KNOX GRAMMAR SCHOOL

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2026 **YEAR 12 TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION**

Mathematics Advanced

General Instructions

- Reading time – 10 minutes
- Working time – 3 hours
- Write using a black or blue pen
- NESA approved calculators may be used
- A reference sheet is provided with this paper

Total marks: 100

Section I – 10 marks (pages 3 – 7)

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section
- Use the multiple-choice answer sheet provided to record your answers

Section II – 90 marks (pages 8 – 39)

- Attempt Questions 11 – 33
- Allow about 2 hours and 45 minutes for this section
- Additional writing paper is provided on page 40

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Section I

10 Marks
Attempt Questions 1–10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1–10

1. What is the solution to $|x-2|<3$?

A. $-1<x<5$

B. $x<-5$ or $x>1$

C. $-5<x<1$

D. $x<-1$ or $x>5$

2. Jacob deposits \$750 every 6 months for 4 years into an account earning interest at 6% p.a compounded biannually.

The table below shows the future value interest factors for an annuity of \$1

Periods (n)	Future Value of an Annuity of \$1				
	Interest Rate				
	2%	3%	4%	5%	6%
2	2.020	2.030	2.040	2.050	2.060
4	4.122	4.184	4.247	4.310	4.375
6	6.308	6.468	6.633	6.802	6.975
8	8.583	8.892	9.214	9.549	9.898
10	10.950	11.464	12.006	12.578	13.181

What is the future value of Jacob's investment?

A. \$3138.00

B. \$3281.25

C. \$6669.00

D. \$7423.50

3. Which of the following is true about $y = 4\sin\left(\frac{x}{2}\right)$?

A. Amplitude $= 4$ and period $= \frac{1}{2}$

B. Amplitude $= 4$ and period $= 2\pi$

C. Amplitude $= \frac{1}{2}$ and period $= 4$

D. Amplitude $= 4$ and period $= 4\pi$

4. A random sample of 84 students was asked how many revision websites they had visited in the past month.

The data is summarised in the table below.

Number of websites	**Frequency**
0	1
1	4
2	18
3	16
4	5
5	37
6	2
7	1

What is the interquartile range of the number of websites visited by these 84 students?

A. 3

B. 4

C. 19

D. 42

5. What is the angle of inclination of the line $3x + 2y = 7$?

A. $33°41'$

B. $56°19'$

C. $123°41'$

D. $146°19'$

6. The probability density function for the continuous random variable X is:

$$f(x) = \begin{cases} kx^3 + \dfrac{3}{4}x & 0 < x < 2 \\ 0 & \text{otherwise} \end{cases}$$

What is the value of k?

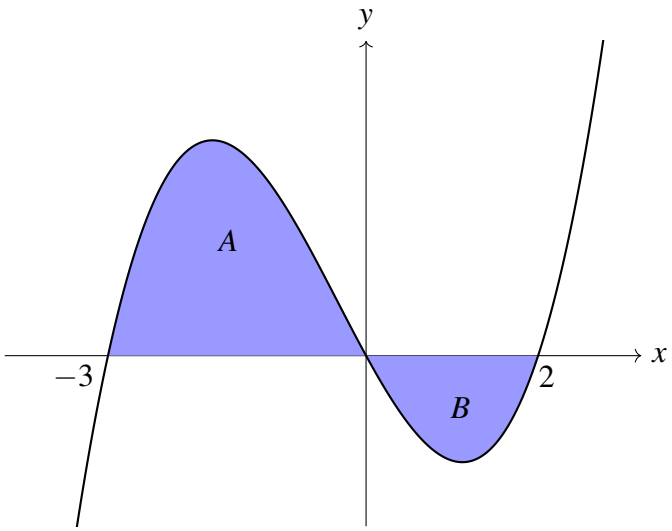
A. $-\dfrac{1}{8}$

B. $-\dfrac{3}{8}$

C. $-\dfrac{3}{16}$

D. $-\dfrac{9}{16}$

7. The graph of $y = f(x)$ intersects the x-axis at $(-3, 0)$, $(0, 0)$ and $(2, 0)$ as shown in the diagram below.



The shaded region A has an area of 189u^2.

The shaded region B has an area of 64u^2.

What is the value of $\displaystyle\int_{-3}^{2} f(x)\,\mathrm{d}x$?

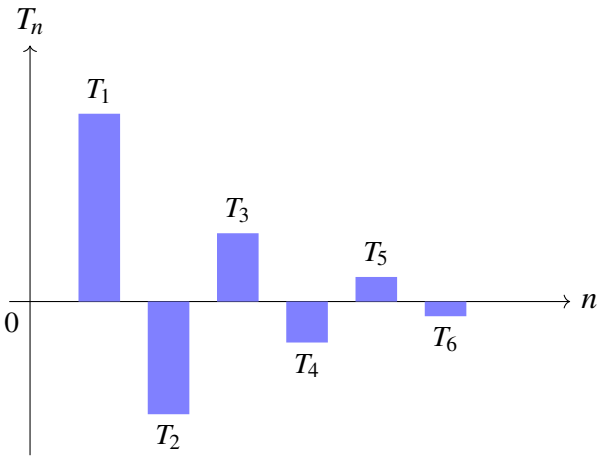
A. -253

B. -125

C. 125

D. 253

8. The values for a geometric sequence $T_n = ar^{n-1}$ are plotted on a bar graph:



Which statement must be true?

A. $a < 0,\ -1 < r < 0$

B. $a < 0,\ 0 < r < 1$

C. $a > 0,\ -1 < r < 0$

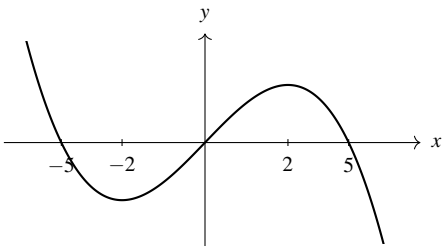
D. $a > 0,\ 0 < r < 1$

9. Consider a curve with the following properties:

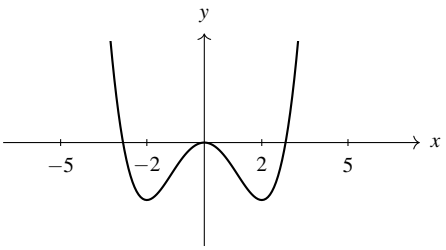
- $g(x)$ is odd.
- $g(5) = 0$ and $g'(2) = 0$.
- $g'(x) > 0$ for $x > 2$.

Which of the following could be the graph of $y = g(x)$?

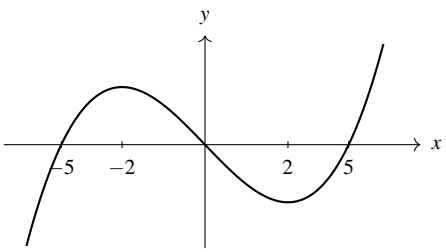
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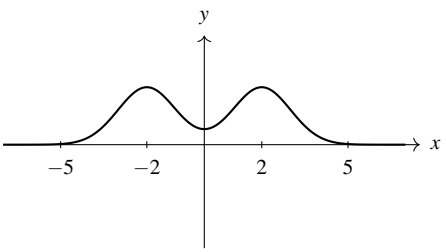
B.



C.



D.



10. The point $P(2,-4)$ lies on $y=f(x)$. A sequence of transformations is applied to $y=f(x)$ to obtain

$$g(x)=2f\left(\frac{4-x}{3}\right)-1$$

Which of the points on the graph of $g(x)$ correspond to the original point P?

A. $(-2,-8)$

B. $\left(\frac{2}{3},-8\right)$

C. $(-2,-9)$

D. $\left(\frac{2}{3},-9\right)$

End of Section I

Section II

90 marks

Attempt Questions 11 – 33
Allow about 2 hours and 45 minutes for this section

Answer the questions in the spaces provided.
Include all relevant mathematical reasoning and/or calculations.

Question 11 (4 marks)

In an experiment, 5 million cells were stored in a container. It was observed that 40% of the cells are dying every minute. The number of cells remaining, N, after t minutes, is given by $N = Ae^{kt}$, where A and k are constants.

(a) Explain why $k = \ln\frac{3}{5}$. **2**

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(b) At what time, to two decimal places, will there be 2000 cells remaining? **2**

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Question 12 (2 marks) **2**

Evaluate $\int_0^{\frac{\pi}{2}} \cos x \, dx$.

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Question 13 (3 marks)

(a) Differentiate $y = xe^{5x}$. **1**

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(b) Hence, find the exact value of $\int_0^3 e^{5x}(20x+4)\, dx$. **2**

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Question 14 (3 marks) **3**

Find the equation of the tangent to the curve $y = x\sin x$ at the point $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.

Question 15 (2 marks) **2**

Find the sum of the arithmetic series $180 + 165 + 150 + \cdots - 15 - 30$.

Question 16 (2 marks) **2**

Differentiate $y = \dfrac{x^2 - x}{x+2}$.

Question 17 (3 marks) **3**

If $y = \ln\left(\dfrac{\cos x}{1-\sin x}\right)$ show that $\dfrac{dy}{dx} = \sec x$.

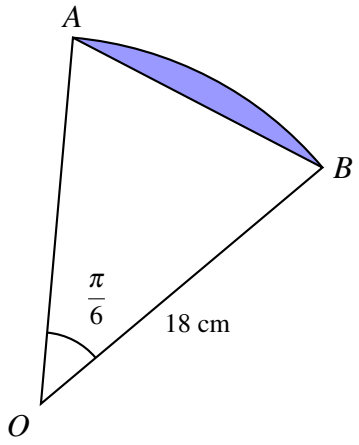
Question 18 (2 marks) **2**

The diagram below shows a sector of a circle OAB.

The chord AB divides the sector into a triangle and a shaded segment.

Angle AOB is $\frac{\pi}{6}$ radians.

The radius of the sector is 18 cm.



The area of the shaded segment can be expressed in the form

$$k(\pi - 3) \text{ cm}^2, \text{where } k \text{ is an integer.}$$

Find the value of k.

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Question 19 (5 marks)

Let A and B be two independent events such that $P(A \cap B') = 0.64$, $P(A' \cap B) = 0.04$ and $P(A \cap B) = x$, where $0 < x < 1$.

