Killara High School

Student Number

2026 **Year 12 Trial Examination**

Mathematics Advanced

07/08/2026

General Instructions

- Reading time – 10 minutes
- Working time – 3 hours
- Write using blue or black pen
- Calculators approved by NESA may be used
- A reference sheet is provided
- For questions in Section II, show relevant mathematical reasoning and/or calculations
- No white-out may be used

Total Marks: 100

Section I - 10 marks (pages 2–7)

- Allow about 15 minutes for this section

Section II - 90 marks (pages 10–34)

- Allow about 2 hours and 45 minutes for this section

This question paper must not be removed from the examination room.

This assessment task constitutes 30% of the course.

Section I

10 marks
Attempt Questions 1–10
Allow about 15 minutes for this section.

Use the multiple-choice sheet for Question 1–10.

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1 What is the solution to the equation $\log_3(x + 5) = 4$?

A. 7

B. 59

C. 61

D. 76

2 The probability distribution for a discrete random variable X is shown.

x	3	5	10
$P(X = x)$	0.5	0.4	

What is $E(X)$?

A. 0.1

B. 3.5

C. 4.5

D. 5

3 The table shows future value interest factors for an annuity of $1.

Table of future value interest factors

Time periods	*Interest rate per period*				
	1%	2%	3%	4%	5%
4	4.0604	4.1216	4.1836	4.2465	4.3101
5	5.1010	5.2040	5.3091	5.4163	5.5256
6	6.1520	6.3081	6.4684	6.6330	6.8019
7	7.2135	7.4343	7.6625	7.8983	8.1420
8	8.2857	8.5830	8.8923	9.2142	9.5491

Eva makes deposits of $1200 at the end of each year into an investment account for 5 years. This account pays interest at a rate of 4% per annum, compounded annually.

What is the future value of Eva's annuity, to the nearest dollar?

A. $5172

B. $6000

C. $6500

D. $7300

4 What is the following anti-derivative?

$$\int \frac{x}{x^2+1}dx$$

A. $\frac{1}{2}\ln(x^2+1)+C$

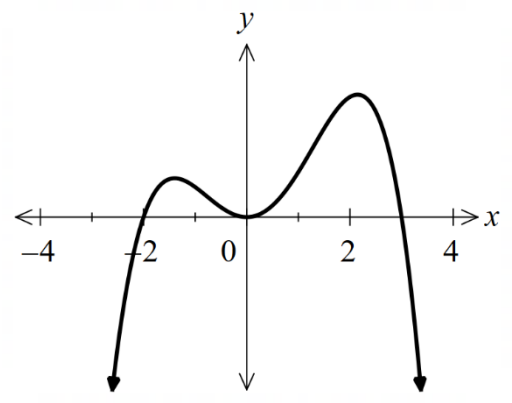
B. $2\ln(x^2+1)+C$

C. $\frac{2}{\ln(x^2+1)}+C$

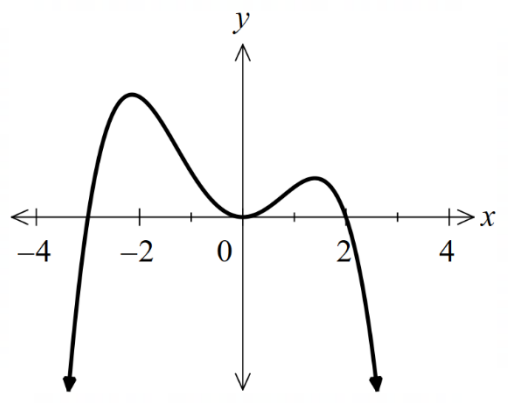
D. $\frac{1}{2\ln(x^2+1)}+C$

5 Which graph shows the function $y = -x^2(x-2)(x+3)$?

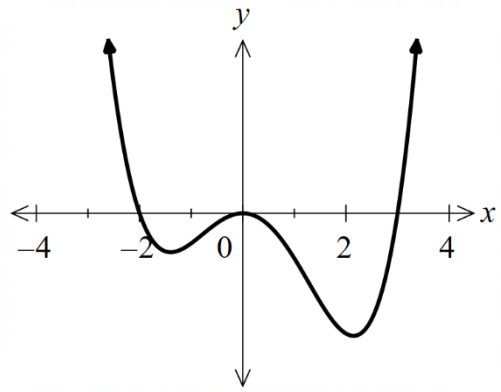
A.



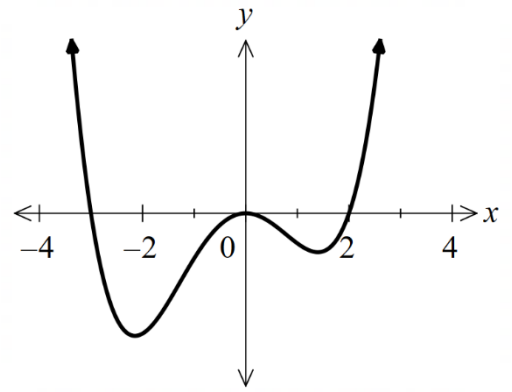
B.



C.



D.



6 A geometric series is given by

$$4 + 2x + x^2 + \frac{x^3}{2} + \ ...$$

If the limiting sum of the series is 10, what is the value of x?

A. $\frac{1}{2}$

B. $\frac{3}{5}$

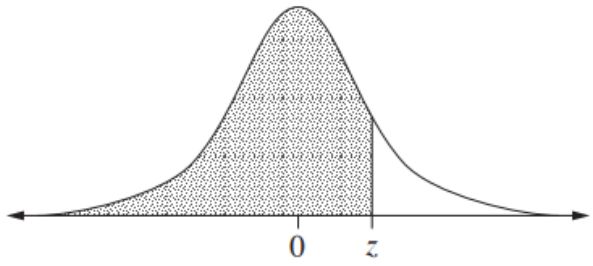
C. $\frac{4}{5}$

D. $\frac{6}{5}$

7 A random variable is normally distributed with mean 0 and standard deviation 1. The table gives the probability that this random variable is less than z.

z	0.7	0.8	0.9	1	1.1	1.2
Probability	0.7580	0.7881	0.8159	0.8413	0.8643	0.8849

The probability values given in the table for different values of z are represented by the shaded area in the following diagram.



What is $P(-0.8 < z < 1.2)$?

A. 0.0968

B. 0.5968

C. 0.673

D. 0.68

8 The function $f(x) = \ln(x + 2)$ has the following transformations applied to it:

- Translated up by $\ln(2)$ units
- Dilated horizontally by a factor of 2

The result is a new function $g(x)$.

Which single transformation could also transform $f(x)$ to $g(x)$?

