Student Number: | | | | | | | | | |

Maths Class: _____

2026 **HSC TRIAL EXAMINATION**

Mathematics Advanced

General Instructions

- Reading time – 10 minutes
- Working time – 3 hours;
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided at the back of this paper
- For questions in Section II, show relevant mathematical reasoning and/or calculations
- Write your Student Number and Maths Class at the top of this page

Total marks: 100

Section I – 10 marks (pages 3–6)

- Attempt Questions 1–10
- Allow about 15 minutes for this section

Section II – 90 marks (pages 7–34)

- Attempt Questions 11–28
- Allow about 2 hours 45 minutes for this section

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Section I

10 marks
Attempt Questions 1–10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1–10.

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1 In a network audit, 58% of devices show a temperature alert, 46% show a humidity alert, and 31% show both alerts. A device is selected from those showing at least one of the two alerts. What is the probability that it shows exactly one alert?

A. $\frac{31}{73}$

B. $\frac{42}{73}$

C. $\frac{42}{104}$

D. $\frac{73}{104}$

2 How many solutions does

$$\sin\theta = \cos(2\theta)$$

have in the interval $0 \leq \theta \leq 2\pi$?

A. 2

B. 4

C. 3

D. 5

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3 A normally distributed random variable X satisfies

$$P(X < 42) = 0.10, \qquad P(X > 58) = 0.10.$$

The variable Y is defined by $Y = 100 - 2X$. What is $P(-16 < Y < 16)$?

A. 0.80

B. 0.20

C. 0.90

D. It cannot be determined

4 Suppose

$$f(x) = \frac{ax + b}{cx + d},$$

where $ad - bc \neq 0$ and $c \neq 0$.

Which has the following properties?

A. The graph has exactly one turning point.

B. The graph has a vertical asymptote but no horizontal asymptote.

C. There is exactly one y-value that is not in the range of the function.

D. The graph has domain equal to all real numbers.

5 Let

$$2^{2x} - 5 \cdot 2^x + 6 = 0.$$

Which of the following could be a value of x?

A. $\log_3 2$

B. 1

C. 2

D. $\log_2 6$

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6 A data logger has recorded n measurements with mean 20 and population standard deviation 2. A further measurement equal to 20 is added to the data. Which statement is correct?

A. The mean and the standard deviation remain the same.

B. The mean remains the same and the standard deviation decreases.

C. The mean increases and the standard deviation decreases.

D. The mean remains the same and the standard deviation increases.

7 A particle moves along a straight line such that its displacement x metres at time t seconds is implicitly defined by the equation

$$t = x^3 + 3x.$$

What is the acceleration of the particle (in m/s^2) at the instant when $x = 1$?

A. $-\frac{1}{6}$

B. $-\frac{1}{36}$

C. $\frac{1}{6}$

D. $\frac{1}{36}$

8 The area bounded by the curve

$$y = \frac{x}{x^2 + 1},$$

the x-axis, the y-axis and the vertical line $x = k$, where $k > 0$, is exactly equal to $\ln(5)$ square units.

What is the exact value of k?

A. $2\sqrt{6}$

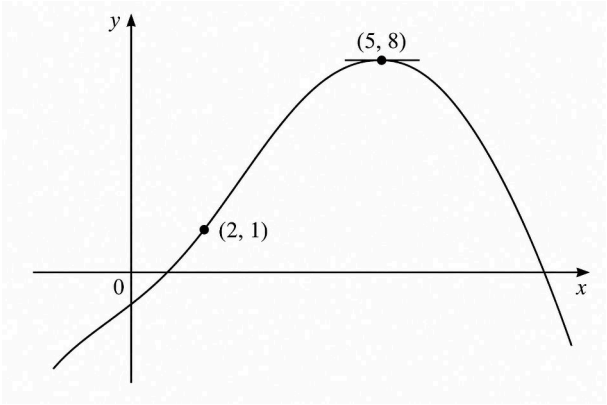
B. 3

C. 24

D. $\sqrt{6}$

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9 The diagram below shows the graph of a differentiable function $y = f(x)$, which passes through the point $(2, 1)$ and has a stationary point at $(5, 8)$.



A new function $g(x)$ is defined by

$$g(x) = [f(x)]^2 f(2x - 2).$$

Find the exact value of $g'(2)$.

A. 0

B. $2f'(2)$

C. $3f'(2)$

D. $4f'(2)$

10 A student deposits \$800 at the end of each month for 18 months into an account earning 6% p.a. compounded monthly. No further deposits are made, but the money remains invested for another 6 months. Which expression gives the value of the account after 24 months?

A. $800\left(\dfrac{(1.005)^{24} - 1}{0.005}\right)$

B. $800(18)(1.005)^6$

C. $800\left(\dfrac{(1.06)^{18} - 1}{0.06}\right)(1.06)^6$

D. $800\left(\dfrac{(1.005)^{18} - 1}{0.005}\right)(1.005)^6$

End of Section I

Section II continues on the next page. . .

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Section II

90 marks
Attempt Questions 11–28
Allow about 2 hours 45 minutes for this section

Answer each question in the appropriate writing booklet. Extra writing booklets are available.

For questions in Section II, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (2 marks)

Using first principles, differentiate **2**

$$f(x) = 2x^2 - 5x + 3.$$

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Question 12 (3 marks)

Evaluate the definite integral, leaving your answer in exact form. **3**

$$\int_1^4 \left(\frac{2}{3x-2} + \frac{1}{\sqrt{x}} \right) dx.$$

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Question 13 (4 marks)

A wildlife camera records a sequence of short video clips over consecutive nights. The number of clips recorded each night forms an arithmetic sequence. Over the first 10 nights, the camera records a total of 730 clips. On the tenth night, it records 100 clips.

a) Show the number of clips recorded on the first night is 46. **1**

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b) Find the common difference of the sequence. **1**

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c) Assuming the same pattern continues, find the first night by which the total number of clips recorded exceeds 1200. **2**

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Question 14 (2 marks)

A smart greenhouse uses an automated misting system. The time T, in seconds, for the humidity sensor to stabilise after each misting cycle is normally distributed with mean 18.5 seconds and standard deviation 2.4 seconds. **2**

A cycle is classified as delayed if the stabilisation time is more than 22.1 seconds.

During a week, the system completes 360 misting cycles. Estimate the number of delayed cycles in that week.

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Question 15 (3 marks)

Solve for x, given **3**

$$\log_{\frac{1}{3}}(2x-1) + \log_3(x-3) = -1.$$

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Question 16 (7 marks)

A box contains one fair die and one special die. The special die has

$$P(6) = \frac{1}{3}, \qquad P(1) = P(2) = P(3) = P(4) = P(5) = \frac{2}{15}.$$

One die is chosen at random and then rolled twice. The same die is used for both rolls.

a) Given that the first roll is a 6, find the probability that the special die was chosen. **2**

b) Given that the first roll is a 6, find the probability that the second roll is even. **2**

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c) A game pays \$18 if both rolls are sixes, \$6 if exactly one roll is a six, and nothing otherwise. Determine the entry fee that will make the game fair. **3**

Question 17 (2 marks)

The graph of the function $f(x) = x^2$ is dilated vertically by a scale factor of k, translated m units to the right, and then translated 3 units up. The resulting graph is then reflected in the x-axis to form the final function **2**

$$g(x) = -2x^2 + 8x - 11.$$

Find the values of m and k, where $k > 0$.

Exam continues on the next page...

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Question 18 (6 marks)

An environmental research team is investigating the effect of wind speed on the sideways drift of seeds released by an aerial seeding drone. During a series of test flights, the wind speed, w, measured in metres per second (m s^{-1}), and the corresponding sideways drift, d, measured in metres, were recorded.

w	1.5	2.0	2.8	3.4	4.1	4.8	5.6	6.2
d	3.2	4.3	5.8	7.5	8.4	10.1	12.2	13.1

a) Determine the least-squares regression line for predicting the sideways drift, d, from the wind speed, w. Hence, state the value of the correlation coefficient, r. Give all coefficients to three significant figures. **2**

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b) In the context of this investigation, explain the meaning of:

i) The gradient of the regression line. **1**

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ii) The correlation coefficient. **1**

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Question 18 Continues on the next page...

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The drone's navigation software is programmed to release seeds only when the predicted sideways drift does not exceed 12 metres.

c) Use the regression model to determine the greatest wind speed for which the drone should release seeds. State whether your prediction is based on interpolation or extrapolation. **1**

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d) Give one reason why the regression model may not provide reliable predictions when the wind speed is $10\,\text{m}\,\text{s}^{-1}$. **1**

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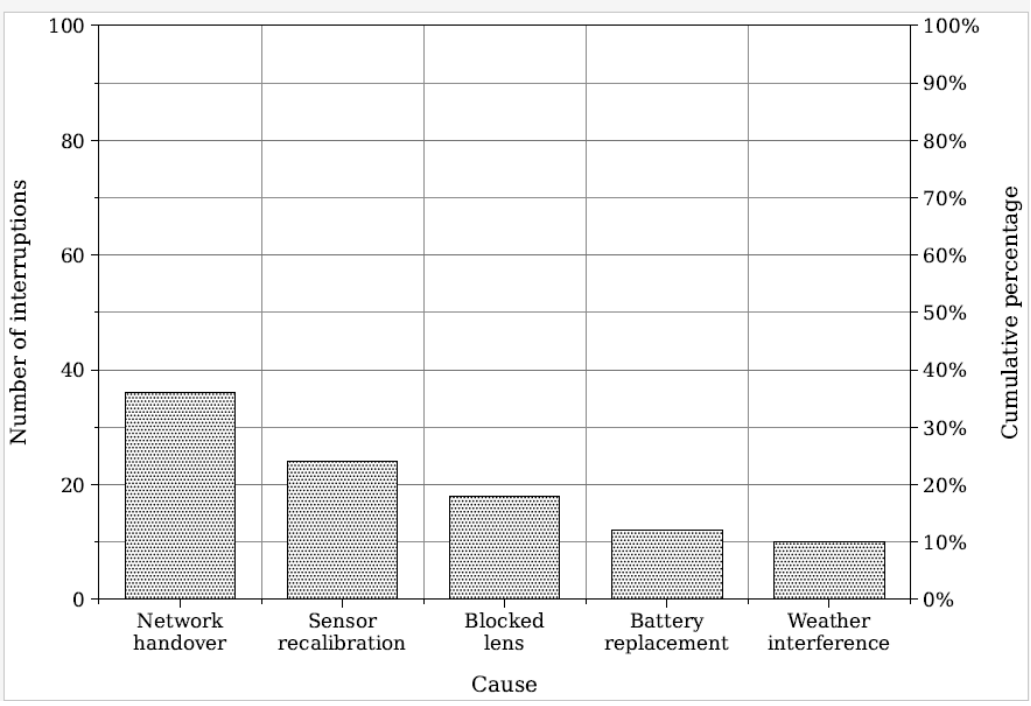
Question 19 (5 marks)