(a) If $x^2 - 0.32x + 0.0256 = (x-k)^2$, find k. **1**

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(b) Given $A = (A \cap B) \cup (A \cap B')$, explain why $P(A) = x + 0.64$. **1**

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Question 19 continues on page 17

Question 19 (continued)

(c) By writing a similar result for $P(B)$, deduce that $P(A \cap B) = (x+0.64)(x+0.04)$ and hence find the value of x. **2**

(d) Hence, or otherwise, find $P(A \mid B)$. **1**

End of Question 19

Question 20 (4 marks)

(a) Prove the identity $\dfrac{\sin^2 x - \cos x - 1}{1 + \cos x} = -\cos x$. **2**

(b) Hence solve the equation $\dfrac{2\sin^2 x - 2\cos x - 2}{1 + \cos x} = 1$ for $0^\circ \leqslant x \leqslant 360^\circ$. **2**

Question 21 (4 marks)

The completion times (in minutes) for a large group of runners in a city half-marathon are normally distributed with a mean of 110 minutes and a standard deviation of 20 minutes.

(a) Estimate the percentage of runners whose completion times are between 110 minutes and 130 minutes. **1**

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(b) Explain why the percentage of runners whose completion times are between 90 minutes and 130 minutes is twice your answer to part (a). **1**

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Question 21 continues on page 17

Question 21 (continued)

(c) By utilising the values from the standard normal distribution table provided below, find an approximate maximum completion time that a runner would need to finish in the fastest 10% of all runners. **2**

z	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4
Probability	0.7257	0.7580	0.7881	0.8159	0.8413	0.8643	0.8849	0.9032	0.9192

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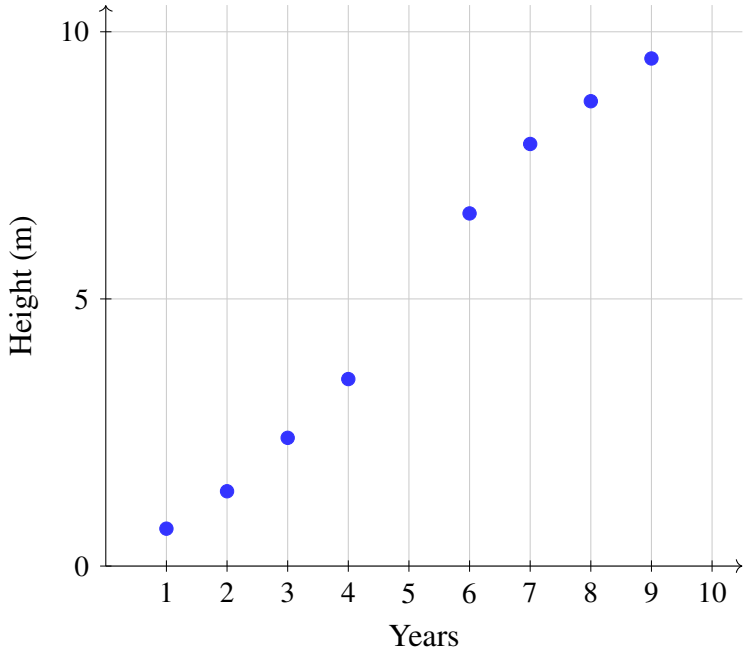
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End of Question 21

Question 22 (4 marks)

Sam is an agricultural scientist studying the growth of a particular tree over several years. The data she recorded is shown in the table and graph below.

Years since planting, t	1	2	3	4	6	7	8	9
Height of tree, H metres	0.7	1.4	2.4	3.5	6.6	7.9	8.7	9.5



(a) What is the correlation coefficient for this data (correct to 4 decimal places)? **1**

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(b) Find the equation of the least-squares line of best fit for the height in terms of years. **1**
Answer using values A and B correct to 2 decimal places, where $H = A + Bt$.

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Question 22 continues on page 19

Question 22 (continued)

(c) Hence, determine approximately, how many years it will take for the tree to reach a height of 20 metres. Give your answer correct to 1 decimal place. **1**

(d) What is the limitation of this model? **1**

End of Question 22

Question 23 (7 marks)

The depth of water in a tidal basin is modelled by the function $d(t) = 3 + 2\cos\left(\frac{\pi}{6}t - \frac{\pi}{3}\right)$ for $0 \leq t \leq 12$, where d is the depth in metres and t is the time in hours after midnight.

(a) On the number plane below, sketch the graph of $d(t)$ for $0 \leq t \leq 12$. **3**

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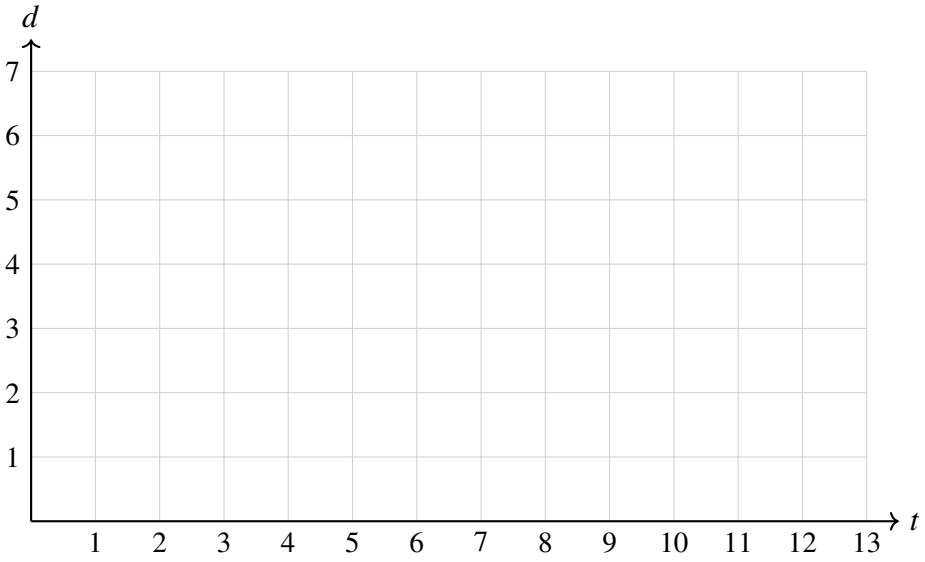
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Question 23 continues on page 21

Question 23 (continued)

(b) At what time is the depth of water decreasing at the fastest rate? **1**

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(c) Solve $3+2\cos\left(\frac{\pi}{6}t-\frac{\pi}{3}\right)=2$ for $0\leq t\leq 12$. **2**

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(d) The basin can only be safely navigated when the water depth is at least two metres. Using the results in part (c) or your graph, find the total length of time during which the basin is safe for navigation over the given domain. **1**

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End of Question 23

Question 24 (5 marks)

A particle moves in a straight line so that its velocity v metres per second at time t is given by $v = 4 - \dfrac{2}{t+1}$. Initially the particle is at the origin.

(a) Find an expression for the position x of the particle at any time t. **2**

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(b) Explain why the velocity of the particle is never 4 ms^{-1}. **1**

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(c) Find the acceleration of the particle when $t = 2$ seconds. **2**

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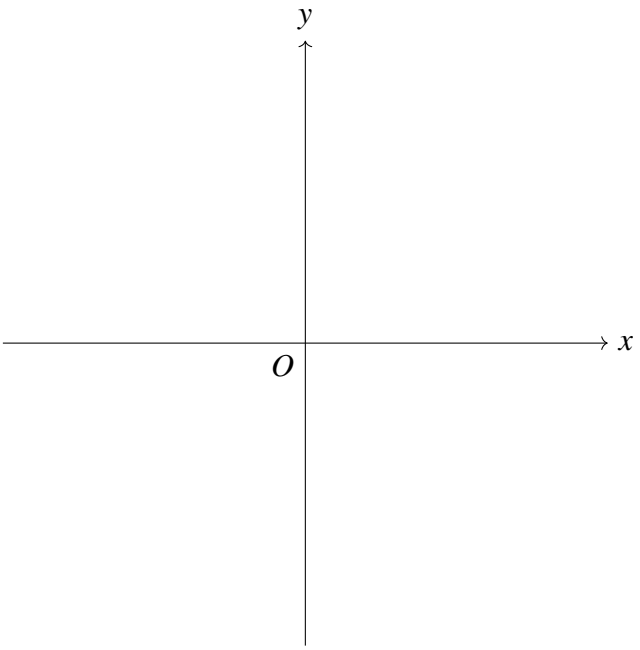
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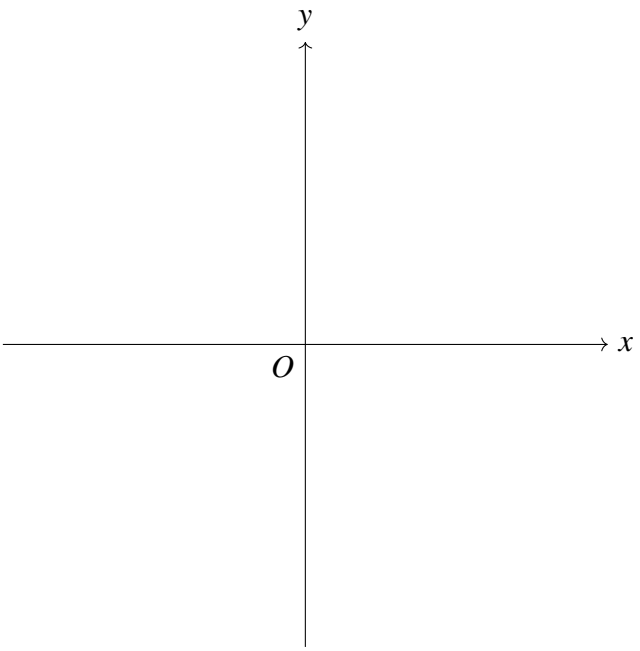
Question 25 (3 marks)

It is given that $f(x) = x(x-a)(x-6)$ where $0 < a < 6$.

(a) Sketch the graph of $y = f(x)$ on the axes below, labelling all x-intercepts. **1**



(b) Sketch the graph of $y = f(-2x)$ on the axes below. **2**



Question 26 (2 marks) **2**

Figure 1 below shows a water trough of length 1500 cm. The symmetrical cross-section of the water trough is shown below, in **Figure 2**, where x and y are measured in centimetres.