A. Translation left 2 units

B. Translation right 2 units

C. Dilate vertically by a factor of 2

D. Dilate vertically by a factor of $\frac{1}{2}$

9 How many solutions are there to the following equation for $x \in [0, 2\pi]$?

$$(\tan x - 1)(\sin^2 x - 1) = 0$$

A. 1

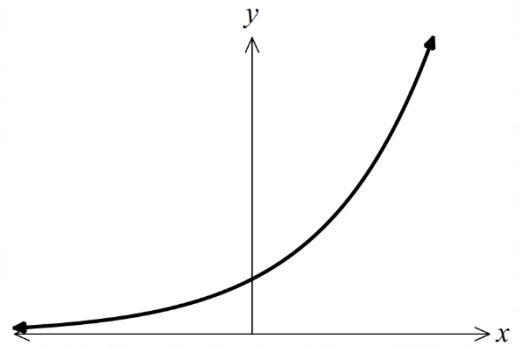
B. 2

C. 3

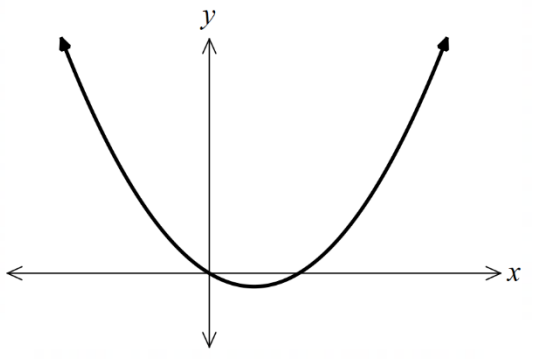
D. 4

10 Consider the two functions shown

$y = f(x)$

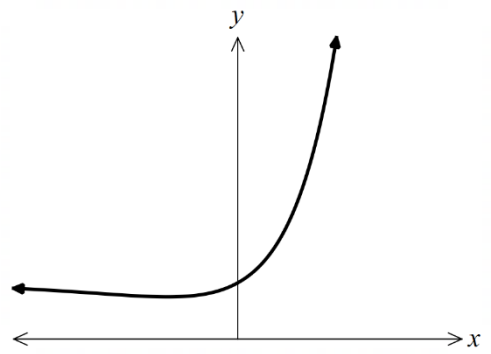


$y = g(x)$

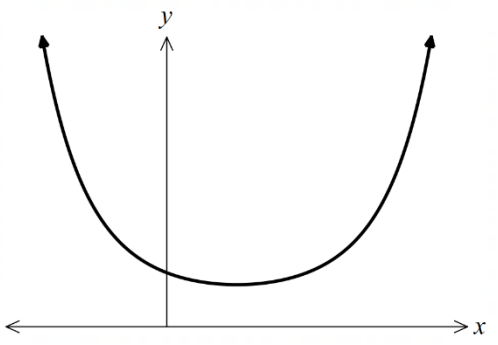


Which graph best represents $y = f\big(g(x)\big)$?

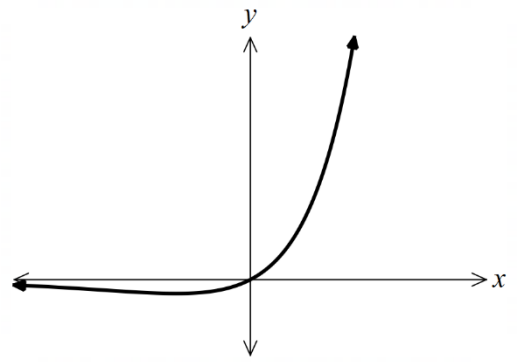
A.



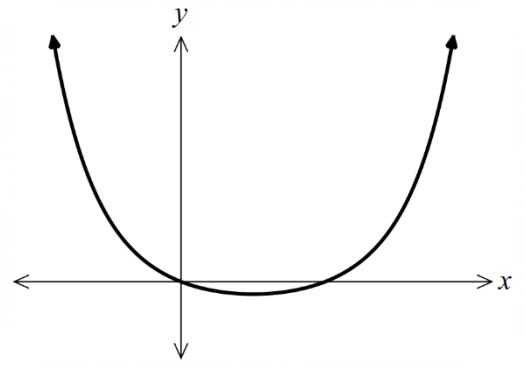
B.



C.



D.



End of Section I

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2026 **Year 12 Trial Examination**

Mathematics Advanced

Section II Answer Booklet

07/08/26

Section II

90 Marks
Attempt Questions 11–34
Allow about 2 hours and 45 minutes for this section

Instructions

- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided at the back of this booklet. If you use this space, clearly indicate which question you are answering.

Please turn over

Do NOT write in this area.

Question 11 (2 marks)

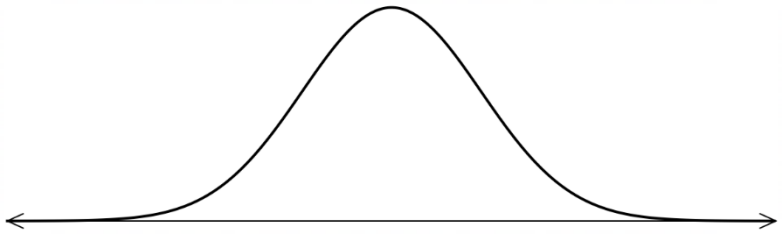
Solve the equation **2**

$$|3x - 4| = 13$$

Question 12 (2 marks)

Pine Meadow High School's Year 12 students sat a Mathematics examination. Their marks were normally distributed with a mean of 75 and standard deviation of 8. **2**

What percentage of students would you expect to have scored between 67 and 91?



Do NOT write in this area.

Question 13 (4 marks)

A certain influencer begins a new social media trend, and their subscribers increase rapidly after a viral post. The numbers of subscribers can be found using the equation

$$S = A(1.075)^n$$

Where S is the number of subscribers, and n is the number of days after the video was released.

(a) After 2 days, the number of subscribers has hit 20 000. Find the value of A, and describe the meaning of this value in context of the question. **2**

(b) How many days would it take for the influencer to reach 50 000 subscribers, correct to the nearest day? **2**

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Question 14 (3 marks)

In the box below, sketch the function **3**

$$y = \frac{3}{x-2} + 1$$

clearly showing all intercepts and asymptotes.

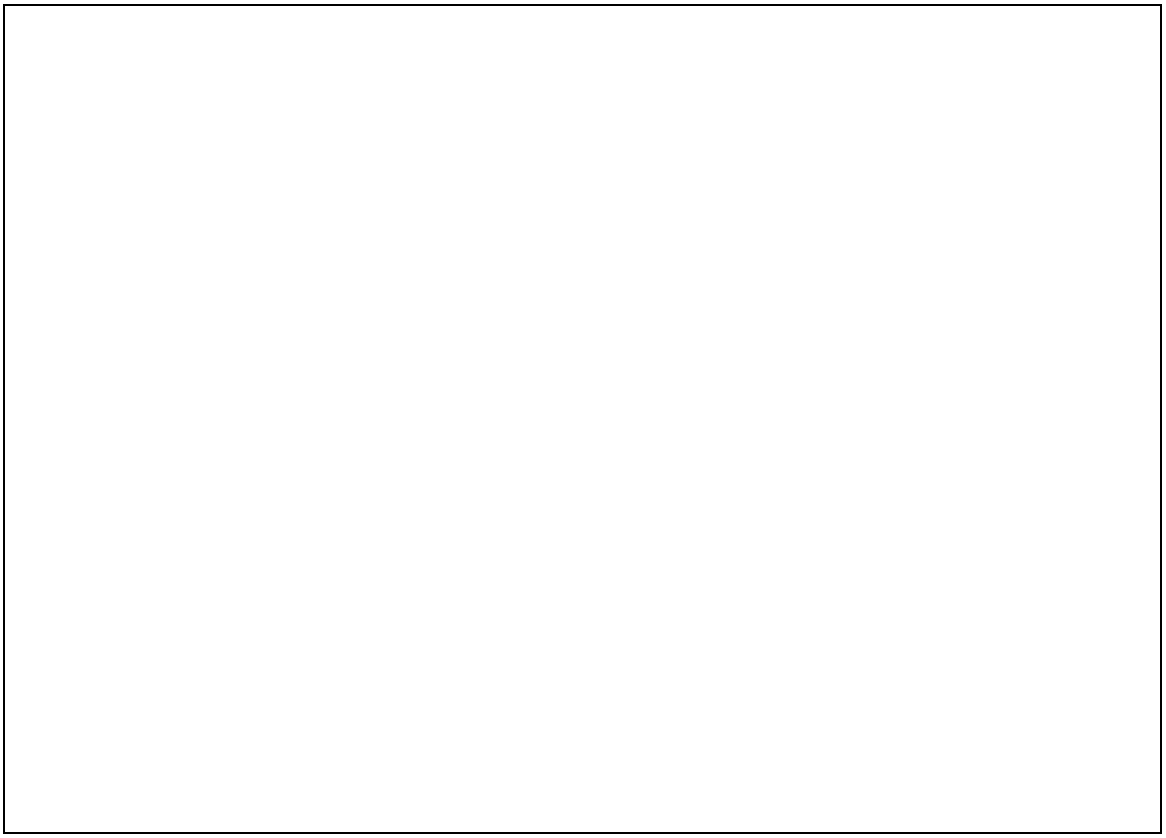
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Question 15 (2 marks)

A drawer contains 6 black socks and 8 white socks. Two socks are selected at random, without replacement. **2**

What is the probability that the two socks are the same colour?

Question 16 (2 marks)

Show that **2**

$$\operatorname{cosec}\theta - \sin\theta = \cos\theta\cot\theta$$

Do NOT write in this area.

Question 17 (3 marks)

Find the equation of the tangent to the curve below when $x = e$. **3**

$$y = (\ln x + 1)^3$$

Do NOT write in this area.

