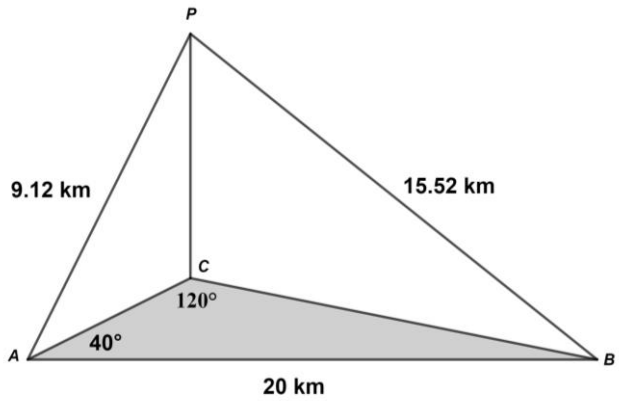
(b) Show that $\int_0^2 3^{-x}\,dx = \frac{8}{9\ln 3}$. **2**

(c) By finding the areas of the four shaded rectangles and using part (b), **1**

show that $\ln 3 < \frac{4}{\sqrt{3}+1}$.

Question 32 (5 marks**)**

Consider the diagram below where A, B and C are three points on level ground and P is a point vertically above C



(i) Find the length of AC, correct to the nearest 0.1 km. **2**

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(ii) Hence, find the angle of elevation of P from A, correct to the nearest minute **1**

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(iii) Find the angle between PA and PB to the nearest minute. **2**

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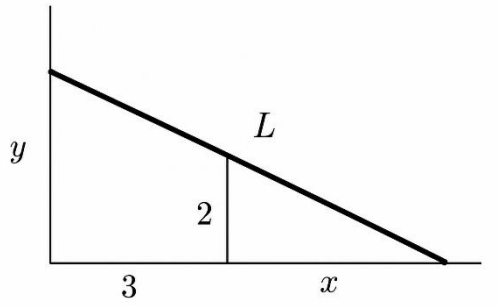
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QUESTION 33 is over the page

Question 33 (6 marks)

A 2-metre vertical fence stands 3 metres from a high vertical wall. A ladder, L, is placed from the horizontal ground to the wall, resting on the fence. The base of the ladder is x metres from the fence.



(a) If L represents the length of the ladder, show that **2**

$$L^2 = (x+3)^2\left(1+\frac{4}{x^2}\right)$$

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Question 33 continues on page 31

Question 33 (continued)

(b) By first finding $\frac{dL^2}{dx}$, calculate the shortest ladder that can reach from the ground outside the fence to the wall, correct to 1 decimal place. **4**

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Section II extra writing space

If you use this space, clearly indicate which question you are answering.

Section II extra writing space

If you use this space, clearly indicate which question you are answering.

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HORNSBY GIRLS HIGH SCHOOL

Mathematics Advanced

Year 12 Higher School Certificate Trial Examination
Term 3 2026

STUDENT NUMBER: Solutions and Markers Feedback.

General Instructions:

- Reading time – 10 minutes
- Working time – 3 hours
- Write using black pen
- Calculators approved by NESA may be used
- For questions in Section II, show relevant mathematical reasoning and/ or calculations
- A NESA reference sheet is provided

Total Marks:
100

Section I – 10 marks (pages 3–7)

- Attempt Questions 1– 10
- Allow about 15 minutes for this section

Section II – 90 marks (pages 9 - 31)

- Attempt Questions 11 - 33
- Allow about 2 hours and 45 minutes for this section

Q1 - 10	Q11	Q12	Q13	Q14	Q15	Q16	Q17	Q18	Q19	Q20	Q21	
/10	/3	/4	/2	/2	/3	/3	/5	/3	/3	/4	/4	
Q22	Q23	Q24	Q25	Q26	Q27	Q28	Q29	Q30	Q31	Q32	Q33	**Total**
/2	/6	/3	/5	/3	/9	/2	/5	/3	/5	/5	/6	/100

Outcomes assessed: MA 12 – 1, 3, 4, 5, 6, 7, 8, 10

This assessment task constitutes 30% of the Higher School Certificate Course School Assessment

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Section I

10 marks
Attempt Questions 1 – 10
Allow about 15 minutes for this section

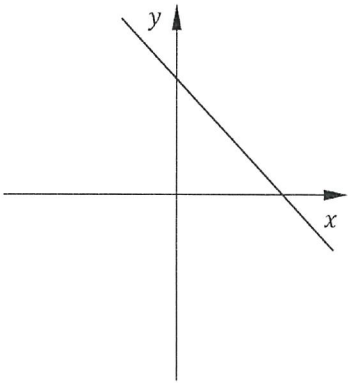
Use the Objective Response answer sheet for Questions 1 – 10

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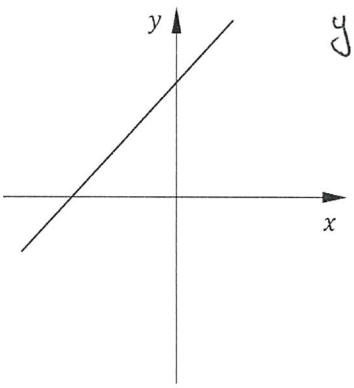
1 Which of the following could be the graph of $y = x - 1$?

m = 1
y int: −1

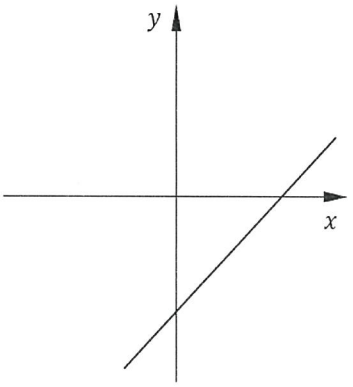
A.



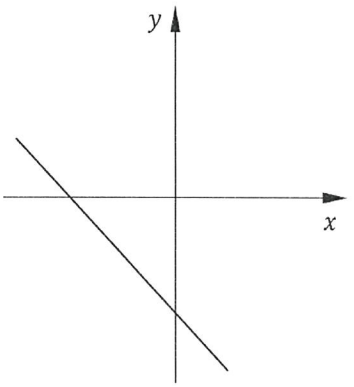
B.



C.



D.



2 Consider the following dataset:

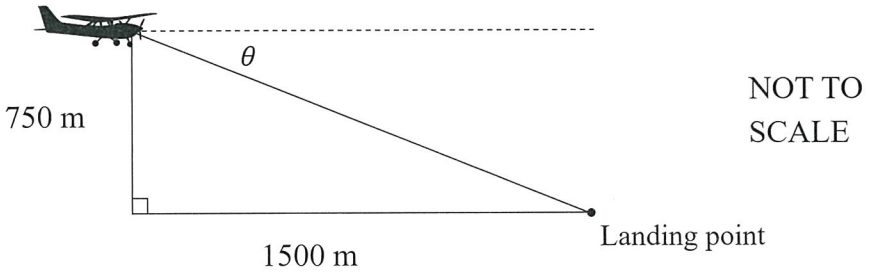
15 18 20 21 22 28

Which row of the table below shows how the mean and standard deviation are affected when each score is increased by 10?

↑ increase by 10 ↑ spread unchanged

	Mean	*Standard Deviation*
A.	Changes	Changes
B.	Stays the same	Stays the same
C.	Changes	Stays the same
D.	Stays the same	Changes

3 A small plane is approaching an airport to land. It is currently 750 metres above the airport and 1500 metres horizontally from its landing point.



NOT TO SCALE

What is the angle of depression, θ, from the plane to its landing point, correct to the nearest degree?

A. 27°
B. 30°
C. 60°
D. 63°

$\tan\theta = \frac{750}{1500}$

$\tan\theta = \frac{1}{2}$

$\theta = \tan^{-1}\left(\frac{1}{2}\right)$

$= 26.565$

$= 27^\circ$

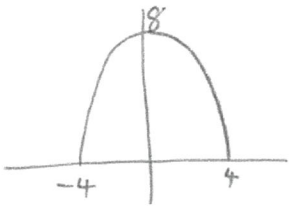
4 Which of the following is the range of the function $f(x) = 2\sqrt{16 - x^2}$?

