Student Number

Hunters Hill High School

2026 TRIAL EXAMINATION

Mathematics Advanced

General Instructions

- Reading time – 10 minutes
- Working time – 3 hours
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided at the back of this paper
- For questions in Section II, show relevant mathematical reasoning and/or calculations
- Write your Student Number at the top of this page

Total marks: 100

Section I – 10 marks (pages 3 – 6)

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section

Section II – 90 marks (pages 7 – 30)

- Attempt Questions 11 – 33
- Allow about 2 hours and 45 minutes for this section

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Section I

10 Marks
Attempt Questions 1–10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1–10

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1. Which of the following functions is neither odd nor even?

(A) $f(x) = e^x$

(B) $f(x) = \frac{1}{x}$

(C) $f(x) = x^6$

(D) $f(x) = 4x^5 + x$

2. The first three terms of an arithmetic sequence are 1, 3, 5, …

Which of the following gives the sum of the first n terms of this sequence, S_n?

(A) $S_n = n^2$

(B) $S_n = 2n$

(C) $S_n = 2n - 1$

(D) $S_n = 2n + 1$

3. The discrete random variable X has the probability distribution function shown.

x	0.1	0.3	0.5	0.9
$p(x)$	0.2	0.3	0.1	a

What is the value of a?

(A) 0

(B) 0.4

(C) 0.6

(D) 1

4. The table shows the function values of $y = f(x)$ and $y = g(x)$.

x	-1	0	1	2	3
$f(x)$	3	0	3	12	27
$g(x)$	4	3	2	1	0

What is the value of $y = f\big(g(1)\big)$?

(A) 0

(B) 1

(C) 6

(D) 12

5. Which of the following expressions is the derivative of $y = \ln\left(\frac{x-2}{4x+1}\right)$?

(A) $\frac{3x+3}{(x-2)(4x+1)}$

(B) $\frac{9}{(x-2)(4x+1)}$

(C) $\frac{3x-1}{(x-2)(4x+1)}$

(D) $\frac{4x+1}{x-2}$

6. If $\log_{10}(x-a) = 3$, what is the value of x?

(A) a^3

(B) $1000a$

(C) $1000 + a$

(D) $30 + a$

7. Consider that $\int_2^8 f(x)dx = C$ and $\int_6^8 f(x)dx = D$.

What is the value of $\int_2^6 f(x)dx$?

(A) $C + D$

(B) $C + D - 2$

(C) $C - D$

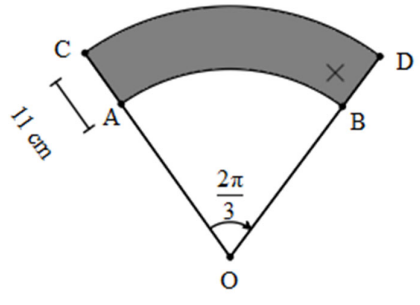
(D) $D - C$

8. Which of the following gives the period and range of $f(x) = 3\cos\left(2x + \frac{\pi}{2}\right) + 1$?

	Period	Range
(A)	π	$[-3, 3]$
(B)	4π	$[-3, 3]$
(C)	π	$[-2, 4]$
(D)	4π	$[-2, 4]$

9. In the figure, OAB and OCD are sectors with centre O. It is given that the area of the shaded region is 143π cm^2

If $AC = 11$ cm and $\angle AOB = \frac{2\pi}{3}$, what is the value of OA?



(A) 12 cm

(B) 14 cm

(C) 20 cm

(D) 25 cm

10. Simplify $\sqrt{x^2 + 2x + 1} - \sqrt{x^2 - 2x + 1}$.

(A) -2

(B) 2

(C) $2x$

(D) $2x + 2$

End of Section I

Section II

90 Marks
Attempt Questions 11–33
Allow about 2 hours and 45 minutes for this section

For questions in Section II, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (2 marks) **Marks**

Solve $\frac{6x+3}{x} = \frac{5}{x} + 4$ **2**

Question 12 (2 marks)

Find the values of k for which the expression $x^2 - 3x + (4 - 2k)$ is always positive. **2**

Question 13 (3 marks) **Marks**

On the first day of the harvest, an orchard produces 1630 kg of fruit. On the next day, the orchard produces 1605 kg, and the amount produced continues to decrease by the same amount each day. **3**

On what day does the daily production first fall below 600 kg?

Question 14 (3 marks)

Find the equation of the tangent to $y = 3x^4 - 2x^2 - 5$ at the point where $x = 1$. **3**

Question 15 (2 marks) **Marks**

The numbers 1280, p, q, 20 form a geometric sequence. **2**
Find the values of p and q.

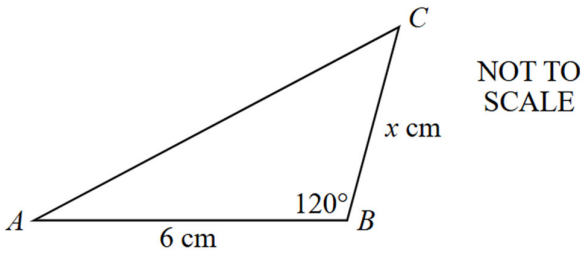
Question 16 (2 marks)

Fifty tickets are sold in a raffle for which there is a first prize and a second prize. After a ticket is drawn for the first prize, it is discarded, and another ticket is then drawn for the second prize. **2**

Tim bought two of the raffle tickets. What is the probability he wins only one of the prizes?