Question 18 (4 marks)

Present value interest factors for an annuity of $1 for various interest rates and time periods are given in the table below.

Table of present value interest factors

Time periods	*Interest rate per period*				
	0.20%	0.22%	0.24%	0.26%	0.28%
6	5.9582	5.9541	5.9499	5.9458	5.9416
12	11.8454	11.8301	11.8149	11.7996	11.7844
18	17.6625	17.6293	17.5961	17.5630	17.5300
24	23.4103	23.3524	23.2947	23.2373	23.1800

Luka deposits a lump sum into an account which earns interest at a rate of 2.4% per annum, compounding monthly.

(a) Luka can make withdrawals of $750 at the end of each month for 2 years. What is the minimum lump sum he must have deposited to be able to do this? Give your answer to the nearest dollar. **2**

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(b) Luka deposited the amount found in part (a). Rather than withdrawing money from the account for the first 6 months, Luka let the money sit and accrue compound interest. He then began withdrawing money for the remaining 18 months. **2**

What is the maximum amount he can withdraw each month, to the nearest dollar?

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Question 19 (3 marks)

Consider the function

$$f(x) = \frac{1}{\sqrt{x-1}}$$

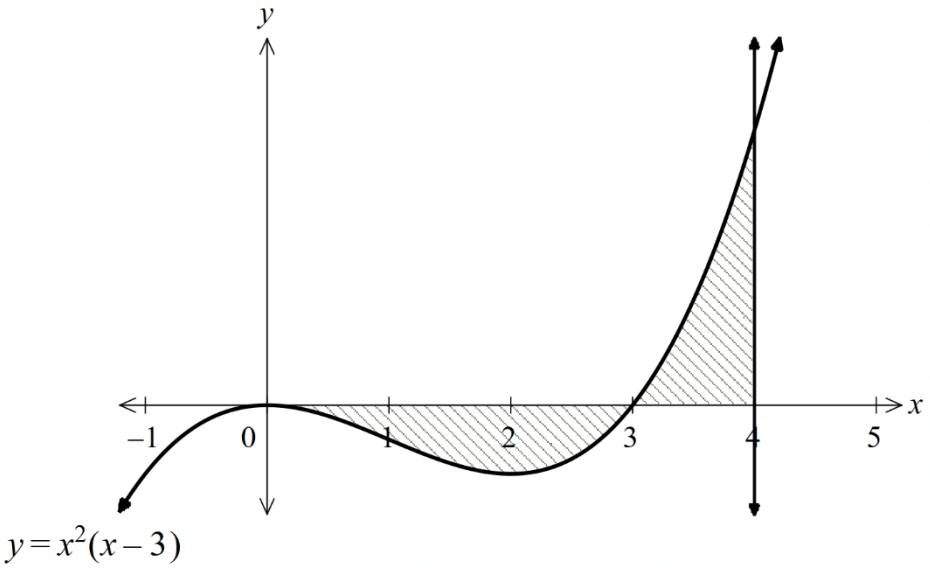
(a) State the domain of $f(x)$. **1**

(b) Find $f'(x)$, and justify why the function is always decreasing. **2**

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Question 20 (3 marks)

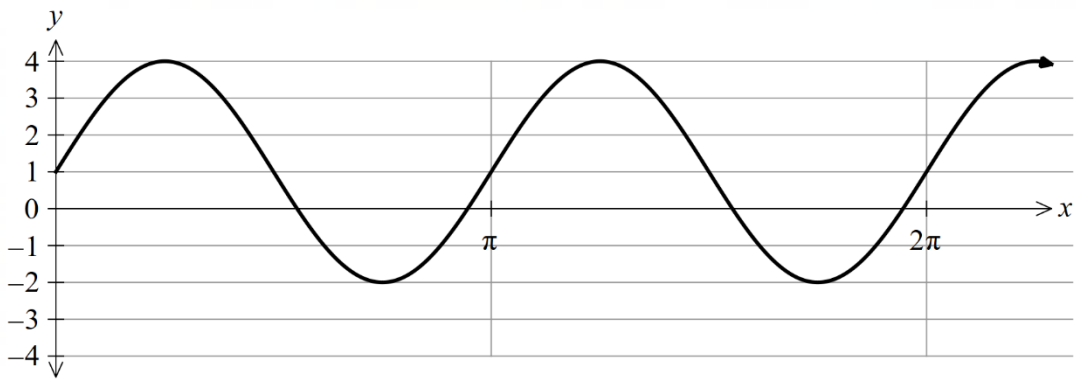
Find the exact area between the curve $y = x^2(x - 3)$ and the x-axis, for $0 \leq x \leq 4$. **3**



Do NOT write in this area.

Question 21 (5 marks)

The following graph shows the function $y = k\sin(ax) + c$



(a) Determine the values of k, a and c. **3**

(b) The function above is transformed into a new function by translating, then dilating. **2**

Fill in the blank spaces to apply two transformations, in the order shown, so that the range of the new function is $y \in [-3, 9]$.

- A translation up _____ units
- A vertical dilation by a factor of _____

Do NOT write in this area.

Question 22 (4 marks)

Find the global maximum and minimum of the curve below on the domain $x \in [-2, 3]$. **4**

$$y = x^3 + x^2 - 5x - 2$$

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Question 23 (6 marks)

A continuous random variable X has a probability density function given by

$$f(x) = \begin{cases} k \sin\left(x + \frac{\pi}{3}\right) & 0 \leq x \leq \frac{2\pi}{3} \\ 0 & \text{otherwise} \end{cases}$$

(a) Show that $k = \frac{2}{3}$. **2**

(b) Find the cumulative density function $F(x)$. **2**

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Question 23 (continued)

(c) Find the median of the probability density function, correct to 2 decimal places. **2**

Question 24 (3 marks)

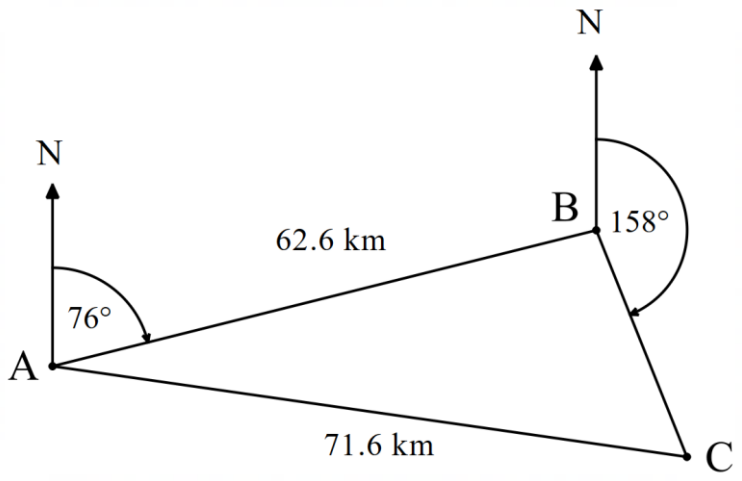
In an arithmetic sequence, the sum of the first six terms is 78. The sum of the first ten terms is 210. **3**

Find the first term and the common difference in the sequence.

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Question 25 (4 marks)

A ship travels from point A to point B on a bearing of 076°T for 62.6 km. It then travels on a bearing of 158°T, until it reaches point C. Point C is 71.6 km from point A.