A. $[-8,8]$

B. $[0,8]$

C. $[0,4]$

D. $[-4,4]$



5 For what values is $y = \dfrac{x^2}{2x^2 + x + 1}$ decreasing?

A. $-2 < x < 0$

B. $x < -2,\ x > 0$

C. $0 < x < 2$

D. $x = 0,\ -2$

$$\frac{d}{dx}\frac{x^2}{2x^2+x+1} = \frac{2x(2x^2+x+1) - x^2(4x+1)}{(2x^2+x+1)^2}$$

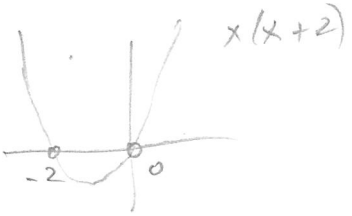
$$= \frac{4x^3 + 2x^2 + 2x - 4x^3 - x^2}{(2x^2+x+1)^2}$$

$$= \frac{x^2 + 2x}{(2x^2+x+1)^2}$$

$$= \frac{x(x+2)}{(2x^2+x+1)^2}$$

for $\dfrac{dy}{dx} < 0$, $x(x+2) < 0$

$\therefore -2 < x < 0$



6 What is $\int \dfrac{1}{(3x+2)^3}\,dx$

A. $\dfrac{-1}{2(3x+2)^2} + C$

B. $\dfrac{-6}{(3x+2)^2} + C$

C. $\dfrac{-1}{6(3x+2)^2} + C$

D. $\dfrac{-1}{3(3x+2)^2} + C$

$$\int (3x+2)^{-3}\,dx$$

$$= \frac{(3x+2)^{-2}}{-6} + c$$

7 In a certain Sydney Suburb, the probability of an adult catching the flu is 0.2 . Two adults living in this suburb are chosen at random. The probability of at least one of them catching the flu is:

A. 0.04

B. 0.32

C. 0.36

D. 0.64

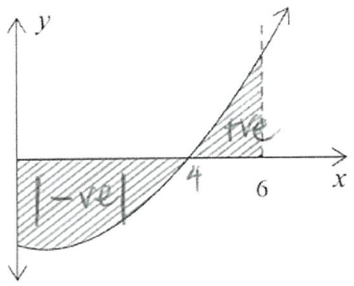
$$P(\text{at least 1}) = 1 - P(\text{none})$$

$$= 1 - 0.8 \times 0.8$$

$$= 1 - 0.64$$

$$= 0.36$$

8 The diagram below is the graph of $y = x^2 + x - 20$.



What is the correct expression for the area bounded by the x-axis and the Graph $y = x^2 + x - 20$ between $0 < x < 6$?

A. $A = \int_0^6 x^2 + x - 20\, dx$

B. $A = \left|\int_0^4 x^2 + x - 20\, dx\right| + \int_4^6 x^2 + x - 20\, dx$

C. $A = \int_0^4 x^2 + x - 20\, dx + \left|\int_4^6 x^2 + x - 20\, dx\right|$

D. $A = \left|\int_0^6 x^2 + x - 20\, dx\right|$

9 A vessel initially contains 100 litres of water.
It is being emptied at the rate of

$$\frac{dv}{dt} = -\left(2 + \frac{20}{t+1}\right)$$ where V is the volume in litres after t minutes.

Which of the following gives the exact volume of water in the vessel after 5 minutes?

A. $10 + 20\ln 6$

B. $90 - 20\ln 6$

C. $90 + 20\ln 6$

D. $110 + 20\ln 6$

$\frac{dv}{dt} = -2 - \frac{20}{t+1}$

$V = -2t - 20\ln|t+1| + c$

$t = 0 \quad V = 100$

$100 = -2\times 0 - 20\ln 1 + c$

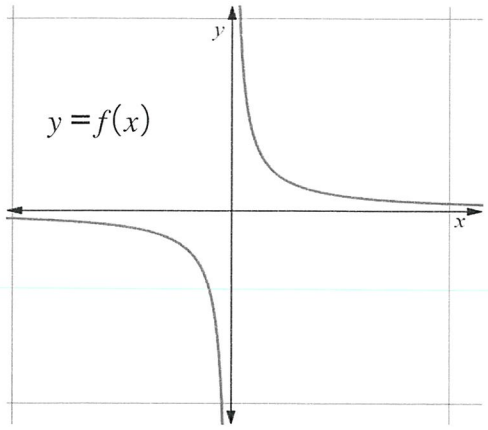
$100 = c$

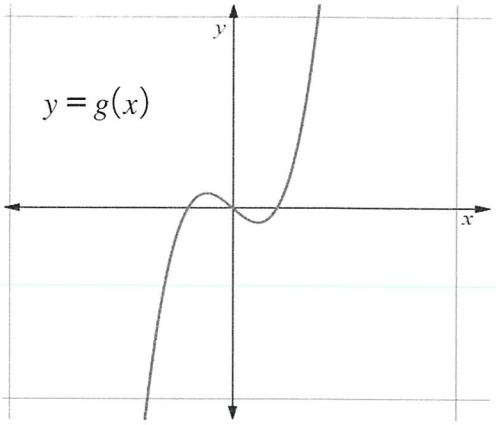
$V = -2t - 20\ln|t+1| + 100$

$t = 5 \quad = -2\times 5 - 20\ln|5+1| + 100$

$= 90 - 20\ln 6$

10 The graphs of $y = f(x)$ and $y = g(x)$ are shown.





Which graph best represents $y = g(f(x))$?

f (x) is odd

g(x) is odd

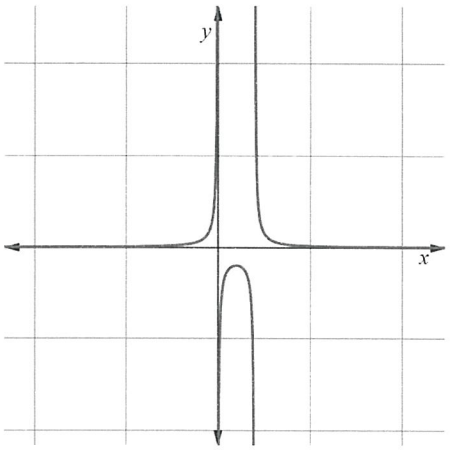
$g(f(-x)) = g(-f(x))$

$= -g(f(x))$

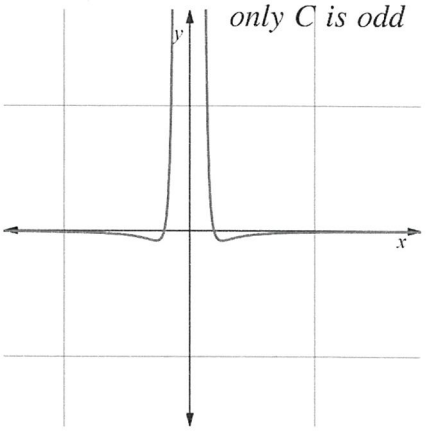
∴ g(f (x)) is also odd

only C is odd

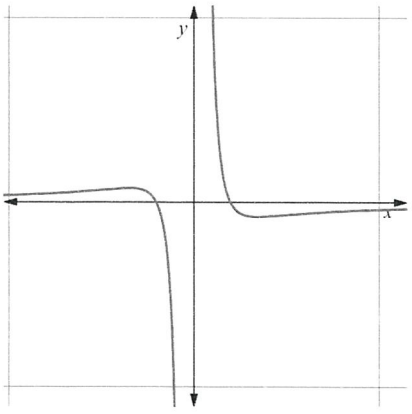
A.



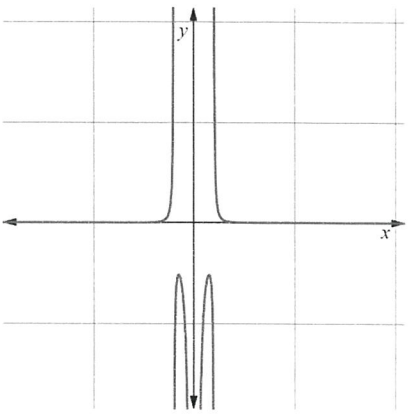
B.



C.



D.



End of Section I

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Section II

90 marks
Attempt questions 11 to 33
Allow about 2 hours and 45 minutes for this section

Answer each question in the spaces provided.
Your responses should include relevant mathematical reasoning and/or calculations.
Extra writing space is provided at the back of the examination paper.