A remote observatory records the causes of 100 interruptions and the mean downtime associated with each cause.

a) Complete the Cumulative (%) column in the table below by calculating the cumulative percentage for each cause. **1**

Cause	**Number of interruptions**	**Mean downtime (minutes)**	**Cumulative (%) No of Interruptions**
Network handover	36	5	
Sensor recalibration	24	12	
Blocked lens	18	25	
Battery replacement	12	40	
Weather interference	10	18	

b) The Pareto chart below shows the frequencies for each cause. Draw the cumulative percentage line on the chart. **1**



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c) The observatory wants to address the smallest number of causes that together account for at least 75% of interruptions. Identify these causes. **1**

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d) A proposed upgrade can eliminate 70% of the interruptions, (downtime) from exactly one cause. Installing the upgrade itself creates 90 minutes of downtime. If the objective is to maximise the net reduction in expected total downtime, determine which cause should be selected and the net reduction. **2**

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Question 20 (3 marks)

Using the trapezoidal rule with four sub-intervals, find an approximate value for **3**

$$f(x) = \int_1^3 \frac{2lnx}{x\sqrt{1 + (ln\, x)^2}} dx,$$

correct to 4 decimal places.

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Question 21 (5 marks)

a) By using the product rule, show that the derivative of $y = x^3 \ln(x)$ is given by: **2**

$$\frac{dy}{dx} = x^2(3\ln(x) + 1)$$

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b) Hence, find the exact value of the definite integral **3**

$$\int_1^e x^2 \ln(x)\, dx$$

in simplified exact form.

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Question 22 (5 marks)

Consider the cubic curve given by the equation

$$y = x^3 - 3kx,$$

where k is a positive constant.

A tangent is drawn to the curve at a point P in the first quadrant. This tangent is parallel to the line

$$y = 9x + 2.$$

A normal to the curve at the same point P intersects the y-axis at the point $(0, b)$.

a) Show that the x-coordinate of point P is given by $x = \sqrt{k+3}$. **2**

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b) Show that the y-coordinate of point P can be expressed as: **1**

$$y = (3 - 2k)\sqrt{k+3}$$

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Question 22 Continues on the next page...

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c) Hence, show the expression for the y- intercept is: **2**

$$b = \left(\frac{28}{9} - 2k\right)\sqrt{k+3}$$

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Question 23 (8 marks)

A loan of \$180000 is to be repaid with equal monthly repayments made at the end of each month. The initial interest rate is 6% p.a., compounded monthly.

a) Show that the balance remaining after 3 months is given by: **2**
$A_3 = 180,000(1.005)^3 - M(1 + 1.005 + 1.005^2)$

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Question 23 continues on the next page...

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b) Hence, or otherwise, find the monthly repayment $\$M$ required to fully repay the loan after 180 months. Give your answer to the nearest cent. **2**

c) Find the outstanding balance immediately after the 120th repayment. **2**

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d) Immediately after the 120th repayment, the interest rate rises to 7.8% p.a., compounded monthly. Find the new monthly repayment required so that the loan is still fully repaid after 180 months. **2**

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Question 24 (4 marks)

The population $P(t)$ of an endangered species in a wildlife reserve is restricted by environmental resources. Biologists model the growth of the population using the logarithmic relationship

$$\ln\left(\frac{P(t)}{3000 - P(t)}\right) = 0.2t - c,$$

where t is the time in years and c is a constant.

Initially, when $t = 0$, the population is 500.

a) Find the exact time t when the population reaches 2500. **2**

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Question 24 Continues on the next page...

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b) Sketch the graph of $P(t)$ against t for $t \geq 0$. Clearly show all important features. **2**

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Question 25 (7 marks)

For values of θ for which it is defined, let

$$E(\theta) = \frac{1 + \sin\theta}{\cos\theta} + \frac{\cos\theta}{1 + \sin\theta}.$$

a) State the values of θ in $0 \leq \theta \leq 2\pi$ for which $E(\theta)$ is not defined, and prove that $E(\theta) = 2\sec\theta$ on its domain. **2**

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b) Hence solve **2**

$$E(\theta) = 4\cos\theta, \qquad 0 \leq \theta \leq 2\pi.$$

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Question 25 Continues on the next page...

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c) For a real number m, determine all values of m for which **3**

$$E(\theta) = m + \cos\theta$$

has exactly two solutions in $0 \leq \theta \leq 2\pi$.

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Question 26 (9 marks)

A dimensionless calibration-drift score X is used to measure how far a sensor has drifted from its target calibration. A score near 1 represents typical drift, while scores near the endpoints represent unusually low or unusually high drift. A physics-based model suggests that X has a probability density function

$$f(x) = \begin{cases} k(x+1), & -1 \leq x \leq 1, \\ k(3-x), & 1 < x \leq 3, \\ 0, & \text{otherwise.} \end{cases}$$

a) Find the value of k. **2**

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Question 26 Continues on the next page...

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b) Find the cumulative distribution function $F(x)$ for all real x. **3**

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Question 26 Continues on the next page...

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c) Find the lower quartile Q_1, the upper quartile Q_3, and hence the inter-quartile range of X, in exact form. **2**

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d) For reporting purposes, an engineering team proposes a normal model Y whose mean is the same as the median of X and interquartile range as the physics-based model. Estimate $P(Y > 2)$, and compare this with $P(X > 2)$ under the physics-based model. **2**

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Question 27 (7 marks)

The depth of water, W metres, inside a tidal shipping canal is modelled by the function

$$W(t) = 8 + \sin(t) + \cos(t),$$

where t is the time in hours after high tide, for $0 \leq t \leq \pi$.

a) Find the coordinates of any stationary points on the curve $W(t)$ within the interval $0 \leq t \leq \pi$, and determine their nature. **3**

Question 27 Continues on the next page...

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b) Find the point(s) of inflection. **2**

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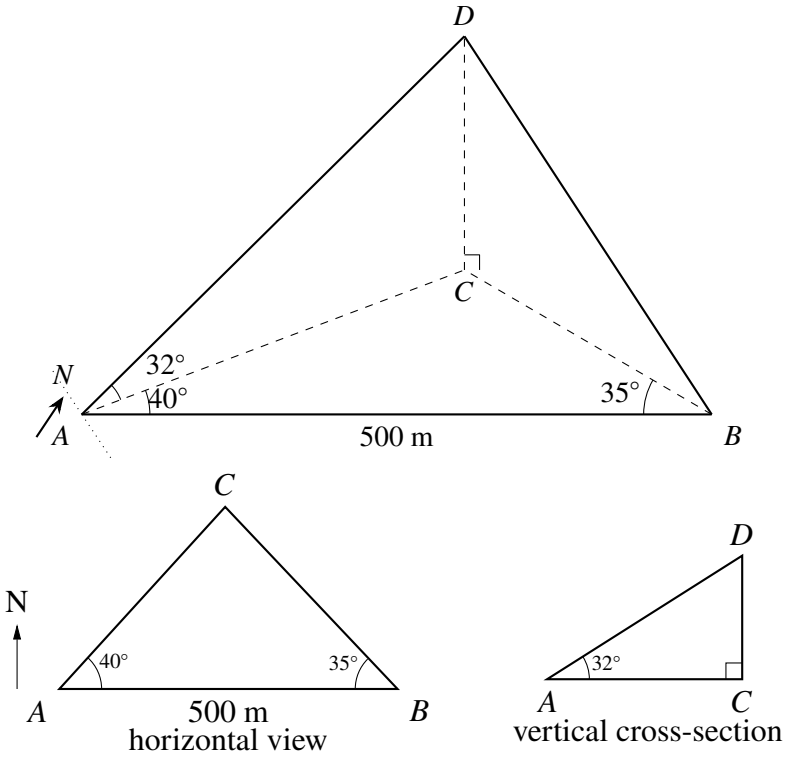
c) A large cargo ship requires the water depth to be dropping at its absolute fastest rate to clear a shallow sandbar safely using the outgoing current. Find the exact value of t when the ship should cross the sandbar. **2**

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Question 28 (8 marks)

Two coastal observation stations A and B are 500 metres apart, with B due east of A. The point C, directly below a rescue drone, has bearing 050° from A and bearing 305° from B. The drone is at D, vertically above C. The angle of elevation of D from A is 32°.



a) Find AD, correct to the nearest metre. **2**

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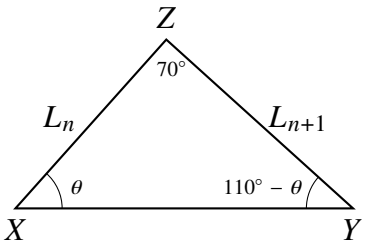
Question 28 Continues on the next page...

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b) After the drone is located, its guidance system makes a sequence of correction flights with lengths **2**

$$L_1, L_2, L_3, \ldots,$$

where $L_1 = AD$. Consecutive correction lengths are related by the calibration triangle shown below.



The angle opposite L_{n+1} is θ, the angle opposite L_n is $110° - \theta$, and $0° < \theta < 55°$.

Show that the correction lengths form a geometric sequence with

$$\frac{L_{n+1}}{L_n} = q, \qquad q = \frac{\sin\theta}{\sin(110° - \theta)},$$

and hence write L_n in terms of AD, q, and n.