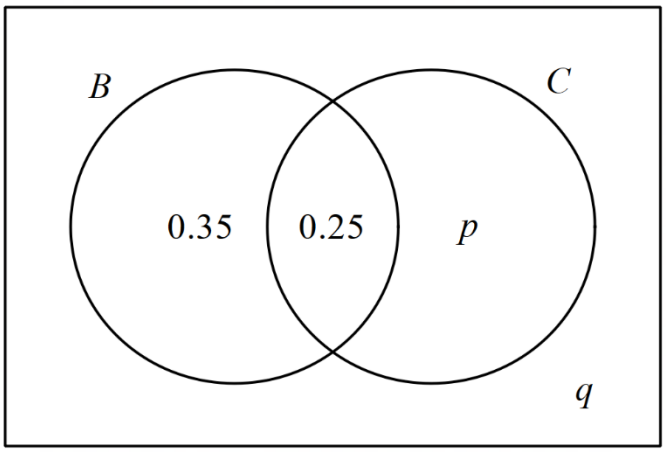
(a) What is the size of $\angle ABC$? **1**

(b) Find the distance between points B and C, correct to 3 significant figures. **3**

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Question 26 (3 marks)

The Venn diagram shows two events, B and C, and some associated probabilities. The variables p and q represent unknown probabilities. **3**

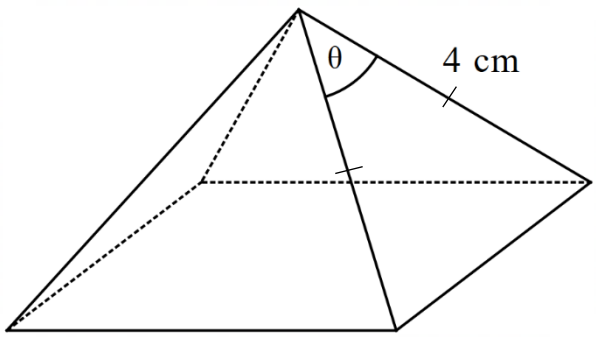


It is known that events B and C are independent. Find the values p and q.

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Question 27 (4 marks)

A square pyramid has 4 isosceles triangular faces, with equal area. These triangles have two side lengths of 4 cm.



The area of one of these triangular faces is 6 cm^2. The angle made at the top of the pyramid for one of these triangles is θ, where θ is acute.

(a) Find the exact value of $\cos\theta$. **2**

(b) Hence, find the exact area of the base of the pyramid in the form $a + b\sqrt{c}$. **2**

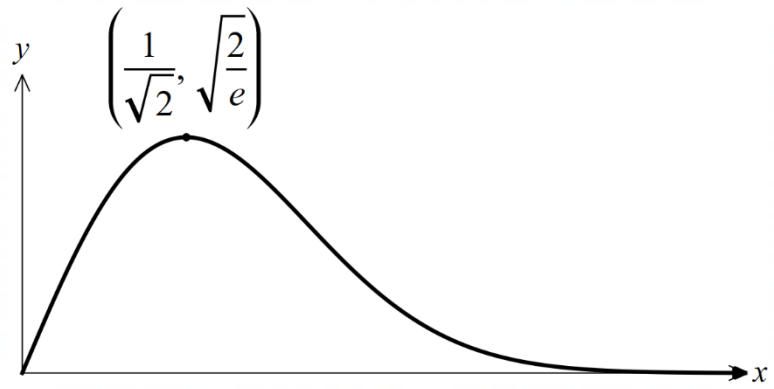
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Question 28 (3 marks)

The probability density function shown in the graph below has the equation: **3**

$$f(x) = \begin{cases} 2xe^{-x^2} & 0 \leq x \leq \infty \\ 0 & \text{otherwise} \end{cases}$$

The maximum turning point of the graph is given.



Find $P(X < \text{mode})$, correct to 3 significant figures.

Do NOT write in this area.

Question 29 (7 marks)

A reducing-balance loan of \$900 000 is taken out. Interest is charged at a monthly rate of 0.4%, compounding monthly.

Interest is added to the balance owing during the month, and at the end of each month, $\$M$ is repaid.

Let A_n be the amount owing at the end of each month.

(a) Show that $A_2 = 900\,000(1.004)^2 - M(1 + 1.004)$ **2**

(b) After 10 years, the amount still owing on the loan is \$660 800. Find the monthly repayments $\$M$ being made, to the nearest dollar. **2**

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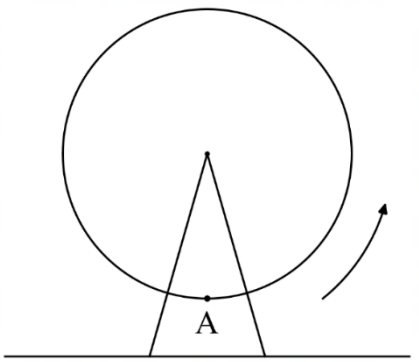
(c) At this point, 10 years after the beginning of the loan, the monthly repayments are increased by \$500. **3**

How many more months will it take to pay off the loan?

Do NOT write in this area.

Question 30 (5 marks)

An observation wheel has a radius of 70 metres and takes passengers from point A for one rotation around the wheel at a constant speed.



The height above the ground of an observer after t minutes is given by the function

$$h(t) = -70\cos\left(\frac{\pi t}{10}\right) + 80$$

(a) What is the maximum height of the observation wheel above the ground? **1**

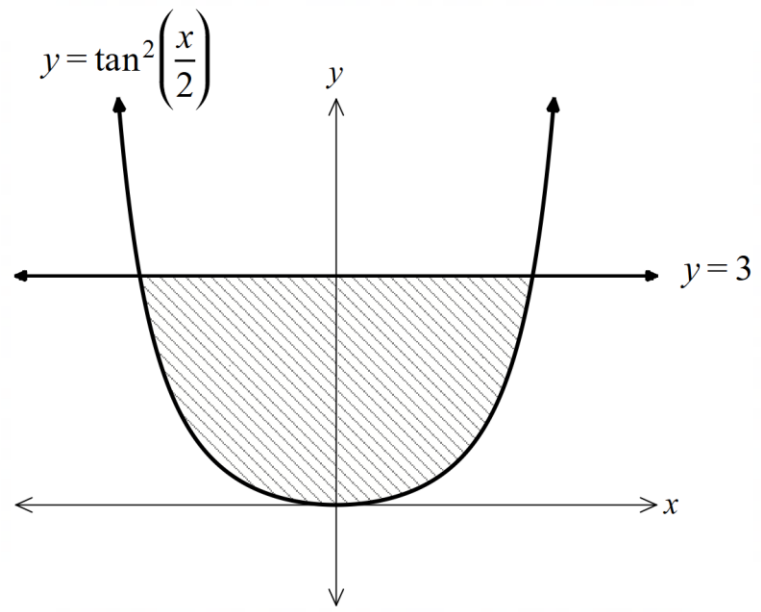
(b) How many minutes does it take for the wheel to complete one rotation? **1**

(c) A landmark can be seen from the observation wheel when its height is 100 metres above the ground. How long will the landmark be visible from the wheel, correct to the nearest minute? **3**

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Question 31 (4 marks)

Find the exact shaded area between the curves $y = \tan^2\left(\frac{x}{2}\right)$ and $y = 3$. **4**

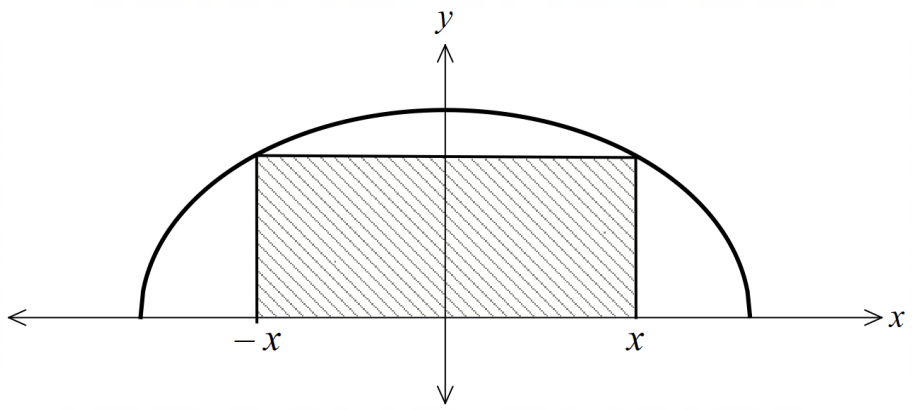


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Question 32 (6 marks)

The curve below shows the function

$$y = \sqrt{1 - \frac{x^2}{a}}$$



A rectangle has been constructed under the function, between two x values, where one is the negative of the other.

(a) Show that the area of the rectangle is given by the function **2**

$$A(x) = \sqrt{4x^2 - \frac{4x^4}{a}}$$

Do NOT write in this area.

(b) There is an x value greater than 0 which gives the maximum area of the rectangle. Show that this is **2**

$$x = \sqrt{\frac{a}{2}}$$

(you do not need to prove that this is a maximum)

(c) The shaded rectangle shown has an area of 10 units^2, and is the largest possible for some a value. **2**

Find the value of a.

