NORMANHURST BOYS HIGH SCHOOL

MATHEMATICS ADVANCED

2026 Year 12 Course Assessment Task 4 (Trial HSC Examination)
Friday, 31 July 2026

General instructions

- Working time – 3 hours.
  (plus 10 minutes reading time)
- Write using blue or black pen. Where diagrams are to be sketched, these may be done in pencil.
- NESA approved calculators may be used.
- Attempt **all** questions.

SECTION I

- Mark your answers on the answer grid provided (on page 33)

SECTION II

- All necessary working should be shown in every question. Marks may be deducted for illegible or incomplete working.

NESA STUDENT #: .....

Class (please ✔)

- ◯ 12MAA.1 – Ms Quek
- ◯ 12MAA.2 – Mr Gan
- ◯ 12MAX.2 – Miss Lee
- ◯ 12MAX.3 – Miss Chaussivert
- ◯ 12MAX.4 – Miss J. Kim

Marker's use only.

QUESTION	1-10	11-15	16-18	19-22	23-26	27-29	30-32	**Total**
MARKS	/10	/17	/15	/19	/12	/11	/16	/100

Section I

10 marks
Attempt Question 1 to 10
Allow approximately 15 minutes for this section

Mark your answers on the answer grid provided (labelled as page 33).

Questions **Marks**

1. Which of the following gives the correct values of a and b? **1**

$$\frac{\sqrt{2}-3}{3\sqrt{2}} = a + b\sqrt{2}$$

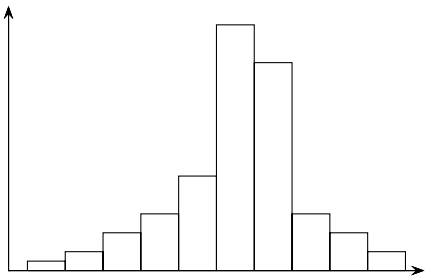
(A) $a = \frac{1}{3}, \quad b = -\frac{1}{2}$

(B) $a = -\frac{1}{2}, \quad b = \frac{1}{3}$

(C) $a = \frac{1}{3}, \quad b = -\frac{1}{3}$

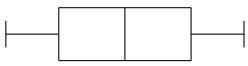
(D) $a = \frac{1}{2}, \quad b = -\frac{1}{2}$

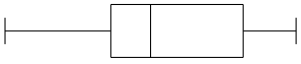
2. The distribution of a set of data is shown below. **1**

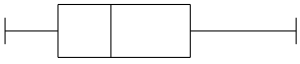


Which of the following box plots best represents the distribution of the data?

(A) 

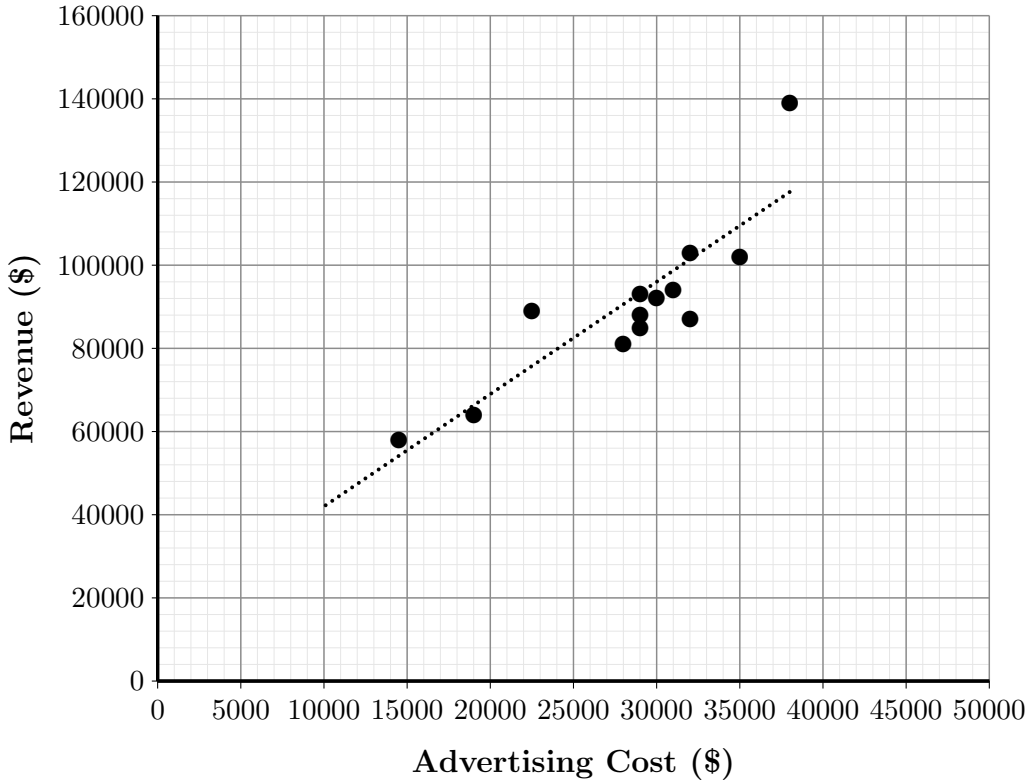
(C) 

(B) 

(D) 

3. A company manager recorded data on the relationship between the amount spent on advertising and revenue from sales. **1**

The bivariate data is shown on the graph below with a line of best fit drawn. For every extra dollar spent on advertising, the company's revenue changed by approximately \$2.70.



Which of the following equations best represents the line of best fit?

(A) $y = -2.7x + 4\,000$

(B) $y = 15\,000x + 2.7$

(C) $y = 2.7x + 15\,000$

(D) $y = 2.7x + 4\,000$

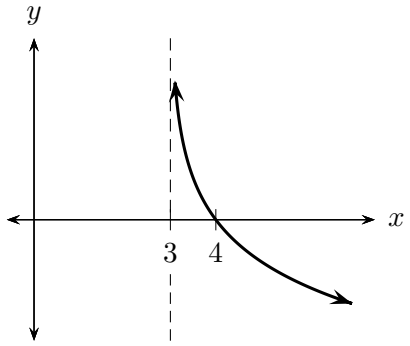
4. What is the solution to the equation $\log_3(2x - 5) = 2$? **1**

(A) $x = 1$ (B) $x = \dfrac{13}{2}$ (C) $x = \dfrac{11}{2}$ (D) $x = 7$

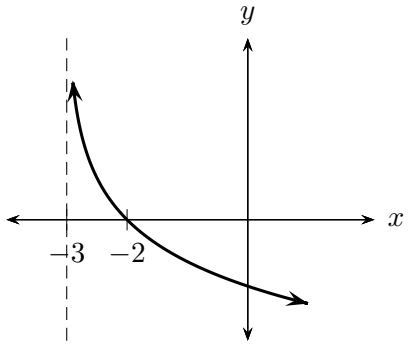
Examination continues overleaf...

5. Which of the following could represent the graph of $y = -\log_e(x-3)$? **1**

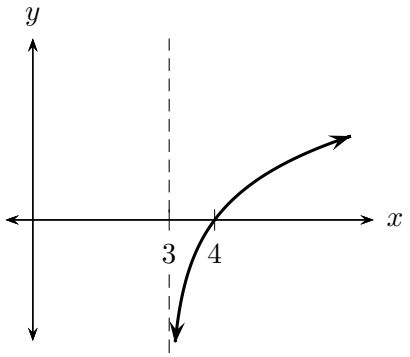
(A)



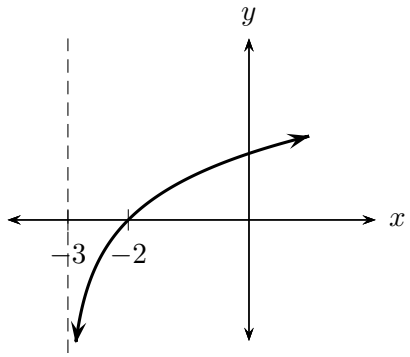
(C)



(B)



(D)