Question 17 (3 marks) **Marks**

The triangle ABC has an exact area of $\left(12-\sqrt{3}\right)$ cm^2. Find the exact value of x. **3**



Question 18 (2 marks)

A circle is defined by $x^2+2x+y^2+y-2=y+1$. **2**
Determine its centre and radius.

Question 19 (2 marks) **Marks**

Differentiate $y = \dfrac{2x}{3x^2 + 1}$ **2**

Question 20 (3 marks)

Find $\int \sqrt[3]{4x + 7}\,dx$ **3**

Question 21 (5 marks) **Marks**

The students in a school were surveyed on the number of hours of sleep they have per week. The results were normally distributed. The survey indicated that 95% of students had between 42 and 54 hours of sleep per week.

(a) Determine the mean number of hours of sleep per week. **1**

(b) What was the standard deviation? **1**

(c) What percentage of students would have indicated they had between 51 and 57 hours of sleep per week? **3**

Question 22 (3 marks) **Marks**

Point $P(-1,-2)$ lies on $f(x)$. Point P' is the image of point P on the function $g(x) = 2f(3x-1)$. **3**

By clearly stating the transformations required to produce $g(x)$ from $f(x)$, show that the coordinates of point P' are $(0,-4)$.

Question 23 (5 marks) **Marks**

A manufacturer stores drums of chemicals. During storage, evaporation takes place. A random sample of 10 drums was taken and the time in storage, x weeks, and the evaporation loss, y ml, are shown in the table below.

$x =$ weeks	4	6	7	9	11	13	14	16	17	19
$y =$ evaporation loss (ml)	35	49	52	60	68	78	81	89	87	95

(a) Find the correlation coefficient r and comment on the strength of the relationship. **2**

(b) Find the equation of the least squares' regression line. **1**

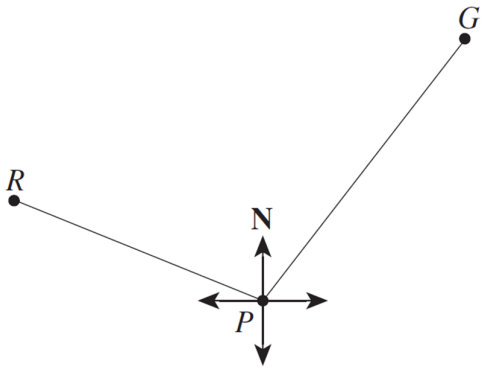
Question 23 continued **Marks**

(c) Using your model, predict the amount of evaporation that would take place after 35 weeks. **1**

(d) Comment on the reliability of this prediction from your answer in part (c). **1**

Question 24 (3 marks) **Marks**

Ringo and Greg start at location P and drive in different directions.
Ringo drives at a speed of 70 km/h on a bearing of 285°, while Greg drives at a speed of 60 km/h on a bearing of 030°, as shown in the diagram.



NOT TO SCALE

(a) Show that $\angle GPR = 105°$. **1**

(b) What will be the distance between Ringo and Greg after travelling for 2 hours? **2**

Give your answer correct to the nearest kilometre.

Question 25 (8 marks) **Marks**

A particle moves in a straight line. At time t seconds, its distance x metres from a fixed point 0 on the line is given by $x = 1 - \cos(2t)$.

(a) Sketch the graph of x as a function of t for $0 \leq t \leq \pi$. **2**

(b) Using your graph, or otherwise, find the times when the particle is at rest and the position of the particle at these times. **2**

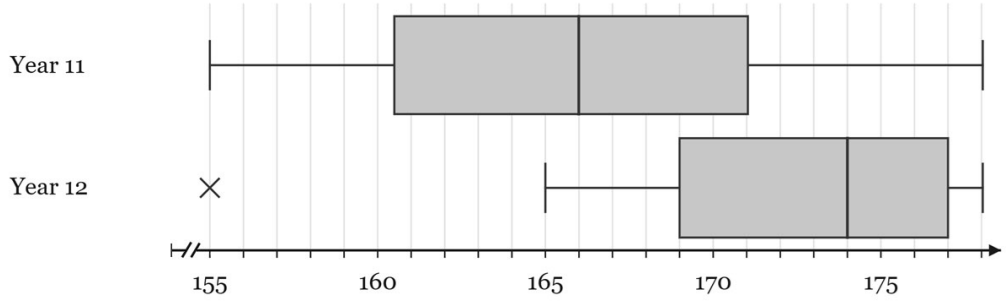
Question 25 continued **Marks**

(c) Find the velocity of the particle when $t = \frac{\pi}{4}$. **2**

(d) When is the particle's velocity greater than 1 m/s? **2**

Question 26 (5 marks) **Marks**

A study was conducted on Year 11 and Year 12 students at one school. Students were asked to report their height in centimetres. The data was used to create the parallel box plots below.



(a) Compare the Year 11 and Year 12 students' distributions of heights, disregarding the data point marked with an × symbol. **3**

(b) One of the Year 12 students reported their height to be 155 cm. Their height was marked with an × symbol on the box plot, indicating their height is an outlier.
Verify with calculations if this student's height is an outlier. **2**

Question 27 (7 marks) **Marks**

Consider the curve $y = x^3 - 6x^2 + 9x + 3$.