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Question 33 (5 marks)

(a) Differentiate the function **1**

$$f(x) = x \ln x$$

(b) Using part (a), find an expression for **2**

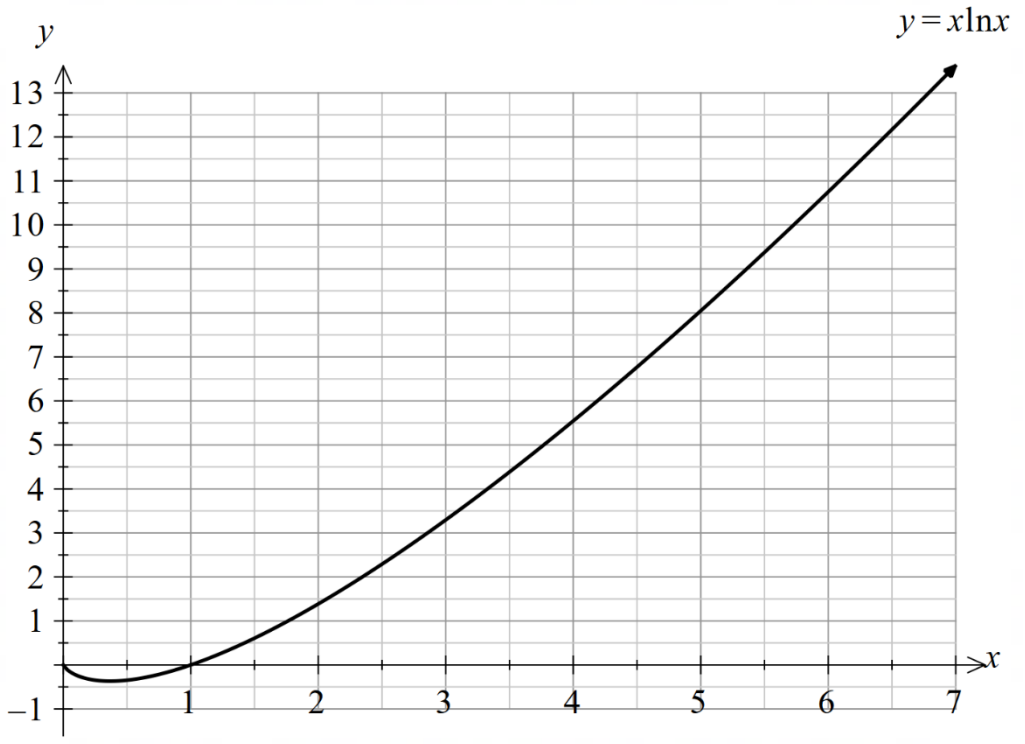
$$\int_1^k \ln x \, dx$$

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(c) The graph shows the function $y = x \ln x$. **2**

Using the graph, estimate the value of k that gives the solution to the equation

$$\int_1^k \ln x \, dx = 4$$



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Question 34 (3 marks)

On a particular farm, the weights of baby cows are recorded, and the means and standard deviations are shown. **3**

	Male	Female
Mean (μ)	39 kg	36 kg
Standard Deviation (σ)	4 kg	5 kg

60% of the baby cows are male, and 40% are female.

One baby cow is chosen at random.

Given that the weight of the cow is less than 31 kg, what is the probability that it is a female? Leave your answer as a percentage.

End of paper

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Section II extra writing space

If you use this space, clearly indicate which question you are answering.

Do NOT write in this area.

Section II extra writing space

If you use this space, clearly indicate which question you are answering.

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2026 – Mathematics Advanced Trial Exam

Solutions and Marking Guidelines

Section I

Multiple-choice Answer Key

Question	Answer
1	D
2	C
3	C
4	A
5	B
6	D
7	C
8	A
9	B
10	B

Section II

Question 11

Criteria	Marks
▪ Provides both solutions with working	2
▪ Provides one correct solution OR ▪ Gets to $3x - 4 = \pm 13$	1

Sample answer:

$$|3x - 4| = 13$$

$$3x - 4 = \pm 13$$

$$3x = 17, -9$$

$$x = \frac{17}{3}, -3$$

Feedback:

Mostly answered well and the most common mistake was to provide only one solution ie. only make $3x - 4 = 13$

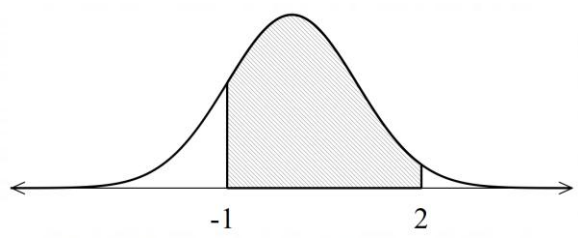
Question 12

Criteria	Marks
▪ Finds the correct percentage	2
▪ Finds the correct z-score OR ▪ Finds one correct z-score and a correct percentage from that	1

Sample answer:

67 – z-score is -1
91 – z-score is 2

$$\text{percentage} = \frac{68\%}{2} + \frac{95\%}{2}$$
$$= 81.5\%$$



Feedback:

Most students were able to find the z scores, however some mistakes in finding the percentages between -1 and 2 standard deviations

Question 13 (a)

Criteria	Marks
▪ Finds A and correctly describes its meaning	2
▪ Find the value of A	1

Sample answer:

$$20000 = A(1.075)^2$$

$$A = \frac{20000}{(1.075)^2}$$

$$= 17307$$

So there are 17307 subscribers on the day of the video.

Feedback:

This question was answered well. Be careful with rounding.

Question 13 (b)

Criteria	Marks
▪ Finds the correct number of days through correct working	2
▪ Substitutes and attempts to find n	1

Sample answer:

$$50000 = 17307(1.075)^n$$

$$(1.075)^n = \frac{50000}{17307}$$

$$n = \log_{1.075}\left(\frac{50000}{17307}\right)$$

$$n = 14.66\ldots$$

$$= 15 \text{ days}$$

Feedback:

Most students were able to rearrange using logarithms, even if they got part (a) wrong

Question 14

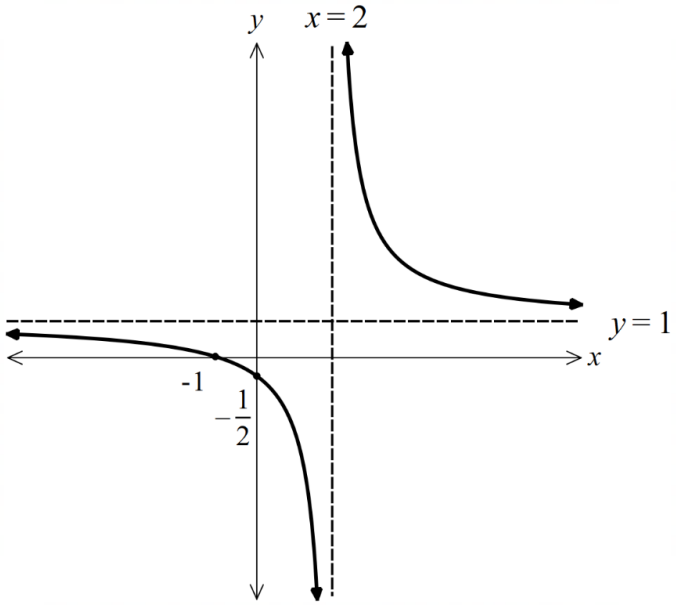
Criteria	Marks
▪ Provides a correct sketch with all features found and shown.	3
▪ Find the intercepts and states the asymptotes OR ▪ A hyperbola sketch with only 2 features correct	2
▪ Finds the correct intercepts or asymptotes OR ▪ Draws a hyperbola with no correct features	1

Sample answer:

x-int: $y = 0 \rightarrow -1 = \frac{3}{x-2}$
$-x + 2 = 3$
$-x = 1$
$x = -1$

y-int: $x = 0 \rightarrow y = \frac{3}{-2} + 1 = -\frac{1}{2}$

Asymptotes: $x = 2$ and $y = 1$



Feedback:

Most students drew a hyperbolic function, however common errors included labelling incorrect asymptotes or not finding the x and y-intercepts.
Some issues with sketching

Question 15

Criteria	Marks
▪ Finds the correct probability	2
▪ Uses a correct method but doesn't change the denominator OR ▪ Draws a tree diagram with correct branches	1

Sample answer:

$$P(BB) + P(WW) = \frac{6}{14} \times \frac{5}{13} + \frac{8}{14} \times \frac{7}{13}$$
$$= \frac{43}{91}$$

Feedback:

Mostly answered well but common mistake was to ignore the without replacement

Question 16

Criteria	Marks
▪ Provides a clear and logical proof	2
▪ Begins a correct proof and attempts to put over a common denominator OR ▪ The correct steps shown but hasn't set out their solution logically (eg works from both sides at the same time)	1

Sample answer:

$$\begin{aligned} LHS &= \operatorname{cosec}\theta - \sin\theta \\ &= \frac{1}{\sin\theta} - \frac{\sin^2\theta}{\sin\theta} \\ &= \frac{\cos^2\theta}{\sin\theta} \\ &= \cos\theta\frac{\cos\theta}{\sin\theta} \\ &= \cos\theta\cot\theta \\ &= RHS \end{aligned}$$

Feedback:

This was answered poorly.