6. What is the amplitude and period for the function $f(x) = 4\sin\left(\dfrac{x+\pi}{3}\right)$? **1**

(A) Amplitude: 3, period: $\dfrac{\pi}{2}$

(B) Amplitude: 3, period: 6π

(C) Amplitude: 4, period: $\dfrac{\pi}{2}$

(D) Amplitude: 4, period: 6π

7. For $0 \leq x \leq \pi$, which of the following is the solution to the equation below? **1**

$$\cos x(\tan x - 1) = 0$$

(A) No solution

(B) $x = \dfrac{\pi}{4}$ only

(C) $x = \dfrac{\pi}{2}$ only

(D) $x = \dfrac{\pi}{4}$ or $x = \dfrac{\pi}{2}$

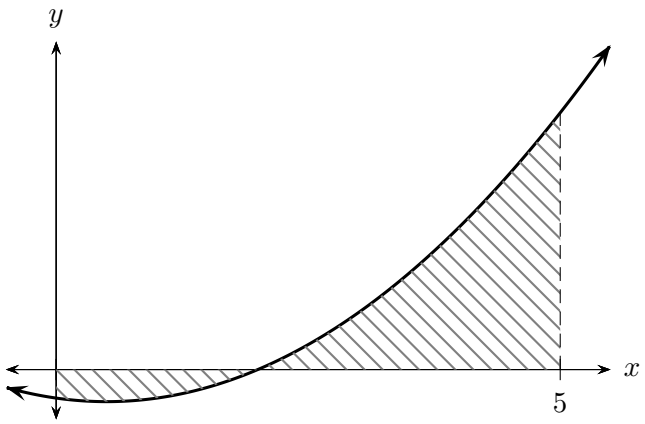
8. For what value of c is the line $y = 4x$ tangent to the parabola $y = 8x - x^2 + c$? **1**

(A) -4 (B) -2 (C) 4 (D) 8

9. Given that $\displaystyle\int_2^a f(x)\,dx = 3$, what is the value of $\displaystyle\int_2^a \left(\frac{1}{3}f(x) - 2\right)\,dx$? **1**

(A) $5 - 2a$ (B) $5 + 2a$ (C) $-2a$ (D) $2a$

10. The graph of $y = x^2 - x - 2$ is shown below. **1**



Which expression will give the area of the shaded region bounded by the curve $y = x^2 - x - 2$, the x-axis and the lines $x = 0$ and $x = 5$?

(A) $A = \left|\int_0^1 x^2 - x - 2\,dx\right| + \int_1^5 x^2 - x - 2\,dx$

(B) $A = \int_0^1 x^2 - x - 2\,dx + \left|\int_1^5 x^2 - x - 2\,dx\right|$

(C) $A = \left|\int_0^2 x^2 - x - 2\,dx\right| + \int_2^5 x^2 - x - 2\,dx$

(D) $A = \int_0^2 x^2 - x - 2\,dx + \left|\int_2^5 x^2 - x - 2\,dx\right|$

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2026 Year 12 Course Assessment Task 4

Mathematics Advanced

Section II Answer Booklet

90 marks
Attempt Questions 11 to 32
Allow approximately 2 hours and 45 minutes for this section

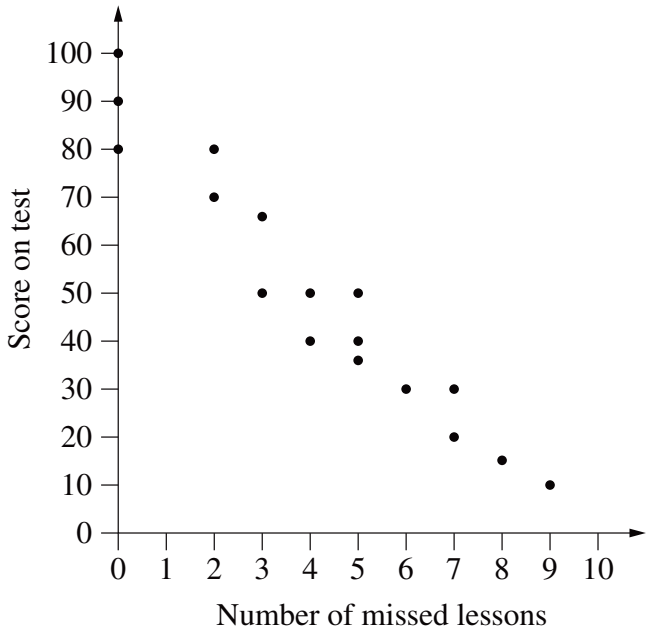
Instructions

- Answer the questions in the spaces provided
- Your responses should include relevant mathematical reasoning and/or calculations
- Extra writing space is provided at the back of this booklet. If you use this space, clearly indicate which question you are answering

Please turn over

Question 11 (6 marks)

A group of students sat a test at the end of term. The number of lessons each student missed during the term and their score on the test are shown on the scatterplot.



(a) Describe the strength and direction of the linear association observed in this dataset. **2**

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(b) Calculate the range of the test scores for the students who missed no lessons. **1**

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(c) Draw a line of best fit by eye on the scatterplot on the previous page. **1**

(d) Meg did not sit the test. She missed five lessons. **1**
Use the line of best fit drawn in part (c) to estimate Meg's score on this test.

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(e) John also did not sit the test and he missed 16 lessons. **1**
Is it appropriate to use the line of best fit to estimate his score on the test? Briefly explain your answer.

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Question 12 (2 marks)

If $\log_a 5 = 1.3$ and $\log_a 7 = 1.5$, find the value of the following: **2**

$$\log_a \left(\frac{25}{7}\right)$$

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Question 13 (2 marks)

Solve for x: **2**

$$(\ln x)^2 - \ln x^3 - 4 = 0$$

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Examination continues overleaf...

Question 14 (3 marks)

The graph $f(x)$ undergoes the following sequence of transformations to produce **3**

$$g(x) = \sqrt{3 - 2x}$$

- Translation 2 units to the right
- Horizontal dilation by a factor of $\frac{1}{2}$
- Reflected along the y axis

Find the equation of the original graph $f(x)$.

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Question 15 (4 marks)

The sales team at Frontier phone company sells 12 000 phones in the first month of operation. They increase their sales by 800 phones each month on the proceeding month's sales.

(a) Find the number of phones sold in the last month of the second year of operation. **1**

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(b) Sampson, another phone company commenced business at exactly the same time as Frontier. Their sales team sells 5 000 phones in their first month of operation and increases their sales by 1 500 each month on the proceeding month's sales. **3**

After how many months will Sampson's total sales exceed Frontier's total sales?

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Question 16 (7 marks)

Differentiate with respect to x:

(a) $f(x) = \dfrac{2x^2 + 7x + 3}{\sqrt{x}}$ **2**

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Examination continues overleaf...

(b) $f(x) = (x^2 + 5)\ln x$ **2**

(c) $f(x) = \dfrac{e^{-2x}}{\sin x - \cos x}$ **3**

Question 17 (4 marks)

Sketch the curve $y = x^3 - 3x^2 - 9x + 15$ by first finding all stationary points and checking their nature. (Do **NOT** find the x-intercepts.) **4**

Examination continues overleaf...

Question 18 (4 marks)

4

A continuous and differentiable function is defined by

$$f(x) = \begin{cases} \frac{1}{9}\left(x^2 + ax + c\right) & , x < 3 \\ b\left(x + 12\right) & , x \geq 3 \end{cases}$$

Given that $f(5) = \frac{17}{3}$, find the values of a, b and c.

Question 19 (3 marks)

(a) Find $\dfrac{d}{dx}\Big((x^2-5)^9\Big)$. **2**

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(b) Hence, or otherwise, find $\displaystyle\int x(x^2-5)^8\,dx$. **1**

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Question 20 (5 marks)

Evaluate:

(a) $\displaystyle\int x(x-2)(x+1)\,dx$ **2**

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(b) $\displaystyle\int_0^{\frac{\pi}{4}} \frac{2\sin x}{1+\cos x}\,dx$ **3**

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Question 21 (6 marks)

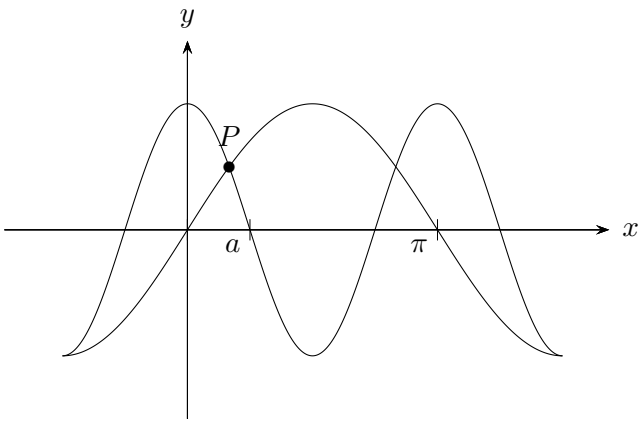
(a) Find the exact value of $\int_0^1 e^{(\ln 3)x}\,dx$. **2**

(b) Using the trapezoidal rule with 3 function values, find an approximation to the integral $\int_0^1 e^{(\ln 3)x}\,dx$, leaving your answer in exact form. **2**

(c) Hence, show that $\ln 3 \approx 8 - 4\sqrt{3}$. **2**

Question 22 (5 marks)

The graphs of $y = \cos 2x$ and $y = \sin x$ intersect at the point P.