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c) For a reduced-battery mission, the limiting total correction distance must not exceed 900 metres.

i) Using parts **a)** and **b)** , find the total limiting correction distance in terms of AD and q. **1**

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ii) Calculate the maximum allowable value of q, correct to four decimal places. **1**

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At the greatest allowable value of θ, the maximum value of q is attained and q is an increasing function for $0° < \theta < 55°$. (DO NOT PROVE THIS).

iii) Hence or otherwise, determine the greatest allowable value of θ to the nearest degree. **2**
You may use:

$$\sin(110° - \theta) = \sin 110° \cos\theta - \cos 110° \sin\theta.$$

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Section II extra writing space

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2026 HSC Trial Examination
Mathematics Advanced

Suggested Solutions and Marking Scheme

Marking principle. Award marks for mathematically valid equivalent methods. Unless a question explicitly requires an exact value, allow normal calculator rounding. Follow-through should be awarded where a student carries a previous numerical value consistently and the later method is correct.

Section I - Multiple Choice

Question	1	2	3	4	5	6	7	8	9	10
Answer	B	C	A	C	B	B	B	A	D	D

Brief checks

1. $P(A \cup B) = 0.58 + 0.46 - 0.31 = 0.73$ and $P(\text{exactly one}) = 0.42$, so $42/73$.
2. $\sin\theta = 1 - 2\sin^2\theta$, so $(2\sin\theta - 1)(\sin\theta + 1) = 0$: three solutions.
3. $-16 < 100 - 2X < 16 \iff 42 < X < 58$, whose probability is 0.80.
4. $y = a/c$ is the single excluded range value of a non-degenerate linear fractional function.
5. Let $u = 2^x$: $u^2 - 5u + 6 = 0$, so $x = 1$ or $x = \log_2 3$; option B is possible.
6. Adding a value equal to the mean leaves the mean unchanged and decreases the population variance denominator effect.
7. $v = \dfrac{dx}{dt} = \dfrac{1}{3x^2+3}$, hence $a = \dfrac{dv}{dt} = -\dfrac{6x}{(3x^2+3)^3}$ and $a(1) = -1/36$.
8. $\int_0^k \dfrac{x}{x^2+1}\,dx = \frac{1}{2}\ln(k^2+1) = \ln 5$, giving $k = 2\sqrt{6}$.
9. Product and chain rules give $g'(2) = 4f'(2)$.
10. The 18-payment annuity accumulates for a further 6 months: $800\dfrac{1.005^{18}-1}{0.005}(1.005)^6$.

Section II

90 marks
Attempt Questions 11–28
Allow about 2 hours 45 minutes for this section

Answer each question in the appropriate writing booklet. Extra writing booklets are available.

For questions in Section II, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (2 marks)

Using first principles, differentiate **2**

$$f(x) = 2x^2 - 5x + 3.$$

$f'(x) = \lim_{h\to 0} \dfrac{[2(x+h)^2 - 5(x+h) + 3] - [2x^2 - 5x + 3]}{h}$

$= \lim_{h\to 0} \dfrac{(2x^2 + 4xh + 2h^2 - 5x - 5h + 3) - (2x^2 - 5x + 3)}{h}$ — (1)

$= \lim_{h\to 0} \dfrac{4xh + 2h^2 - 5h}{h}$

$= \lim_{h\to 0} 4x + 2h - 5 = 4x - 5$ — (1)

Question 12 (3 marks)

Evaluate the definite integral, leaving your answer in exact form. **3**

$$\int_1^4 \left(\frac{2}{3x-2} + \frac{1}{\sqrt{x}}\right) dx.$$

$= \left[\frac{2}{3}\ln|3x-2| + 2\sqrt{x}\right]_1^4$ — (1)

(1)

$= \left(\frac{2}{3}\ln 10 + 4\right) - (0 + 2)$

$= \frac{2}{3}\ln 10 + 2$ — (1)

Exam continues on the next page...

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Question 13 (4 marks)

A wildlife camera records a sequence of short video clips over consecutive nights. The number of clips recorded each night forms an arithmetic sequence. Over the first 10 nights, the camera records a total of 730 clips. On the tenth night, it records 100 clips.

a) Show the number of clips recorded on the first night is 46. **1**

$T_{10} = 100, S_{10} = 730 \Rightarrow \frac{10}{2}[T_1 + 100] = 730$

$5T_1 + 500 = 730$

$5T_1 = 230$

$T_1 = 230 \div 5$

$= 46$

b) Find the common difference of the sequence. **1**

$T_{10} = 100 \Rightarrow 46 + (10-1)d = 100$

$46 + 9d = 100$

$9d = 54$

$d = 6$

c) Assuming the same pattern continues, find the first night by which the total number of clips recorded exceeds 1200. **2**

$S_n = 1200 \Rightarrow \frac{n}{2}[2(46) + (n-1) \times 6] = 1200$

$\frac{n}{2}[92 + 6n - 6] = 1200$

$3n^2 + 43n - 1200 = 0$ — (1)

$n = \frac{-43 \pm \sqrt{1849 + 4(3)(1200)}}{6}$

$\approx \frac{-44 \pm 127.5}{6}$

$= 14.08$

$\therefore$ after the 15th night. — (1)

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Question 14 (2 marks)

A smart greenhouse uses an automated misting system. The time T, in seconds, for the humidity sensor to stabilise after each misting cycle is normally distributed with mean 18.5 seconds and standard deviation 2.4 seconds. **2**

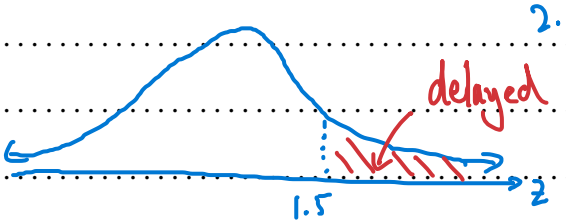
A cycle is classified as delayed if the stabilisation time is more than 22.1 seconds.

During a week, the system completes 360 misting cycles. Estimate the number of delayed cycles in that week.

$\mu = 18.5 \quad \sigma = 2.4$

Let X = the stabilisation time of any misting cycle.

$x = 22.1 \Rightarrow z = \dfrac{22.1 - 18.5}{2.4} = 1.5$ — (1)



$\therefore$ number of delayed cycles is equal to $360 \times (1 - 0.9332)$

≈ 24 — (1)

Question 15 (3 marks)

Solve for x, given **3**

$$\log_{\frac{1}{3}}(2x-1) + \log_3(x-3) = -1.$$

$\dfrac{\log_3(2x-1)}{\log_3\left(\frac{1}{3}\right)} + \log_3(x-3) = \log_3\left(\frac{1}{3}\right)$ — (1) change all to common base

$-\log_3(2x-1) + \log_3(x-3) = \log_3\left(\frac{1}{3}\right)$

$\dfrac{x-3}{2x-1} = \dfrac{1}{3}$ — (1)

$3x - 9 = 2x - 1$

$x = 8$ — (1)

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Question 16 (7 marks)

A box contains one fair die and one special die. The special die has

$$P(6) = \frac{1}{3}, \qquad P(1) = P(2) = P(3) = P(4) = P(5) = \frac{2}{15}.$$

One die is chosen at random and then rolled twice. The same die is used for both rolls.

a) Given that the first roll is a 6, find the probability that the special die was chosen. **2**

$P(6) = \left(\frac{1}{2} \times \frac{1}{3}\right) + \left(\frac{1}{2} \times \frac{1}{6}\right) = \frac{1}{4}$ — (1)

$P(\text{special die} \mid 6) = \dfrac{P(\text{special die and } 6)}{P(6)}$

$= \dfrac{\frac{1}{2} \times \frac{1}{3}}{\frac{1}{4}}$

$= \frac{2}{3}$ — (1)

Note: Many students got the right answer with incorrect methods. No marks.

b) Given that the first roll is a 6, find the probability that the second roll is even. **2**

$P(\text{special die}) = \frac{2}{3}$, $P(\text{fair die}) = \frac{1}{3}$ — (1)

$\therefore P(\text{2nd roll being even}) = \left(\frac{2}{3} \times \frac{9}{15}\right) + \left(\frac{1}{3} \times \frac{1}{2}\right)$

$= \frac{6}{15} + \frac{1}{6}$

$= \frac{17}{30}$ — (1)

Question 16 Continues on the next page...

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c) A game pays \$18 if both rolls are sixes, \$6 if exactly one roll is a six, and nothing otherwise. Determine the entry fee that will make the game fair. **3**

$P(\text{No 6}) = \left(\frac{1}{2} \times \frac{2}{3} \times \frac{2}{3}\right) + \left(\frac{1}{2} \times \frac{5}{6} \times \frac{5}{6}\right) = \frac{4}{18} + \frac{25}{72} = \frac{41}{72}$

special die, fair die

$P(\text{one 6}) = \left(\frac{1}{2} \times \frac{2}{3} \times \frac{1}{3}\right) \times 2 + \left(\frac{1}{2} \times \frac{1}{6} \times \frac{5}{6}\right) \times 2$

$= \frac{2}{9} + \frac{5}{36}$

$= \frac{13}{36}$ — (1)

$P(\text{two 6's}) = \left(\frac{1}{2} \times \frac{1}{3} \times \frac{1}{3}\right) + \left(\frac{1}{2} \times \frac{1}{6} \times \frac{1}{6}\right)$

$= \frac{1}{18} + \frac{1}{72}$

$= \frac{5}{72}$ — (1)

let Z be the return of the game. $E(Z) = \left(0 \times \frac{41}{72}\right) + \left(\frac{13}{36} \times 6\right) + \left(\frac{5}{72} \times 18\right)$

$\doteq \$3.42$

$\approx \$3.42$ — (1)

Question 17 (2 marks)

The graph of the function $f(x) = x^2$ is dilated vertically by a scale factor of k, translated m units to the right, and then translated 3 units up. The resulting graph is then reflected in the x-axis to form the final function **2**

$$g(x) = -2x^2 + 8x - 11.$$

Find the values of m and k, where $k > 0$.

x^2

↓

kx^2

↓

$k(x-m)^2$

↓

$k(x-m)^2 + 3$

↓

$-k(x-m)^2 - 3$

Equate:

$-2x^2 + 8x - 11 = -k(x-m)^2 - 3$

$= -kx^2 + 2kxm - km^2 - 3$

$k = 2$ — (1)

$2km = 8$

$2m = 4$

$m = 2$ — (1)

Note: Students do not get full marks if correct answer is obtained by incorrect working.