WORK DOWN THE PAGE

Do not work both sides of the equation without separating the LHS and RHS

Question 17

Criteria	Marks
▪ Finds the equation of the tangent with a correct derivative	3
▪ Find the derivative correctly and substitutes to find the gradient OR ▪ A mistake with the derivative but the rest of the working correct	2
▪ Finds the correct derivative ▪ OR ▪ Finds the y-coordinates and follows a correct process with several errors	1

Sample answer:

$y(e) = 2^3 = 8$

$y'(x) = 3(\ln x + 1)^2 \times \frac{1}{x}$

$m = y'(e) = 3 \times 4 \times \frac{1}{e} = \frac{12}{e}$

$y - 8 = \frac{12}{e}(x - e)$

$y = \frac{12}{e}x - 4$

Feedback:

Most students knew to find the derivative but had issues with finding the tangent.

- Some issues with understanding $\ln e = 1$

Question 18 (a)

Criteria	Marks
▪ Finds the correct present value	2
▪ Multiplies 750 by the incorrect interest factor	1

Sample answer:

$$PV = 750 \times 23.4103$$
$$= \$17558$$

Feedback:

Pooly done, Too many students did not know how to look up a value from the table and multiply two numbers. A small % of student created a GP etc and did a page of writing.

Question 18 (b)

Criteria	Marks
▪ Finds the correct withdrawal amount	2
▪ Finds the correct future value OR ▪ Finds an incorrect future value, but uses the correct factor in the table	1

Sample answer:

$$FV = 17558(1 + 0.2\%)^6$$
$$= \$17769.75$$

$$\text{Withdrawal} = 17769.75 \div 17.6625$$
$$= \$1006$$

Feedback:

Poorly done, too many did not show compound interest for 6 months.

Question 19 (a)

Criteria	Marks
▪ Provides the correct domain	1

Sample answer:

$$x > 1$$

Feedback:

Well done! Some students think it is OK to divide by zero

Question 19 (b)

Criteria	Marks
▪ Finds the derivative and concludes that it is negative	2
▪ Find the correct derivative	1

Sample answer:

$$f(x) = (x-1)^{-\frac{1}{2}}$$

$$f'(x) = -\frac{1}{2}(x-1)^{-\frac{3}{2}}$$

$$= -\frac{1}{2\sqrt{(x-1)^3}}$$

Which is negative for all x in the domain.

Feedback:

Derivative generally well done.

Indicating $f'(x)$ was negative ==> decreasing sound to poor.

Question 20

Criteria	Marks
▪ Finds a correct area with correct working	3
▪ Correctly sets up the integral, finds the correct anti-derivative, and attempts to substitute the bounds	2
▪ Finds the correct anti-derivative and attempts to find the integral	1

Sample answer:

$$\begin{aligned}\text{Area } &= -\int_0^3 x^3 - 3x^2\,dx + \int_3^4 x^3 - 3x^2\,dx \\ &= \left[\frac{x^4}{4} - x^3\right]_3^0 + \left[\frac{x^4}{4} - x^3\right]_3^4 \\ &= \left(0 - \left(\frac{81}{4} - 27\right)\right) - \left((64 - 64) - \left(\frac{81}{4} - 27\right)\right) \\ &= \frac{27}{2}\ \text{u}^2\end{aligned}$$

Feedback:

Generally well done. Most students need practice setting up integral with absolute values for the area below the x-axis.

Question 21 (a)

Criteria	Marks
▪ Provides 3 correct values	3
▪ Provides 2 correct values	2
▪ Provides 1 correct value	1

Sample answer:

$c = 1$
$k = 3$
$a = 2$

Feedback:

Well done.

Question 21 (b)

Criteria	Marks
▪ Provides both correct numbers	2
▪ Provides one correct number	1

Sample answer:

- Up $\underline{\mathbf{0.5}}$ units
- Vertical dilation by a factor of $\underline{\mathbf{2}}$

Feedback:

- Translation - Many students did not answer correctly.
- Dilation - Most students did well.

Question 22

Criteria	Marks
▪ Correctly finds the global min and max with logical working	4
▪ Finds the turning points and end points, and provides the max and min from this, with a minor error	3
▪ Finds the coordinates of the turning points	2
▪ Finds the x-coordinates of the turning points	1

Sample answer:

$$y' = 3x^2 + 2x - 5 = 0$$
$$(3x + 5)(x - 1) = 0$$
$$x = -\frac{5}{3},\ 1$$

$$y(1) = -5$$

$$y\left(-\frac{5}{3}\right) = 4\frac{13}{27} = 4.48\ldots$$

Checking end points:

$$y(-2) = 4$$

$$y(3) = 19$$

- The global max is 19
- The global min is -5

Feedback:

Some missed out Global maximum and/or Global minimum. They just calculated local maximum and/ or local minimum.

About 1/3 of students did not check end points.

Question 23 (a)

Criteria	Marks
▪ Finds k from integrating the function	2
▪ Correctly integrates the function and attempts to find k OR ▪ Finds k with an error in the integral	1

Sample answer:

$$k\int_0^{\frac{2\pi}{3}} \sin\left(x+\frac{\pi}{3}\right) = 1$$

$$k\left[-\cos\left(x+\frac{\pi}{3}\right)\right]_0^{\frac{2\pi}{3}} = 1$$

$$k\left(-\cos\pi - \left(-\cos\frac{\pi}{3}\right)\right) = 1$$

$$k\left(1+\frac{1}{2}\right) = 1$$

$$k = 1 \div \frac{3}{2} = \frac{2}{3}$$

Feedback:

Small number of students did not show steps.

Question 23 (b)

Criteria	Marks
▪ Finds the correct CDF with correct working	2
▪ Attempts to find the CDF with integration but doesn't find c correctly	1

Sample answer:

$$F(x) = \int \frac{2}{3}\sin\left(x + \frac{\pi}{3}\right) dx$$
$$= -\frac{2}{3}\cos\left(x + \frac{\pi}{3}\right) + c$$

$$F(0) = 0$$

$$-\frac{2}{3}\cos\left(\frac{\pi}{3}\right) + c = 0$$
$$c = \frac{1}{3}$$

$$F(x) = -\cos\left(x + \frac{\pi}{3}\right) + \frac{1}{3}$$

Feedback:

About 20% of students did not evaluate $F(0) = 0$ to find c.

Question 23 (c)

Criteria	Marks
▪ Finds the median	2
▪ Let's $F(x) = 0.5$ and attempts to solve the trigonometric equation	1

Sample answer:

Let $F(x) = 0.5$

$$0.5 = -\frac{2}{3}\cos\left(x + \frac{\pi}{3}\right) + \frac{1}{3}$$

$$\frac{2}{3}\cos\left(x + \frac{\pi}{3}\right) = -\frac{1}{6}$$

$$\cos\left(x + \frac{\pi}{3}\right) = -\frac{1}{4}$$

$$x + \frac{\pi}{3} = \cos^{-1}\left(-\frac{1}{4}\right)$$

$$x = 0.78$$

Feedback:

Small number of students did not evaluate median occurs when CDF = 0.5

Question 24

Criteria	Marks
▪ Finds the correct first term and common difference	3
▪ Sets up two correct simultaneous equations to solve, but a mistake is made when solving simultaneously	2
▪ Sets up one correct equation correctly	1

Sample answer:

$$S_6 = \frac{6}{2}(2a + 5d) = 78 \rightarrow \quad 2a + 5d = 26 \quad (A)$$

$$S_{10} = \frac{10}{2}(2a + 9d) = 210 \rightarrow \quad 2a + 9d = 42 \quad (B)$$

$$(B) - (A): \quad 4d = 16$$
$$d = 4$$

$$\text{Sub into } (A): \quad 2a + 20 = 26$$
$$2a = 6$$
$$a = 3$$

The first term is 3 and the common difference is 4.