(a) Find the coordinates of P. **2**

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(b) Show that the area bounded by the curves $y = \cos(2x)$, $y = \sin x$ and the x-axis from $x = 0$ to $x = a$ is given by **3**

$$A = \frac{3}{2} - \frac{3\sqrt{3}}{4}$$

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Examination continues overleaf...

Question 23 (2 marks)

Prove that $\dfrac{\cos\theta}{1+\sin\theta} + \tan\theta = \sec\theta$. **2**

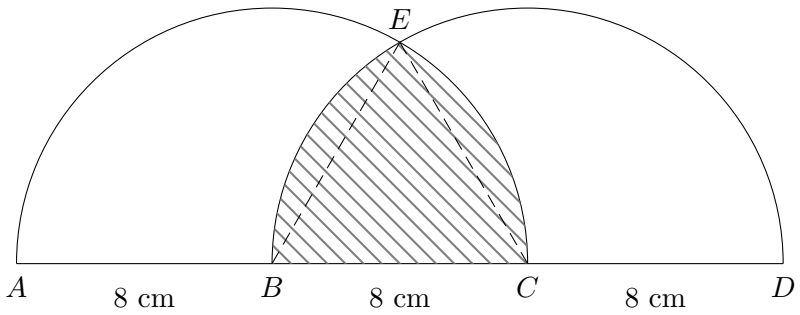
Question 24 (3 marks)

If $a \neq 0$ and $\sec\theta = a + \dfrac{1}{4a}$, prove that $\sec\theta + \tan\theta = 2a$ or $\dfrac{1}{2a}$. **3**

Question 25 (3 marks)

In the diagram, A, B, C and D are collinear, where $AB = BC = CD = 8$ cm. **3**
The semicircles on diameters AC and BD intersect at E so that $\triangle BCE$ is equilateral.

Find the area of the region common to the two semicircles in simplest exact form.



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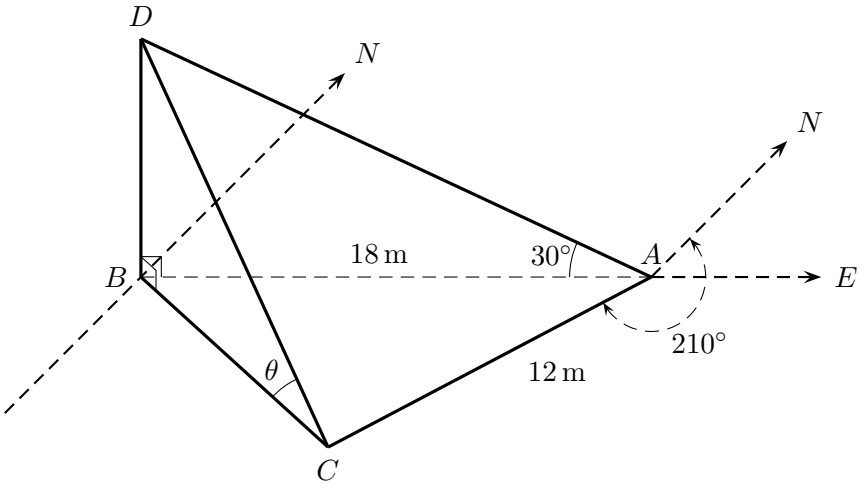
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Examination continues overleaf. . .

Question 26 (4 marks)

A flagpole, BD, stands on flat level ground and it is supported by two cables, AD and CD, as shown below. **4**

The cable AD is inclined at an angle of 30° to the horizontal and is tied to the ground at point A, 18 m east of the foot of the pole.
The second cable is tied to the ground at point C, where C is 12 m from A and on a bearing of 210°T.



Find θ, correct to the nearest degree.

Question 26 continued

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Question 27 (3 marks)

Events A and B are not mutually exclusive.
It is given that $P(A) = 0.35$, $P(B) = 0.55$, and $P\left(\overline{A \cup B}\right) = 0.2$.

(a) Find $P(A \cap B)$. **1**

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(b) Find $P(B|A)$. Hence determine whether events A and B are independent. Justify your answer. **2**

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Examination continues overleaf...

Question 28 (5 marks)

The random variable X represents the amount of money won, in dollars, per game on a poker machine.

The probability distribution table is shown below.

x	b	2	3	4	5
$P(X = x)$	0.35	0.20	a	0.17	0.09

(a) Calculate the value of a. **1**

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(b) Given that the expected loss is \$1.40 per game, calculate the value of b. **2**

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(c) Hence, calculate the variance of X. **2**

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Question 29 (3 marks)

A tenpin bowler has 28% chance of scoring a strike in bowling each time they bowl.

(a) The bowler bowls twice. Calculate the probability that the bowler does not get a strike in both bowls. **1**

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(b) What is the minimum number of bowls required for the bowler to have a 90% chance of bowling at least one strike? **2**

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Examination continues overleaf...

Question 30 (5 marks)

Euan invented a dice game to play with his brother. They take turns rolling a standard pair of six-sided dice. The game continues until the difference between the two numbers rolled is either 2 or 3.

- If the difference is 2, the player who made the roll wins immediately.
- If the difference is 3, the other player wins immediately.
- Otherwise, play passes to the other player, who rolls the dice next.

Assume that Euan rolls first.

(a) Show that the probability that Euan wins on his first throw of the dice is $\frac{2}{9}$. **1**

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(b) What is the probability that Euan wins before the fourth throw of the game? **2**

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(c) Calculate the probability that Euan wins the game eventually. **2**

Examination continues overleaf...

Question 31 (4 marks)

(a) State the centre and radius of the circle $x^2 + y^2 - 4x = 0$. **2**

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(b) Hence, evaluate $\int_{-4}^{-2} \sqrt{-4x - x^2}\, dx$. **2**

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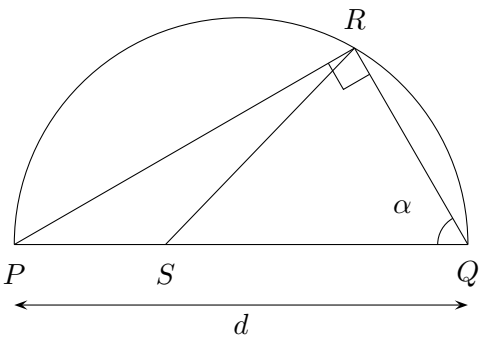
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Question 32 (7 marks)

$\triangle PQR$ is a right angled triangle inscribed in a semicircle.

The point S lies on PQ such that $SQ = kQR$, where k is a positive constant. Let $PQ = d$ cm, $\angle PQR = \alpha$ radians, where d is a constant.



(a) Show that the area of $\triangle SQR$ is $A = \dfrac{1}{2}kd^2\cos^2\alpha\sin\alpha$. **2**

(b) Find the greatest possible exact area of $\triangle SQR$ in terms of k and d. **5**

Examination continues overleaf...

Question 32(b) continued

End of paper.

Section II extra writing space

If you use this space, clearly indicate which question you are answering.

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Friday, 31 July 2026

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  (plus 10 minutes reading time)
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NESA STUDENT #: Sample solutions

Class (please ✔)

- ◯ 12MAA.1 – Ms Quek
- ◯ 12MAA.2 – Mr Gan
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Marker's use only.