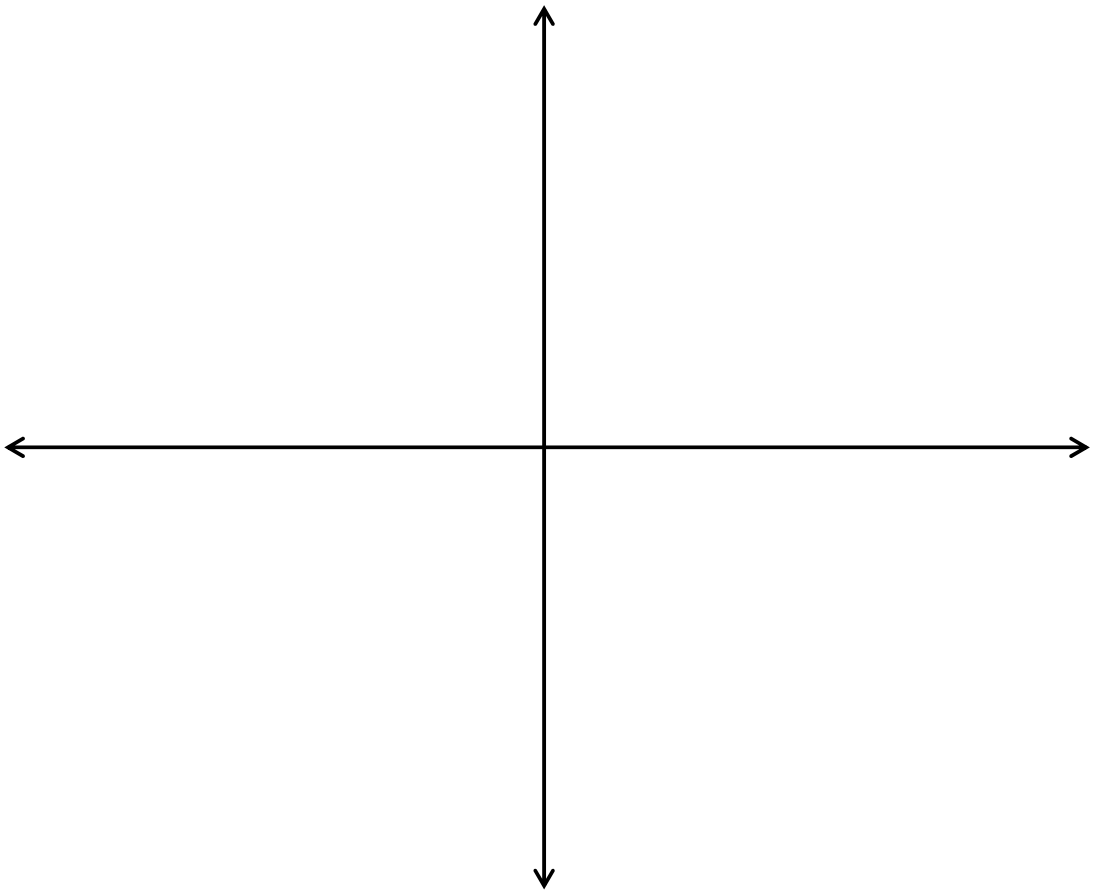
(a) Find the location and nature of any stationary points. **3**

(b) Find the point of inflection. **2**

Question 27 continued **Marks**

(c) Sketch the curve $y = x^3 - 6x^2 + 9x + 3$ labelling the stationary points, the point of inflection and y-intercepts. **2**

The x-intercepts are not required.



Question 28 (4 marks) **Marks**

(a) Prove that $\sec x + \tan x = \dfrac{\cos x}{1 - \sin x}$. **2**

(b) Hence, or otherwise, find the exact value of $\int_0^{\frac{\pi}{3}} (\sec x + \tan x) dx$. **2**

Question 29 (5 marks) **Marks**

ABC is a triangle.

The numbers $1 + \sin A, \frac{1}{2}, 1 - \sin A$ are the first three terms of a geometric sequence.

(a) Find the possible values of A. **3**

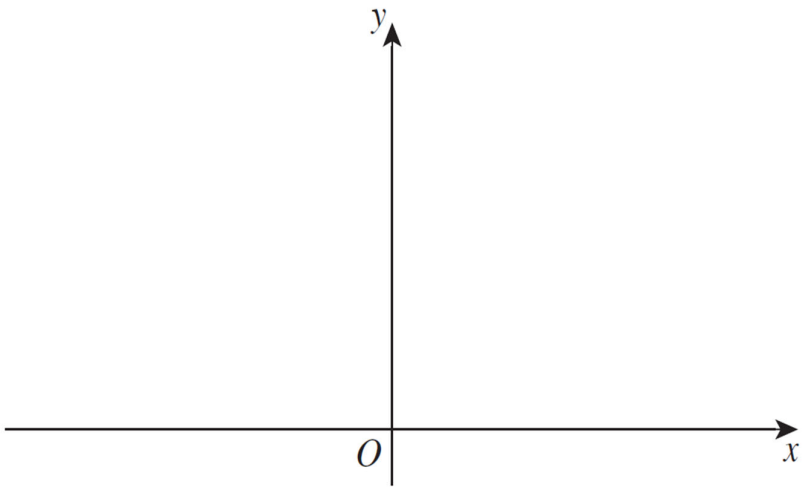
(b) Show that the limiting sum of the series $(1 + \sin A) + \frac{1}{2} + (1 - \sin A) + \cdots$ exists. **2**

Question 30 (7 marks) **Marks**

The lifetime, X, in **tens** of hours, of a battery has a cumulative distribution function $F(x)$ that is given by

$$F(x) = \begin{cases} 0, & x < 1 \\ \frac{25}{116}(x^2 + 3x - 4), & 1 < x < 1.8 \\ 1, & x \geq 1.8 \end{cases}$$

(a) Sketch the cumulative distribution function. **2**



Question 30 continued **Marks**

(b) An outdoor lantern requires three batteries, all of which must be working, to operate. **3**
Three new batteries are inserted into the lantern.
Find the probability that the lantern will be working after 15 hours. Answer as a percentage, rounded to two decimal places.