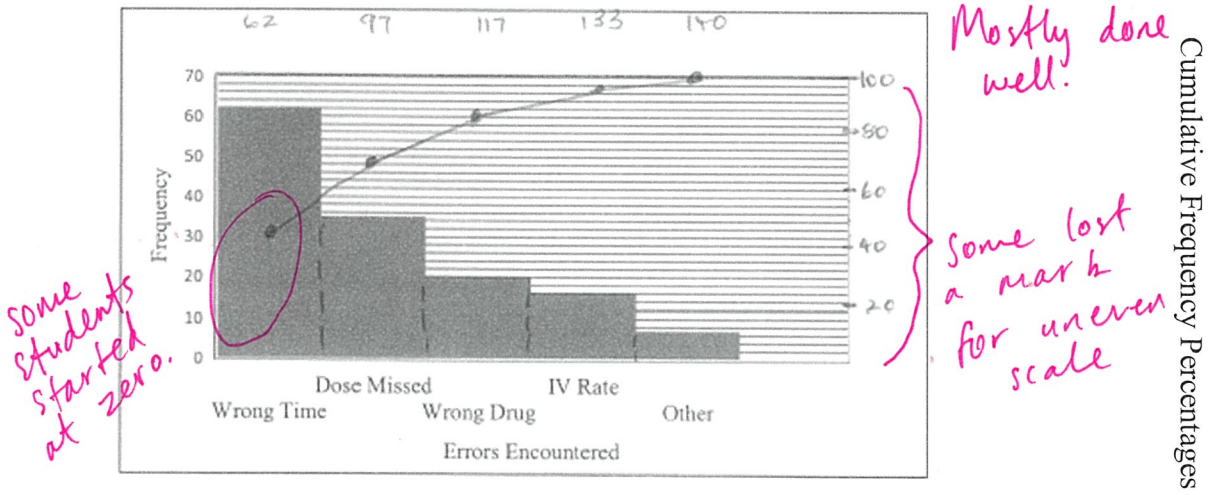
Question 11 (3 marks)

A hospital wants to identify the most significant errors encountered when it comes to administering medications.
A random survey was conducted, and the results are shown in the table below.

Error	Frequency
Dose missed	35
Wrong Drug	20
Wrong Time	62
IV rate	16
Other	7

The receptionist was asked to construct a Pareto chart, to illustrate the data collected from the survey.

Below is a partially constructed Pareto chart.



a) Complete the Pareto chart above, by choosing an appropriate scale for the vertical axis on the right-hand side of the Pareto chart and hence drawing a cumulative frequency percentage line graph. **2**

b) What percentage of errors will be solved if there is more focus put on improving the two main errors when administering medication? (Give your answer to 2 decimal places) **1**

62+35
=97

$\frac{97}{140} \times 100 = 69.2857...\%$

$= 69.29\%$ (2dp)

Done well.

Question 12 (4 marks)

The time (H hours) it takes to travel to a location varies inversely with the speed (S km/hour) travelled. It takes 4 hours driving at an average speed of 60 km/hour to reach a location.

(a) Find the equation relating H and S. 2

$H \propto \frac{1}{S}$

$H = \frac{k}{S}$

$4 = \frac{k}{60}$

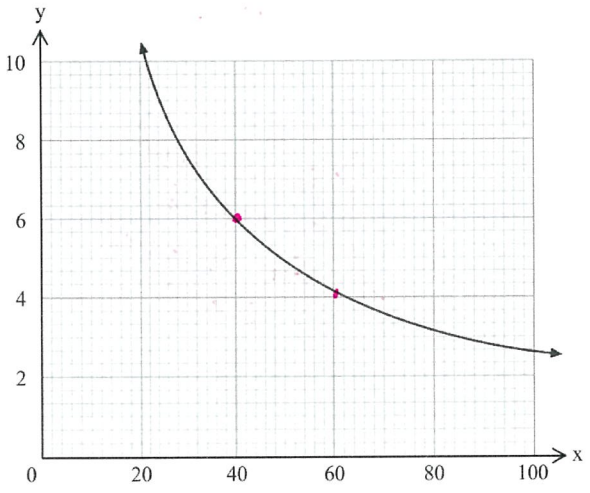
$k = 240$

$\therefore H = \frac{240}{S}$

A number of students made mistakes by getting $H = \frac{1}{k}S$ which turns out to be a positive linear relationship which is not inverse variation.

(b) By first completing the table of values, graph the relationship between H and S. 2
Express values correct to the nearest hour.

S	100	60	40
$H \approx$	2	4	6



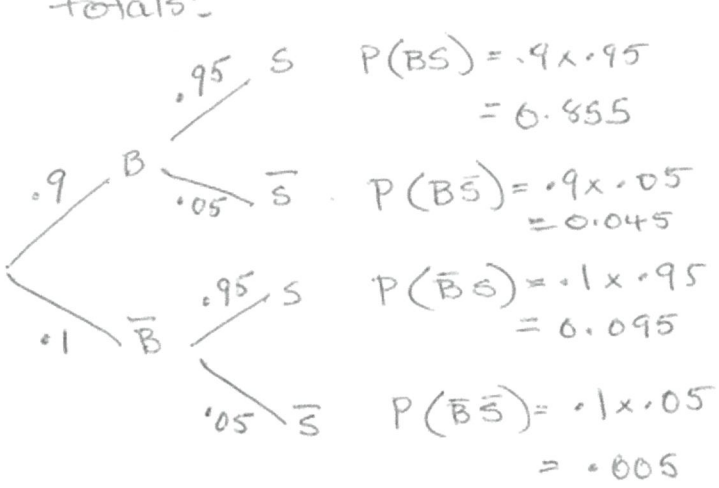
The graph needs to be consistent with the equation in (a).

Question 15 (3 marks)

Remy orders a burger and a shake from the drive-through at her favourite fast-food restaurant. From previous experiences, she knows that there is a 90% chance that she will receive the burger she ordered and there is a 95% chance she will get the shake she ordered.

(a) Using the above information, construct a probability tree. **2**

Showing the prob on each branch. totals?



$P(BS) = .9 \times .95 = 0.855$

$P(B\bar{S}) = .9 \times .05 = 0.045$

$P(\bar{B}S) = .1 \times .95 = 0.095$

$P(\bar{B}\bar{S}) = .1 \times .05 = .005$

Done well!

(b) Hence, or otherwise, find the probability that exactly one of the items she receives does not match the items she ordered. **1**

$P(B\bar{S}) + P(\bar{B}S) = 0.045 + 0.095$

$= 0.14$ or 14% or $\frac{14}{100}$ or $\frac{7}{50}$

Done well!

Question 13 (2 marks)

Use 2 applications of the trapezoidal rule to find an approximate value of $\int_2^4 \ln x \, dx$. Give your answer to 2 decimal places. **2**

$\int_2^4 \ln x \, dx \approx \frac{1}{2}[1 \times \ln 2 + 2 \times \ln 3 + 1 \times \ln 4]$

$= \frac{1}{2}[\ln 2 + \ln 9 + \ln 4]$

$= 2.138\ldots$

$= 2.14$ (2 d.p.)

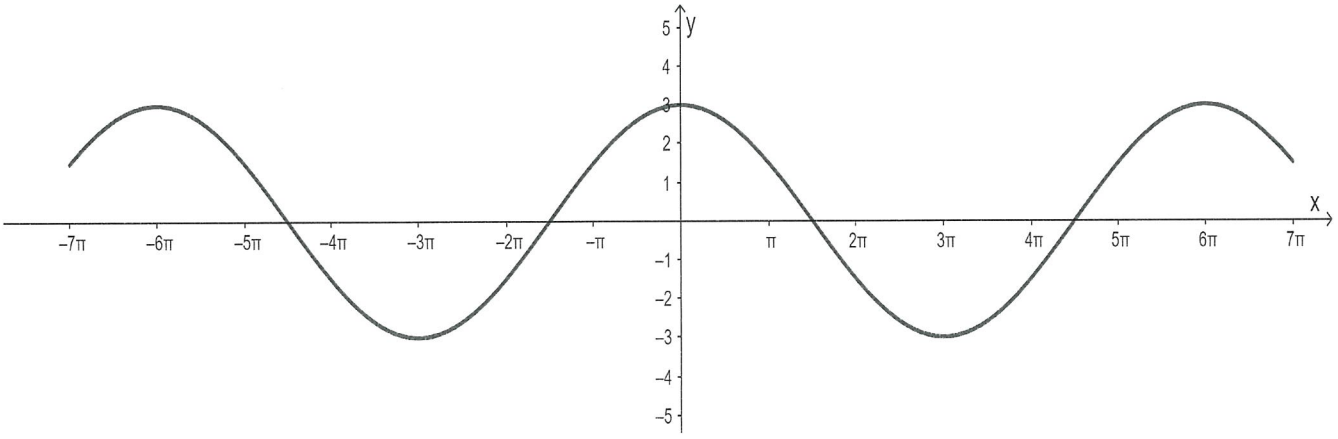
$h = 1$

x	2	3	4
$f(x)$	$\ln 2$	$\ln 3$	$\ln 4$

Generally well done however there were a few students who confused the number of applications and thought they needed 4 intervals rather than 2.