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Section I

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Questions **Marks**

1. Which of the following gives the correct values of a and b? **1**

$$\frac{\sqrt{2}-3}{3\sqrt{2}} = a + b\sqrt{2}$$

$\frac{\sqrt{2}-3}{3\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{2-3\sqrt{2}}{3\cdot 2}$

$= \frac{1}{3} - \frac{\sqrt{2}}{2}$

$\therefore a = \frac{1}{3},\ b = -\frac{1}{2}$

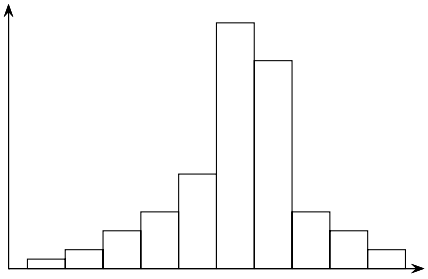
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(B) $a = -\frac{1}{2}, \quad b = \frac{1}{3}$

(C) $a = \frac{1}{3}, \quad b = -\frac{1}{3}$

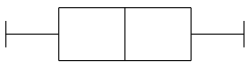
(D) $a = \frac{1}{2}, \quad b = -\frac{1}{2}$

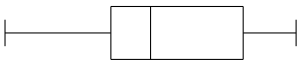
2. The distribution of a set of data is shown below. **1**

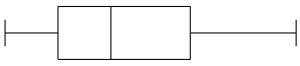


Which of the following box plots best represents the distribution of the data?

(A) 

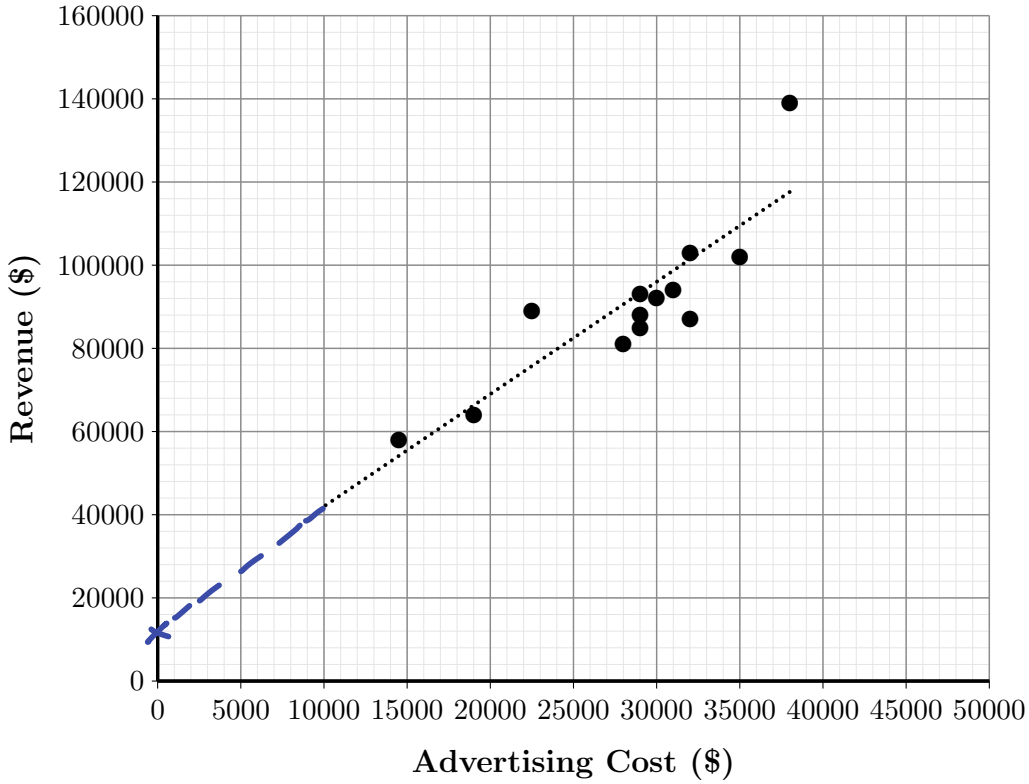
(C) 

(B) 

(D) 

3. A company manager recorded data on the relationship between the amount spent on advertising and revenue from sales. **1**

The bivariate data is shown on the graph below with a line of best fit drawn. For every extra dollar spent on advertising, the company's revenue changed by approximately \$2.70.



Which of the following equations best represents the line of best fit?

(A) $y = -2.7x + 4\,000$

(C) $y = 2.7x + 15\,000$

(B) $y = 15\,000x + 2.7$

(D) $y = 2.7x + 4\,000$

4. What is the solution to the equation $\log_3(2x - 5) = 2$? **1**

(A) $x = 1$ (B) $x = \frac{13}{2}$ (C) $x = \frac{11}{2}$ (D) $x = 7$

$\log_3(2x-5)=2$

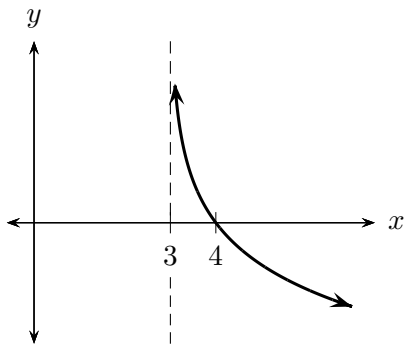
$3^2 = 2x-5$

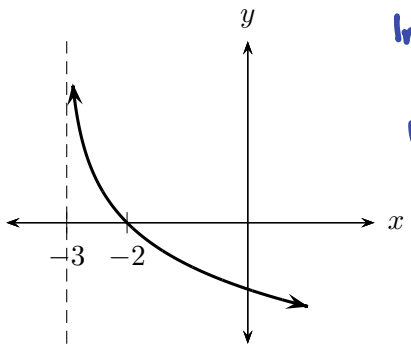
$2x = 14$

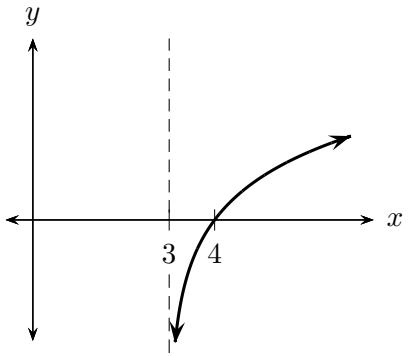
$x = 7$

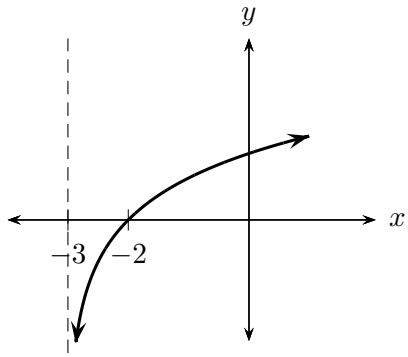
Examination continues overleaf...

5. Which of the following could represent the graph of $y = -\log_e(x-3)$? **1**

(A)


(C)


(B)


(D)


6. What is the amplitude and period for the function $f(x) = 4\sin\left(\dfrac{x+\pi}{3}\right)$? **1**

(A) Amplitude: 3, period: $\dfrac{\pi}{2}$

(C) Amplitude: 4, period: $\dfrac{\pi}{2}$

(B) Amplitude: 3, period: 6π

(D) Amplitude: 4, period: 6π

7. For $0 \le x \le \pi$, which of the following is the solution to the equation below? **1**

$$\cos x(\tan x - 1) = 0$$

(A) No solution

(C) $x = \dfrac{\pi}{2}$ only

(B) $x = \dfrac{\pi}{4}$ only

(D) $x = \dfrac{\pi}{4}$ or $x = \dfrac{\pi}{2}$

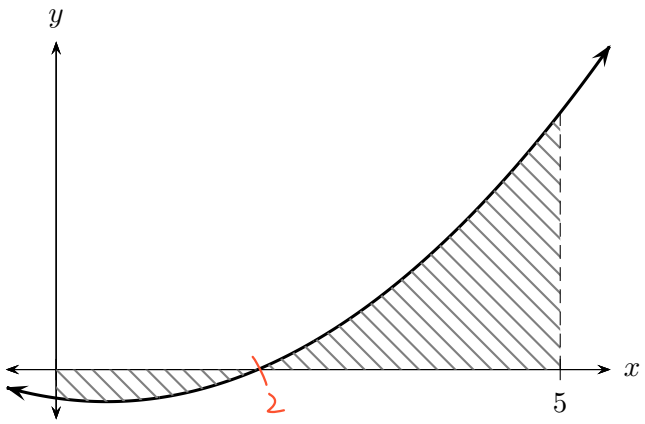
8. For what value of c is the line $y_1 = 4x$ tangent to the parabola $y_2 = 8x - x^2 + c$? **1**

(A) -4 (B) -2 (C) 4 (D) 8

9. Given that $\displaystyle\int_2^a f(x)\,dx = 3$, what is the value of $\displaystyle\int_2^a \left(\frac{1}{3}f(x) - 2\right)dx$? **1**

(A) $5 - 2a$ (B) $5 + 2a$ (C) $-2a$ (D) $2a$

10. The graph of $y = x^2 - x - 2$ is shown below. **1**



Which expression will give the area of the shaded region bounded by the curve $y = x^2 - x - 2$, the x-axis and the lines $x = 0$ and $x = 5$?