Figure 1

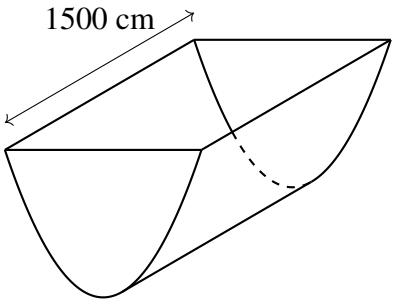
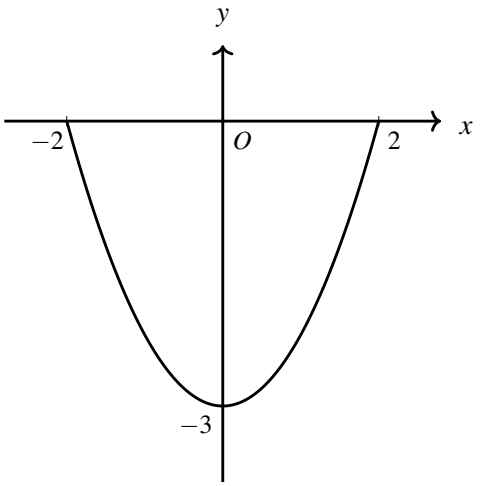


Figure 2



Use the trapezoidal rule, with the values shown in the table below, to find an estimate for the water trough's capacity. You may assume 1 $\text{cm}^3 = 1$ mL.

x	0	0.4	0.8	1.2	1.6	2
y	-3	-2.943	-2.752	-2.353	-1.572	0

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Question 27 (6 marks)

Michael invests \$50 000 in an account which earns 8% interest, compounded annually. He intends to withdraw \$$M$ at the end of each year, immediately after the interest has been paid. He wishes to be able to do this for exactly 20 years, so that the account will then be empty.

(a) How much money does he have in the account immediately after he has made his first withdrawal? Give your answer in terms of M. **1**

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(b) Let A_n represent the amount of money in the account immediately after his n^{th} withdrawal. **3**

Show that $A_{20} = 50000(1.08)^{20} - M\left[\dfrac{1.08^{20}-1}{0.08}\right]$.

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Question 27 continues on page 27

Question 27 (continued)

(c) Calculate the value of M which leaves his account empty after the 20^{th} withdrawal. **2**

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End of Question 27

Question 28 (6 marks)

Consider the function $f(x) = \dfrac{x}{x^2+1}$.

(a) Show that $f(x)$ is odd. **1**

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(b) Find any stationary points and determine their nature. **3**

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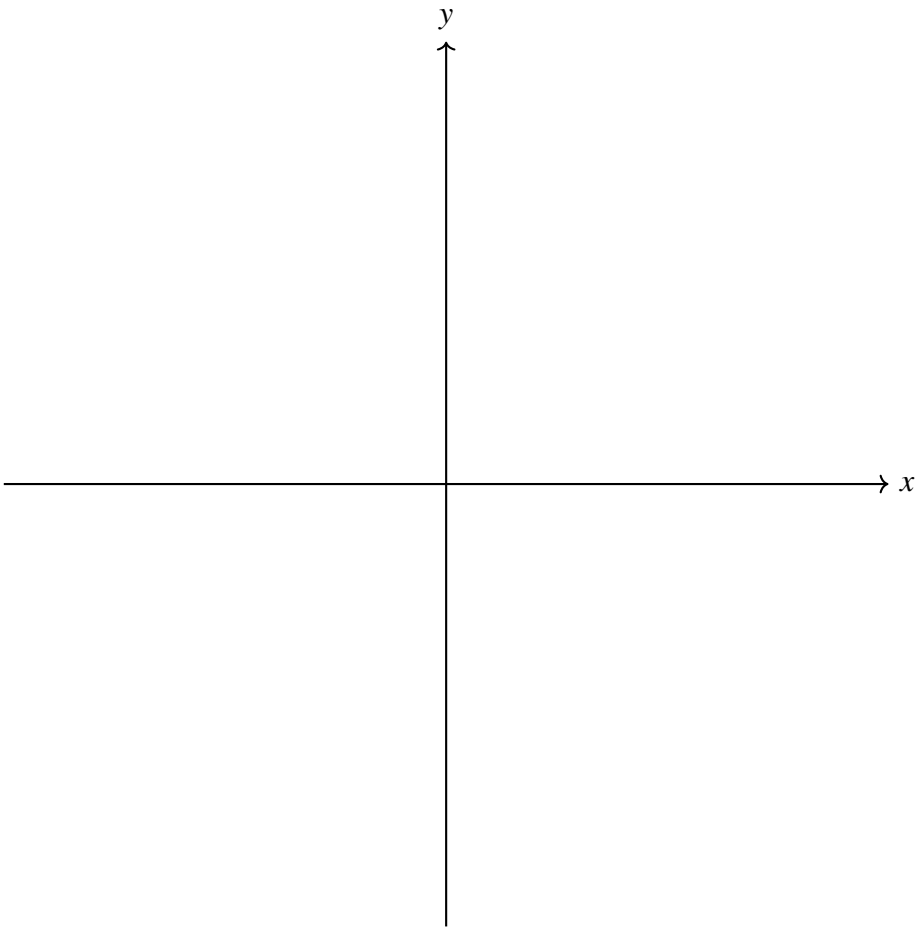
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Question 28 continues on page 29

Question 28 (continued)

(c) Sketch $y = f(x)$ showing intercepts, asymptotes and stationary points. **2**



End of Question 28

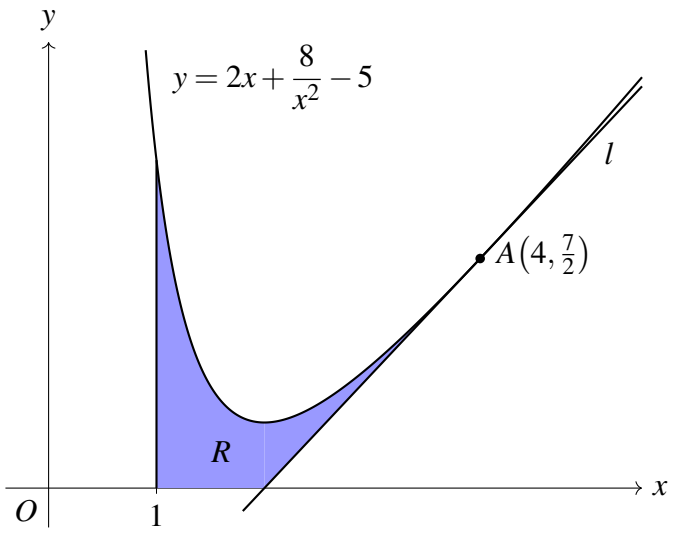
Question 29 (4 marks) **4**

The diagram below shows a sketch of part of the curve C with equation $y = 2x + \dfrac{8}{x^2} - 5,\ x > 0$.

The point $A\left(4, \dfrac{7}{2}\right)$ lies on C. The line l is the tangent to C at the point A.

The shaded region R is bounded by the line l, the curve C, the line with equation $x = 1$ and the x-axis.

Find the exact area of R.



Question 30 (4 marks)

A continuous random variable X has probability density function below.

$$f(x) = \begin{cases} ke^{-2x}, & x \geq 0 \\ 0, & \text{otherwise} \end{cases}$$

(a) Given $\int_0^{\infty} ke^{-2x}\,dx = \lim_{N\to\infty} \int_0^{N} ke^{-2x}\,dx$, show that $k = 2$. **1**

(b) Find $\mathrm{P}(X \leq 1)$. **1**

Question 30 continues on page 33

Question 30 (continued)

(c) Find the median of the distribution. **2**

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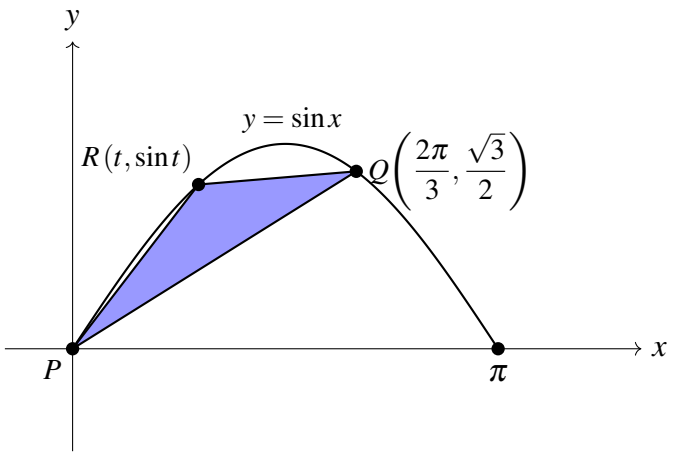
End of Question 30

Question 31 (6 marks)

The diagram below shows the graph of the function $f(x) = \sin x$ for $0 \leq x \leq \pi$.

The points $P(0, 0)$ and $Q\left(\frac{2\pi}{3}, \frac{\sqrt{3}}{2}\right)$ both lie on the curve.

The point $R(t, \sin t)$ is also a point on the curve between P and Q, such that $0 \leq t \leq \frac{2\pi}{3}$.



(a) Show that A, the area of $\triangle PQR$, is given by **3**

$$A = \frac{1}{12}\left(4\pi \sin t - 3\sqrt{3}t\right) \text{ where } 0 \leq t \leq \frac{2\pi}{3}.$$

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Question 31 continues on page 35

Question 31 (continued)

(b) Find the maximum value of A, the area of $\triangle PQR$. **3**
Justify your answer, leaving your answer correct to two decimal places.

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End of Question 31

Question 32 (5 marks)

Two arithmetic progressions, A and B, each have 100 terms denoted by a_i and b_i respectively, where $i = 1, 2, 3, \ldots 100$.

The common difference of A is d, where d is a positive integer.

The two progressions have the following properties.

- $a_1 = b_{100} = 4$
- $b_1 = a_{100}$

(a) You are given that there is at least one value of i for which $b_i = 10 + a_i$. **3**

Show that, in this case,

$$i = \frac{101}{2} - \frac{5}{d}.$$

Question 32 continues on page 37

Question 32 (continued)

(b) Hence, determine the value(s) of d for which $b_i = 10 + a_i$ is true. **2**

End of Question 32

Question 33 (4 marks)

(a) For a certain value of the constant p, the random variables X and Y have the probability distributions given in the table. **2**

x	1	2	3	4
$\mathrm{P}(X = x)$	p	$\frac{1}{6}p$	p^2	$\frac{1}{2}$
$\mathrm{P}(Y = x)$	p	$\frac{1}{6}p$	p^2	$\frac{1}{2}$

Determine the value of p.

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Question 33 continues on page 39

Question 33 (continued)

(b) X and Y are independent random variables so that, for example: **2**

$$P(X = 1 \cap Y = 2) = P(X = 1) \times P(Y = 2)$$

Calculate $\mathrm{P}(Y = 2X \mid Y > X)$.