Question 14 (2 marks)

The graph of $y = k\cos(ax)$ is shown. **2**



What are the values of k and a?

k is the amplitude $\therefore k = 3$

a is associated with period. Period $= 6\pi$ $\left(P = \frac{2\pi}{n}\right)$ here $n = a$

i.e. $6\pi = \frac{2\pi}{a}$

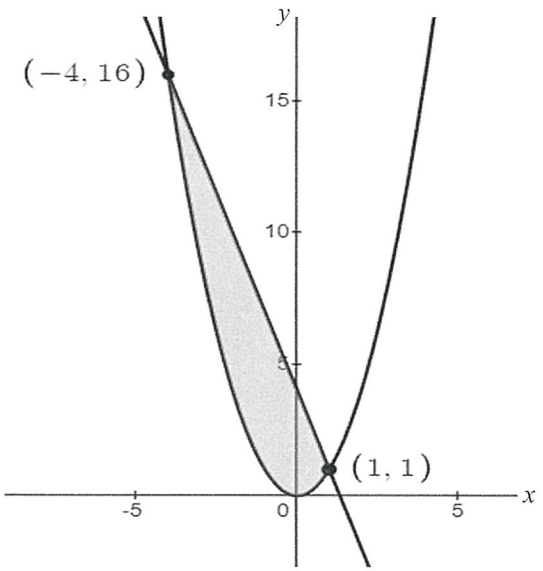
$a = \frac{2\pi}{6\pi}$

$= \frac{1}{3}$

Mostly well done. Some students need to check their knowledge of the period of a sine or cosine curve

Question 16 (3 marks)

The parabola $y = x^2$ meets the line $y = 4 - 3x$ at the points $(-4,16)$ and $(1,1)$, as shown in the diagram below. **3**



Find the area enclosed by the parabola and the line.

$$\text{Area} = \int_{-4}^{1} (4 - 3x - x^2)\,dx$$

$$= \left[4x - \frac{3x^2}{2} - \frac{x^3}{3}\right]_{-4}^{1}$$

$$= \left[4(1) - \frac{3}{2}(1)^2 - \frac{1^3}{3}\right] - \left[4(-4) - \frac{3}{2}(-4)^2 - \frac{(-4)^3}{3}\right]$$

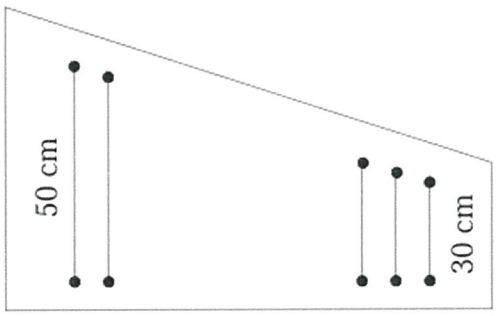
$$= 4 - \frac{3}{2} - \frac{1}{3} - \left(-16 - 24 + \frac{64}{3}\right)$$

$$= \frac{125}{6}\ u^2$$

Well done!
Some students reversed the functions.

Question 17 (5 marks)

A musical instrument has many strings attached, as shown in the diagram below. The difference between the lengths of adjacent strings is a constant so that the lengths of the strings are the terms of an arithmetic sequence. The shortest string is 30cm long and the longest string is 50cm. The sum of the lengths of all the strings is 1 240 cm.



(a) Find the number of strings. **2**

$S_n = \frac{n}{2}(a+l)$ Well done!

$1240 = \frac{n}{2}(30+50)$

$1240 = 40n$

$n = 31$ ∴ There are 31 strings

(b) Find the difference in length between adjacent strings. **2**

$T_{31} = 50$, $a = 30$

$50 = 30 + (31-1)d$

$50 = 30 + 30d$

$20 = 30d$

$\therefore d = \frac{2}{3}$ cm

or

$30 = 50 + (31-1)d$ Well done!

$\frac{-20}{30} = d$

$\therefore d = -\frac{2}{3}$

(c) What is the length of the tenth string? **1**

$T_{10} = 30 + (10-1)\left(\frac{2}{3}\right)$

$= 30 + 9 \times \frac{2}{3}$

$= 36$ cm

or $T_{10} = 50 + (10-1)\left(-\frac{2}{3}\right)$

$T_{10} = 44$ cm Well done!

Question 18 (3 marks)

(a) Differentiate $(\ln x)^2$. **1**

$y = (\ln x)^2$

$\frac{dy}{dx} = 2(\ln x)^1 \times \frac{1}{x} = \frac{2\ln x}{x}$

Done well!

(b) Hence evaluate $\int_1^4 \frac{\ln x}{x}\,dx$, keeping answer in exact form. **2**

$\int_1^4 \frac{\ln x}{x}\,dx = \frac{1}{2}\int_1^4 \frac{2\ln x}{x}\,dx$

$= \frac{1}{2}\left[(\ln x)^2\right]_1^4$

$= \frac{1}{2}\left[(\ln 4)^2 - (\ln 1)^2\right]$

$= \frac{1}{2}(\ln 4)^2$

Mostly done well

students got full marks if they got to here.

Question 19 (3 marks)

The graph of the function $f(x) = x^3$ is translated m units to the left, dilated vertically by a scale factor of k, and then translated 4 units up. The equation of the transformed function is $g(x) = 2x^3 + 18x^2 + 54x + 58$. Find the values of m and k. **3**

$y = k(x+m)^3 + 4$

$y = k(x^3 + 3x^2m + 3xm^2 + m^3) + 4$

$y = kx^3 + 3kx^2m + 3kxm^2 + km^3 + 4$

$\therefore k = 2$ coefficient of x^3

and

$km^3 + 4 = 58$ constant term

$2m^3 = 54$

$m^3 = 27$

$m = 3$

$\therefore m = 3 \quad k = 2$

Done well!

Question 20 (4 marks)

A zoologist is studying the growth of a meerkat colony. The zoologist models the number of meerkats, $N(t)$, by the equation

$$N(t) = 46e^{0.082t},$$

where t is the number of months after starting the experiment.

(a) What is the initial number of meerkats in the experiment? **1**

$N(0) = 46$ well done!

(b) What is the number of meerkats one year after starting the experiment? **1**

$N(12) = 46e^{0.082(12)}$
$= 123.05622...$
$= 123$ Meerkats.

Generally well done but some students forgot to convert the year to months.

(c) What is the rate of increase in the number of meerkats one year after starting the experiment? **2**

$N'(t) = 0.082 \times 46e^{0.082t}$
$N'(12) = 0.082 \times 46e^{0.082(12)}$
$= 0.082 \times 123.0562289...$
$= 10.1$ meerkats/yr

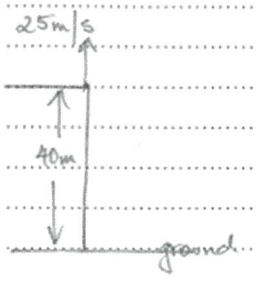
Well done but same as above (b).