(C) Find the probability density function of the random variable X. **2**

Question 31 (3 marks) **Marks**

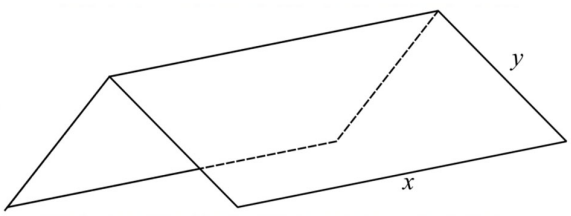
The average life cycle of an insect is one month. A viable nest of this insect has between 100 000 to 500 000 insects. The population P of a nest of this insect grows exponentially so that: **3**

$$\frac{dP}{dt} = 1200e^{0.3t}$$

A nest of these insects had a population of 5000 after one month.
Determine how long it will take the nest to reach the viable stage of at least 100 000.
Answer correct to the nearest month.

Question 32 (6 marks) **Marks**

Harvey is designing an open-ended tent as shown below.



The frame of the tent is made using rods of two different lengths:

- x metres for the top and bottom edges
- y metres for each sloping edge.

The rods cost \$9.50 per metre.
The frame is covered by one rectangular sheet of fabric whose area is 24 m^2.

(a) Show that the total length, l m, of rods is given by $l = 3x + \frac{48}{x}$. **2**

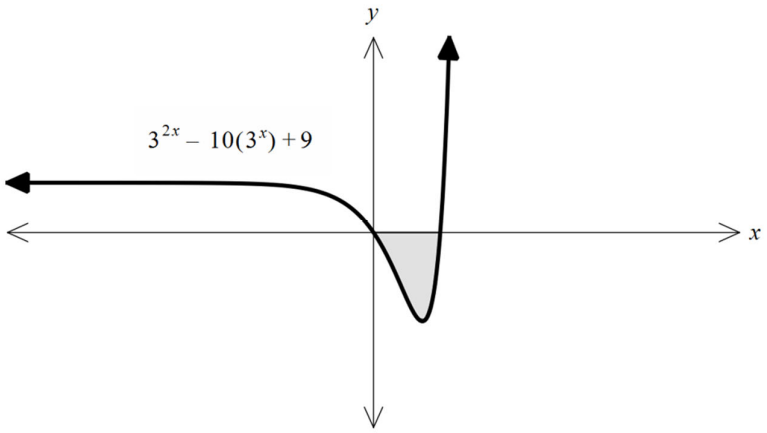
Question 32 continued

(b) Find the value of x for which l is a minimum. **2**

(c) What is the minimum cost of the frame? **2**

Question 33 (5 marks) **Marks**

The graph of $f(x) = 3^{2x} - 10(3^x) + 9$ is shown below. Find the exact value of the area shaded. **5**



Question 33 continued

End of examination

Section II Extra writing space

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Section II Extra writing space

If you use this space, clearly indicate which question you are answering.

2026 HHHS – Mathematics Advanced – Trial Solutions

Section I

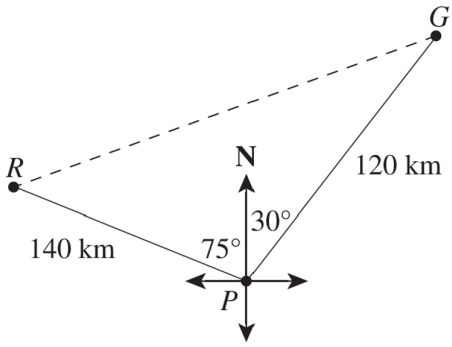
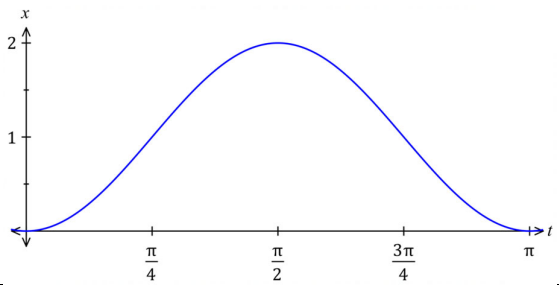
1	$f(-x) = e^{-x}$, which is neither e^x or $-e^x$ Hence the function is neither odd nor even.	A
2	Sequence is 1, 3, 5,..... AP wherw $a = 1, and\ d = 2$ $S_n = \frac{n}{2}[2a + (n-1)d]$ $= \frac{n}{2}[2 + (n-1)2]$ $= \frac{n}{2}[2 \times 1 + (n-1)2]$ $= \frac{n}{2}[2 + 2n - 2]$ $= n^2$	A
3	$a = 1 - (0.2 + 0.3 + 0.1)$ $= 0.4$	B
4	$g(1) = 2$ $f(2) = 12$	D
5	$\ln\left(\frac{x-2}{4x+1}\right) = \ln(x-2) - \ln(4x+1)$ $y' = \frac{1}{x-2} - \frac{4}{4x+1}$ $= \frac{4x+1-4(x-2)}{(x-2)(4x+1)}$ $= \frac{4x+1-4x+8}{(x-2)(4x+1)}$ $= \frac{9}{(x-2)(4x+1)}$	B
6	$x - a = 10^3$ $x = 1000 + a$	C
7	$\int_2^6 f(x)dx = \int_2^8 f(x)dx - \int_6^8 f(x)dx$ $= C - D$	C
8	Period $\frac{2\pi}{2} = \pi$ Range $= [1-3, 1+3]$ $= [-2, 4]$	C