Exam continues on the next page...

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Question 18 (6 marks)

An environmental research team is investigating the effect of wind speed on the sideways drift of seeds released by an aerial seeding drone. During a series of test flights, the wind speed, w, measured in metres per second (m s^{-1}), and the corresponding sideways drift, d, measured in metres, were recorded.

w	1.5	2.0	2.8	3.4	4.1	4.8	5.6	6.2
d	3.2	4.3	5.8	7.5	8.4	10.1	12.2	13.1

a) Determine the least-squares regression line for predicting the sideways drift, d, from the wind speed, w. Hence, state the value of the correlation coefficient, r. Give all coefficients to three significant figures. **2**

$r = 0.998$ — (1)

$d = 2.13w - 0.0168$ — (1)

b) In the context of this investigation, explain the meaning of:

i) The gradient of the regression line. **1**

For each m/s the wind speed increases, the seeds drift by an extra 2.13 m.

ii) The correlation coefficient. **1**

there is a close to perfect linear correlation between the wind speed and the sideways drift of the seed.

Question 18 Continues on the next page...

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The drone's navigation software is programmed to release seeds only when the predicted sideways drift does not exceed 12 metres.

c) Use the regression model to determine the greatest wind speed for which the drone should release seeds. State whether your prediction is based on interpolation or extrapolation. **1**

$d < 12 \Rightarrow 2.13w - 0.0168 < 12$

$2.13w < 11.9832$

$w < 5.63$

$\therefore$ greatest wind speed is $5.63\ m/s$

This is an interpolation.

Need both

d) Give one reason why the regression model may not provide reliable predictions when the wind speed is $10\,m\,s^{-1}$. **1**

When the windspeed is $10\ m/s$, this data point is too far outside the data points in which the model was based off.

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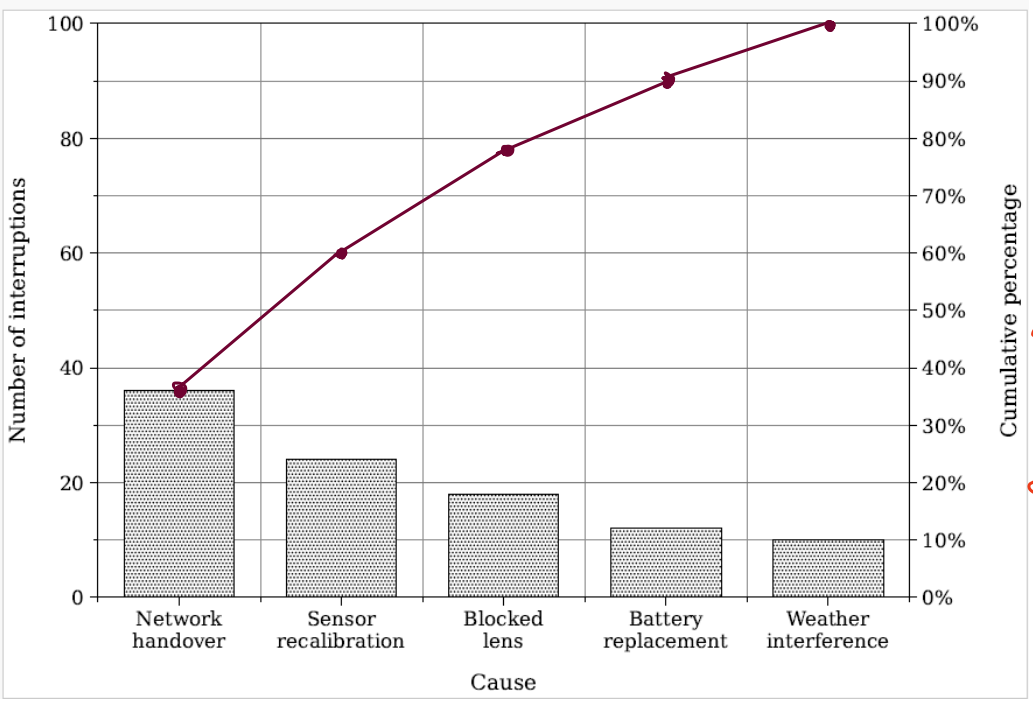
Question 19 (5 marks)

A remote observatory records the causes of 100 interruptions and the mean downtime associated with each cause.

a) Complete the Cumulative (%) column in the table below by calculating the cumulative percentage for each cause. **1**

Cause	Number of interruptions	Mean downtime (minutes)	Cumulative (%) No of Interruptions
Network handover	36	5	36
Sensor recalibration	24	12	60
Blocked lens	18	25	78
Battery replacement	12	40	90
Weather interference	10	18	100

b) The Pareto chart below shows the frequencies for each cause. Draw the cumulative percentage line on the chart. **1**



This is how it is presented in HSC papers!

Mark was allocated for all !!! Please ensure how to present the graph!

Question 19 Continues on the next page...

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c) The observatory wants to address the smallest number of causes that together account for at least 75% of interruptions. Identify these causes. **1**

Network handover
Sensor recalibration
Blocked lens

(1)

d) A proposed upgrade can eliminate 70% of the interruptions, (downtime) from exactly one cause. Installing the upgrade itself creates 90 minutes of downtime. If the objective is to maximise the net reduction in expected total downtime, determine which cause should be selected and the net reduction. **2**

N => 36 x 5 = 180
S => 24 x 12 = 288
L => 18 x 25 = 450
B => 12 x 40 = 480 —— the max cause is Battery
W => 10 x 18 = 180

Net reduction = 12 x 40 = 480 x 70%
= 336 (1) — 90
= 246 (1)

Stating Battery & 336 (1)
Getting the answer 246 (1)

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Question 20 (3 marks)

Using the trapezoidal rule with four sub-intervals, find an approximate value for **3**

$$f(x) = \int_1^3 \frac{2lnx}{x\sqrt{1+(ln\,x)^2}}dx,$$

correct to 4 decimal places.

x	$f(x)$	w	$w \cdot f(x)$
1	0	1	1 × 0
1·5	0·50100035	2	2 × ....
2	0·56967629 3	2	2 × ....
2·5	0·54045999	2	2 × ....
3	0·49301149 8	1	1 × ....

$\Sigma w = 8$

$$\int_1^3 \frac{2\ln x}{x\sqrt{1+(\ln x)^2}}\,dx = \frac{h}{2}\left(f(1)+f(3)+2\left(f(1{\cdot}5)+f(2)+f(2{\cdot}5)\right)\right)$$

(Save these in calculator)

$h = \frac{b-a}{n}$

$= \frac{3-1}{4}$

$= \frac{1}{2}$

1st mark $\frac{h}{2} = \frac{\frac{1}{2}}{2} = \frac{1}{4}$

2nd mark 5 func and weighting provided

3rd mark getting an answer of 0·9288

Exam continues on the next page...

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Question 21 (5 marks)

a) By using the product rule, show that the derivative of $y = x^3 \ln(x)$ is given by: **2**

$$\frac{dy}{dx} = x^2(3\ln(x) + 1)$$

$y = x^3 \ln(x)$ (1) Must show!

$= (\ln x \cdot 3x^2 + (x^3 \cdot (\frac{1}{x}))$ (1)

$= 3x^2 \ln x + x^2$ many students didn't show this step!

$= x^2(3\ln x + 1)$ It's a show question!!

$u = x^3 \quad u' = 3x^2$

$v = \ln(x), \quad v' = \frac{1}{x}$

b) Hence, find the exact value of the definite integral **3**

$$\int_1^e x^2 \ln(x)\, dx$$

in simplified exact form.

$\frac{d}{dx} x^3 \ln x = 3x^2 \ln(x) + x^2$ (from pt (a))

$\int_1^e \frac{d}{dx}(x^3 \ln x) = \int_1^e 3x^2 \ln(x)\, dx + \int_1^e x^2\, dx$

$[x^3 \ln x]_1^e - \int_1^e x^2\, dx = 3\int_1^e x^2 \ln(x)\, dx$

$\int_1^e x^2 \ln(x)\, dx = \frac{1}{3}\{[x^3 \ln(x)]_1^e - [\frac{x^3}{3}]_1^e\}$

$= \frac{1}{3}\{[e^3 \ln(e) - 1^3 \ln(1)] - (\frac{e^3}{3} - \frac{1^3}{3})\}$

$= \frac{1}{3}[e^3 \ln(e) - (\frac{e^3}{3} - \frac{1}{3})]$

$= \frac{1}{3}e^3 - \frac{1}{3}(\frac{e^3}{3}) + \frac{1}{3}(\frac{1}{3})$

$= \frac{1}{3}e^3 - \frac{1}{9}e^3 + \frac{1}{9}$

$= \frac{2}{9}e^3 + \frac{1}{9}$ — (1)

There are many ways to get first 2 marks since the question says 'Hence'; This is just 1 method!! If students didn't get the final answer; max mark is '1' provided in exact form.

Exam continues on the next page...

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Question 22 (5 marks)

Consider the cubic curve given by the equation

$$y = x^3 - 3kx,$$

where k is a positive constant.

A tangent is drawn to the curve at a point P in the first quadrant. This tangent is parallel to the line

$$y = 9x + 2.$$

A normal to the curve at the same point P intersects the y-axis at the point $(0, b)$.

a) Show that the x-coordinate of point P is given by $x = \sqrt{k+3}$. **2**

$y = 9x + 2$

$y' = 9 = m$ since parallel, $m = 9$

we know $y = x^3 - 3kx$

$y' = 3x^2 - 3k$

$9 = 3x^2 - 3k$

$3 = x^2 - k$

$x^2 = 3 + k$

$x = \pm\sqrt{3+k}$ (point P is in 1st quadrant)

$\therefore x = \sqrt{3+k}$

b) Show that the y-coordinate of point P can be expressed as: **1**

$$y = (3 - 2k)\sqrt{k+3}$$

$x = \sqrt{3+k}$

$y = x^3 - 3kx$

$y = (\sqrt{k+3})^3 - 3k(\sqrt{k+3})$

$= \sqrt{k+3}(k+3) - 3k(\sqrt{k+3})$

$= \sqrt{k+3}(k+3-3k)$

$= \sqrt{k+3}(3-2k)$

MUST SHOW!! As it is a show question!!!