Question 21 (4 marks)

From the top of a 40 m high building, a ball is projected vertically upwards with an initial velocity of $25\,ms^{-1}$. On its return, it just misses the edge of the building and continues to the ground. If the acceleration due to gravity is $10\,ms^{-2}$, find

(a) The velocity and displacement functions of the ball. **2**

$t=0 \quad y=40 \quad \dot{y}=25 \qquad a=\ddot{y}=-10$



$V=\dot{y}=-10t+C$

$t=0 \quad \dot{y}=25 \therefore 25=-10\times0+C$

$C=25$

$\therefore V=\dot{y}=-10t+25$

$\text{disp}=y=-5t^2+25t+k$

$t=0 \quad y=40 \qquad 40=-5\times0^2+25\times0+k$

$\therefore k=40 \qquad \therefore y=-5t^2+25t+40$

In general this was not well done. This is vertical straight line motion NOT projectile motion. For these questions you must choose up as positive and down as negative in direction. You should choose the ground as zero displacement

(b) The greatest height that the ball reaches above the ground. **2**

greatest height when $\dot{y}=0$

$0=-10t+25$

$t=2.5$ sec.

$\therefore y=-5\times2.5^2+25\times2.5+40$

$=71.25$ m above the ground.

A lot of carried forward errors here.

Question 22 (2 marks)

The probability that a student studies Physics given they study Mathematics is 0.6. Using the following table, find the probability that a student chosen at random studies both Physics and Maths. **2**

	PHYSICS	NOT PHYSICS
MATHS	A	32
NOT MATHS	28	16

TOTAL $A+76$

$P(\text{Physics}\,|\,\text{Maths})=\dfrac{P(\text{Physics}\cap\text{Maths})}{P(\text{Maths})}$

$\dfrac{6}{10}=\dfrac{A}{\frac{76+A}{A+32}\times\frac{76+A}{1}}$

$\therefore \dfrac{6}{10}=\dfrac{A}{A+32}$

$6A+192=10A$

$192=4A$

$A=48$

$\therefore P(\text{Physics}\cap\text{Maths})=\dfrac{48}{124}=\dfrac{12}{31}$ //

About 70% answered this correctly. Please check method if you got it wrong.

Question 23 (6 marks)

Between 6 am and 6 pm on 7th April 2026, the height, h, of the tide in a harbour was given by

$$h = 2 + 0.7\sin\left(\frac{\pi}{6}t\right), \text{ for } 0 \le t \le 12, \text{ where } h \text{ is in metres and } t \text{ is in hours.}$$

Taking $t = 0$ at 6 am,

(a) What is the period of the function h? **1**

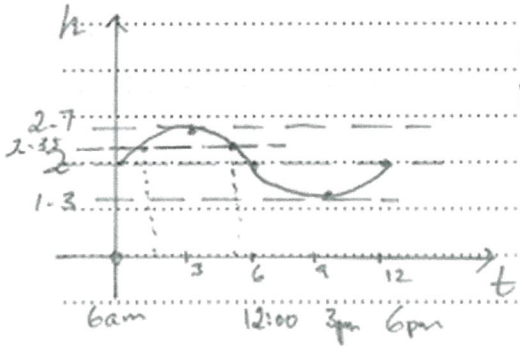
Period $= \frac{2\pi}{n}$ $\quad n = \frac{\pi}{6}$

$= \frac{2\pi}{\frac{\pi}{6}}$ i.e 12 hours

$= 12$

Well done!

(b) What was the value of h at low tide and at what time did low tide occur? **2**



Low tide is 1 amplitude length below equilibrium height of 2m

$\therefore$ low tide $= 2 - 0.7$

$= 1.3$ m

This occurs at 3 pm

generally well done

(c) A ship is able to enter the harbour only if the height of the tide is at least 2.35 m. Find all times between 6 am and 6 pm on 7th April 2026 during which the ship was able to enter the harbour. **3**

$2.35 = 2 + 0.7\sin\left(\frac{\pi}{6}t\right)$

$0.35 = 0.7\sin\left(\frac{\pi}{6}t\right)$

$\frac{1}{2} = \sin\left(\frac{\pi}{6}t\right)$

$\therefore \frac{\pi}{6}t = \frac{\pi}{6}, \frac{5\pi}{6}$ (1st/2nd Quads)

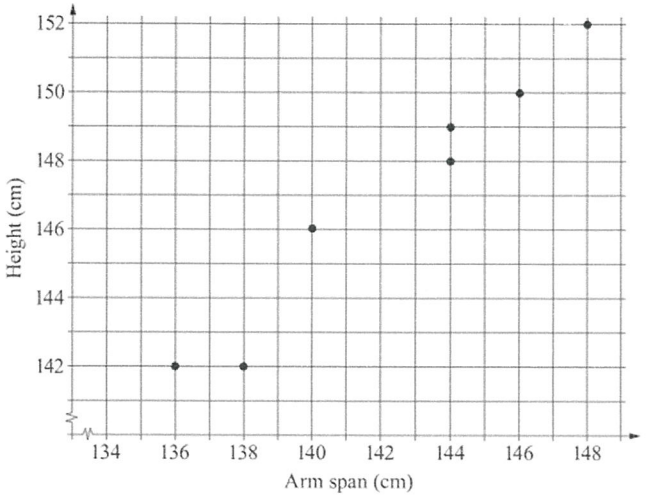
$t = 1$h and 5h after 6.00 am

$\therefore$ Ship can enter at or after 7am and leave before 11am.

Mostly well done.

Question 24 (3 marks)

The height and arm span of seven children is measured. The information is displayed in the scatterplot below.



(a) Calculate Pearson's Correlation Coefficient, correct to 2 decimal places. **1**

0.9811199...
0.98 (2 dp)

Done well

(b) Describe the correlation between height and arm span, in terms of form and direction. **1**

Strong positive linear correlation which is indicated by the high Pearson's Coefficient.

Mostly done well
Some didn't comment on form.

(c) Using technology, determine the least squares regression line that allows height to be predicted from arm span. **1**

gint $A = 23.733$ ∴ height = .866 arm span + 23.733
grad $B = 0.866$ $h = .866a + 23.733$

Done well

Question 25 (5 marks)

Water is flowing into an empty tank which has a hole in its base. The rate of change of the level of water in the tank is given by $\frac{dh}{dt} = 3^t - 5$, where h is the depth of the water in the tank in cm and t is the time in minutes after the water starts flowing in.

(a) Show that the depth of the water in the tank is **2**

$$h = \frac{3^t}{\ln 3} - 5t - \frac{1}{\ln 3}$$

$$h = \int (3^t - 5)\,dt$$

$$= \frac{3^t}{\ln 3} - 5t + C$$

$t = 0,\ h = 0$

$$0 = \frac{3^{(0)}}{\ln 3} - 5(0) + C$$

$$0 = \frac{1}{\ln 3} + C$$

$$C = \frac{-1}{\ln 3}$$

$$\therefore\ h = \frac{3^t}{\ln 3} - 5t - \frac{1}{\ln 3}$$

Generally well done. Students need to show how to find C.

(b) Calculate the depth of the water in the tank after 2 minutes? **3**

Correct answer to two decimal places.

Sub. $t = 2$,

$h = \frac{3^2}{\ln 3} - 5(2) - \frac{1}{\ln 3}$ (Aw 1)

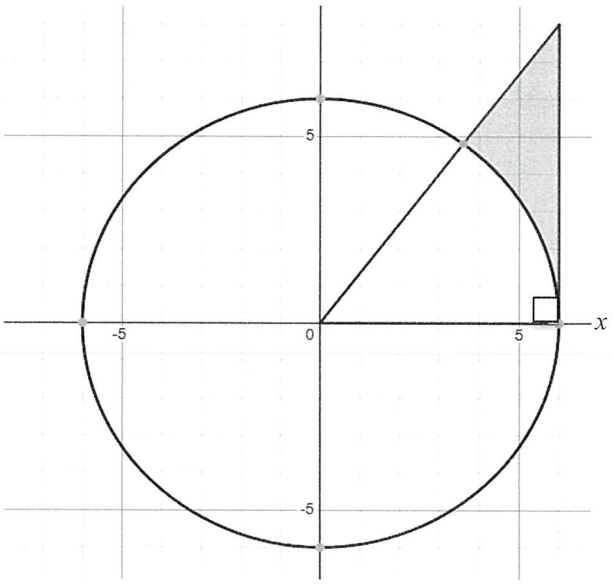
$= -2.71808\ldots$ (Aw 1)

The depth of the water is negative, indicating that the water flowing in all went out of the hole, therefore the ~~tank~~ tank is still empty after 2 minutes. (Aw 1)

The negative value of h need to be interpreted correctly.