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End of paper

Section II extra writing space

If you use this space, clearly indicate which question you are answering.

Solutions

KNOX GRAMMAR SCHOOL

PLACE STICKER HERE

2026 **YEAR 12 TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION**

Mathematics Advanced

General Instructions

- Reading time – 10 minutes
- Working time – 3 hours
- Write using a black or blue pen
- NESA approved calculators may be used
- A reference sheet is provided with this paper

Total marks: 100

Section I – 10 marks (pages 3 – 7)

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section
- Use the multiple-choice answer sheet provided to record your answers

Section II – 90 marks (pages 8 – 39)

- Attempt Questions 11 – 33
- Allow about 2 hours and 45 minutes for this section
- Additional writing paper is provided on page 40

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Section I

10 Marks
Attempt Questions 1–10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1–10

1. What is the solution to $|x-2|<3$?

A. $-1<x<5$

B. $x<-5$ or $x>1$

C. $-5<x<1$

D. $x<-1$ or $x>5$

2. Jacob deposits \$750 every 6 months for 4 years into an account earning interest at 6% p.a compounded biannually.

The table below shows the future value interest factors for an annuity of \$1

Periods (n)	Future Value of an Annuity of \$1				
	Interest Rate				
	2%	3%	4%	5%	6%
2	2.020	2.030	2.040	2.050	2.060
4	4.122	4.184	4.247	4.310	4.375
6	6.308	6.468	6.633	6.802	6.975
8	8.583	8.892	9.214	9.549	9.898
10	10.950	11.464	12.006	12.578	13.181

What is the future value of Jacob's investment?

A. \$3138.00

B. \$3281.25

C. \$6669.00

D. \$7423.50

3. Which of the following is true about $y = 4\sin\left(\frac{x}{2}\right)$?

A. Amplitude $= 4$ and period $= \frac{1}{2}$

B. Amplitude $= 4$ and period $= 2\pi$

C. Amplitude $= \frac{1}{2}$ and period $= 4$

D. Amplitude $= 4$ and period $= 4\pi$

period $= \frac{2\pi}{n}$ amplitude $= 4$

$= \frac{2\pi}{\frac{1}{2}}$

$= 4\pi$

4. A random sample of 84 students was asked how many revision websites they had visited in the past month.

The data is summarised in the table below.

Number of websites	Frequency
0	1
1	4
2	18
3	16
4	5
5	37
6	2
7	1

What is the interquartile range of the number of websites visited by these 84 students?

A. 3

B. 4

C. 19

D. 42

5. What is the angle of inclination of the line $3x + 2y = 7$?

A. $33°41'$

B. $56°19'$

C. $123°41'$

D. $146°19'$

$2y = -3x + 7$

$y = -\frac{3}{2}x + \frac{7}{2}$

$\tan\theta = -\frac{3}{2}$

$\theta = -56.31°$

$\therefore \theta = 123.69°$ (2nd quadrant)

6. The probability density function for the continuous random variable X is:

$$f(x) = \begin{cases} kx^3 + \frac{3}{4}x & 0 < x < 2 \\ 0 & \text{otherwise} \end{cases}$$

What is the value of k?

(A.) $-\frac{1}{8}$

B. $-\frac{3}{8}$

C. $-\frac{3}{16}$

D. $-\frac{9}{16}$

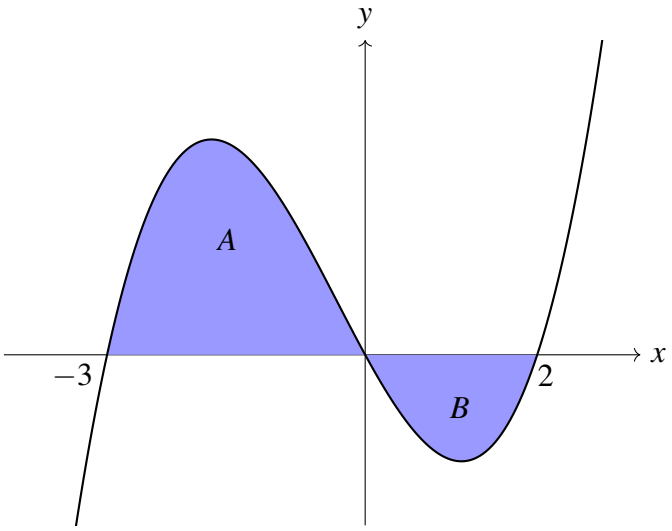
$$\int_0^2 \left(kx^3 + \frac{3}{4}x\right) dx = 1$$

$$\left[\frac{kx^4}{4} + \frac{3x^2}{8}\right]_0^2 = 1$$

$$4k + \frac{3}{2} = 1$$

$$k = -\frac{1}{8}$$

7. The graph of $y = f(x)$ intersects the x-axis at $(-3, 0)$, $(0, 0)$ and $(2, 0)$ as shown in the diagram below.



The shaded region A has an area of 189u^2.

The shaded region B has an area of 64u^2.

What is the value of $\int_{-3}^{2} f(x)\,dx$?

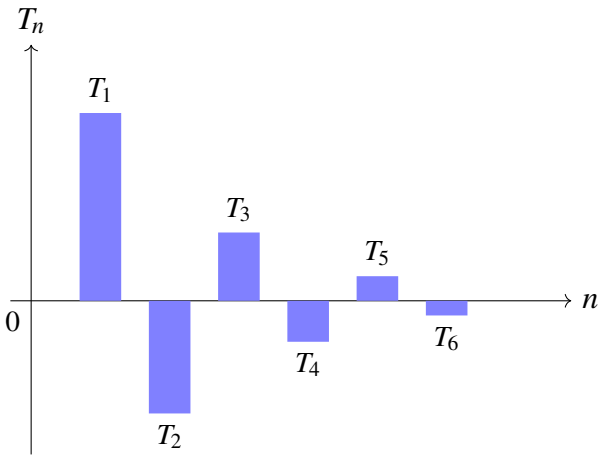
A. -253

B. -125

(C.) 125

D. 253

8. The values for a geometric sequence $T_n = ar^{n-1}$ are plotted on a bar graph:



Which statement must be true?

A. $a < 0,\ -1 < r < 0$

B. $a < 0,\ 0 < r < 1$

C. $a > 0,\ -1 < r < 0$

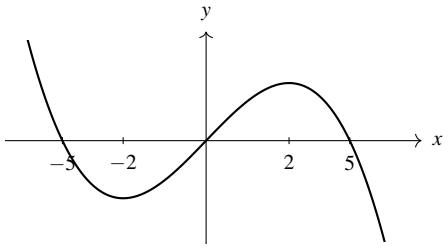
D. $a > 0,\ 0 < r < 1$

9. Consider a curve with the following properties:

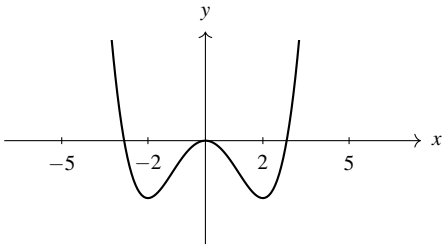
- $g(x)$ is odd.
- $g(5) = 0$ and $g'(2) = 0$.
- $g'(x) > 0$ for $x > 2$.

Which of the following could be the graph of $y = g(x)$?

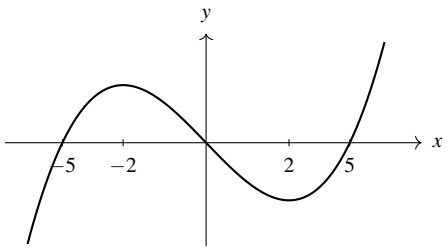
A.



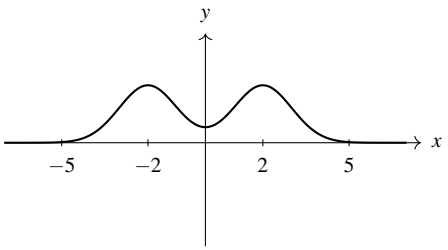
B.



C.



D.



10. The point $P(2,-4)$ lies on $y=f(x)$. A sequence of transformations is applied to $y=f(x)$ to obtain

$$g(x)=2f\left(\frac{4-x}{3}\right)-1$$

Which of the points on the graph of $g(x)$ correspond to the original point P?

A. $(-2,-8)$

B. $\left(\frac{2}{3},-8\right)$

C. $(-2,-9)$

D. $\left(\frac{2}{3},-9\right)$

we know $f(2)=-4$.

Find x:

$$\frac{4-x}{3}=2$$

$$x=-2$$

find $g(-2)$:

$$g(-2)=2f\left(\frac{4-(-2)}{3}\right)-1$$

$$=2f(2)-1$$

$$=-9$$

$\therefore (-2,-9)$

End of Section I

Section II

90 marks

Attempt Questions 11 – 33

Allow about 2 hours and 45 minutes for this section

Answer the questions in the spaces provided.
Include all relevant mathematical reasoning and/or calculations.

Question 11 (4 marks)

In an experiment, 5 million cells were stored in a container. It was observed that 40% of the cells are dying every minute. The number of cells remaining, N, after t minutes, is given by $N = Ae^{kt}$, where A and k are constants.

(a) Explain why $k = \ln\frac{3}{5}$. **2**

when $t = 1$:

$Ae^k = \frac{60}{100}A$

$e^k = \frac{3}{5}$

$k = \ln\frac{3}{5}$

① Mark for progress toward answer

① Mark for completed proof.

(b) At what time, to two decimal places, will there be 2000 cells remaining? **2**

$2000 = 5000000\,e^{(\ln\frac{3}{5})t}$

$(\ln\frac{3}{5})t = \ln\frac{2000}{5000000}$

$t = \frac{\ln\left(\frac{1}{2500}\right)}{\ln\left(\frac{3}{5}\right)}$

$= 15.32 \text{ min}$

① Mark for progress toward answer

① Mark for solution.

Question 12 (2 marks) **2**

Evaluate $\int_0^{\frac{\pi}{2}} \cos x\,dx$.

$= [\sin x]_0^{\pi/2}$

$= \sin\left(\frac{\pi}{2}\right) - \sin 0$

$= 1$

(1) Mark for correct integration.

(1) Mark for correct answer.

Question 13 (3 marks)

(a) Differentiate $y = xe^{5x}$. **1**

$u = x \qquad v = e^{5x}$

$u' = 1 \qquad v' = 5e^{5x}$

$y' = e^{5x} + 5xe^{5x}$

$y' = e^{5x}(1 + 5x)$

$y' = e^{5x}(5x + 1)$

(1) Mark for correct differentiation.