Feedback:

Most students did well setting up 2 equations and solve simultaneously.

Question 25 (a)

Criteria	Marks
▪ States the correct angle	1

Sample answer:

Angle ABN is $180° - 76° = 104°$

$\angle ABC = 360° - 104° - 158 = 98°$

Feedback:

Well done.

Question 25 (b)

Criteria	Marks
▪ Finds BC	3
▪ Finds all angles in the triangle and attempts to find BC	2
▪ Uses the sine rule to find a correct angle	1

Sample answer:

$$\frac{\sin(\angle BCA)}{62.6} = \frac{\sin(98°)}{71.6}$$

$$\sin(\angle BCA) = 0.86579\ldots$$

$$\angle BCA = 59.9733°$$

So $\angle BAC = 180° - 98° - 59.973° = 22.0266°$

$$\frac{BC}{\sin 22.0266°} = \frac{71.6}{\sin(98°)}$$

$$BC = 27.1 \text{ km}$$

Feedback:

Many students did not correct the answer to 3 significant figures as required. However, no mark has been deducted.

Question 26

Criteria	Marks
▪ Finds both p and q	3
▪ Finds p using the fact that the events are independent	2
▪ Uses an identity related to independent events	1

Sample answer:

Independent means $P(C) = P(C|B)$

$$p + 0.25 = \frac{0.25}{0.6}$$

$$p = \frac{1}{6}$$

$$q = 1 - \frac{1}{6} - 0.6$$

$$= \frac{7}{30}$$

Feedback:

Quite lot of students used p as $P(C)$, instead of $p + 0.25$.

Question 27 (a)

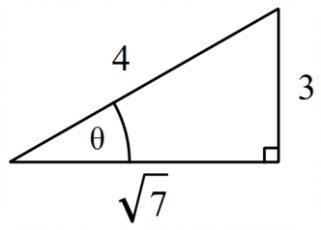
Criteria	Marks
▪ Finds $\cos\theta$ with correct working	2
▪ Finds and expression for $\sin\theta$ from the area formula	1

Sample answer:

$$\text{Area} = \frac{1}{2}4^2 \sin\theta = 6$$

$$8\sin\theta = 6$$

$$\sin\theta = \frac{3}{4}$$



$$\cos\theta = \frac{\sqrt{7}}{4}$$

Feedback:

A few students used 4 ad height of a triangle. Some students did not answer the question by finding angle instead of cosine ratio.

Question 27 (b)

Criteria	Marks
▪ Uses the cosine rule to find the base area	2
▪ Uses the cosine rule to attempt to find the side length	1

Sample answer:

Base area $= c^2$

$$c^2 = 4^2 + 4^2 - 2(4^2)\cos\theta$$

$$= 32 - 32 \times \frac{\sqrt{7}}{4}$$

$$= 32 - 8\sqrt{7}$$

Feedback:

Question 28

Criteria	Marks
▪ Finds the correct probability	3
▪ Sets up and integrates the function correctly	2
▪ Recognises the mode and attempts to integrate	1

Sample answer:

$$P(X < \text{mode}) = -\int_0^{\frac{1}{\sqrt{2}}} -2xe^{-x^2}dx$$

$$= -\left[e^{-x^2}\right]_0^{\frac{1}{\sqrt{2}}}$$

$$= e^0 - e^{-\frac{1}{2}}$$

$$= 0.393$$

Feedback:

Some students calculated P(X> mode), instead of P(X<mode). Quite a few students did differentiation instead of integration.

Question 29 (a)

Criteria	Marks
▪ Shows the results clearly going through A_1	2
▪ Sets up an expression for A_1	1

Sample answer:

$A_1 = 900\,000(1.004) - M$

$$A_2 = (900\,000(1.004) - M)(1.004) - M$$
$$= 900\,000(1.004)^2 - M(1.004) - M$$
$$= 900\,000(1.004)^2 - M(1 + 1.004)$$

Feedback:

Show that questions need to be *shown* explicitly. Students cannot assume a general formula for these reducing balance loans. The general formula comes from showing A_1, A_2 etc.

Question 29 (b)

Criteria	Marks
▪ Sets up an expression for A_{120} to solve for the monthly repayment	2
▪ Sets up an expression for A_{120} OR ▪ Solves for M using A_{10} with a series formula	1

Sample answer:

$A_3 = 900\,000(1.004)^3 - M(1 + 1.004 + 1.004^2)$

$A_{120} = 900\,000(1.004)^{120} - M(1 + 1.004 + 1.004^2 + \ \ldots + 1.004^{119}) = 660\,800$

$M(1 + 1.004 + 1.004^2 + \ \ldots + 1.004^{119}) = 900\,000(1.004)^{120} - 660\,800$

$$M\frac{(1.004)^{120} - 1}{1.004 - 1} = 792275.05$$

$$M = \$5157$$

Feedback:

The main mistake here was students finding A_{10} instead of A_{120}. Students needed to use the series set up to receive 1 mark. Students who tried to use a formula without any series sum were not awarded any marks.

Question 29 (c)

Criteria	Marks
▪ Provides a correct solution and rounds the final answer up	3
▪ Sets up a correct and equation and makes progress solving for n OR ▪ Completes a correct process using \$900 000 instead of \$660 800	2
▪ Sets up an equation with the new monthly payment, and the unknown n	1

Sample answer:

New monthly payments are \$5657

$$A_n = 660\,800(1.004)^n - 5657\left(\frac{(1.004)^n - 1}{1.004 - 1}\right) = 0$$

$$660\,800(1.004)^n - 1\,414\,250(1.004^n - 1) = 0$$

$$660\,800(1.004)^n - 1\,414\,250(1.004)^2 + 1\,414\,250 = 0$$

$$753450(1.004)^n = 1\,414\,250$$

$$(1.004)^n = 1.87703\ldots$$

$$n = \log_{1.004}(1.87703\ldots)$$

$$= 157.73\ldots$$

$$= 158 \text{ months}$$

Feedback:

This was done reasonably well by students who completed part (b). A bit of care in the algebra is needed.

Question 30 (a)

Criteria	Marks
▪ States the correct height	1

Sample answer:

Max height is 150 m

Feedback:

Done well

Question 30 (b)

Criteria	Marks
▪ Finds the correct time with working and units	1

Sample answer:

$$\text{Time} = \frac{2\pi}{\frac{\pi}{10}} = 20 \text{ min}$$

Feedback:

Students needed working and correct units here to receive the mark.

Question 30 (c)

Criteria	Marks
▪ Solves for t and correctly states the 8 minutes	3
▪ Attempts to find 2 values for t using the different quadrants	2
▪ Rearranges up to the point $\cos\left(\frac{\pi t}{10}\right) = -\frac{2}{7}$	1

Sample answer:

$$-70\cos\left(\frac{\pi t}{10}\right) + 80 = 100$$
$$-70\cos\left(\frac{\pi t}{10}\right) = 20$$
$$\cos\left(\frac{\pi t}{10}\right) = -\frac{2}{7}$$

Related angle $= \cos^{-1}\left(\frac{2}{7}\right) = 1.2810446\ldots$

Quadrants 2 and 3:

$$\frac{\pi t}{10} = \pi - 1.281\ldots, \pi + 1.281\ldots$$
$$= 1.86054\ldots\ , 4.422637$$
$$t = 5.922\ldots, 14.077\ldots$$

So the landmark is visible for about 8 minutes.

Feedback:

Many students got up to showing $\cos\left(\frac{\pi t}{10}\right) = -\frac{2}{7}$. However, many student need to work on finding two solutions in the correct quadrants from that point.