Question 26 (3 marks)

The diagram below shows a circle of radius 6cm and a right triangle with a perpendicular height of length 8cm. **3**



NOT TO SCALE

Calculate the area of the shaded region, correct to 2 decimal places.

$A_{\Delta} = \frac{1}{2}(6)(8)$

$= 24 cm^2$

$\tan\theta = \frac{8}{6}$

$\theta = \tan^{-1}\left(\frac{8}{6}\right)$

$\theta = 0.9273^c$

$\therefore$ Area sector $= \frac{1}{2}(6)^2(0.9273)$

$= 16.6913 cm^2$

$\therefore$ shaded area $= 24 - 16.6913$

$= 7.31 cm^2$

Very well done!

Question 27 (9 marks)

Consider the function $f(x) = x^4 - 8x^2$.

(a) Find the coordinates of the stationary points of the curve $y = f(x)$ and determine their nature. **3**

T.P's ($f'(x)=0$) $f'(x) = 4x^3 - 16x$

$= 4x(x^2 - 4)$

$= 0$ when $x = 0, \quad x = -2, \quad x = 2$

$y = 0 \quad y = -16, \quad y = -16$

test nature

$f''(x) = 12x^2 - 16$

at $(0,0)$ $f''(0) = -16$ ∩ $\therefore$ max. t.p.

at $(-2,-16)$ $f''(-2) = 32$ ∪ $\therefore$ min t.p

at $(2,-16)$ $f''(2) = 32$ ∪ $\therefore$ min t.p.

Mostly done well.

1 – stationary points coordinates

1 – for nature

1 – for correct deriving.

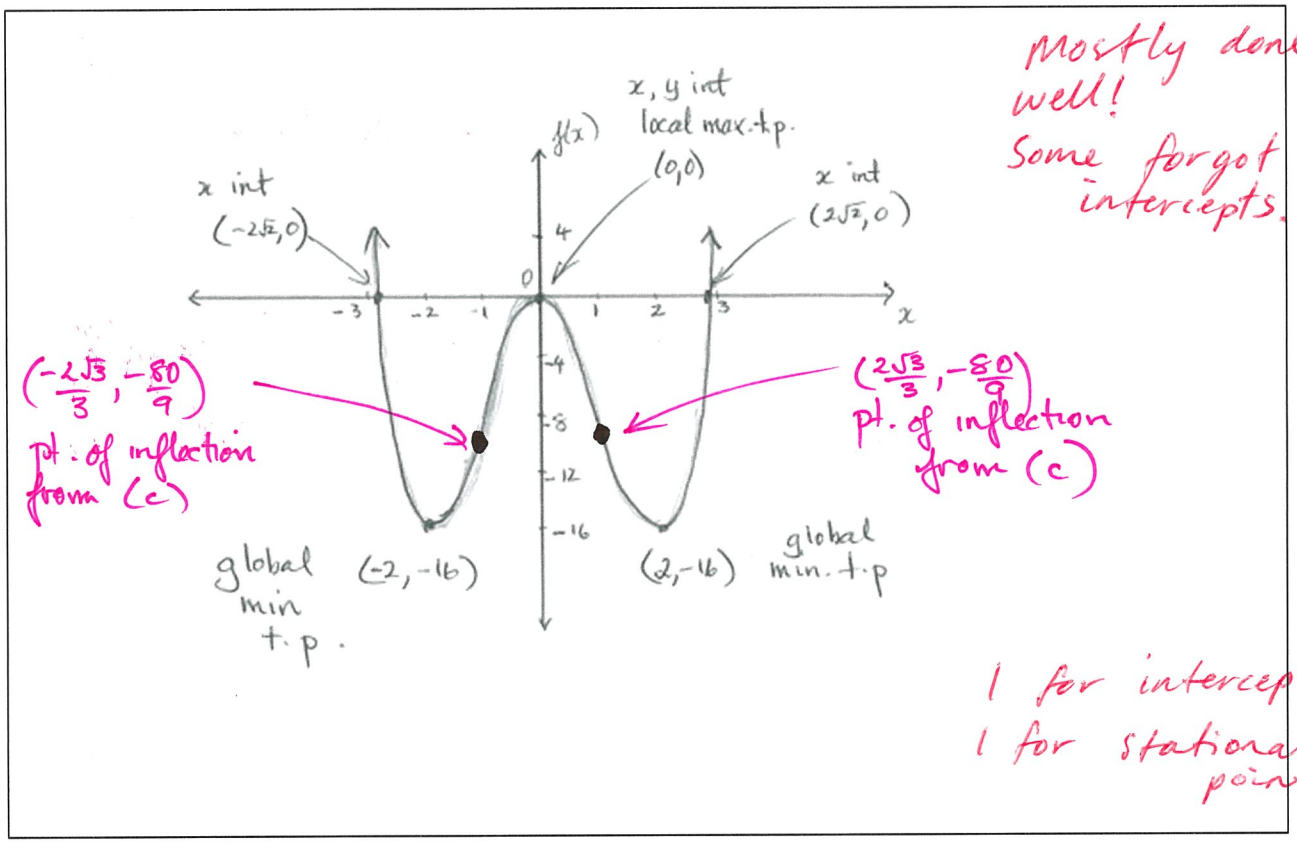
x int ($y(x)=0$)

$0 = x^4 - 8x^2$

$0 = x^2(x^2 - 8)$

$x = 0, \quad x = \pm 2\sqrt{2}$

(b) Sketch the curve showing where it meets the axes and any stationary points. **2**



Mostly done well!

Some forgot intercepts.

1 for intercepts

1 for stationary points

Question 27 continues on page 23

Question 27 (continued)

(c) Find and test for the point(s) of inflection for the function $f(x)$. Mark these on your sketch. **3**

Pts of inflexion $f''(x) = 0$

$f''(x) = 12x^2 - 16 = 4(3x^2 - 4)$

$= 0$ when $x = \pm\frac{2}{\sqrt{3}} = \pm\frac{2\sqrt{3}}{3}$, $y = \left(\frac{2\sqrt{3}}{3}\right)^4 - 8\left(\frac{2\sqrt{3}}{3}\right)^2 = \frac{144}{81} - 8\times\frac{12}{9} = -\frac{80}{9}$

test $x = -\frac{2}{\sqrt{3}}$: $x < -\frac{2}{\sqrt{3}}$ $f''(x) > 0$ ∪; $f''(x) = 0$; $x > -\frac{2}{\sqrt{3}}$ $f''(x) < 0$ ∩ ∴ change in concavity ∴ $\left(-\frac{2}{\sqrt{3}}, -\frac{80}{9}\right)$ is a pt. of inflection

test $x = \frac{2}{\sqrt{3}}$: $x < \frac{2}{\sqrt{3}}$ $f''(x) < 0$ ∩; $f''(x) = 0$; $x > \frac{2}{\sqrt{3}}$ $f''(x) > 0$ ∪ ∴ change in concavity ∴ $\left(\frac{2}{\sqrt{3}}, -\frac{80}{9}\right)$ is a pt. of inflexion

1 for finding coordinate
1 for testing
1 for marked on sketch.

(d) For which values of x is the curve $y = f(x)$ concave up? **1**

$f''(x) = 12x - 16$

$= 0$ when $x = \pm\frac{2}{\sqrt{3}} = \pm\frac{2\sqrt{3}}{3}$

∴ concave up when $\left(-\infty, -\frac{2\sqrt{3}}{3}\right) \cup \left(\frac{2\sqrt{3}}{3}, \infty\right)$

Mostly done well!

If answered $x < -\frac{2}{\sqrt{3}}$ and $x > \frac{2}{\sqrt{3}}$, had to match graph.

Question 28 (2 marks)

The distance between the curves $y = \log_2(x-3)$ and $y = \log_2 x$ at $x = k$ is 2 units. **2**
Find the value of k.