Do not penalise for not factorising

(b) Hence, find the exact value of $\int_0^3 e^{5x}(20x+4)\,dx$. **2**

$= \int_0^3 4e^{5x}(5x+1)\,dx$

$= 4\int_0^3 e^{5x}(5x+1)\,dx$

$= 4\left[xe^{5x}\right]_0^3$

$= 4\left(3e^{15} - 0\right)$

$= 12e^{15}$

(1) Mark for factorising out 4.

(1) Mark for correct answer.

Question 14 (3 marks) **3**

Find the equation of the tangent to the curve $y = x\sin x$ at the point $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.

$y = x\sin x$

$y' = \sin x + x\cos x$

sub in $x = -\frac{\pi}{2}$ for gradient:

$m = \sin\left(-\frac{\pi}{2}\right) + -\frac{\pi}{2}\times\cos\left(-\frac{\pi}{2}\right)$

$m = -1$

$\therefore\ y - \frac{\pi}{2} = -1\left(x + \frac{\pi}{2}\right)$

$y = -x$

① Mark for correct differentiation.

① Mark for correct gradient

① Mark for correct equation.

Question 15 (2 marks) **2**

Find the sum of the arithmetic series $180 + 165 + 150 + \cdots - 15 - 30$.

$a = 180 \quad d = -15$

$T_n = a + (n-1)d$

$-30 = 180 + (n-1)\times -15$

$-210 = -15n + 15$

$n = 15$

$\therefore\ S_{15} = \frac{15}{2}(180 - 30)$

$= 1125$

① Mark for finding n.

① Mark for finding sum.

Question 16 (2 marks) **2**

Differentiate $y = \dfrac{x^2 - x}{x+2}$.

$u = x^2 - x$ $\quad v = x+2$

$u' = 2x - 1$ $\quad v' = 1$

$$y' = \frac{(x+2)(2x-1) - (x^2 - x)}{(x+2)^2}$$

$$= \frac{2x^2 + 3x - 2 - x^2 + x}{(x+2)^2}$$

$$= \frac{x^2 + 4x - 2}{(x+2)^2}$$

(1) Mark for correct values of u and v.

(1) Mark for correct substitution into quotient rule.

Question 17 (3 marks) **3**

If $y = \ln\left(\dfrac{\cos x}{1 - \sin x}\right)$ show that $\dfrac{dy}{dx} = \sec x$.

$$y = \ln \cos x - \ln(1 - \sin x)$$

$$y' = \frac{-\sin x}{\cos x} - \frac{-\cos x}{1 - \sin x}$$

$$y' = -\frac{\sin x}{\cos x} + \frac{\cos x}{1 - \sin x}$$

$$y' = \frac{\cos x}{1 - \sin x} - \frac{\sin x}{\cos x}$$

$$y' = \frac{\cos^2 x - \sin x(1 - \sin x)}{\cos x(1 - \sin x)}$$

$$y' = \frac{\cos^2 x - \sin x + \sin^2 x}{\cos x(1 - \sin x)}$$

$$y' = \frac{1 - \sin x}{\cos x(1 - \sin x)}$$

$$y' = \frac{1}{\cos x} = \sec x$$

$$\therefore \frac{dy}{dx} = \sec x$$

(1) Mark for correct differentiation

(1) Mark for combining fractions

(1) Mark for cancelling out $(1 - \sin x)$

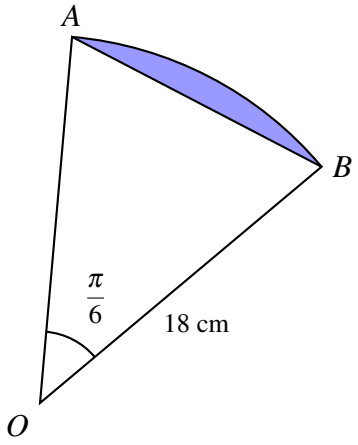
Question 18 (2 marks) **2**

The diagram below shows a sector of a circle OAB.

The chord AB divides the sector into a triangle and a shaded segment.

Angle AOB is $\frac{\pi}{6}$ radians.

The radius of the sector is 18 cm.



The area of the shaded segment can be expressed in the form

$$k(\pi - 3) \text{ cm}^2, \text{where } k \text{ is an integer.}$$

Find the value of k.

$A = \frac{1}{2} r^2 (\theta - \sin\theta)$ for shaded area:

$A = \frac{1}{2} \times 18^2 \left(\frac{\pi}{6} - \sin\frac{\pi}{6}\right)$

(1) Mark for correct use of formula OR alternate formula.

$A = 27(\pi - 3) \text{ cm}^2$ (1) Mark for correct answer

$\therefore k = 27$

Question 19 (5 marks)

Let A and B be two independent events such that $P(A \cap B') = 0.64$, $P(A' \cap B) = 0.04$ and $P(A \cap B) = x$, where $0 < x < 1$.

(a) If $x^2 - 0.32x + 0.0256 = (x-k)^2$, find k. **1**

$x^2 - 0.32x + 0.0256 = x^2 - 2k + k^2$

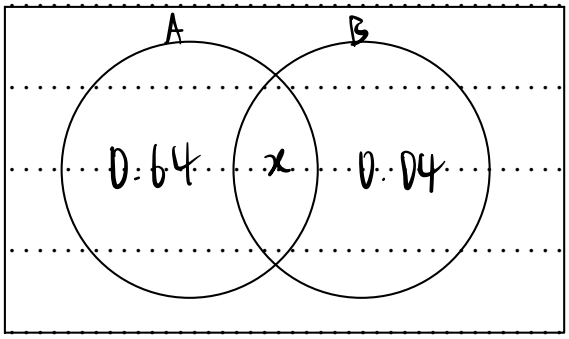
Matching coefficients:

$-0.32 = -2k$

$k = 0.16$ ✓

(b) Given $A = (A \cap B) \cup (A \cap B')$, explain why $P(A) = x + 0.64$. **1**

Using a venn diagram:



∴ From venn diagram, $P(A) = x + 0.64$

either method accepted.

Alternatively:

$A \cap B$ and $A \cap B'$ are mutually exclusive, since no outcome can lie in both B and B'. Their union is A, so by the addition rule for mutually exclusive events:

$P(A) = P(A \cap B) + P(A \cap B') = x + 0.64$.

Question 19 continues on page 17

Question 19 (continued)

(c) By writing a similar result for $P(B)$, deduce that $P(A \cap B) = (x+0.64)(x+0.04)$ and hence find the value of x. **2**

By the same argument, $B = (A \cap B) \cup (A' \cap B)$ with two parts mutually exclusive, so $P(B) = x + 0.04$

Note: student also could have used venn diagram from (b).

Since A and B are independent, $P(A \cap B) = P(A)P(B)$,

hence $x = (x+0.64)(x+0.04)$ ① Mark for establishing $P(B) = x + 0.04$

Expanding: $x = x^2 + 0.04x + 0.64x + 0.0256$

$0 = x^2 - 0.32x + 0.0256$ ① Mark for correct answer.

$0 = (x - 0.16)^2 \quad x = 0.16$

(d) Hence, or otherwise, find $P(A \mid B)$. **1**

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$= \frac{0.16}{0.2}$$

$$= 0.8 \quad \checkmark$$

End of Question 19

Question 20 (4 marks)

(a) Prove the identity $\dfrac{\sin^2 x - \cos x - 1}{1 + \cos x} = -\cos x$. **2**

L.H.S:

$= \dfrac{\sin^2 x - \cos x - 1}{1 + \cos x}$

$= \dfrac{1 - \cos^2 x - \cos x - 1}{1 + \cos x}$

$= \dfrac{-\cos^2 x - \cos x}{1 + \cos x}$

$= \dfrac{-\cos x(1 + \cos x)}{1 + \cos x} = -\cos x = $ R.H.S.

$\star \sin^2 x + \cos^2 x = 1$

$\sin^2 x = 1 - \cos^2 x$

① Mark for use of $\sin^2 x + \cos^2 x = 1$

① Mark for factorising

(b) Hence solve the equation $\dfrac{2\sin^2 x - 2\cos x - 2}{1 + \cos x} = 1$ for $0° \leqslant x \leqslant 360°$. **2**

$\dfrac{2(\sin^2 x - \cos x - 1)}{1 + \cos x} = 1$

① Mark for connection to part (a)

from part (a):

① Mark for correct answer.

$2x - \cos x = 1$

$\cos x = -\dfrac{1}{2}$

$x = 120, 240$

Question 21 (4 marks)

The completion times (in minutes) for a large group of runners in a city half-marathon are normally distributed with a mean of 110 minutes and a standard deviation of 20 minutes.

(a) Estimate the percentage of runners whose completion times are between 110 minutes and 130 minutes. **1**

$z = \frac{130 - 110}{20} = 1$

$\therefore P(110 < X < 130) = P(0 < Z < 1) = 84\% - 50\% = 34\%$ ✓

(b) Explain why the percentage of runners whose completion times are between 90 minutes and 130 minutes is twice your answer to part (a). **1**

The normal distribution is symmetric about the mean 110, and 90 and 130 are each 20 minutes from the mean, so:

$P(90 < X < 110) = P(110 < X < 130)$.

$\therefore P(90 < X < 130)$ is twice the answer to (a).

① Mark for making specific mention of symmetry

Question 21 continues on page 17

Question 21 (continued)

(c) By utilising the values from the standard normal distribution table provided below, find an approximate maximum completion time that a runner would need to finish in the fastest 10% of all runners. **2**

z	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4
Probability	0.7257	0.7580	0.7881	0.8159	0.8413	0.8643	0.8849	0.9032	0.9192

The fastest 10% corresponds to the lowest 10% of completion times, so use:

$z \approx -1.3$ since $P(Z < -1.3) = 0.0968$ is close to 0.10.

Then, $t = 110 + (-1.3)(20) = 84$.