Question 22 Continues on the next page...

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c) Hence, show the expression for the y- intercept is: **2**

$$b = \left(\frac{28}{9} - 2k\right)\sqrt{k+3}$$

from (a) $x = \sqrt{3+k}$, from (b) $y = (3-2k)(\sqrt{k+3})$ } (1)

for normal, $m = -\frac{1}{9}$

$y - (3-2k)(\sqrt{k+3}) = -\frac{1}{9}\left(x - (\sqrt{k+3})\right)$

$y = -\frac{1}{9}x + \frac{1}{9}\sqrt{k+3} + (3-2k)(\sqrt{k+3})$

$= -\frac{1}{9}x + \sqrt{k+3}\left(\frac{1}{9} + (3-2k)\right)$

$= -\frac{1}{9}x + \sqrt{k+3}\left(\frac{1+27-18k}{9}\right)$ —— (1)

$= -\frac{1}{9}x + \left(\sqrt{k+3}\right)\left(\frac{28-2k}{9}\right)$

y intercept

Question 23 (8 marks)

A loan of \$180000 is to be repaid with equal monthly repayments made at the end of each month. The initial interest rate is 6% p.a., compounded monthly.

a) Show that the balance remaining after 3 months is given by: **2**
$A_3 = 180,000(1.005)^3 - M(1 + 1.005 + 1.005^2)$

$A_1 = 180000(1+0.005) - M$

$= 180000(1.005) - M$

$A_2 = \{180000(1.005) - M)\} \times 1.005 - M$

$= 180000(1.005)^2 - M(1.005) - M$

$A_3 = 180000(1.005)^3 - M(1.005)^2 - M(1.005) - M$

$= 180\,000(1.005)^3 - M((1.005)^2 + (1.005) + (1))$

$= 180\,000(1.005)^3 - M(1 + 1.005 + 1.005^2)$

$a = 1$, $r = 0.005$

Many students decided to write whatever to show... Follow the proper working out!! was given shown full marks... generous!!!

Question 23 continues on the next page...

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b) Hence, or otherwise, find the monthly repayment $\$M$ required to fully repay the loan after 180 months. Give your answer to the nearest cent. **2**

If students got full marks in pt a) ... then pt (b - d) will be penalised! if the number of years for 'a' & gp is not correct!!

$A_{180} = 180000(1.005)^{180} - M\left[\frac{1(1.005)^{180}-1}{1.005-1}\right]$ (A)

To pay full loan; $A_{180} = 0$

$0 - 180000(1.005)^{180} = -M$ (A) Save this in your calc

$M = \frac{-180000(1.005)^{180}}{}$ (A)

≈ 1518.94

GP formula $a\left[\frac{(1+r)^n-1}{(1+r)-1}\right]$

c) Find the outstanding balance immediately after the 120th repayment. **2**

$A_{120} = 180000(1.005)^{120} - 1518.94\left\{\frac{1(1.005^{120}-1)}{1.005-1}\right\}$ (1)

Save in calc

≈ 78578.14 (outstanding balance) (1)

Question 23 Continues on the next page...

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d) Immediately after the 120th repayment, the interest rate rises to 7.8% p.a., compounded monthly. Find the new monthly repayment required so that the loan is still fully repaid after 180 months. **2**

$I = \frac{7.8\%}{12} = 0.0065$; $n = 60$

(Save in calc)

$\therefore A_{60} = 7858.14(1.0065)^{60} - M\left(\frac{(1.0065)^{60}-1}{1.0065-1}\right)$ ①

$M = \frac{-7858.14(1.0065)^{60}}{-\left(\frac{(1.0065)^{60}-1}{1.0065-1}\right)}$

≈ 1585.57 ①

Question 24 (4 marks)

The population $P(t)$ of an endangered species in a wildlife reserve is restricted by environmental resources. Biologists model the growth of the population using the logarithmic relationship

$$\ln\left(\frac{P(t)}{3000 - P(t)}\right) = 0.2t - c,$$

where t is the time in years and c is a constant.

Initially, when $t = 0$, the population is 500.

a) Find the exact time t when the population reaches 2500. **2**

when $t = 0$, $P = 500$

$\frac{\ln 500}{3000 - 500} = 0.2(0) - c$

$c = \ln 5$ ①

Find exact time when $P = 2500$ $\therefore \ln \frac{2500}{3500 - 2500} = 0.2t - \ln 5$

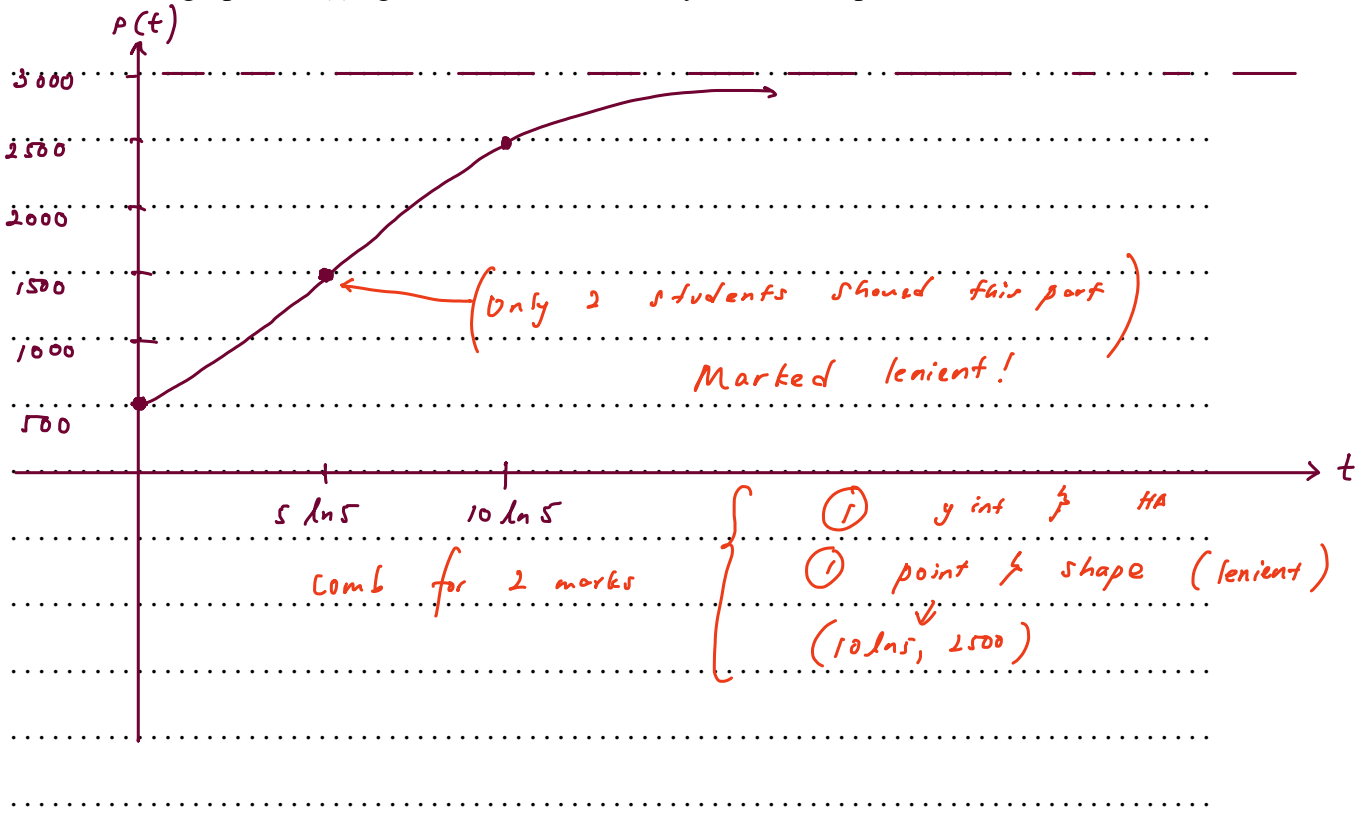
$\ln 5 = 0.2t - \ln 5$

$t = 10\ln 5,\ 16.09,\ \frac{\ln 25}{0.2},\ 5\ln 25$ ①

Question 24 Continues on the next page...

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b) Sketch the graph of $P(t)$ against t for $t \geq 0$. Clearly show all important features. **2**



Exam continues on the next page...

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Year 12 2026 Mathematics Advanced Trial

Suggested Solutions, Marking Scheme and Feedback

Questions 25-28

Question 25(a) **[2 marks]**

For values of θ for which it is defined, let

$$E(\theta) = \frac{1+\sin\theta}{\cos\theta} + \frac{\cos\theta}{1+\sin\theta}.$$

State the values of θ in $0 \leq \theta \leq 2\pi$ for which $E(\theta)$ is not defined, and prove that $E(\theta) = 2\sec\theta$ on its domain.

Solution

The denominators are zero when

$$\cos\theta = 0 \quad \text{or} \quad 1+\sin\theta = 0.$$

Hence, in $0 \leq \theta \leq 2\pi$,

$$\boxed{\theta = \frac{\pi}{2} \text{ or } \theta = \frac{3\pi}{2}}.$$

On the domain of E,

$$\begin{aligned} E(\theta) &= \frac{(1+\sin\theta)^2 + \cos^2\theta}{\cos\theta(1+\sin\theta)} \\ &= \frac{2+2\sin\theta}{\cos\theta(1+\sin\theta)} = \frac{2}{\cos\theta} = 2\sec\theta. \end{aligned}$$

Marking Scheme and Feedback

1 mark: States both undefined values of θ.
1 mark: Correctly proves $E(\theta) = 2\sec\theta$ on its domain.

Feedback

Some students wrote $\theta \neq \frac{\pi}{2}, \frac{3\pi}{2}$. The question asks where E is undefined, so the required statements use equality.