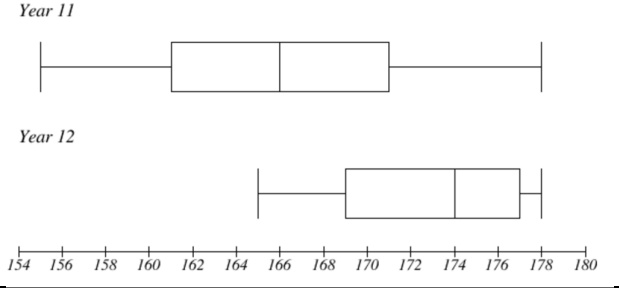
9	$\frac{1}{2}\times\frac{2\pi}{3}[(11+r)^2-r^2]=143\pi$ $\frac{1}{3}(121+22r+r^2-r^2)=143$ $\frac{1}{3}(121+22r)=143$ $r=14$ cm	B
10	$\sqrt{x^2+2x+1}-\sqrt{x^2-2x+1}$ $=\sqrt{(x+1)^2}-\sqrt{(x-1)^2}$ $=x+1-(x-1)$ $=2$	B

Section II		
11	$\frac{6x+3}{x}=\frac{5}{x}+4$ $6x+3=5+4x$ $2x=2$ $x=1$	1-Simplifying to no denominator 2-Correct solution with working
12	$x^2-3x+(4-2k)>0$ concave up Always positive (no roots) if $\Delta<0$ $b^2-4ac<0$ $(-3)^2-4\times1\times(4-2k)<0$ $9-16+8k<0$ $8k<7$ $k<\frac{7}{8}$	1-Attempt to use the discriminant 2-Correct solution with working
13	Find n such that $T_n<600$ $a=1630\ and\ d=-25$ $T_n=a+(n-1)d<600$ $1630-25(n-1)<600$ $1630-25n+25<600$ $-25n<-1055$ $n>42.2$ $\therefore$ on the 43th day	1-Finds a and d Attempts to use rule for Tn 2-Write correct inequality to progress to the solution 3-Correct solution with working
14	$y=3x^4-2x^2-5$ $y'=12x^3-4x$ $m=12(1)^3-4(1)$ $=8$ When $x=1, y=-4$ $y-(-4)=8(x-1)$ $y=8x-12$	1-Find the derivative of the function 2-Finds gradient 3-Correct solution with working
15	$a=1280$	1- Find expression for T_4

	$T_4 = 20$ $1280r^3 = 20$ $r = \frac{1}{4}$ $P = 1280 \times \frac{1}{4}$ $= 320$ $q = 320 \times \frac{1}{4}$ $= 80$	2-Correct solution with working
16	$P(Win\ then\ lose) or\ P(Lose\ then\ Win)$ $= \left(\frac{2}{50} \times \frac{48}{49}\right) + \left(\frac{48}{50} \times \frac{2}{49}\right)$ $= \frac{96}{1225}$	1-Correct for $P(win\ then\ loses) = \frac{48}{1225}$ 2-Correct solution with working
17	$\frac{1}{2} \times 6 \times x \times \sin 120^o = 12 - \sqrt{3}$ $3x \sin 120^o = 12 - \sqrt{3}$ $\frac{3\sqrt{3}x}{2} = 12 - \sqrt{3}$ $x = \frac{2(12 - \sqrt{3})}{3\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$ $= \frac{24\sqrt{3} - 6}{9}$ $= \frac{8\sqrt{3} - 2}{3}$	1-Applies area formula with one error 2-Finds x but not as an exact answer 3-Correct solution with working
18	$x^2 + 2x + y^2 + y - 2 = y + 1$ $x^2 + 2x + y^2 = 3$ $x^2 + 2x + 1 + y^2 = 3 + 1$ $(x + 1)^2 + y^2 = 4$ centre is $(-1,0)$ and radius is $\sqrt{4} = 2$	1-Writes the equation in the form of a completed square 2-Correct solution with working
19	$f(x) = \frac{2x}{3x^2 + 1}$ $u = 2x \qquad v = 3x^2 + 1$ $u' = 2 \qquad v' = 6x$ $f'(x) = \frac{(3x^2 + 1)(2) - (2x)(6x)}{(3x^2 + 1)^2}$ (2 marks for achieving above) $= \frac{2 - 6x^2}{(3x^2 + 1)^2}$	1- attempt to use quotient rule 2-Correct solution with working