∴ Approximate maximum completion time is 84 minutes

① Mark for converting to correct completion time

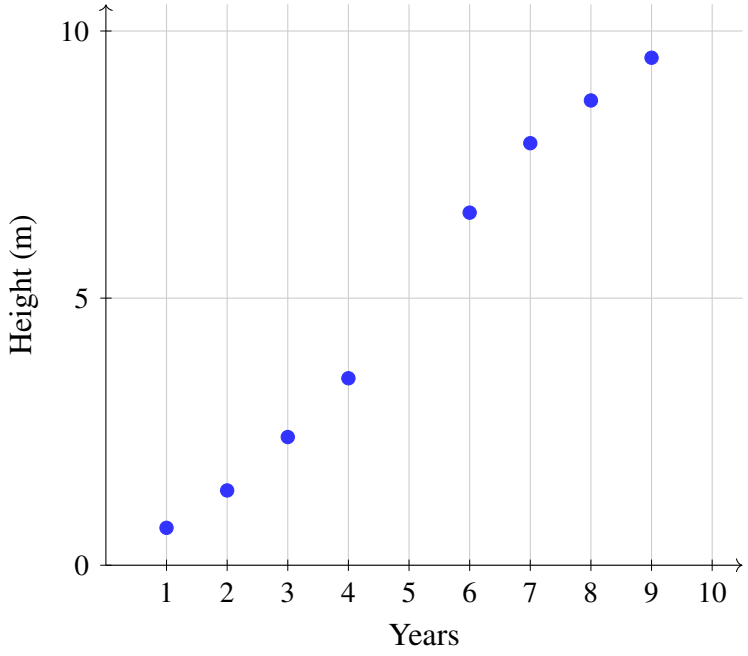
① Mark for identifying correct z-value.

End of Question 21

Question 22 (4 marks)

Sam is an agricultural scientist studying the growth of a particular tree over several years. The data she recorded is shown in the table and graph below.

Years since planting, t	1	2	3	4	6	7	8	9
Height of tree, H metres	0.7	1.4	2.4	3.5	6.6	7.9	8.7	9.5



(a) What is the correlation coefficient for this data (correct to 4 decimal places)? **1**

$r = 0.9952$

(b) Find the equation of the least-squares line of best fit for the height in terms of years. **1**
Answer using values A and B correct to 2 decimal places, where $H = A + Bt$.

$H = -0.85 + 1.19t$

Question 22 continues on page 19

Question 22 (continued)

(c) Hence, determine approximately, how many years it will take for the tree to reach a height of 20 metres. Give your answer correct to 1 decimal place. **1**

When $H = 20$

$20 = -0.85 + 1.19t$

$t = \frac{20.85}{1.19}$

$= 17.52$

$\therefore$ After 17.5 years ✓

(d) What is the limitation of this model? **1**

As t increases, its growth may not continue in a linear fashion (or equivalent reasoning). ✓

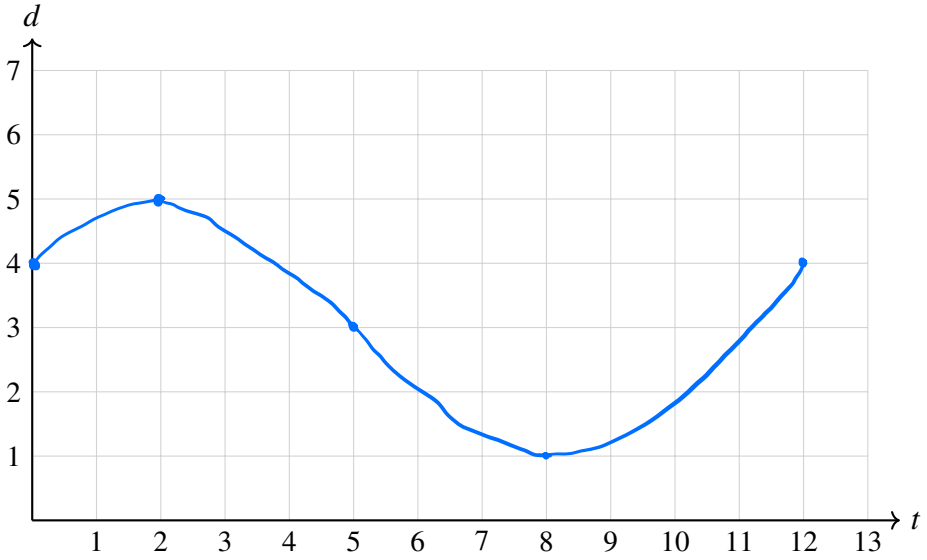
End of Question 22

Question 23 (7 marks)

The depth of water in a tidal basin is modelled by the function $d(t) = 3 + 2\cos\left(\frac{\pi}{6}t - \frac{\pi}{3}\right)$ for $0 \leq t \leq 12$, where d is the depth in metres and t is the time in hours after midnight.

(a) On the number plane below, sketch the graph of $d(t)$ for $0 \leq t \leq 12$. **3**

Amplitude is 2 so from -2 to 2, with 0 in middle.
moved up by 3, so now its 1 to 5 with 3 in middle.
$d(t) = 3 + 2\cos\left[\frac{\pi}{6}(t-2)\right]$
$\therefore$ period $= \frac{2\pi}{\frac{\pi}{6}}$ phase shift to right by 2
$= 12$



① Mark for correct shape
① Mark for correct range
① Mark for correct endpoints and stationary points.

Question 23 continues on page 21

Question 23 (continued)

(b) At what time is the depth of water decreasing at the fastest rate? **1**

At inflexion point: $t = 5 \rightarrow$ 5:00 am ✓

(c) Solve $3 + 2\cos\left(\frac{\pi}{6}t - \frac{\pi}{3}\right) = 2$ for $0 \leq t \leq 12$. **2**

$3 + 2\cos\left(\frac{\pi t}{6} - \frac{\pi}{3}\right) = 2$

$0 \leq t \leq 12$

$\cos\left(\frac{\pi t}{6} - \frac{\pi}{3}\right) = -\frac{1}{2}$ $\quad -\frac{\pi}{3} \leq \frac{\pi t}{6} - \frac{\pi}{3} \leq \frac{5\pi}{3}$

$\frac{\pi t}{6} - \frac{\pi}{3} = \frac{2\pi}{3}$ $\quad t = 6$

$\frac{\pi t}{6} - \frac{\pi}{3} = \frac{4\pi}{3}$ $\quad t = 10$

(1) Mark for one correct solution to trig method.

(1) Mark for correct answer.

★ If using graph, graph must reflect correct values.

(d) The basin can only be safely navigated when the water depth is at least two metres. Using the results in part (c) or your graph, find the total length of time during which the basin is safe for navigation over the given domain. **1**

From (c), at $\frac{2\pi}{3}$, $t = 6$ and $\frac{4\pi}{3}$, $t = 10$

$\therefore$ total time $= 6 + (12 - 10)$

$= 8$ hours

(1) Mark for correct answer

End of Question 23

Question 24 (5 marks)

A particle moves in a straight line so that its velocity v metres per second at time t is given by $v = 4 - \frac{2}{t+1}$. Initially the particle is at the origin.

(a) Find an expression for the position x of the particle at any time t. **2**

$x = \int v\,dt$

$= 4t - 2\ln(t+1) + C$

when $t = 0$, $x = 0$

$x = 4t - 2\ln(x+1)$

(1) Mark for correct integration

(1) Mark for correct final equation.

(b) Explain why the velocity of the particle is never 4 ms^{-1}. **1**

$v = 4 - \frac{2}{t+1}$

when $v = 4$ $\quad 4 = 4 - \frac{2}{t+1}$

$\therefore \frac{2}{t+1} = 0$ which is not possible.

(c) Find the acceleration of the particle when $t = 2$ seconds. **2**

$v = 4 - 2(t+1)^{-1}$

$a = 2(t+1)^{-2}$

$= \frac{2}{(t+1)^2}$

sub in $t = 2$:

$a = \frac{2}{9}$

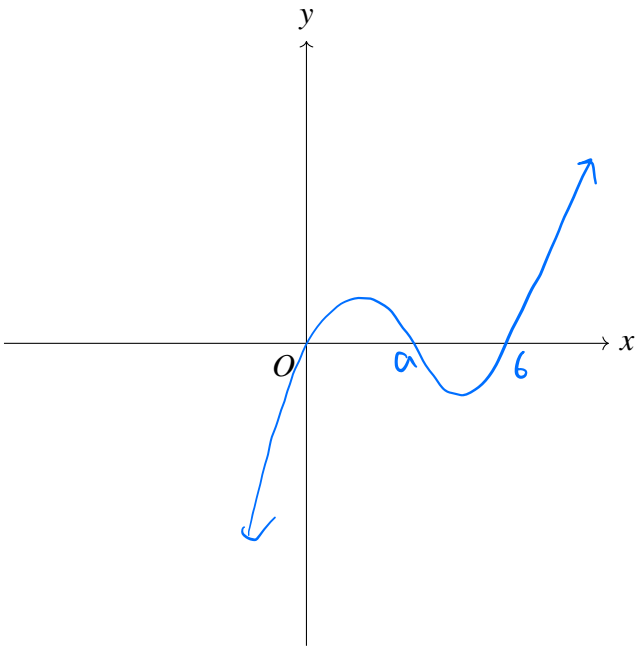
(1) Mark for correct differentiation

(1) Mark for correct answer.

Question 25 (3 marks)

It is given that $f(x) = x(x-a)(x-6)$ where $0 < a < 6$.

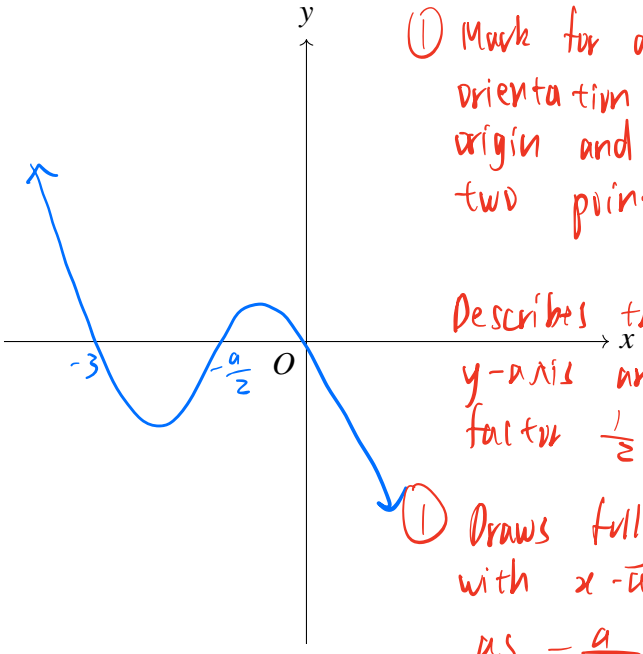
(a) Sketch the graph of $y = f(x)$ on the axes below, labelling all x-intercepts. **1**



① Mark for drawing a cubic with two turning points.