Question 25(b) **[2 marks]**

Hence solve

$$E(\theta) = 4\cos\theta, \qquad 0 \leq \theta \leq 2\pi.$$

Solution

Using part (a),

$$\frac{2}{\cos\theta} = 4\cos\theta \quad \Longrightarrow \quad \cos^2\theta = \frac{1}{2}.$$

Therefore

$$\boxed{\theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}}.$$

Marking Scheme and Feedback

1 mark: Obtains $\cos^2\theta = \frac{1}{2}$.
1 mark: Gives all four exact solutions.

Feedback

Use exact values and include all four solutions.

Question 25(c) **[3 marks]**

For a real number m, determine all values of m for which

$$E(\theta) = m + \cos\theta$$

has exactly two solutions in $0 \leq \theta \leq 2\pi$.

Solution

Method 1 - Make m the subject (recommended)

Let

$$x = \cos\theta, \qquad -1 \leq x \leq 1, \qquad x \neq 0.$$

The case $x = 0$ is excluded because $E(\theta)$ is undefined when $\cos\theta = 0$.
The equation becomes

$$\frac{2}{x} = m + x \quad \Longrightarrow \quad m = g(x) = \frac{2}{x} - x.$$

Since

$$g'(x) = -\frac{2}{x^2} - 1 < 0,$$

g is decreasing on each part of its domain.
For $0 < x \leq 1$,

$$g(1) = 1, \qquad \lim_{x\to 0^+} g(x) = \infty,$$

so $m \geq 1$. The endpoint $x = 1$ gives $\theta = 0, 2\pi$, so $m = 1$ is included.
For $-1 \leq x < 0$,

$$g(-1) = -1, \qquad \lim_{x\to 0^-} g(x) = -\infty.$$

The endpoint $x = -1$ gives only $\theta = \pi$, so $m = -1$ is excluded. Hence $m < -1$.
Thus

$$\boxed{m < -1 \quad \text{or} \quad m \geq 1}.$$

Marking Scheme and Feedback

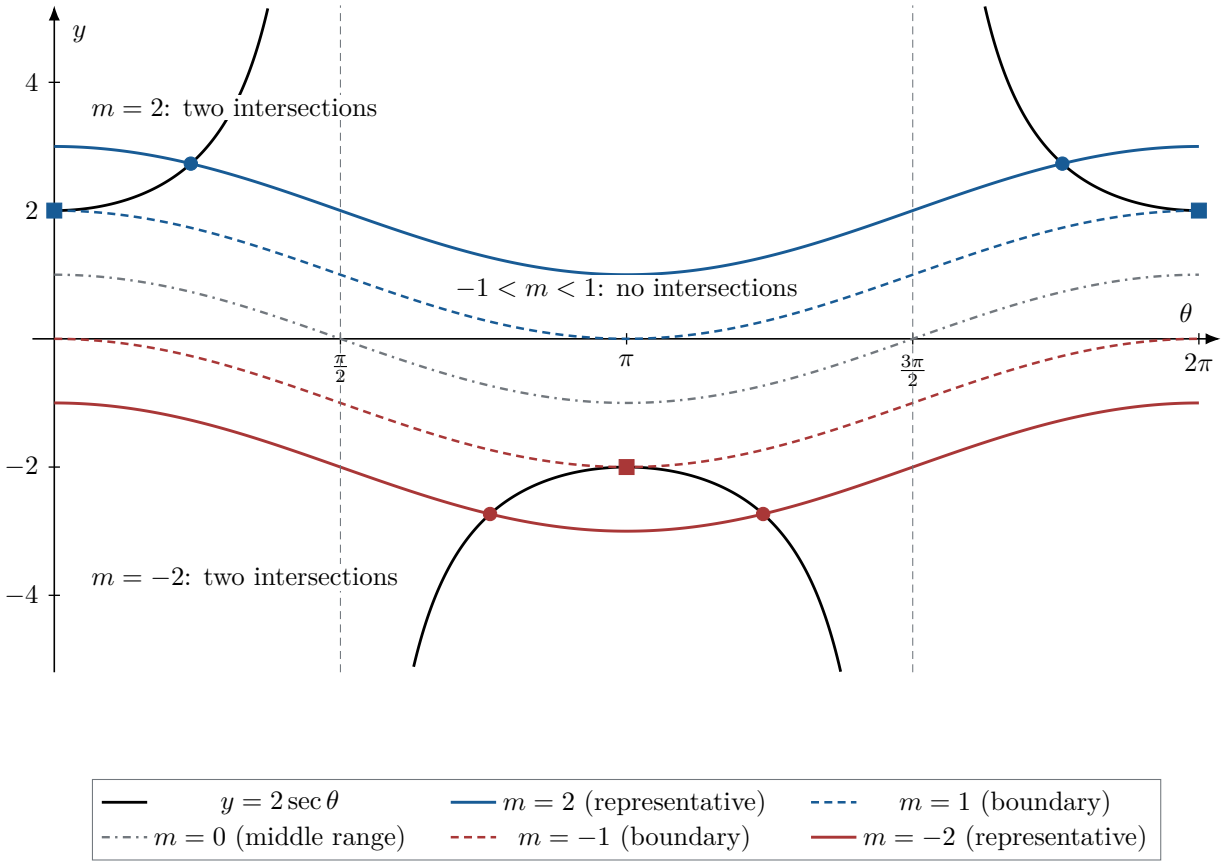
1 mark: Establishes $x^2 + mx - 2 = 0$, or an equivalent valid setup. A correct quadratic-formula setup including $\Delta = m^2 + 8$ earns this mark.
1 mark: Correctly obtains either $m \geq 1$ or $m < -1$, with relevant domain reasoning.
1 mark: Obtains both intervals and treats the endpoints correctly.

Feedback

This was a challenging part. Very few students obtained the correct range of m; the only fully correct response used a graphical approach.
Many students found $\Delta = m^2+8$ but did not restrict $x = \cos\theta$ or count the corresponding θ-values.

Question 25(c) - Method 2: Graphical solution

The graph of $y = 2\sec\theta$ is compared with vertical translations of $y = m + \cos\theta$ on $0 \leq \theta \leq 2\pi$. The solid blue and red curves are representative cases; the dashed curves are the boundaries $m = 1$ and $m = -1$.



The square markers show the boundary cases: $m = 1$ gives the two endpoint solutions $\theta = 0, 2\pi$, whereas $m = -1$ gives only $\theta = \pi$. Therefore

$$\boxed{m < -1 \quad \text{or} \quad m \geq 1}.$$

Question 25(c) - Further alternative methods

Method 3 - Quadratic formula (Extension 1-style)
From

$$x^2 + mx - 2 = 0,$$

the roots are

$$x_{\pm} = \frac{-m \pm \sqrt{m^2+8}}{2}.$$

Since $\sqrt{m^2+8} > |m|$, $x_+ > 0$ and $x_- < 0$.
For the positive root, require $x_+ \leq 1$:

$$\sqrt{m^2+8} \leq m + 2.$$

This inequality forces $m + 2 \geq 0$, so both sides are non-negative and may be squared equivalently:

$$m^2 + 8 \leq (m+2)^2 \quad \Longrightarrow \quad m \geq 1.$$

For the negative root to give two θ-values, require $x_- > -1$:

$$\sqrt{m^2+8} < 2 - m.$$

Here $2 - m > \sqrt{m^2+8} > 0$, so both sides are positive and may be squared equivalently:

$$m^2 + 8 < (2-m)^2 \quad \Longrightarrow \quad m < -1.$$

Hence $m < -1$ or $m \geq 1$.

Method 4 - Vieta's formulas (Extension 1-style)
Let

$$p(x) = x^2 + mx - 2.$$

If its roots are x_1, x_2, then

$$x_1 x_2 = -2.$$

Thus the roots have opposite signs and cannot both lie in $[-1, 1]$. At most one root is a valid cosine value.
Also $p(0) = -2 < 0$, so 0 lies between the roots.
The positive root lies in $(0, 1]$ exactly when

$$p(1) = m - 1 \geq 0.$$

Hence $m \geq 1$.
The negative root lies in $(-1, 0)$ exactly when

$$p(-1) = -m - 1 > 0.$$

Hence $m < -1$. At $m = -1$, the valid root is $x = -1$, which gives only $\theta = \pi$.
Therefore

$$\boxed{m < -1 \quad \text{or} \quad m \geq 1}.$$

These alternatives are not required for full marks.

Question 26(a) **[2 marks]**

A random variable X has probability density function

$$f(x) = \begin{cases} k(x+1), & -1 \leq x \leq 1, \\ k(3-x), & 1 < x \leq 3, \\ 0, & \text{otherwise.} \end{cases}$$

Find the value of k.

Solution

Method 1 - Integration

$$\int_{-1}^{1} k(x+1)\,\mathrm{d}x + \int_{1}^{3} k(3-x)\,\mathrm{d}x = 1$$

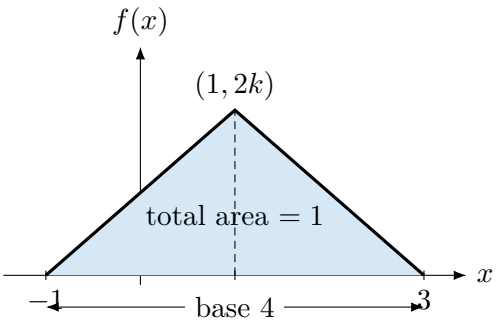
$$4k = 1.$$

Hence $\boxed{k = \frac{1}{4}}$.

Method 2 - Area

The density is an isosceles triangle with base 4 and height $2k$.

$$\frac{1}{2}(4)(2k) = 1 \quad \Longrightarrow \quad \boxed{k = \frac{1}{4}}.$$



Marking Scheme and Feedback

1 mark: Uses the total area under both pieces.
1 mark: Obtains $k = \frac{1}{4}$.

Feedback

Some students set the integral over only one piece equal to 1. Both pieces are required.

Question 26(b) **[3 marks]**
Find the cumulative distribution function $F(x)$ for all real x.