<table>
<tr>
<td>20</td>
<td>$$\int \sqrt[3]{4x+7}\, dx$$
$$= \int (4x+7)^{\frac{1}{3}}\, dx$$
$$= \frac{(4x+7)^{\frac{4}{3}}}{\frac{4}{3}} \times \frac{1}{4} + C$$
$$= \frac{3(4x+7)^{\frac{4}{3}}}{16} + C$$</td>
<td>1- attempt to use the reverse chain rule.
2-Correctly integrates but forgets the C
-Attempt to use reverse chain rule and includes C
3- solution with working</td>
</tr>
<tr>
<td>21a</td>
<td>95% of the data lie within two standard deviations of the mean.
42 hours is a z-score of –2 and 54 hours has a z-score of 2.
The mean is midway between 42 and 54 hours.
$\therefore$ Mean is 48 hours.</td>
<td>1-Correct solution</td>
</tr>
<tr>
<td>b</td>
<td>There are 4 standard deviations between 42 and 54 hours.
$$\text{Standard deviation} = \frac{54-42}{4}$$
$$= 3 \text{ hours}$$</td>
<td>1-Correct solution with working</td>
</tr>
<tr>
<td>c</td>
<td>To find the z-score of 51 and 57
$$z = \frac{x-\mu}{s} = \frac{51-48}{3} = 1$$
$$z = \frac{x-\mu}{s} = \frac{57-48}{3} = 3$$
$$\text{Percentage} = \frac{99.7\%}{2} - \frac{68\%}{2}$$
$$= 15.85\%$$
$\therefore$ 15.85% have sleep between 51 and 57 hours of sleep per week</td>
<td>1-Uses the z – score formula
2-Find the correct z −score for both
3-Correct solution with working</td>
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<tr>
<td>22</td>
<td>The original function is $y = f(x)$ and $P(-1,-2)$ lies on the graph of $f(x)$. The require transformations to produce $g(x)$ from $f(x)$ are as follows.
<table>
<tr><th>New function</th><th>Transformation</th><th>New point</th></tr>
<tr><td>$\frac{y}{2} = f(x)$
$\rightarrow y = 2f(x)$</td><td>Vertical dilation with a scale factor of 2</td><td>$(-1, -2 \times 2)$
$= (-1, -4)$</td></tr>
<tr><td>$y = 2f(x-1)$</td><td>Translation to the right by 1 unit</td><td>$(-1+1, -4)$
$= (0, -4)$</td></tr>
<tr><td>$y = 2f(3x-1)$</td><td>Horizontal dilation with a scale factor of $\frac{1}{3}$</td><td>$\left(0 \times \frac{1}{3}, -4\right)$
$= (0, -4)$</td></tr>
</table>
</td>
<td>1- Correctly describes 1 transformation
2- Correctly describes 2 transformations
3- Correctly describes 3 transformations</td>
</tr>
</table>

23a	$r = 0.9901098193 \ldots\ldots$ $r = 0.99$ $\therefore$ strong positive linear relationship	1- Finds r 2-Correct solution
b	$y = A + Bx$ $A = 24.117338$ $B = 3.903677758$ $y = 24.12 + 3.90x$	1- Correct solution with working
c	$y = 24.12 + 3.9(35)$ $= 160.62$ $= 160$	1- Correct solution with working
d	Although there is a strong relationship between the variables our x value of 35 is far away from our data set. i.e. we are extrapolating and the value is less reliable the farther away from the data set we used in our calculations	1- Correct explanation
24a	 $\angle GPR = 75° + 30°$ $= 105°$	1- Correct solution
b	After 2 hours, Greg has travelled a distance of 120 km and Ringo has travelled a distance of 140 km. Let d be the distance apart. $d^2 = 140^2 + 120^2 - 2(140)(120)\cos 105^o$ $= 42696.1992 \ldots\ldots$ $d = 206.63$ ≈ 207 km Therefore, they are 207 km apart.	1 - applies the cos rule 2 -correct solution with working
25a	Amplitude = 1 Period $= \frac{2\pi}{2} = \pi$ 	1 - Finds the general shape of the curve 2 -Correct solution

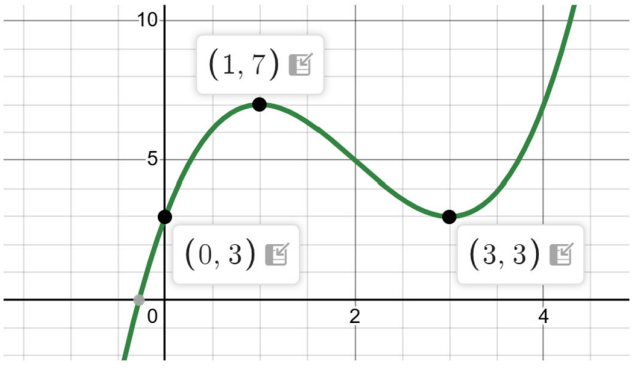
b	The particle is a rest when $v = 0$ or $\frac{dx}{dt} = 0$ $\therefore when\ t = 0, \frac{\pi}{2}, \pi$ $\therefore$ Position of the particle at these times: $x = 0, 2, 0$	1- Find 2 times or 2 positions 2 -Correct solution with working
c	$x = 1 - \cos 2t$ $v = \frac{dx}{dt} = 2\sin 2t$ At $t = \frac{\pi}{4}$. $v = 2\sin\left(2 \times \frac{\pi}{4}\right)$ $= 2$ m/s $\therefore$ Velocity of the particle is 2 m/s.	1 - Finds velocity function 2 -Correct solution with working
d	$2\sin 2t = 1$ $\sin 2t = \frac{1}{2}$ $2t = \frac{\pi}{6}, \frac{5\pi}{6}$ $t = \frac{\pi}{12}, \frac{5\pi}{12}$ Using the graph $\therefore \frac{\pi}{12} < t < \frac{5\pi}{12}$	1 – Solves equation and finds $t = \frac{\pi}{12}$ 2 -Correct solution with working
26a	The Year 11 students' heights have a lower median compared to the Year 12 students' heights. The Year 11 students' heights have more variation (or spread) compared to the Year 12 students. The Year 11 students' heights are more symmetrically distributed while the Year 12 students' heights are negatively skewed.	1 - Compares one of the below 2 -Compares two of the below 3 - Provides a correct comparison of the centre, spread, and shape of the distributions
b	Year 12 students IQR: $177 - 169 = 8$ $Q_1 - 1.5 \times IQR = 169 - 1.5 \times 8 = 157$ Any score below 157 is an outlier, so 155 is an outlier. 	1- Finds IQR 2-Correct solution with working