$\log_2 x - \log_2(x-3) = 2$ at $x = k$

$\log_2\left(\frac{k}{k-3}\right) = 2$

$\frac{k}{k-3} = 4$

$4 = k - 3$

$4k - 12 = k$

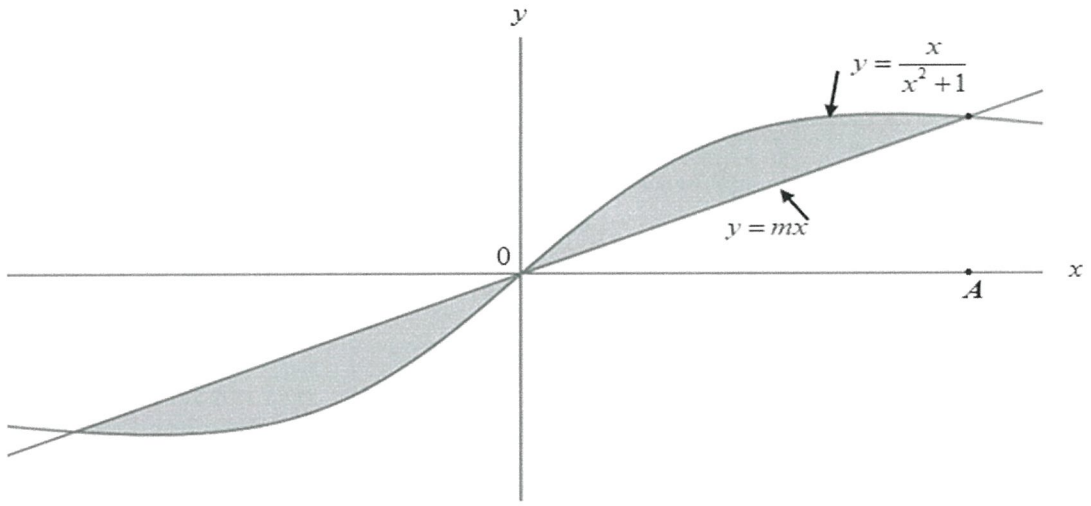
$3k = 12$

$\therefore k = 4$

Generally well done but some students had the two log graphs the other way around producing a negative value.

Question 29 (5 marks)

The diagram below shows the areas between two odd functions: $y = \dfrac{x}{x^2+1}$ and $y = mx$.



(a) Show that the x-coordinate, A, of the point of intersection of the functions in the first quadrant is given by $\sqrt{\dfrac{1}{m}-1}$. **2**

$y = mx$ — ① $\quad y = \dfrac{x}{x^2+1}$ — ②

① = ② $\quad mx = \dfrac{x}{x^2+1}$

$mx^3 + mx = x$

$mx^3 + mx - x = 0$

$x(mx^2 + m - 1) = 0$

$\therefore x = 0, \quad mx^2 = 1 - m$

$x^2 = \dfrac{1-m}{m}$

$x^2 = \dfrac{1}{m} - 1$

$\therefore x = \pm\sqrt{\dfrac{1}{m}-1}$

$\therefore x = \sqrt{\dfrac{1}{m}-1}$ in Q1

Many students did not account for $x \neq 0$. Also a mark was deducted if a student left out the '$\pm$' or didn't mention Q1.

(b) Calculate the area of the shaded region in terms of m. **3**

$A = 2\displaystyle\int_0^A \left(\frac{x}{x^2+1} - mx\right) dx$

$A = \displaystyle\int_0^A \frac{2x}{x^2+1}\,dx - \int_0^A 2mx\,dx$

$= \ln|x^2+1|\Big]_0^A - mx^2\Big]_0^A$

$= \ln|A^2+1| - \ln 0 - \left[mA^2 - m(0)\right]$

$= \ln|A^2+1| - mA^2$

$= \ln\left|\dfrac{1}{m} - 1 + 1\right| - m\left(\dfrac{1}{m} - 1\right)$

$= \ln\dfrac{1}{m} - 1 + m$

$= -\ln m + m$

$= (m - \ln m)u^2$

So, so! Many students became confused with the substitution.

Question 30 (3 marks)

Solve $\tan\left(2x-\frac{\pi}{4}\right)=\frac{1}{\sqrt{3}}$ for $x\in[-\pi,\pi]$

3

Domain

$-\pi \le x \le \pi$

$\therefore\ -2\pi \le 2x \le 2\pi$

$\therefore\ -2\pi-\frac{\pi}{4} \le 2x-\frac{\pi}{4} \le 2\pi-\frac{\pi}{4}$

$\therefore\ -\frac{9\pi}{4} \le 2x-\frac{\pi}{4} \le \frac{7\pi}{4}$

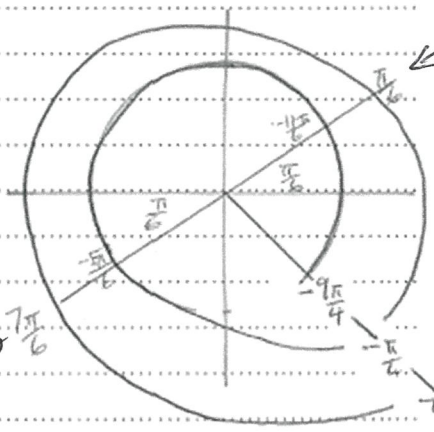
$\tan\left(2x-\frac{\pi}{4}\right)=\frac{1}{\sqrt{3}}$ 1st, 3rd quads

$2x-\frac{\pi}{4} = -\frac{11\pi}{6}, -\frac{5\pi}{6}, \frac{\pi}{6}, \frac{7\pi}{6}$

$2x-\frac{6\pi}{24} = -\frac{44\pi}{24}, -\frac{20\pi}{24}, \frac{4\pi}{24}, \frac{28\pi}{24}$

$2x = -\frac{38\pi}{24}, -\frac{14\pi}{24}, \frac{10\pi}{24}, \frac{34\pi}{24}$

$x = -\frac{19\pi}{24}, -\frac{7\pi}{24}, \frac{5\pi}{24}, \frac{17\pi}{24}$



solutions 1st Quad

solutions 3rd quad.

domain start & finish

This was generally well done if students adjusted their domain correctly.

Most students received 2 or 3 marks

2 marks (at least 2 correct answers)

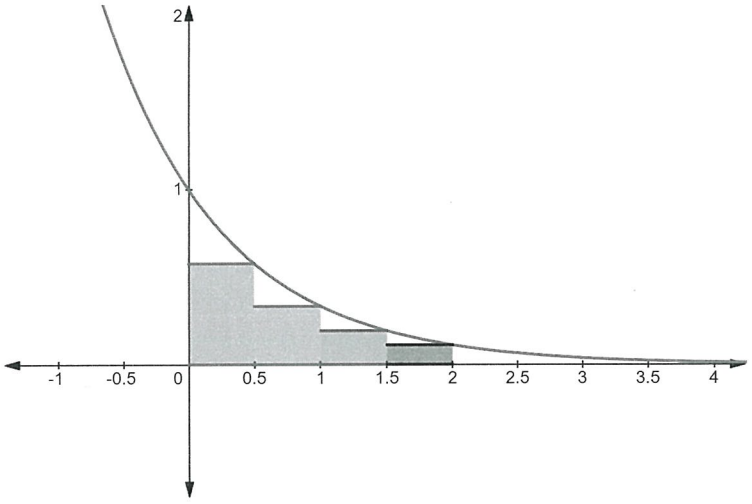
3 marks (4 correct answers)

1 mark (some indication of knowing the base angles)

Question 31 (5 marks)

The diagram shows the graph of $y = 3^{-x}$. Also shown on the diagram are the first 4 of an infinite number of rectangular strips of width $\frac{1}{2}$ unit and height $y = 3^{-x}$, for non-negative integer values of x.

NOT TO SCALE



(a) The sum of the areas of the rectangles forms a geometric series. **2**

Show that the limiting sum of this series is $\frac{1}{2\sqrt{3}-2}$.

$$\text{Area of the rectangles} = \frac{1}{2}\times 3^{\frac{-1}{2}} + \frac{1}{2}\times 3^{-1} + \frac{1}{2}\times 3^{\frac{-3}{2}} + \frac{1}{2}\times 3^{-2} + \cdots$$

$$= \frac{1}{2}\left(3^{\frac{-1}{2}} + 3^{-1} + 3^{\frac{-3}{2}} + 3^{-2} + \cdots\right)$$

The GP has first term $a = 3^{\frac{-1}{2}}$, common ratio $r = 3^{\frac{-1}{2}}$,

$$\therefore \text{The limiting sum } S_\infty = \frac{3^{\frac{-1}{2}}}{1-3^{\frac{-1}{2}}} \times \frac{1}{2}$$

$$= \frac{\frac{1}{\sqrt{3}}}{\frac{\sqrt{3}-1}{\sqrt{3}}} \times \frac{1}{2}$$

$$= \frac{1}{\sqrt{3}-1} \times \frac{1}{2} = \frac{1}{2\sqrt{3}-2}$$

Generally well done. Some mistakes identifying the wrong common ratio.