Solution

Method 1 - Integration
For $-1 \leq x \leq 1$,

$$F(x) = \int_{-1}^{x} \frac{1}{4}(t+1)\,\mathrm{d}t = \frac{(x+1)^2}{8}.$$

For $1 < x \leq 3$,

$$\begin{aligned} F(x) &= \underbrace{F(1)}_{1/2} + \int_{1}^{x} \frac{1}{4}(3-t)\,\mathrm{d}t \\ &= \frac{1}{2} + \int_{1}^{x} \frac{1}{4}(3-t)\,\mathrm{d}t \\ &= \frac{-x^2+6x-1}{8}. \end{aligned}$$

Therefore

$$F(x) = \begin{cases} 0, & x < -1, \\ \dfrac{(x+1)^2}{8}, & -1 \leq x \leq 1, \\ 1 - \dfrac{(3-x)^2}{8}, & 1 < x \leq 3, \\ 1, & x > 3. \end{cases}$$

Marking Scheme and Feedback

1 mark: Correct first non-zero branch.
1 mark: Includes the existing probability $F(1) = \frac{1}{2}$ in the second branch.
1 mark: Correct complete CDF and intervals.

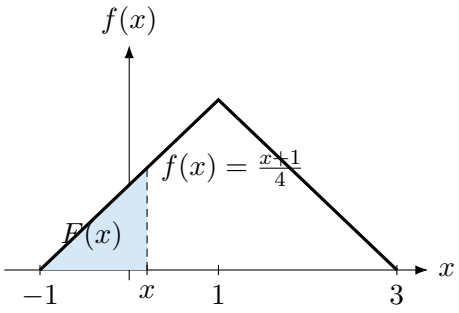
Feedback
Many students need to review that a CDF is cumulative. For $x > 1$, $F(x)$ must include the probability already accumulated over $[-1, 1]$, namely $F(1) = \frac{1}{2}$.
All responses used integration; no students used the area method or recognised the symmetry of the triangular density.

Question 26(b) - Method 2: Geometric area

For $-1 \leq x \leq 1$

The cumulative probability is the area of the shaded triangle:

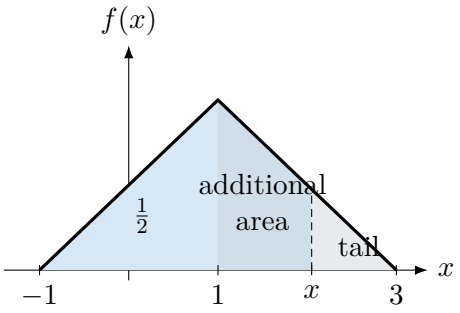
$$F(x) = \frac{1}{2}(x+1)\left(\frac{x+1}{4}\right) = \frac{(x+1)^2}{8}.$$



For $1 < x \leq 3$

The area to the left of x consists of the left half, $\frac{1}{2}$, plus the additional shaded region. Equivalently, subtract the right-tail triangle from 1:

$$F(x) = 1 - \frac{1}{2}(3-x)\left(\frac{3-x}{4}\right) = 1 - \frac{(3-x)^2}{8}.$$



The complete CDF is the same piecewise function obtained by integration.

Question 26(c) **[2 marks]**
Find the lower quartile Q_1, the upper quartile Q_3, and hence the interquartile range of X, in exact form.

Solution

Method 1 - Solve the CDF equations

$$\frac{(Q_1+1)^2}{8} = \frac{1}{4} \quad \Longrightarrow \quad Q_1 = -1 \pm \sqrt{2}.$$

Reject $-1-\sqrt{2}$ because it is outside $[-1,1]$. Hence

$$Q_1 = \sqrt{2} - 1.$$

Similarly,

$$\frac{-Q_3^2 + 6Q_3 - 1}{8} = \frac{3}{4} \quad \Longrightarrow \quad Q_3 = 3 \pm \sqrt{2}.$$

Reject $3+\sqrt{2}$ because it is greater than 3; in the complete CDF, $F(3+\sqrt{2}) = 1$, not 0.75. Thus

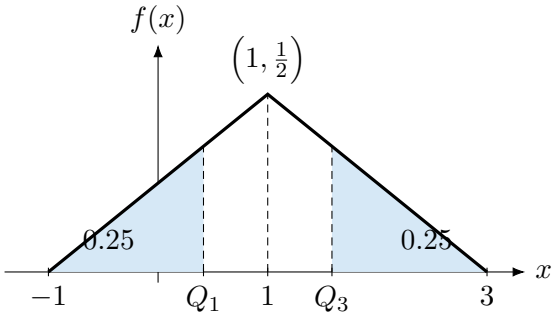
$$Q_3 = 3 - \sqrt{2}.$$

Therefore

$$\boxed{\text{IQR} = Q_3 - Q_1 = 4 - 2\sqrt{2}}.$$

Method 2 - Area and symmetry
The density is symmetric about $x = 1$. Thus

$$Q_3 = 2 - Q_1 = 3 - \sqrt{2}.$$



Marking Scheme and Feedback

1 mark: Correct Q_1.
1 mark: Correct Q_3 and hence the IQR.

Feedback
Most students found Q_1, but many struggled to determine Q_3.

Question 26(d) **[2 marks]**
A normal model Y has mean equal to the median of X and the same interquartile range as the physics-based model. Estimate $P(Y > 2)$, and compare this with $P(X > 2)$.

Solution

By symmetry, the median of X is 1, so $\mu_Y = 1$.
From the standard normal table,

$$z_{0.25} \approx -0.675, \qquad z_{0.75} \approx 0.675.$$

Hence

$$\text{IQR}(Y) = 1.35\sigma = 4 - 2\sqrt{2},$$

so

$$\sigma = \frac{4 - 2\sqrt{2}}{1.35} \approx 0.868.$$

Therefore

$$P(Y > 2) = P\left(Z > \frac{2-1}{0.868}\right) \approx P(Z > 1.15) \approx 0.1251.$$

For the triangular model,

$$P(X > 2) = 1 - F(2) = 1 - \frac{7}{8} = \frac{1}{8} = 0.125.$$

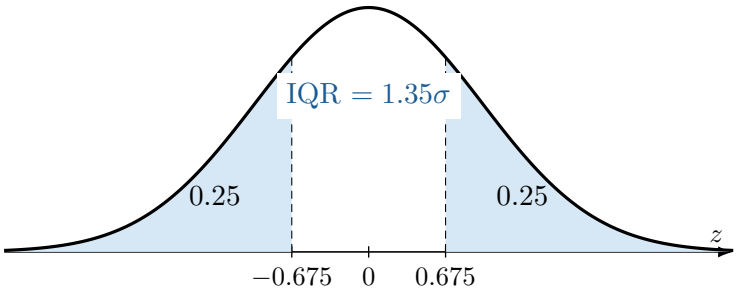
The two models give nearly identical estimates for this particular tail probability.

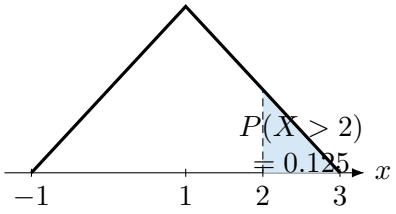
Marking Scheme and Feedback

1 mark: Correctly establishes the normal model and estimates $P(Y > 2)$.
1 mark: Calculates $P(X > 2)$ and makes a valid comparison.

Feedback

This part was poorly done. Many students could not use the IQR to determine σ.





Question 27(a) **[3 marks]**

The depth of water is

$$W(t) = 8 + \sin t + \cos t, \qquad 0 \leq t \leq \pi.$$

Find the coordinates of any stationary points and determine their nature.

Solution

$$W'(t) = \cos t - \sin t.$$

If $\cos t = 0$, then $\sin t = \cos t$ is impossible. Otherwise,

$$\tan t = 1.$$

Using $0 \leq t \leq \pi$ gives

$$t = \frac{\pi}{4}, \qquad W\left(\frac{\pi}{4}\right) = 8 + \sqrt{2}.$$

Also

$$W''(t) = -\sin t - \cos t, \qquad W''\left(\frac{\pi}{4}\right) = -\sqrt{2} < 0.$$

Hence

$$\boxed{\left(\frac{\pi}{4}, 8 + \sqrt{2}\right) \text{ is a local maximum.}}$$

Alternative: first derivative table

Since $t = \frac{\pi}{4}$ is the only zero of W' in the interval, use concrete test values:

t	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$
$W'(t)$	1	0	-1
$W(t)$	$\nearrow$	maximum	$\searrow$

Marking Scheme and Feedback

1 mark: Finds $t = \frac{\pi}{4}$ within the stated interval.
1 mark: Finds $W\left(\frac{\pi}{4}\right) = 8 + \sqrt{2}$.
1 mark: Correctly determines the nature.

Feedback

Generally well done. Some students used $t = \frac{7\pi}{4}$, which is outside $0 \leq t \leq \pi$.
A sign table was valid, but the second derivative test was more natural because W'' was also required in parts (b) and (c).

Question 27(b) **[2 marks]**
Find the point(s) of inflection.

Solution

$$W''(t) = -\sin t - \cos t.$$

Thus

$$W''(t) = 0 \quad \Longrightarrow \quad \tan t = -1.$$

Within $0 \leq t \leq \pi$,

$$t = \frac{3\pi}{4}, \qquad W\left(\frac{3\pi}{4}\right) = 8.$$

The sign table is

t	0	$\frac{3\pi}{4}$	π
$W''(t)$	-1	0	1
concavity	down		up

so the concavity changes. Therefore

$$\boxed{\left(\frac{3\pi}{4}, 8\right)}$$

is a point of inflection.

Marking Scheme and Feedback

1 mark: Finds the correct candidate and coordinate.
1 mark: Verifies a change in concavity.

Feedback
$W''(t) = 0$ alone is not sufficient; a change in concavity must be shown.
For example, $y = x^4$ has $y''(0) = 0$, but $(0, 0)$ is not a point of inflection.

Question 27(c) **[2 marks]**
A large cargo ship requires the water depth to be dropping at its absolute fastest rate. Find the exact value of t when the ship should cross the sandbar.

Solution

The depth is dropping fastest when $W'(t)$ is most negative.