Question 31

Criteria	Marks
▪ Finds the area through correct and logical working	4
▪ Attempts to find the area and uses the $\tan^2\left(\frac{x}{2}\right) = \sec^2\left(\frac{x}{2}\right) - 1$	3
▪ Finds the intersection points and sets up an integral OR ▪ Sets up a correct integral expression and converts $\tan^2\left(\frac{x}{2}\right)$ to $\sec^2\left(\frac{x}{2}\right)$	2
▪ Finds the x-coordinate points of intersection OR ▪ Sets up a correct integral expression with incorrect bounds	1

Sample answer:

Points of intersection

$$\tan^2\left(\frac{x}{2}\right) = 3$$

$$\tan\left(\frac{x}{2}\right) = \pm\sqrt{3}$$

$$\frac{x}{2} = \pm\frac{\pi}{3}$$

$$x = \pm\frac{2\pi}{3}$$

$$\text{Area} = \int_{-\frac{2\pi}{3}}^{\frac{2\pi}{3}} 3 - \tan^2\left(\frac{x}{2}\right) dx$$

$$= 2\int_{0}^{\frac{2\pi}{3}} 3 - \left(\sec^2\left(\frac{x}{2}\right) - 1\right)\, dx$$

$$= 2\int_{0}^{\frac{2\pi}{3}} 4 - \sec^2\left(\frac{x}{2}\right)\, dx$$

$$= 2\left[4x - 2\tan\left(\frac{x}{2}\right)\right]_0^{\frac{2\pi}{3}}$$

$$= 2\left[\frac{8\pi}{3} - 2\tan\frac{\pi}{3} - (0 - 0)\right]$$

$$= \frac{16\pi}{3} - 4\sqrt{3}\ \text{units}^2$$

Feedback:

Students need to make sure they get more comfortable with the longer 4/5 mark questions like this. Not enough students were able to confidently find the x bounds, and set up the correct integral. Doing this would have awarded 2 marks easily.

Question 32 (a)

Criteria	Marks
▪ Shows the area correctly	2
▪ Attempts to multiply the base with the height	1

Sample answer:

Height $= \sqrt{1 - \frac{x^2}{a}}$

Base $= 2x$

$$\begin{aligned} \text{Area} &= 2x\sqrt{1 - \frac{x^2}{a}} \\ &= \sqrt{4x^2}\sqrt{1 - \frac{x^2}{a}} \\ &= \sqrt{4x^2 - \frac{4x^4}{a}} \\ &= A(x) \end{aligned}$$

Feedback:

Not well done. Show that questions need to be clear, and students cannot just jump to the answer given.

Question 32 (b)

Criteria	Marks
▪ Shows the value through differentiation	2
▪ Differentiates the function correctly	1

Sample answer:

$$A'(x) = \frac{1}{2}\left(4x^2 - \frac{4x^4}{a}\right)^{-\frac{1}{2}} \times \left(8x - \frac{16x^3}{a}\right) = \frac{8x - \frac{16x^3}{a}}{2\sqrt{4x^2 - \frac{4x^4}{a}}}$$

$A'(x) = 0$ when $8x - \frac{16x^3}{a} = 0$

$$8x\left(1 - \frac{2x^2}{a}\right) = 0 \qquad \text{but } x \neq 0$$

$$\frac{2x^2}{a} = 1$$

$$x^2 = \frac{a}{2}$$

$$x = \sqrt{\frac{a}{2}} \qquad (x > 0)$$

Feedback:

Reasonably well done.

Question 32 (c)

Criteria	Marks
▪ Finds the value of a	2
▪ Substitutes $x = \sqrt{\frac{a}{2}}$ into the area formula and attempts to solve for a	1

Sample answer:

$$A\left(\sqrt{\frac{a}{2}}\right) = \sqrt{4\left(\sqrt{\frac{a}{2}}\right)^2 - \frac{4\left(\sqrt{\frac{a}{2}}\right)^4}{a}}$$

$$= \sqrt{2a - a} = 10$$

$$a = 100$$

Feedback:

Quite well done. It was good to see that student who did not receive any marks for part (a) and (b) still received marks for this part.

Question 33 (a)

Criteria	Marks
▪ Differentiates the function	1

Sample answer:

$$f'(x) = x \times \frac{1}{x} + \ln x$$
$$1 + \ln x$$

Feedback:

Question was done well by all students

Question 33 (b)

Criteria	Marks
▪ Finds an expression for the integral in terms of k	2
▪ Uses part (a) to set up an integral to solve	1

Sample answer:

$$\int_1^k \ln x \, dx = \int_1^k 1 + \ln x \, dx - \int_1^k 1 \, dx$$
$$= [x \ln x]_1^k - [x]_1^k$$
$$= k \ln k - 1 \ln 1 - (k - 1)$$
$$= k \ln k - k + 1$$

Feedback:

Question was done well by all students

Question 33 (c)

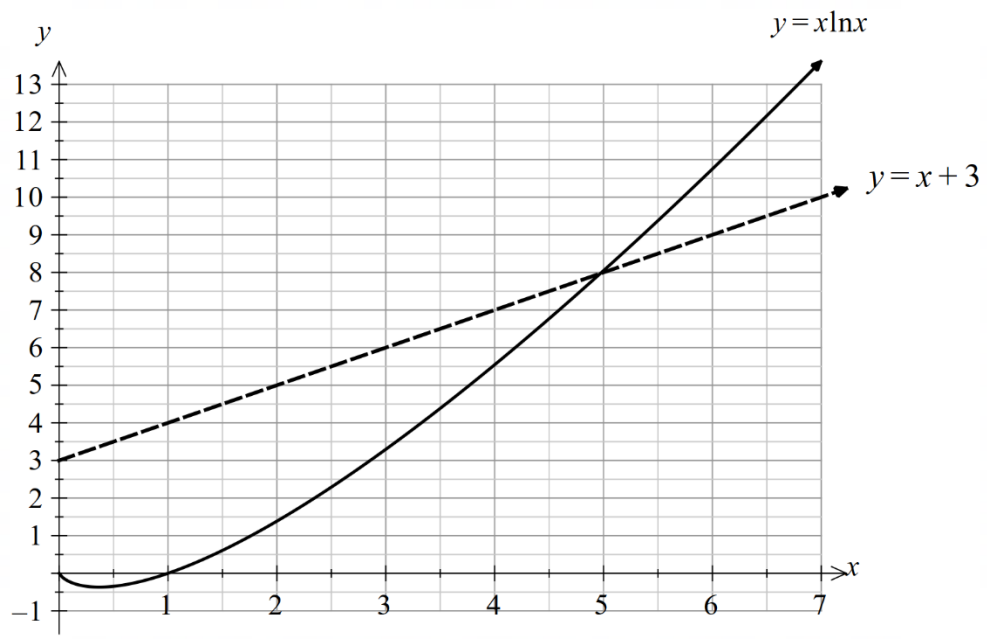
Criteria	Marks
▪ Drawing a graph to solve the equation and estimating the solution	2
▪ Attempting to solve the equation $k \ln k - k + 1 = 4$ using the graph	1

Sample answer:

$\int_1^k \ln x \, dx = 4$ means $k \ln k - k + 1 = 4$

So solving $k \ln k = k + 3$

Drawing $y = x + 3$



The solution is approximately $k = 5$

Feedback:

Question was very poorly done by all students. Many tried to use trapezoidal rule to estimate the value of k, but the the graph is for $x \ln x$ but the integral is $\ln(x)$

Question 34

Criteria	Marks
▪ Finding the correct probability with reasoning	3
▪ Finds the probabilities for each, and attempts to use conditional probability	2
▪ Finding the z-scores and associated probability for them	1

Sample answer:

Finding the z-score of 31 for males and females:

Male: $z = -2$

Female: $z = -1$

The probability of getting a weight of less than 31 kg (using the empirical rules):

Male: $P(z < 31) = 0.025$
Female: $P(z < 31) = 0.16$

$$P(< 31\text{kg} \mid \text{Female}) = \frac{0.16 \times 0.4}{0.025 \times 0.6 + 0.16 \times 0.4}$$
$$\approx 81\%$$

Feedback:

Almost all students were awarded 1 mark because they found the correct z-score and the corresponding percentages for the z-scores. But many were not able to find the correct value for P(less than 31kg) or $\text{P(Female} \cap \text{less than 31 kg)}$