Question 30 continues on page 27

Question 31 (continued)

(b) Show that $\int_0^2 3^{-x}\,dx = \dfrac{8}{9\ln 3}$. **2**

$$\int_0^2 3^{-x}\,dx = \left[\frac{-3^{-x}}{\ln 3}\right]_0^2$$

$$= -\left[\frac{1}{3^x \ln 3}\right]_0^2$$

$$= -\left(\frac{1}{3^2\ln 3} - \frac{1}{3^0\ln 3}\right)$$

$$= -\frac{1}{9\ln 3} + \frac{1}{\ln 3}$$

$$= -\frac{1}{9\ln 3} + \frac{9}{9\ln 3}$$

$$= \frac{-1+9}{9\ln 3}$$

$$= \frac{8}{9\ln 3} \quad \textit{as required}$$

well done.

(c) By finding the areas of the four shaded rectangles and using part (b), **1**

show that $\ln 3 < \dfrac{4}{\sqrt{3}+1}$. *

$$\textit{Area of the rectangles} = \frac{1}{2}\times 3^{\frac{-1}{2}} + \frac{1}{2}\times 3^{-1} + \frac{1}{2}\times 3^{\frac{-3}{2}} + \frac{1}{2}\times 3^{-2}$$

$$= \frac{1}{2}\left(3^{\frac{-1}{2}} + 3^{-1} + 3^{\frac{-3}{2}} + 3^{-2}\right)$$

$$= \frac{1}{2}\left(\frac{1}{\sqrt{3}} + \frac{1}{3} + \frac{1}{3\sqrt{3}} + \frac{1}{3^2}\right)$$

$$= \frac{1}{2}\left(\frac{4}{3\sqrt{3}} + \frac{4}{9}\right)$$

$$= \frac{1}{2}\left(\frac{4\sqrt{3}}{9} + \frac{4}{9}\right)$$

$$= \frac{2}{9}(\sqrt{3}+1)$$

$$\int_0^2 3^{-x}\,dx > \frac{2}{9}(\sqrt{3}+1)$$

$$\frac{8}{9\ln 3} > \frac{2}{9}(\sqrt{3}+1)$$

$$\frac{4}{\ln 3} > (\sqrt{3}+1)$$

$$\therefore \ln 3 < \frac{4}{\sqrt{3}+1}$$

Award mark for attaining this line. due to typo.

Some students approached this using GP:

$r = \frac{1}{\sqrt{3}}$, $a = \frac{1}{\sqrt{3}}$

$S_4 = \frac{\frac{1}{\sqrt{3}}\left(1-\left(\frac{1}{\sqrt{3}}\right)^4\right)}{1-\frac{1}{\sqrt{3}}} \times \frac{1}{2}$

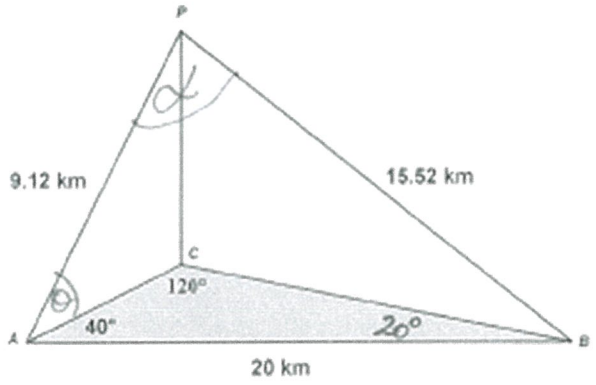
$= \frac{1-\frac{1}{9}}{\sqrt{3}-1} \times \frac{1}{2}$

$= \frac{4}{9}\cdot\frac{\sqrt{3}+1}{2}$

$= \frac{2}{9}(\sqrt{3}+1)$.

Question 32 (5 marks)

Consider the diagram below where A, B and C are three points on level ground and P is a point vertically above C



(i) Find the length of AC, correct to the nearest 0.1 km. **2**

$\frac{AC}{\sin 20} = \frac{20}{\sin 120}$ Using Sine Rule

$AC = \frac{20 \sin 20}{\sin 120}$

$= 7.8986\ldots$ km

$= 7.9$ km (nearest 0.1 km)

Very well done

(ii) Hence, find the angle of elevation of P from A, correct to the nearest minute **1**

$\cos \theta = \frac{7.9}{9.12}$

$\theta = \cos^{-1}\left(\frac{7.9}{9.12}\right)$

$= 29° 59'$ (nearest minute)

well done

(iii) Find the angle between PA and PB to the nearest minute. **2**

Using Cosine Rule

$\cos \alpha = \frac{9.12^2 + 15.52^2 - 20^2}{2 \times 9.12 \times 15.52}$

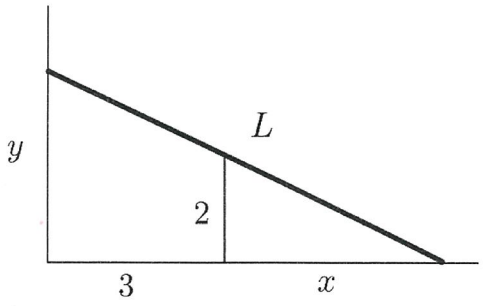
$\alpha = 105° 34'$

Mostly well done
Some where confused as to which angle or used wrong rules.

QUESTION 33 is over the page

Question 33 (6 marks)

A 2-metre vertical fence stands 3 metres from a high vertical wall. A ladder, L, is placed from the horizontal ground to the wall, resting on the fence. The base of the ladder is x metres from the fence.



(a) If L represents the length of the ladder, show that **2**

$$L^2=(x+3)^2\left(1+\frac{4}{x^2}\right)$$

$\frac{2}{y}=\frac{x}{x+3}$ from similar Δ's.

$y=\frac{2(x+3)}{x}$

$L^2=y^2+(x+3)^2$ by Pythagoras

$L^2=\left(\frac{2(x+3)}{x}\right)^2+(x+3)^2$

$L^2=\frac{4}{x^2}(x+3)^2+(x+3)^2$

$L^2=(x+3)^2\left(\frac{4}{x^2}+1\right)$ *as required*

Generally well done. Sufficient steps need to be shown.

Question 33 continues on page 31

Question 33 (continued)

(b) By first finding $\frac{dL^2}{dx}$, calculate the shortest ladder that can reach from the ground outside the fence to the wall, correct to 1 decimal place. **4**

$$\frac{dL^2}{dx} = 2(x+3)\left(\frac{4}{x^2}+1\right)+(x+3)^2\left(\frac{-8}{x^3}\right)$$

$$= 2(x+3)\left(\frac{4}{x^2}+1\frac{-4(x+3)}{x^3}\right)$$

$$= 2(x+3)\left(\frac{4x+x^3-4x-12}{x^3}\right)$$

$$= 2(x+3)\left(\frac{x^3-12}{x^3}\right)$$

Many students differentiated correctly but couldn't solve for $\frac{dL^2}{dx}=0$ correctly.

The best way is to keep the expression in factorised form rather than expand it.

Let $\frac{dL^2}{dx}=0$,

$$2(x+3)\left(\frac{x^3-12}{x^3}\right)=0$$

$x+3=0.\ x^3-12=0$

$x=-3,\ x=\sqrt[3]{12}\approx 2.3$ ($x>0$ as it is a length)

Test the nature,

x	2	$\sqrt[3]{12}$	3
$\frac{dL^2}{dx}$	-5	0	6.666..
	\	—	/

Some forgot to test the nature.

There is a change of gradient from negative to positive, therefore when $x=\sqrt[3]{12}$, the ladder is the shortest. The length of the ladder:

$$L=\sqrt{(x+3)^2\left(1+\frac{4}{x^2}\right)}\quad (L>0)$$

$$=\sqrt{(\sqrt[3]{12}+3)^2\left(1+\frac{4}{\sqrt[3]{12^2}}\right)}$$

$= 7.0234\ldots$

≈ 7.0

Some forgot to find the value of L.

End of paper