① Mark for labelling intercepts and passing through origin

(b) Sketch the graph of $y = f(-2x)$ on the axes below. **2**



① Mark for drawing cubic of correct orientation passing through the origin and negative x-axis at two points.

OR

Describes the reflection in the y-axis and a stretch of scale factor $\frac{1}{2}$ in the x-direction.

① Draws fully correct sketch with x-axis labelled correctly as $-\frac{a}{2}$ and -3.

Question 26 (2 marks) **2**

Figure 1 below shows a water trough of length 1500 cm. The symmetrical cross-section of the water trough is shown below, in **Figure 2**, where x and y are measured in centimetres.

Figure 1

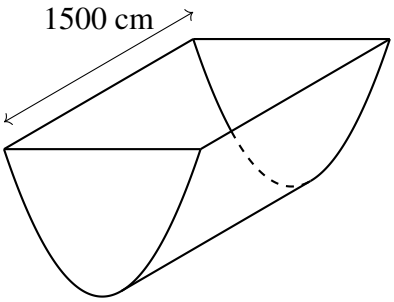
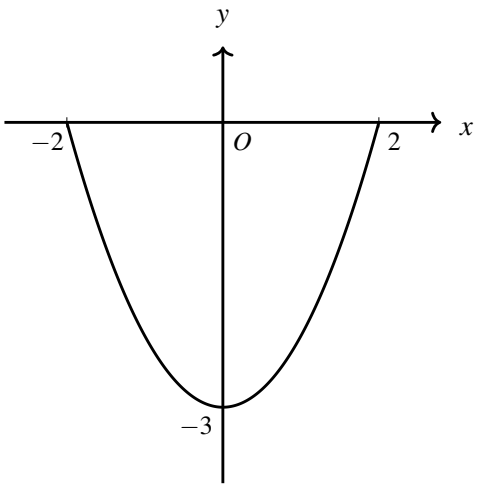


Figure 2



Use the trapezoidal rule, with the values shown in the table below, to find an estimate for the water trough's capacity. You may assume 1 cm^3 = 1 mL.

x	0	0.4	0.8	1.2	1.6	2
y	−3	−2.943	−2.752	−2.353	−1.572	0

$A = \frac{0.4}{2}\left(3 + 0 + 2(2.943 + 2.752 + 2.353 + 1.572)\right)$

$= 4.448$

Total area $= 8.896$

Volume $= 8.896 \times 1500$

$= 13344\ \text{cm}^3$

$= 13344\ \text{mL} \approx 13.3\ \text{L}$

(1) Mark awarded for correct substitution

(1) Mark awarded for correct answer

. . . . . .

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Question 27 (6 marks)

Michael invests \$50 000 in an account which earns 8% interest, compounded annually. He intends to withdraw \$$M$ at the end of each year, immediately after the interest has been paid. He wishes to be able to do this for exactly 20 years, so that the account will then be empty.

(a) How much money does he have in the account immediately after he has made his first withdrawal? Give your answer in terms of M. **1**

$A_1 = (50000 \times 1.08) - M$

$= 54000 - M$ ✓

(b) Let A_n represent the amount of money in the account immediately after his n^{th} withdrawal. **3**

Show that $A_{20} = 50000(1.08)^{20} - M\left[\dfrac{1.08^{20}-1}{0.08}\right]$.

① Mark for establishing the third series and/or series

$A_2 = A_1 \times 1.08 - M$

$= 50000(1.08)^2 - 1.08M - M$ ① Mark for establishing A_n series

$A_3 = 50000(1.08)^3 - M(1.08)^2 - M(1.08) - M$ ① Mark for substitution into GP sum.

$= 50000(1.08)^3 - M(1 + 1.08 + 1.08^2)$

$A_{20} = 50000(1.08)^{20} - M(1 + 1.08 + 1.08^2 + \ldots + 1.08^{19})$

$a = 1 \quad r = 1.08 \quad n = 20$

$= 50000(1.08)^{20} - M\left(\dfrac{1.08^{20}-1}{1.08-1}\right)$ ← GP sum formula

$= 50000(1.08)^{20} - M\left(\dfrac{1.08^{20}-1}{0.08}\right)$

Question 27 continues on page 27

Question 27 (continued)

(c) Calculate the value of M which leaves his account empty after the 20th withdrawal. **2**

$M\left(\frac{1.08^{20}-1}{0.08}\right) = 50000(1.08)^{20}$

$45.7196M = 50000(1.08)^{20}$

$M = \frac{50000(1.08)^{20}}{45.7196}$

$= \$5092.61$

(1) Mark for using part (b) and progressing

(1) Mark for correct final answer.

End of Question 27

Question 28 (6 marks)

Consider the function $f(x) = \dfrac{x}{x^2+1}$.

(a) Show that $f(x)$ is odd. **1**

$f(-x) = \dfrac{-x}{(-x)^2+1}$

$= -\dfrac{x}{x^2+1}$

$\therefore f(-x) = -f(x)$ $\therefore$ odd. ✓

(b) Find any stationary points and determine their nature. **3**

let $f'(x) = 0$: $u = x$ $v = x^2+1$

$u' = 1$ $v' = 2x$

(1) mark for differentiating

$f'(x) = \dfrac{x^2+1-2x^2}{(x^2+1)^2}$ $0 = \dfrac{-x^2+1}{(x^2+1)^2}$ (1) mark for correct st. pts

$f'(x) = \dfrac{-x^2+1}{(x^2+1)^2}$ $0 = -x^2+1$ (1) mark for correct nature.

$x^2 = 1$

let $f'(x) = 0$ for st. pts: $x = 1$ $x = -1$

For nature, use table:

x	0	1	2
y'	1	0	$-\frac{3}{25}$
	/	−	\

x	−2	−1	0
y'	$-\frac{3}{25}$	0	1
	\	−	/

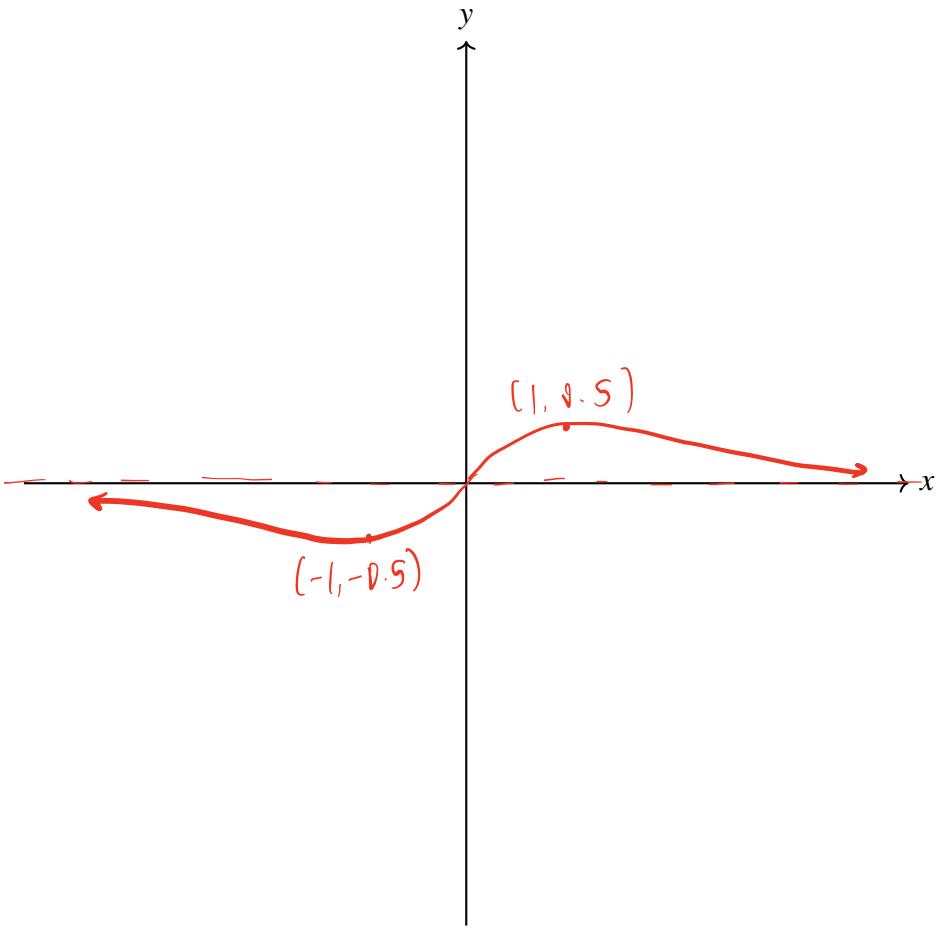
at $x=1$, concave down

at $x=-1$, concave up.

Question 28 continues on page 29

Question 28 (continued)

(c) Sketch $y = f(x)$ showing intercepts, asymptotes and stationary points. **2**



① Mark for correct shape

① Mark for evidently approaching asymptotes.

End of Question 28

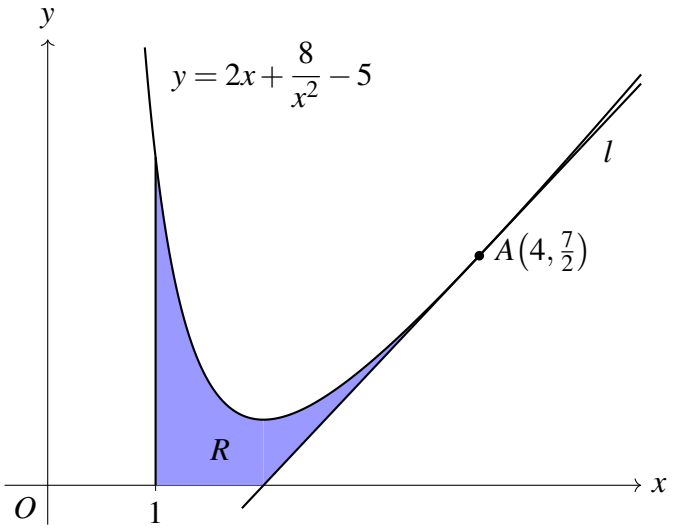
Question 29 (4 marks) **4**

The diagram below shows a sketch of part of the curve C with equation $y = 2x + \frac{8}{x^2} - 5,\ x > 0$.

The point $A\left(4, \frac{7}{2}\right)$ lies on C. The line l is the tangent to C at the point A.

The shaded region R is bounded by the line l, the curve C, the line with equation $x = 1$ and the x-axis.

Find the exact area of R.