<table>
<tr>
<td>27a</td>
<td>$y = x^3 - 6x^2 + 9x + 3$
$y' = 3x^2 - 12x + 9$
$y'' = 6x - 12$

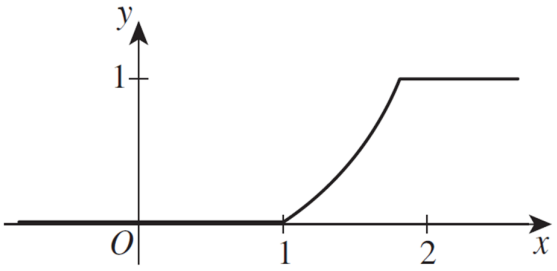
Stationary points occur when $y' = 0$
$3x^2 - 12x + 9 = 0$
$x^2 - 4x + 3 = 0$
$(x - 1)(x - 3) = 0$
$\therefore x = 1 \; or \; x = 3$

For $x = 1$
$y = 1^3 - 6(1)^2 + 9(1) + 3$
$= 7$
$y'' = 6(1) - 12$
$= -6 < 0 \rightarrow$ concave down

For $x = 3$
$y = 3^3 - 6(3)^2 + 9(3) + 3$
$= 3$
$y'' = 6(3) - 12$
$= 24 > 0 \rightarrow$ concave up

$\therefore$ Stationary points at $(1, 7)$ and $(3, 3)$ are a local maximum and a local minimum, respectively.</td>
<td>1- Finds $\frac{dy}{dx}$ and equates it to zero
2- Finds the location of the two stationary points
3- Correct solution with working</td>
</tr>
<tr>
<td>b</td>
<td>Possible inflection when $y'' = 0$
$6x - 12 = 0$
$x = 2$
<table><tr><td>x</td><td>1</td><td>2</td><td>3</td></tr><tr><td>y''</td><td>-6
< 0</td><td>0</td><td>6
> 0</td></tr></table>When $x = 2$, $y = 5$
As concavity changes across the point, (2, 5) is a point of inflection</td>
<td>1- correct point obtained
2- proves point of inflection</td>
</tr>
<tr>
<td>c</td>
<td>
</td>
<td>1-Correct answer including end points.
2- Basic shape of the curve</td>
</tr>
</table>

28a	$LHS = \sec x + \tan x$ $= \frac{1}{\cos x} + \frac{\sin x}{\cos x}$ $= \frac{1 + \sin x}{\cos x}$ $= \frac{(1 + \sin x)(1 - \sin x)}{\cos x\,(1 - \sin x)}$ $= \frac{1 - \sin^2 x}{\cos x(1 - \sin x)}$ $= \frac{\cos^2 x}{\cos x(1 - \sin x)}$ $= \frac{\cos x}{1 - \sin x}$ $= RHS$	1- Simplifies to $\frac{1 + \sin x}{\cos x}$ 2-Correct solution with working
b	$\int_{\mathbf{0}}^{\frac{\boldsymbol{\pi}}{\mathbf{3}}} \frac{\cos x}{1 - \sin x}\,dx = -\left[\ln\lvert 1 - \sin x\rvert\right]_0^{\frac{\pi}{3}}$ $= -\ln\left\lvert 1 - \sin\frac{\pi}{3}\right\rvert + \ln\lvert 1 - \sin 0\rvert$ $= -\ln\left\lvert 1 - \frac{\sqrt{3}}{2}\right\rvert$ $= -\ln\frac{2 - \sqrt{3}}{2}$ $= \ln\frac{2}{2 - \sqrt{3}}$	1- Correct integration 2-Correct solution with working Accept $= -\ln\left\lvert 1 - \frac{\sqrt{3}}{2}\right\rvert$
29a	for geometric sequence, $r_1 = r_2$ $\frac{\frac{1}{2}}{1 + \sin A} = \frac{1 - \sin A}{\frac{1}{2}}$ $(1 - \sin A)(1 + \sin A) = \frac{1}{4}$ $1 - \sin^2 A = \frac{1}{4}$ $\cos^2 A = \frac{1}{4}$ $\cos A = \pm\frac{1}{2}$ $\therefore 0 < A < \pi$ $A = \frac{\pi}{3}$ or $A = \frac{2\pi}{3}$	1 - Writes an equation for $\sin A$ 2 -Finds $\cos A$ or $\sin A$ 3 -Correct solution with working