$y = x^2 - x - 2$
$= (x-2)(x+1)$
when $y=0$, $x = 2, -1$.

(A) $A = \left|\int_0^1 x^2 - x - 2\,dx\right| + \int_1^5 x^2 - x - 2\,dx$

(B) $A = \int_0^1 x^2 - x - 2\,dx + \left|\int_1^5 x^2 - x - 2\,dx\right|$

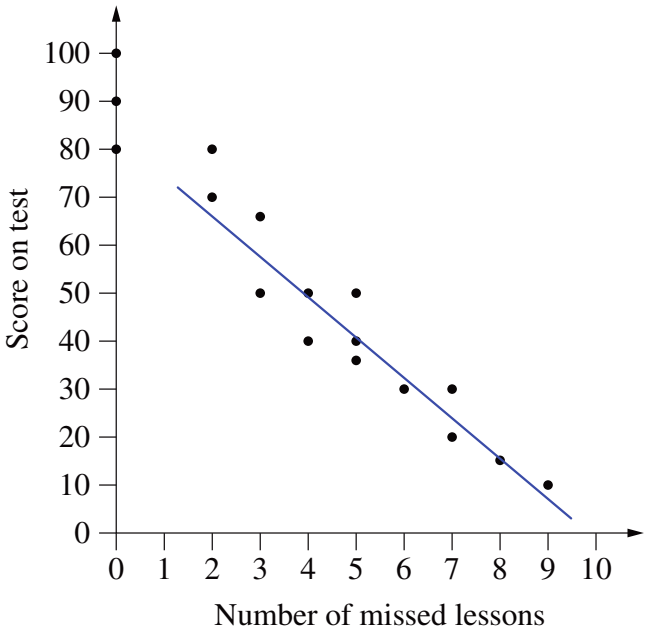
(C) $A = \left|\int_0^2 x^2 - x - 2\,dx\right| + \int_2^5 x^2 - x - 2\,dx$

(D) $A = \int_0^2 x^2 - x - 2\,dx + \left|\int_2^5 x^2 - x - 2\,dx\right|$

Examination continues overleaf...

Question 11 (6 marks) (Chaussivert)

A group of students sat a test at the end of term. The number of lessons each student missed during the term and their score on the test are shown on the scatterplot.



(a) Describe the strength and direction of the linear association observed in this dataset. **2**

The linear association is strong ✓ and the direction is negative ✓

(b) Calculate the range of the test scores for the students who missed no lessons. **1**

$100 - 80 = 20$ ✓

(c) Draw a line of best fit by eye on the scatterplot on the previous page. **1**

(d) Meg did not sit the test. She missed five lessons. **1**
Use the line of best fit drawn in part (c) to estimate Meg's score on this test.

40 ✓

(e) John also did not sit the test and he missed 16 lessons. **1**
Is it appropriate to use the line of best fit to estimate his score on the test? Briefly explain your answer.

No as it is an extrapolation.
It is beyond the range given and can't assume that the trend will continue. ✓

Question 12 (2 marks)

If $\log_a 5 = 1.3$ and $\log_a 7 = 1.5$, find the value of the following: **2**

$$\log_a\left(\frac{25}{7}\right)$$

$\log_a\left(\frac{25}{7}\right) = \log_a 25 - \log_a 7$ ✓
$= 2\log_a 5 - \log_a 7$
$= 2 \times 1.3 - 1.5$
$= 1.1$ ✓

Question 13 (2 marks)

Solve for x: **2**

$$(\ln x)^2 - \ln x^3 - 4 = 0$$

$(\ln x)^2 - 3\ln x - 4 = 0$ ✓
$(\ln x - 4)(\ln x + 1) = 0$
$\ln x = 4$ or $\ln x = -1$
$x = e^4$ or $x = e^{-1}$ ✓

Examination continues overleaf...

Question 14 (3 marks)

The graph $f(x)$ undergoes the following sequence of transformations to produce **3**

$$g(x) = \sqrt{3-2x}$$

- Translation 2 units to the right
- Horizontal dilation by a factor of $\frac{1}{2}$
- Reflected along the y axis

Find the equation of the original graph $f(x)$.

$g(-x) = \sqrt{3+2x} = h(x)$ ✓

$h\left(\frac{x}{2}\right) = \sqrt{3+x} = k(x)$ ✓

$k(x+2) = \sqrt{3+x+2}$

$= \sqrt{x+5}$

$\therefore f(x) = \sqrt{x+5}$ ✓

Question 15 (4 marks)

The sales team at Frontier phone company sells 12 000 phones in the first month of operation. They increase their sales by 800 phones each month on the proceeding month's sales.

(a) Find the number of phones sold in the last month of the second year of operation. **1**

$T_{24} = 12\,000 + (24-1) \times 800$

$= 30\,400$ ✓

$\therefore$ Sold 30 400 phones in the last month of 2nd year.

(b) Sampson, another phone company commenced business at exactly the same time as Frontier. Their sales team sells 5 000 phones in their first month of operation and increases their sales by 1 500 each month on the proceeding month's sales. **3**

After how many months will Sampson's total sales exceed Frontier's total sales?

let S_S = Sampson's total sales $\quad S_S > S_F$

S_F = Frontier's total sales $\quad \frac{n}{2}(23200 + 800n) > \frac{n}{2}(3500 + 1500n)$ ✓

$19700 > 700n$

$S_S = \frac{n}{2}(2\times 12000 + (n-1)\times 800)$ $\quad n < 28.14...$ ✓

$= \frac{n}{2}(23200 + 800n)$ $\quad \therefore$ after 29th month ✓

$S_F = \frac{n}{2}(5000 + (n-1)\times 1500)$

$= \frac{n}{2}(3500 + 1500n)$

Question 16 (7 marks) (Kim)

Differentiate with respect to x:

(a) $f(x) = \dfrac{2x^2 + 7x + 3}{\sqrt{x}}$ **2**

$= 2x^{\frac{3}{2}} + 7x^{\frac{1}{2}} + 3x^{-\frac{1}{2}}$ ✓

$f'(x) = 2\times\frac{3}{2}x^{\frac{1}{2}} + 7\times\frac{1}{2}x^{-\frac{1}{2}} + 3\times-\frac{1}{2}x^{-\frac{3}{2}}$

$= 3\sqrt{x} + \frac{7}{2\sqrt{x}} - \frac{3}{2\sqrt{x^3}}$ ✓

Examination continues overleaf...