Method 1 - Calculus and a sign table (recommended)
Let $V(t) = W'(t)$. Then

$$V'(t) = W''(t).$$

From part (b), the only interior candidate is $t = \frac{3\pi}{4}$. Using concrete test values,

t	0	$\frac{3\pi}{4}$	π
$W''(t)$	-1	0	1
$W'(t)$	$\searrow$	minimum	$\nearrow$

Also

$$W'(0) = 1, \quad W'\left(\frac{3\pi}{4}\right) = -\sqrt{2}, \quad W'(\pi) = -1.$$

Hence W' has its absolute minimum at $t = \frac{3\pi}{4}$.

Method 2 - Trigonometric range (Extension 1)
This auxiliary-angle method is Extension 1 content and is not required for Mathematics Advanced.

$$W'(t) = \cos t - \sin t = \sqrt{2}\cos\left(t + \frac{\pi}{4}\right).$$

Its minimum value $-\sqrt{2}$ occurs when

$$t + \frac{\pi}{4} = \pi,$$

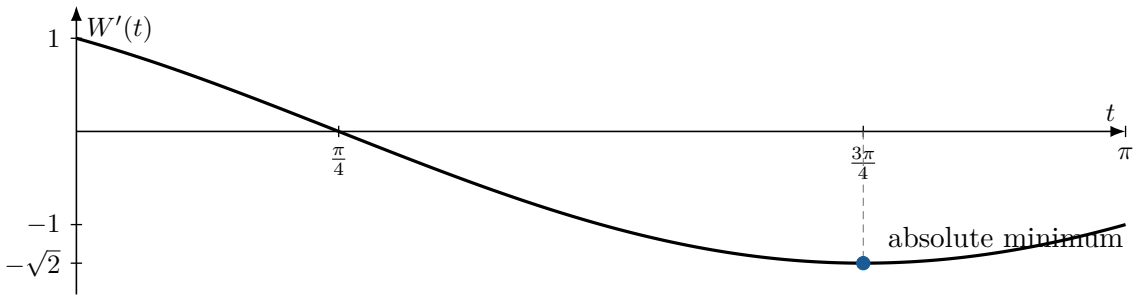
so $t = \frac{3\pi}{4}$.

Marking Scheme and Feedback

1 mark: Identifies $t = \frac{3\pi}{4}$.
1 mark: Shows that $W'(t)$ has an absolute minimum there.

Feedback

Some students identified $t = \frac{3\pi}{4}$ but did not show that $W'(t)$ was minimised.
A point of inflection is not automatically the point of steepest change; it is so here because W' has its absolute minimum there.



Question 28(a) **[2 marks]**

Stations A and B are 500 metres apart, with B due east of A. The point C has bearing 050° from A and bearing 305° from B. The drone is at D, vertically above C, and the angle of elevation of D from A is 32°. Find AD, correct to the nearest metre.

Solution

In the horizontal triangle,

$$\angle A = 40^\circ, \quad \angle B = 35^\circ, \quad \angle C = 105^\circ.$$

By the sine rule,

$$\frac{AC}{\sin 35^\circ} = \frac{500}{\sin 105^\circ},$$

so

$$AC = \frac{500 \sin 35^\circ}{\sin 105^\circ}.$$

In the vertical cross-section,

$$\cos 32^\circ = \frac{AC}{AD}.$$

Hence

$$AD = \frac{500 \sin 35^\circ}{\sin 105^\circ \cos 32^\circ} \approx 350.104.$$

Therefore

$$\boxed{AD = 350 \text{ m}}.$$

Marking Scheme and Feedback

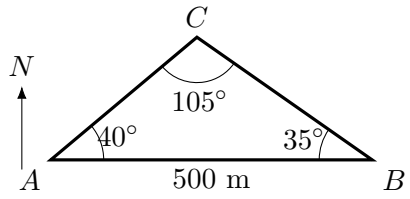
1 mark: Finds AC using the sine rule.

1 mark: Finds $AD = 350$ m.

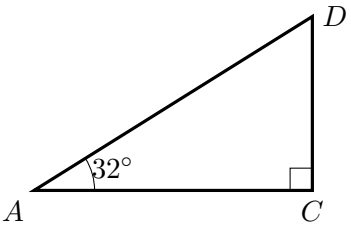
Feedback

Well done.

Horizontal view



Vertical cross-section



Question 28(b) **[2 marks]**

The guidance system makes a sequence of correction flights $L_1, L_2, L_3, \ldots$, where $L_1 = AD$. Consecutive lengths L_n and L_{n+1} form a triangle in which the angles opposite them are $110^\circ - \theta$ and θ, respectively, with $0^\circ < \theta < 55^\circ$. Show that the lengths form a geometric sequence with

$$\frac{L_{n+1}}{L_n} = q, \qquad q = \frac{\sin\theta}{\sin(110^\circ - \theta)},$$

and hence write L_n in terms of AD, q and n.

Solution

By the sine rule,

$$\frac{L_n}{\sin(110^\circ - \theta)} = \frac{L_{n+1}}{\sin\theta}.$$

Therefore

$$\frac{L_{n+1}}{L_n} = \frac{\sin\theta}{\sin(110^\circ - \theta)} = q.$$

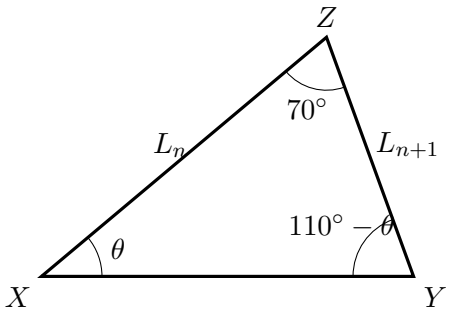
The common ratio is constant, so the lengths form a geometric sequence. Since $L_1 = AD$,

$$\boxed{L_n = ADq^{n-1}}.$$

Marking Scheme and Feedback

1 mark: Establishes the common ratio using the sine rule.
1 mark: Obtains $L_n = ADq^{n-1}$.

Feedback
Well done.



Question 28(c)(i) **[1 mark]**

For a reduced-battery mission, the limiting total correction distance must not exceed 900 metres. Using parts (a) and (b), find the total limiting correction distance in terms of AD and q.

Solution

Since $0° < \theta < 55°$,

$$110° - \theta > \theta.$$

In a triangle, the larger angle is opposite the larger side. Thus

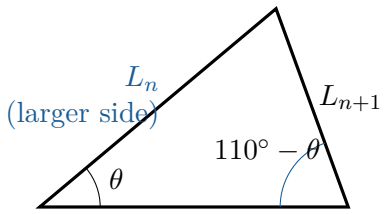
$$L_n > L_{n+1} > 0.$$

Therefore

$$0 < q = \frac{L_{n+1}}{L_n} < 1,$$

so $q \neq 0$ and $|q| < 1$.
The limiting sum exists and is

$$\boxed{S_\infty = \frac{AD}{1-q}}.$$



$$0 < q = \frac{L_{n+1}}{L_n} < 1$$

Marking Scheme and Feedback

1 mark: Writes the limiting total as $\frac{AD}{1-q}$.

Feedback

Well done. Writing $\frac{AD}{1-q} < 900$ was also accepted, although ≤ 900 matches "must not exceed".

Question 28(c)(ii) **[1 mark]**

Calculate the maximum allowable value of q, correct to four decimal places.

Solution

The restriction is

$$\frac{AD}{1-q} \leq 900.$$

Since $0 < q < 1$, $1 - q > 0$, so multiplication preserves the inequality:

$$AD \leq 900(1-q).$$

Using $AD = 350$,

$$350 \leq 900(1-q) \quad \Longrightarrow \quad q \leq \frac{11}{18}.$$

Hence

$$\boxed{q_{\max} = 0.6111}.$$

Marking Scheme and Feedback

1 mark: Correctly forms and solves the inequality for q.

Feedback

The final two parts were not well answered. Students found it difficult to form the inequality for q and then solve for θ. A value of 0.6110 obtained by carrying the unrounded value of AD was also accepted.

Question 28(c)(iii) **[2 marks]**

At the greatest allowable value of θ, the maximum value of q is attained, and q is increasing for $0^\circ < \theta < 55^\circ$ (you are not required to prove this). Hence or otherwise, determine the greatest allowable value of θ to the nearest degree. You may use

$$\sin(110^\circ - \theta) = \sin 110^\circ \cos\theta - \cos 110^\circ \sin\theta.$$

Solution

At the greatest allowable angle,

$$\frac{\sin\theta}{\sin(110^\circ - \theta)} = 0.6111.$$

Using the supplied identity,

$$\sin\theta = q(\sin 110^\circ \cos\theta - \cos 110^\circ \sin\theta),$$
$$\sin\theta(1 + q\cos 110^\circ) = q \sin 110^\circ \cos\theta.$$

Thus

$$\tan\theta = \frac{q\sin 110^\circ}{1 + q\cos 110^\circ}.$$

With $q = 0.6111$,

$$\theta \approx 35.98^\circ.$$

Hence

$$\boxed{\theta = 36^\circ}.$$

Marking Scheme and Feedback

1 mark: Correct trigonometric equation or rearrangement for θ.

1 mark: Obtains the greatest allowable angle 36°.

Feedback

The final two parts were not well answered. The inequality for q and the rearrangement for θ both required careful multi-step reasoning.

Teaching note - Why is q increasing? (Not required in the examination)

Angles are given in degrees, so first write

$$\phi = \frac{\pi\theta}{180}, \qquad 110^\circ = \frac{11\pi}{18}.$$

Then

$$q(\theta) = \frac{\sin\phi}{\sin\left(\frac{11\pi}{18} - \phi\right)}, \qquad \frac{\mathrm{d}\phi}{\mathrm{d}\theta} = \frac{\pi}{180}.$$

Using the quotient rule,

$$\frac{\mathrm{d}q}{\mathrm{d}\theta} = \frac{\pi}{180} \frac{\cos\phi \sin\left(\frac{11\pi}{18} - \phi\right) + \sin\phi\cos\left(\frac{11\pi}{18} - \phi\right)}{\sin^2\left(\frac{11\pi}{18} - \phi\right)}$$

$$= \frac{\pi}{180} \frac{\sin\left(\frac{11\pi}{18}\right)}{\sin^2\left(\frac{11\pi}{18} - \phi\right)} > 0.$$

Therefore q is increasing for $0^\circ < \theta < 55^\circ$.

End of Suggested Solutions