Find equation of line l:

$y' = 2 - 16x^{-3}$

sub in $x = 4$ for m

$m = 2 - 16(4)^{-3} = \frac{7}{4}$

$\therefore y - \frac{7}{2} = \frac{7}{4}(x - 4)$

$y = \frac{7}{4}x - \frac{7}{2}$

Intersection of l with x-axis:

$0 = \frac{7}{4}x - \frac{7}{2}$

$x = 2$.

$\therefore$ Area R = $\int_1^4 2x + 8x^{-2} - 5\,dx$ − Triangle.

$= \int_1^4 2x + 8x^{-2} - 5\,dx$

$= \left[\frac{2x^2}{2} + \frac{8x^{-1}}{-1} - 5x\right]_1^4$

$= \left(\frac{2(16)}{2} + \frac{8(4)^{-1}}{-1} - 5(4)\right)$

$- \left(\frac{2(1)^2}{2} + \frac{8(1)^{-1}}{-1} - 5(1)\right)$

$= 6$

Area of triangle $= \frac{1}{2} \times 2 \times \frac{7}{2} = \frac{7}{2}$

$\therefore$ Area $= 6 - \frac{7}{2} = \frac{5}{2}$

(1) Mark for equation of line

(1) Mark for x-axis intersection

(1) Mark for area under the curve between the bounds of 1 and 4.

(1) Mark for answer.

Question 30 (4 marks)

A continuous random variable X has probability density function below.

$$f(x) = \begin{cases} ke^{-2x}, & x \geq 0 \\ 0, & \text{otherwise} \end{cases}$$

(a) Given $\int_0^{\infty} ke^{-2x}\,dx = \lim_{N\to\infty} \int_0^{N} ke^{-2x}\,dx$, show that $k = 2$. **1**

$\int_0^{\infty} ke^{-2x}\,dx = 1$

$k\left[-\frac{1}{2}e^{-2x}\right]_0^{\infty} = 1$

$k\left(0 - \left(-\frac{1}{2}\right)\right) = 1$

$\frac{k}{2} = 1$

$k = 2$

(b) Find $P(X \leq 1)$. **1**

$P(X \leq 1) = \int_0^{1} 2e^{-2x}\,dx$

$= \left[-e^{-2x}\right]_0^{1}$

$= -e^{-2} - (-1)$

$= 1 - e^{-2}$

Question 30 continues on page 33

Question 30 (continued)

(c) Find the median of the distribution. **2**

$P(X \leq m) = \frac{1}{2}$

$\int_0^m 2e^{-2x}\,dx = \frac{1}{2}$

$\left[-e^{-2x}\right]_0^m = \frac{1}{2}$

$-e^{-2m} + 1 = \frac{1}{2}$

$e^{-2m} = \frac{1}{2}$

$m = \frac{1}{2}\ln 2$

① Mark for integrating correctly

① Mark for correct median.

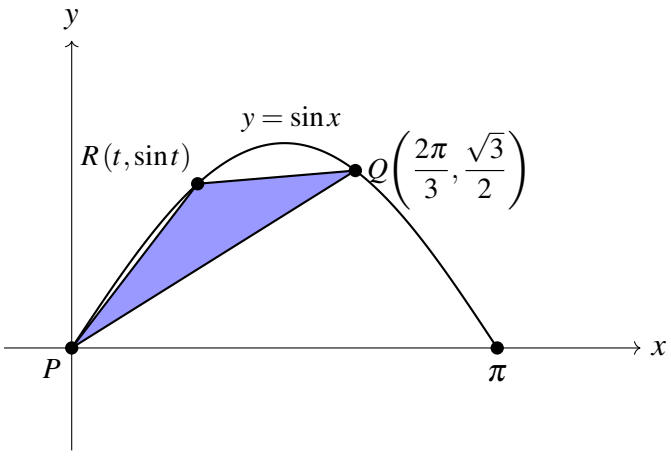
End of Question 30

Question 31 (6 marks)

The diagram below shows the graph of the function $f(x) = \sin x$ for $0 \leq x \leq \pi$.

The points $P(0, 0)$ and $Q\left(\frac{2\pi}{3}, \frac{\sqrt{3}}{2}\right)$ both lie on the curve.

The point $R(t, \sin t)$ is also a point on the curve between P and Q, such that $0 \leq t \leq \frac{2\pi}{3}$.



(a) Show that A, the area of $\triangle PQR$, is given by **3**

$$A = \frac{1}{12}\left(4\pi \sin t - 3\sqrt{3}t\right) \text{ where } 0 \leq t \leq \frac{2\pi}{3}.$$

A = Area below PR + Area below RQ − Area below PQ.

$A = \left(\frac{1}{2} \times t \times \sin t\right) + \left[\frac{1}{2} \times \left(\frac{2\pi}{3} - t\right) \times \left(\sin t + \frac{\sqrt{3}}{2}\right)\right) - \left(\frac{1}{2} \times \frac{2\pi}{3} \times \frac{\sqrt{3}}{2}\right)$

$A = \frac{1}{2} t \sin t + \frac{1}{2}\left(\frac{2\pi}{3} \sin t + \frac{\sqrt{3}\pi}{3} - t\sin t - \frac{\sqrt{3}}{2}t\right) - \frac{\pi\sqrt{3}}{6}$

$A = \frac{\pi}{3}\sin t - \frac{\sqrt{3}}{4}t$

$A = \frac{1}{12}\left(4\pi \sin t - 3\sqrt{3}\, t\right)$

(1) Mark for establishing correct area

(1) Mark for correctly expanding first line.

(1) Mark for correct simplification toward $\triangle PQR$

Question 31 continues on page 35

Question 31 (continued)

(b) Find the maximum value of A, the area of $\triangle PQR$. **3**
Justify your answer, leaving your answer correct to two decimal places.

$A = \frac{1}{12}(4\pi \sin t - 3\sqrt{3}\, t)$

$\frac{dA}{dt} = \frac{1}{12}[4\pi \cos t - 3\sqrt{3})$

let $\frac{dA}{dt} = 0$ for stat point.

$\cos t = \frac{3\sqrt{3}}{4\pi}$ $t = 1.145\ldots$

$\frac{d^2A}{dt^2} = \frac{1}{12}(-4\pi \sin t)$

when $t = 1.145\ldots$

$= -0.9534\ldots$

$\therefore$ curve down [maximum)

$\therefore A = \frac{1}{12}(4\pi \sin 1.145 - 3\sqrt{3} \times 1.145)$

$= 0.46$

① Mark for finding t

① Mark for testing concavity

① Mark for A.

End of Question 31

Question 32 (5 marks)

Two arithmetic progressions, A and B, each have 100 terms denoted by a_i and b_i respectively, where $i = 1, 2, 3, \ldots 100$.

The common difference of A is d, where d is a positive integer.

The two progressions have the following properties.

- $a_1 = b_{100} = 4$
- $b_1 = a_{100}$

(a) You are given that there is at least one value of i for which $b_i = 10 + a_i$. **3**

Show that, in this case,

$$i = \frac{101}{2} - \frac{5}{d}.$$

A: $a_1 = 4$, common difference d, so

$a_i = 4 + (i-1)d$, $a_{100} = 4 + 99d$

B: $b_1 = a_{100} = 4 + 99d$ and $b_{100} = 4$. This means B runs "backwards". Its common difference is:

$$\frac{b_{100} - b_1}{99} = \frac{4 - (4 + 99d)}{99} = -d$$

B is A reversed, so: $b_i = (4 + 99d) - (i-1)d$

$b_i = 10 + a_i$

$\therefore 4 + 99d - (i-1)d = 10 + 4 + (i-1)d$

$99d - 2(i-1)d = 10$

$2(i-1) = 99 - \frac{10}{d}$

$i = 1 + \frac{99}{2} - \frac{5}{d} = \frac{101}{2} - \frac{5}{d}$

(1) Mark for expressing a_i correctly in terms of d.

(1) Mark for identifying $b_1 = 4 + 99d$ AND that B's common difference is "$-d$"

(1) Mark for substituting $b_i = 10 + a_i$ and solves.

Question 32 continues on page 37

Question 32 (continued)

(b) Hence, determine the value(s) of d for which $b_i = 10 + a_i$ is true. **2**

From (a), $i = \frac{101}{2} - \frac{5}{d}$ where i must be a positive integer from $1 \leq i \leq 100$.

Rearranging: $\frac{10}{d} = 101 - 2i$.

Since i is an integer, $101 - 2i$ is odd.

$\therefore \frac{10}{d}$ must be an odd integer.

Divisors of 10 give $\frac{10}{d} = 10, 5, 2, 1$ for $d = 1, 2, 5, 10$.

Only $\frac{10}{d} = 5$ and $\frac{10}{d} = 1$ are odd, so

$d = 2$ or $d = 10$.

(1) Mark for recognising $\frac{10}{d}$ must be an odd integer.

(1) concludes $d = 2, 10$.

End of Question 32

Question 33 (4 marks)

(a) For a certain value of the constant p, the random variables X and Y have the probability distributions given in the table. **2**

x	1	2	3	4
$\mathrm{P}(X=x)$	p	$\frac{1}{6}p$	p^2	$\frac{1}{2}$
$\mathrm{P}(Y=x)$	p	$\frac{1}{6}p$	p^2	$\frac{1}{2}$

Determine the value of p.

$p + \frac{1}{6}p + p^2 + \frac{1}{2} = 1$

(1) Mark for establishing equation

$p^2 + \frac{7}{6}p - \frac{1}{2} = 0$

$6p^2 + 7p - 3 = 0$

$p = \frac{1}{3} \quad p \neq -\frac{3}{2}$

(1) Mark for correct value of p.

Question 33 continues on page 39

Question 33 (continued)

(b) X and Y are independent random variables so that, for example: **2**

$$P(X = 1 \cap Y = 2) = P(X = 1) \times P(Y = 2)$$

Calculate $\mathrm{P}(Y = 2X \mid Y > X)$.

$\therefore \left(\dfrac{P[Y = 2X \cap Y > X)}{P[Y > X)} \right) = \dfrac{P[Y = 2X)}{P(Y > X)}$

$= \dfrac{p \times \frac{p}{6} + \frac{p}{6} \times \frac{1}{2}}{p \times \frac{p}{6} + p \times p^2 + p \times \frac{1}{2} + \frac{p}{6} \times p^2 + \frac{p}{6} \times \frac{1}{2} + p^2 \times \frac{1}{2}}$

$= \dfrac{15}{101} \simeq 0.149$

(1) mark for simplifying conditional probability

(1) mark for correct solution.

End of paper

Section II extra writing space

If you use this space, clearly indicate which question you are answering.