(b) $f(x) = (x^2 + 5)\ln x$ **2**

$u = x^2+5 \qquad v = \ln x$

$u' = 2x \qquad v' = \frac{1}{x}$

$f'(x) = 2x\ln x + \frac{x^2+5}{x}$ ✓

✓

(c) $f(x) = \dfrac{e^{-2x}}{\sin x - \cos x}$ **3**

$u = e^{-2x} \qquad v = \sin x - \cos x$

$u' = -2e^{-2x} \qquad v' = \cos x + \sin x$

✓

$f'(x) = \dfrac{-2e^{-2x}(\sin x - \cos x) - e^{-2x}(\cos x + \sin x)}{(\sin x - \cos x)^2}$ ✓

$= \dfrac{-e^{-2x}(2\sin x - 2\cos x + \cos x + \sin x)}{(\sin x - \cos x)^2}$

$= \dfrac{-e^{-2x}(3\sin x - \cos x)}{(\sin x - \cos x)^2}$ ✓

Question 17 (4 marks)

Sketch the curve $y = x^3 - 3x^2 - 9x + 15$ by first finding all stationary points and checking their nature. (Do **NOT** find the x-intercepts.) **4**

$y = x^3 - 3x^2 - 9x + 15$

$y' = 3x^2 - 6x - 9$

$y'' = 6x - 6$

When $y' = 0$,

$3x^2 - 6x - 9 = 0$

$x^2 - 2x - 3 = 0$

$(x - 3)(x + 1) = 0$

$x = 3, -1$

at $x = 3$, $y'' = 6(3) - 6$

$= 12$

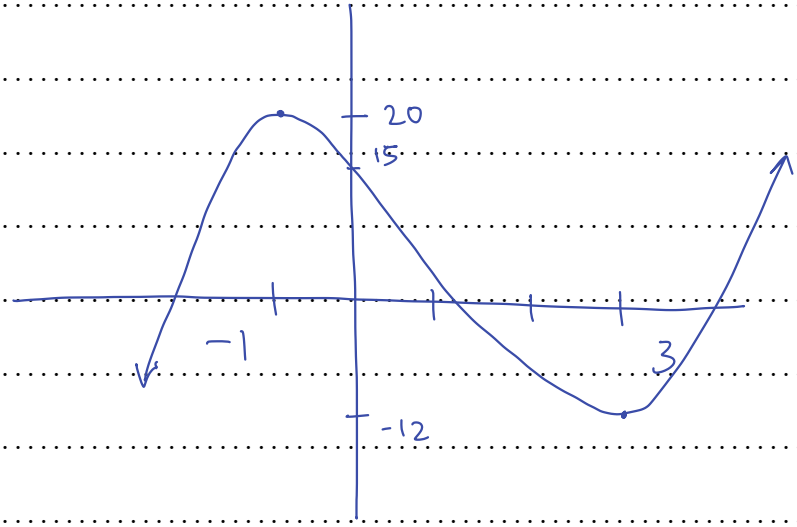
> 0 $\therefore$ local min at $(3, -12)$

at $x = -1$, $y'' = 6(-1) - 6$

$= -12$

< 0 $\therefore$ local max at $(-1, 20)$

When $x = 0$, $y = 15$



Examination continues overleaf...

Question 18 (4 marks) **4**

A continuous and differentiable function is defined by

$$f(x) = \begin{cases} \frac{1}{9}\left(x^2 + ax + c\right) & , x < 3 \\ b\left(x + 12\right) & , x \geq 3 \end{cases}$$

Given that $f(5) = \frac{17}{3}$, find the values of a, b and c.

$f(5) = \frac{17}{3}$;

$\frac{17}{3} = b(5+12)$

$b = \frac{1}{3}$ ✓

at $x = 3$,

$y_1 = y_2$

$\frac{1}{9}(3^2 + 3a + c) = b(3+12)$

$9 + 3a + c = 135 \times \frac{1}{3}$

$3a + c = 36$

Sub $a = -3$,

$c = 36 + 9$

$= 45$ ✓

let $y_1 = \frac{1}{9}(x^2 + ax + c)$

$y_2 = b(x+12)$

$y_1' = \frac{1}{9}(2x + a)$

$y_2' = b = \frac{1}{3}$

$m_1 = m_2$

$\frac{1}{9}(2 \times 3 + a) = \frac{1}{3}$

$6 + a = 3$

$a = -3$ ✓

$\therefore a = -3, b = \frac{1}{3}, c = 45$

Question 19 (3 marks) (Gan)

(a) Find $\frac{d}{dx}\left((x^2-5)^9\right)$. **2**

$\frac{d}{dx}\left((x^2-5)^9\right) = 9(x^2-5)^8 \times 2x$ ✓

$= 18x(x^2-5)^8$ ✓

(b) Hence, or otherwise, find $\int x(x^2-5)^8\,dx$. **1**

$\frac{1}{18}\int 18x(x^2-5)^8\,dx$

$= \frac{1}{18}(x^2-5)^9 + C$ ✓

Question 20 (5 marks)

Evaluate:

(a) $\int x(x-2)(x+1)\,dx$ **2**

$= \int x(x^2-x-2)\,dx$ ✓

$= \int x^3 - x^2 - 2x\,dx$

$= \frac{x^4}{4} - \frac{x^3}{3} - x^2 + C$ ✓

(b) $\int_0^{\frac{\pi}{4}} \frac{2\sin x}{1+\cos x}\,dx$ **3**

$= -2\int_0^{\frac{\pi}{4}} \frac{-\sin x}{1+\cos x}\,dx$ ✓

$= -2\left[\ln|1+\cos x|\right]_0^{\frac{\pi}{4}}$

$= -2\left[\ln\left|1+\frac{1}{\sqrt{2}}\right| - \ln|1+1|\right]$ ✓

$= -2\left[\ln\frac{\frac{\sqrt{2}+1}{\sqrt{2}}}{2}\right]$

$= -2\ln\frac{\sqrt{2}+1}{2\sqrt{2}}$ ✓

Examination continues overleaf...

Question 21 (6 marks)

(a) Find the exact value of $\int_0^1 e^{(\ln 3)x}\,dx$. **2**

$\int_0^1 e^{(\ln 3)x}\,dx = \frac{1}{\ln 3}\left[e^{(\ln 3)x}\right]_0^1$ ✓

$= \frac{1}{\ln 3}\left[e^{\ln 3} - e^0\right]$

$= \frac{2}{\ln 3}$ ✓

(b) Using the trapezoidal rule with 3 function values, find an approximation to the integral $\int_0^1 e^{(\ln 3)x}\,dx$, leaving your answer in exact form. **2**

x	0	0.5	1
$f(x)$	1	$\sqrt{3}$	3

✓

$\int_0^1 e^{(\ln 3)x}\,dx \approx \frac{0.5}{2}\left(1 + 2\times\sqrt{3} + 3\right)$

$= 1 + \frac{\sqrt{3}}{2}$ ✓

(c) Hence, show that $\ln 3 \approx 8 - 4\sqrt{3}$. **2**

$\frac{2}{\ln 3} \approx 1 + \frac{\sqrt{3}}{2}$ ✓

$\frac{2}{\ln 3} = \frac{2+\sqrt{3}}{2}$

$\ln 3 = \frac{4}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}}$

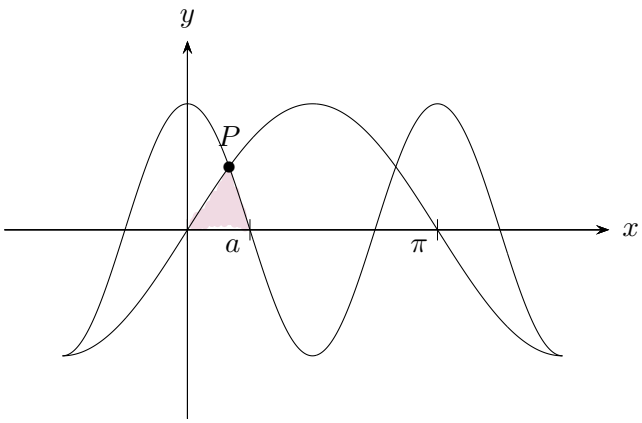
$= \frac{4(2-\sqrt{3})}{4-3}$

$= 8 - 4\sqrt{3}$

$\therefore \ln 3 \approx 8 - 4\sqrt{3}$ ✓

Question 22 (5 marks)

The graphs of $y = \cos 2x$ and $y = \sin x$ intersect at the point P.



(a) Find the coordinates of P. **2**

$\cos 2x = \sin x$ ✓

$1 - 2\sin^2 x = \sin x$

$2\sin^2 x + \sin x - 1 = 0$

$(2\sin x - 1)(\sin x + 1) = 0$

$\sin x = \frac{1}{2}$ or -1 $\quad \because 0 < x < \frac{\pi}{4}$ ✓

$x = \frac{\pi}{6}$ only

(b) Show that the area bounded by the curves $y = \cos(2x)$, $y = \sin x$ and the x-axis from $x = 0$ to $x = a$ is given by **3**

$$A = \frac{3}{2} - \frac{3\sqrt{3}}{4}$$

$A = \int_0^{\frac{\pi}{6}} \sin x \, dx + \int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \cos 2x \, dx$ ✓

$= \left[-\cos x\right]_0^{\frac{\pi}{6}} + \left[\frac{1}{2}\sin 2x\right]_{\frac{\pi}{6}}^{\frac{\pi}{4}}$

$= -\cos\frac{\pi}{6} + \cos 0 + \frac{1}{2}\sin\frac{\pi}{2} - \frac{1}{2}\sin\frac{\pi}{3}$ ✓

$= -\frac{\sqrt{3}}{2} + 1 + \frac{1}{2}\times 1 - \frac{1}{2}\times\frac{\sqrt{3}}{2}$

$= \frac{3}{2} - \frac{2\sqrt{3}+\sqrt{3}}{4}$

$= \frac{3}{2} - \frac{3\sqrt{3}}{4}$ ✓

Examination continues overleaf...