b	$A = \frac{\pi}{3}, \frac{2\pi}{3}$ $\sin A = \frac{\sqrt{3}}{2}$ Common ratio $r = \frac{1 - \sin A}{\frac{1}{2}}$ $= 2\left(1 - \frac{\sqrt{3}}{2}\right)$ $= 2 - \sqrt{3}$ ≈ 0.268 $\therefore \lvert r \rvert < 1$ and hence limiting sum exists	1 - writes an equation for the common ratio in surd form 2 -Correct solution with working
30a		1 - 1 part of graph correct 2 -Correct sketch
b	Finding the probability that one battery will last more than 15 hours gives: $P(X > 1.5) = 1 - P(X \leq 1.5)$ $= 1 - F(1.5)$ $= 1 - \frac{25}{116}(1.5^2 + 3 \times 1.5 - 4)$ $= \frac{189}{464}$ Finding the probability that all three batteries will last more than 15 hours gives: $\left(\frac{189}{464}\right)^3 = 0.06758$ $= 6.76\%$	1- Finding $F(1.5)$ 2-Correctly finding probability for one battery. 3 -correct solution with working
c	$f(x) = \frac{d}{dx}F(x)$ $= \frac{d}{dx}\left(\frac{25}{116}(x^2 + 3x - 4)\right)$ $= \frac{25}{116}(2x + 3)$ Therefore, the probability distribution function is $f(x) = \begin{cases} \frac{25}{116}(2x + 3) & 1 < x < 1.8 \\ 0 & \text{elsewhere} \end{cases}$	1- Attempts to differentiate $F(x)$ 2-Correct solution with working

31	$\frac{dP}{dt} = 1200e^{0.3t}$ $P = \frac{1200}{0.3}e^{0.3t} + C$ $P = 4000e^{0.3t} + C$ Given $P = 5000$ when $t = 1$ $5000 = 4000e^{0.3(1)} + C$ $C = -399.4352 \ldots \approx -399.4$ To find t when $P = 100\,000$ $P = 4000e^{0.3t} - 399.4352 \ldots$ $100\,000 = 4000e^{0.3t} - 399.435 \ldots$ $e^{0.3t} = 25.09985881 \ldots$ $\ln 25.099 \ldots = 0.3t$ $t = 10.74287 \ldots$ ≈ 11 $\therefore$ The nest reaches a viable stage after 11 months.	1- Finds the anti-derivative of the differential equation. 2- Makes significant progress. 3-Correct solution with working
32a	One rectangular sheet with dimensions of x and $2y$. Area $= x \times 2y = 24$ $y = \frac{24}{2x}$ $= \frac{12}{x}$ Length (l) of the frame $l = 3x + 4y$ $= 3x + 4 \times \frac{12}{x}$ $= 3x + \frac{48}{x}$	1- Finds expression for y 2-Correct solution with working
b	Stationary points occur when $\frac{dl}{dx} = 0$ $3 - \frac{48}{x^2} = 0$ $x^2 = 16$ $x = 4$ m (x is length, ignore -4) Test whether $x = 4$ is a minimum. $\frac{d^2l}{dx^2} = \frac{96}{x^3}$ $\frac{96}{4^3} > 0$ at $x = 4$ $\therefore$ Minimum at $x = 4$	1- Find value of x 2-Correct solution with working

c	Minimum length of rods for the frame. $l = 3 \times 4 + \frac{48}{4}$ $= 24 \text{ m}$ $\text{Cost} = 24 \times 9.50$ $= \$228$ The minimum cost of the frame is \$228	1- finds the minimum length of rod 2-Correct solution with working
33	$Let\ 3^x = u$ $0 = u^2 - 10u + 9$ $0 = (u-9)(u-1)$ $u = 1, 9$ $\therefore 3^x = 1 \ \& \ 3^x = 9$ $\therefore x = 0, 2$ As area is below the curve $A = \left\lvert \int_0^2 3^{2x} - 10(3^x) + 9 \, dx \right\rvert$ $A = \left\lvert \left[\frac{3^{2x}}{2\ln 3} - \frac{10(3^x)}{\ln 3} + 9x \right]_0^2 \right\rvert$ $A = \left\lvert \left(\frac{3^4}{2\ln 3} - \frac{10(3^2)}{\ln 3} + 18 \right) - \left(\frac{3^0}{2\ln 3} - \frac{10(3^0)}{\ln 3} + 0 \right) \right\rvert$ $A = \left\lvert \left(\frac{81 - 180 + 36\ln 3 - 1 + 20}{2\ln 3} \right) \right\rvert$ $A = \left\lvert \frac{18\ln 3 - 40}{\ln 3} \right\rvert$ $\therefore A = \frac{40 - 18\ln 3}{\ln 3}$	1 - Finds intercept $x = 2$ - Correct integral with abs value – ignore bounds - Correct primitive function of $3^{2x} - 10(3^x) + 9$ 2 - Correct bounds and integral - must have abs value - Correct primitive, incl abs val, but incorrect or missing bounds - Subst correct bounds into correct primitive function of $3^{2x} - 10(3^x) + 9$ 3 - Correct primitive function with correct bounds. - Correctly subst incorrect bounds into correct primitive (incl abs val) 4 - Correctly substitutes bounds into correct primitive - Appr answer from incorrect bounds - Correct non-exact answer from appropriate working 5 - Correct exact answer from appropriate working