Question 23 (2 marks) (Kim)

Prove that $\dfrac{\cos\theta}{1+\sin\theta} + \tan\theta = \sec\theta$. **2**

$$\text{LHS} = \frac{\cos\theta}{1+\sin\theta} + \tan\theta$$
$$= \frac{\cos\theta}{1+\sin\theta} + \frac{\sin\theta}{\cos\theta} \quad \checkmark$$
$$= \frac{\cos^2\theta + \sin\theta(1+\sin\theta)}{\cos\theta(1+\sin\theta)}$$
$$= \frac{\sin^2\theta + \cos^2\theta + \sin\theta}{\cos\theta(1+\sin\theta)}$$
$$= \frac{1+\sin\theta}{\cos\theta(1+\sin\theta)}$$
$$= \frac{1}{\cos\theta}$$
$$= \sec\theta \quad \checkmark$$
$$= \text{RHS}$$

Question 24 (3 marks)

If $a \neq 0$ and $\sec\theta = a + \dfrac{1}{4a}$, prove that $\sec\theta + \tan\theta = 2a$ or $\dfrac{1}{2a}$. **3**

$1 + \tan^2\theta = \sec^2\theta$

$\tan^2\theta = \sec^2\theta - 1$ ✓

$= \left(a + \frac{1}{4a}\right)^2 - 1$

$= a^2 + \frac{1}{2} + \frac{1}{16a^2} - 1$

$= a^2 - \frac{1}{2} + \frac{1}{16a^2}$

$= \left(a - \frac{1}{4a}\right)^2$

$\tan\theta = \pm\left(a - \frac{1}{4a}\right)$ ✓

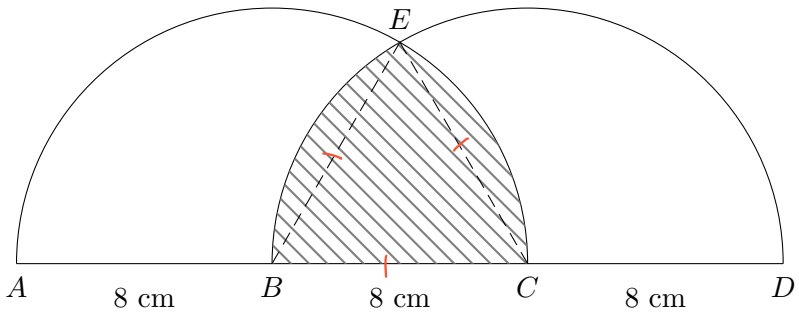
$\therefore \sec\theta + \tan\theta = a + \frac{1}{4a} \pm \left(a - \frac{1}{4a}\right)$

$= 2a$ or $\frac{1}{2a}$ ✓

Question 25 (3 marks) (Lee)

In the diagram, A, B, C and D are collinear, where $AB = BC = CD = 8$ cm. **3**
The semicircles on diameters AC and BD intersect at E so that $\triangle BCE$ is equilateral.

Find the area of the region common to the two semicircles in simplest exact form.



$BE = EC = BC$ (radii) $\therefore$ $\triangle BEC$ is an equilateral ✓

$A = 2 \times \frac{1}{2} \times 8^2 \times \frac{\pi}{3} - \frac{1}{2} \times 8 \times 8 \times \sin\frac{\pi}{3}$ ✓

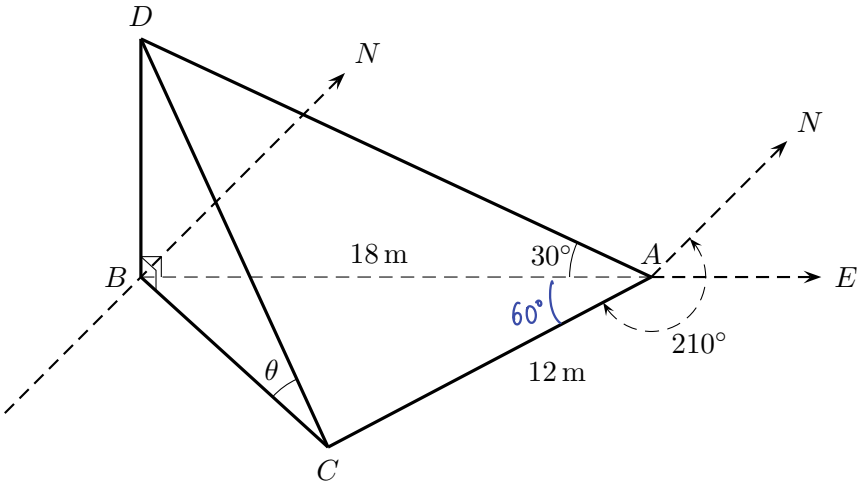
$= \frac{64\pi}{3} - 16\sqrt{3}$ cm^2 ✓

Examination continues overleaf...

Question 26 (4 marks)

A flagpole, BD, stands on flat level ground and it is supported by two cables, AD and CD, as shown below. **4**

The cable AD is inclined at an angle of 30° to the horizontal and is tied to the ground at point A, 18 m east of the foot of the pole.
The second cable is tied to the ground at point C, where C is 12 m from A and on a bearing of 210°T.



Find θ, correct to the nearest degree.

$\tan 30 = \frac{DB}{18}$

$DB = \frac{18}{\sqrt{3}}$

$= 6\sqrt{3}$ ✓

$\angle BAC = 60°$ ✓

$BC^2 = 18^2 + 12^2 - 2 \times 18 \times 12 \cos 60$

$= 252$

$BC = \sqrt{252}$ ✓

$= 6\sqrt{7}$

$\tan \theta = \frac{6\sqrt{3}}{6\sqrt{7}}$

$= \sqrt{\frac{3}{7}}$

$\theta = 33°12'39.28''$ ✓

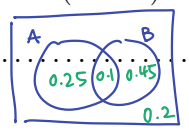
$= 33°$ (nearest degree)

Question 26 continued

Question 27 (3 marks)

Events A and B are not mutually exclusive.
It is given that $P(A) = 0.35$, $P(B) = 0.55$, and $P\left(\overline{A \cup B}\right) = 0.2$.

(a) Find $P(A \cap B)$. **1**



$P(A \cap B) = 0.1$ ✓

(b) Find $P(B|A)$. Hence determine whether events A and B are independent. Justify your answer. **2**

$P(B|A) = \frac{0.1}{0.35}$

$= \frac{2}{7}$

$= 0.2857\ldots$ ✓

$\neq P(B)$

$\therefore$ not independent ✓

Examination continues overleaf...

Question 28 (5 marks)

The random variable X represents the amount of money won, in dollars, per game on a poker machine.

The probability distribution table is shown below.

x	b	2	3	4	5
$P(X = x)$	0.35	0.20	a	0.17	0.09

(a) Calculate the value of a. **1**

$0.35 + 0.2 + a + 0.17 + 0.09 = 1$ ✓

$a = 0.19$

(b) Given that the expected loss is \$1.40 per game, calculate the value of b. **2**

$E(X) = -1.4$; ✓

$-1.4 = 0.35b + 2 \times 0.2 + 3 \times 0.19 + 4 \times 0.17 + 5 \times 0.09$

$b = -3.5$ ✓

(c) Hence, calculate the variance of X. **2**

$\text{Var}(X) = E(X^2) - [E(X)]^2$ ✓

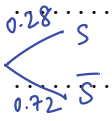
$= (-3.5)^2 \times 0.35 + 4 \times 0.2 + 9 \times 0.19 + 16 \times 0.17 + 25 \times 0.09 - (-1.4)^2$

$= 9.8075$ ✓

Question 29 (3 marks)

A tenpin bowler has 28% chance of scoring a strike in bowling each time they bowl.

(a) The bowler bowls twice. Calculate the probability that the bowler does not get a strike in both bowls. **1**



$P(\bar{S}\bar{S}) = 0.72^2$

$= 0.5184$ ✓

(b) What is the minimum number of bowls required for the bowler to have a 90% chance of bowling at least one strike? **2**

$P(\text{at least one } S) = 1 - (P(\bar{S}))^n$

$= 1 - (0.72)^n$

$\therefore 1 - (0.72)^n > 0.9$ ✓

$(0.72)^n < 0.1$

$n \ln(0.72) < \ln 0.1$

$n > \frac{\ln 0.1}{\ln 0.72}$

$n > 7.009\ldots$

$\therefore$ 8 number of bowls ✓

Examination continues overleaf...

Question 30 (5 marks) (Quek)

Euan invented a dice game to play with his brother. They take turns rolling a standard pair of six-sided dice. The game continues until the difference between the two numbers rolled is either 2 or 3.

- If the difference is 2, the player who made the roll wins immediately.
- If the difference is 3, the other player wins immediately.
- Otherwise, play passes to the other player, who rolls the dice next.

Assume that Euan rolls first.

(a) Show that the probability that Euan wins on his first throw of the dice is $\frac{2}{9}$. **1**

(6,4)
(5,3) ×2
(4,2)
(3,1)

P(win on 1st throw) = $\frac{4\times2}{36}$ = $\frac{2}{9}$ ✓

(b) What is the probability that Euan wins before the fourth throw of the game? **2**

P(wins 1st round) + P(wins 2nd round) + P(wins 3rd round) ✓

= $\frac{2}{9} + \frac{11}{18}\times\frac{1}{6} + \frac{11}{18}\times\frac{11}{18}\times\frac{2}{9}$

= $\frac{1187}{2916}$ ✓

(6,3) (5,2) (4,1) ×2 → $\frac{6}{36}=\frac{1}{6}$

$\frac{2}{9}$ getting a difference of 2

$\frac{1}{6}$ getting a difference of 3

$\frac{11}{18}$ other

(c) Calculate the probability that Euan wins the game eventually. **2**

$P(\text{Euan wins eventually}) = \frac{2}{9} + \frac{11}{18} \times \frac{1}{6} + \left(\frac{11}{18}\right)^2 \times \frac{2}{9} + \left(\frac{11}{18}\right)^3 \times \frac{1}{6} + \left(\frac{11}{18}\right)^4 \times \frac{2}{9} + \left(\frac{11}{18}\right)^5 \times \frac{1}{6} + \cdots$

$= \frac{2}{9} + \left(\frac{11}{18}\right)^2 \times \frac{2}{9} + \left(\frac{11}{18}\right)^4 \times \frac{2}{9} + \cdots + \frac{11}{18} \times \frac{1}{6} + \left(\frac{11}{18}\right)^3 \times \frac{1}{6} + \left(\frac{11}{18}\right)^5 \times \frac{1}{6} + \cdots$

$a = \frac{2}{9}, \ r = \left(\frac{11}{18}\right)^2$ $\qquad a = \frac{11}{18} \times \frac{1}{6}, \ r = \left(\frac{11}{18}\right)^2$

$= \frac{\frac{2}{9}}{1 - \left(\frac{11}{18}\right)^2} + \frac{\frac{11}{18} \times \frac{1}{6}}{1 - \left(\frac{11}{18}\right)^2}$

$= \frac{72}{203} + \frac{33}{203}$

$= \frac{15}{29}$

Examination continues overleaf...

Question 31 (4 marks)

(a) State the centre and radius of the circle $x^2 + y^2 - 4x = 0$. **2**

$x^2 - 4x + y^2 = 0$

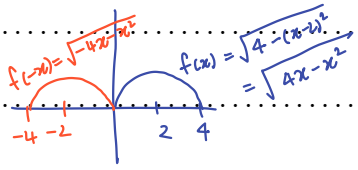
$x^2 - 4x + 2^2 + y^2 = 2^2$

$(x-2)^2 + y^2 = 4$ ✓

centre $(2, 0)$ & radius $= 2$ ✓

(b) Hence, evaluate $\int_{-4}^{-2} \sqrt{-4x - x^2}\, dx$. **2**

From a)



$f(x) = \sqrt{4x - x^2}$

$f(-x) = \sqrt{4(-x) - (-x)^2}$ ✓

$= \sqrt{-4x - x^2}$

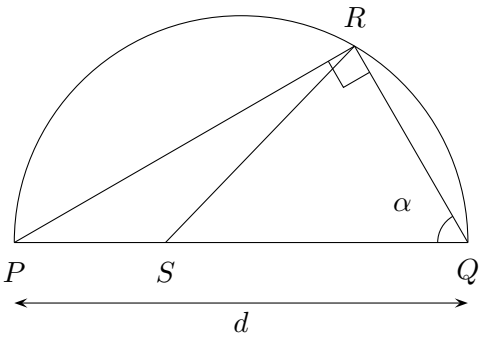
$\therefore \int_{-4}^{-2} \sqrt{-4x - x^2}\, dx = \frac{1}{4} \times \pi \times 2^2$

$= \pi$ ✓

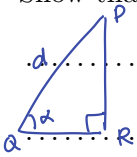
Question 32 (7 marks)

$\triangle PQR$ is a right angled triangle inscribed in a semicircle.

The point S lies on PQ such that $SQ = kQR$, where k is a positive constant. Let $PQ = d$ cm, $\angle PQR = \alpha$ radians, where d is a constant.

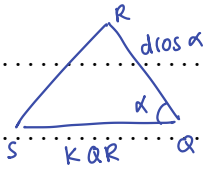


(a) Show that the area of $\triangle SQR$ is $A = \frac{1}{2}kd^2\cos^2\alpha\sin\alpha$. **2**



$\cos\alpha = \frac{QR}{d}$

$QR = d\cos\alpha$



$A = \frac{1}{2}\,kd\cos\alpha \times d\cos\alpha \times \sin\alpha$

$= \frac{1}{2}kd^2\cos^2\alpha\sin\alpha$

(b) Find the greatest possible exact area of $\triangle SQR$ in terms of k and d. **5**

$A = \frac{1}{2}kd^2\cos^2\alpha\sin\alpha$

$u = \cos^2\alpha \qquad v = \sin\alpha$

$u' = 2\cos\alpha \times -\sin\alpha \qquad v' = \cos\alpha$

$\frac{dA}{d\theta} = \frac{1}{2}kd^2\left[-2\cos\alpha\sin\alpha\cdot\sin\alpha + \cos^3\alpha\right]$

$= \frac{1}{2}kd^2\left(-2\cos\alpha\sin^2\alpha + \cos^3\alpha\right)$

when $\frac{dA}{d\theta} = 0$,

$\cos^3\alpha - 2\cos\alpha\sin^2\alpha = 0$

$\cos\alpha\left(\cos^2\alpha - 2\sin^2\alpha\right) = 0$

$\cos\alpha = 0 \quad$ or $\quad \cos^2\alpha = 2\sin^2\alpha$

$\alpha = \frac{\pi}{2}, \frac{3\pi}{2} \qquad \tan^2\alpha = \frac{1}{2}$

$\tan\alpha = \pm\frac{1}{\sqrt{2}}$

$\therefore\ 0 < \alpha < \frac{\pi}{2}$

$\tan\alpha = \frac{1}{\sqrt{2}}$ only

Examination continues overleaf...

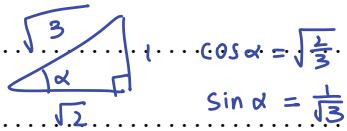
Question 32(b) continued

θ	34°	35°	36°
$\frac{dA}{d\theta}$	0.05	0	−0.03

$\tan\alpha = \frac{1}{\sqrt{2}}$

$\alpha \approx 35^\circ$ (nearest degree)

$\therefore$ max at $\alpha = 35^\circ$



$\cos\alpha = \sqrt{\frac{2}{3}}$

$\sin\alpha = \frac{1}{\sqrt{3}}$

$A_{max} = \frac{1}{2}kd^2\cos^2\alpha\sin\alpha$

$= \frac{1}{2}kd^2\left(\sqrt{\frac{2}{3}}\right)^2\left(\frac{1}{\sqrt{3}}\right)$

$= \frac{1}{2}kd^2\left(\frac{2}{3}\right)\left(\frac{1}{\sqrt{3}}\right)$

$= \frac{kd^2}{3\sqrt{3}}$

End of paper.