NORTH SYDNEY BOYS HIGH SCHOOL

2026 HSC ASSESSMENT TASK 3 (TRIAL HSC)

Mathematics Advanced

General Instructions

- Reading time – 10 minutes
- Working time – 3 hours
- Write in the space provided in the booklet provided
- Write using blue or black pen
- Board approved calculators may be used
- All necessary working should be shown in every question
- Extra working space is provided at the end of examination.
- Attempt all questions

Class Teacher:
(Please tick or highlight)

- ○ Mr Lin
- ○ Ms Everingham
- ○ Ms Cai
- ○ Ms Sarofim

(To be used by the exam markers only.)

Question Numbers	1-10	11-16	17-21	22-25	26-29	30-32	33-35	Total
Mark	$\overline{10}$	$\overline{14}$	$\overline{16}$	$\overline{14}$	$\overline{15}$	$\overline{16}$	$\overline{15}$	$\overline{100}$

Section I

10 Marks
Attempt Questions 1-10
Allow about 15 minutes for this section

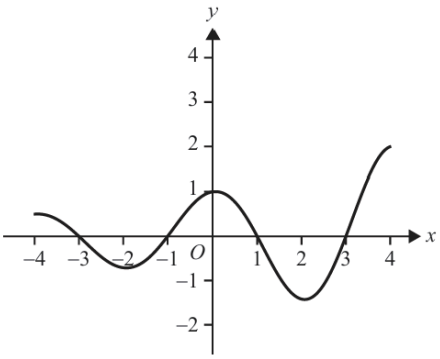
Use the multiple-choice sheet for Questions 1-10

1 What is the 20th term of the following arithmetic sequence

$$-7, -3, 1, 5, \ldots$$

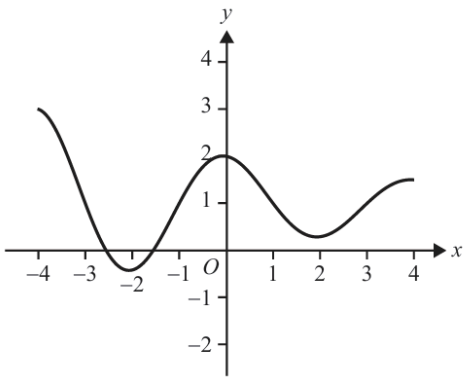
A. 65
B. 69
C. 73
D. 77

2 The graph of $y = f(x)$ is shown below.

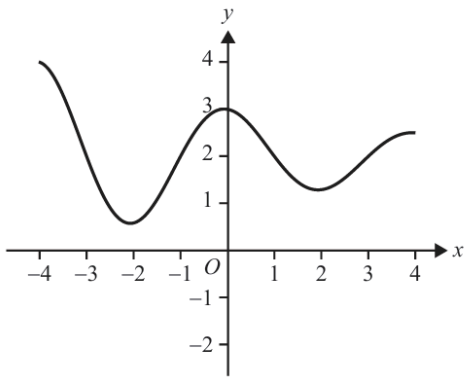


Which of the following options best represents the graph of $y = f(-x) + 2$?

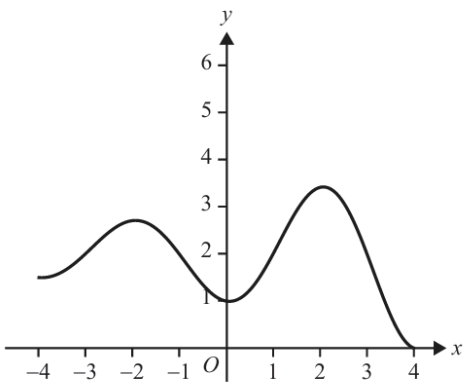
A.



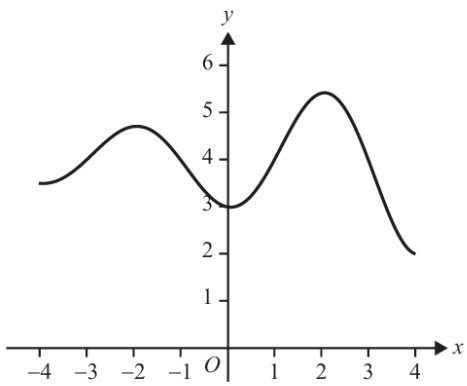
B.



C.



D.



3 The equation $x^2 + y^2 - 2x + 6y + 6 = 0$ represents a circle with centre C and radius r

Which of the following is correct?

A. $C = (1, -3)$ and $r = 2$

B. $C = (1, -3)$ and $r = 4$

C. $C = (-1, 3)$ and $r = 2$

D. $C = (-1, 3)$ and $r = 4$

4 Which of the following is equivalent to $-\log_a\left(\frac{x}{y}\right) - \log_a y$?

A. $\log_a\left(\frac{1}{x}\right)$

B $\log_a\left(\frac{1}{y}\right)$

C. $\log_a\left(\frac{x}{y^2}\right)$

D. $\log_a\left(\frac{y}{x}\right)$

5 A fair spinner numbered 1 to 12 is spun once.

Let A be the event that the number spun is odd, and let B be the event that the number spun is greater than 6.

Which statement is true?

A. A and B are independent.

B. A and B are mutually exclusive.

C. A and B are neither independent nor mutually exclusive.

D. A and B are both independent and mutually exclusive.

6 A camera uses the exposure value $E = \log_2\left(\frac{1}{t}\right)$ where t is the shutter time.

If the shutter time is halved, E increases by

A. 0.5

B. 1

C. 2

D. $\log_2(2t)$

7 What is the number of solutions to the equation $\sin 2x\,(\sec x - 1) = 0$ for $0 \leq x \leq 2\pi$

A. 2

B. 3

C. 5

D. 7

8 One day, at a particular school, m students walked to school and the remaining n students travelled to school using a different form of transport.

Of the m students who walked, 20% took at least 30 minutes to get to school.
Of the n students who used a different form of transport, 40% took at least 30 minutes to get to school.

Given that a randomly selected student took at least 30 minutes to get to school, the probability they walked to school is given by

A. $\dfrac{m}{m+n}$

B. $\dfrac{2n}{m+n}$

C. $\dfrac{m}{5(m+n)}$

D. $\dfrac{m}{m+2n}$

9 Let $f(x) = ae^{kx}$, where $a > 0$ and $k > 0$

Suppose that $f(p) = 3$ and $f(q) = 12$.

What is the value of $f\left(\dfrac{p+q}{2}\right)$?

A. 6

B. 7.5

C. 6.5

D. Cannot be determined with the given information.

10 What is the value of $\int_0^3 f(x)\,dx$ given that $f(x) + f(3-x) = 10$, for all $x \in [0,3]$?

A. 10

B. 15

C. 20

D. 30

SECTION II

90 Marks
Attempt Questions 11-35
Allow about 2 hours and 45 minutes for this section

Question 11

Find the sum of the following arithmetic series **2**

$$2 + 5 + 8 + \ldots + 53$$

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Question 12

Solve $|2x - 5| = 3$ **2**

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Question 13 **2**

Solve $2x^2 + 5x - 12 > 0$

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Question 14

Find the equation of the tangent to the curve $f(x) = 6\sqrt{x+1} + 5$ at the point where $x = 8$ **3**

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Question 15

Stations A and B are 40km apart where station B is due east of A.

A flare at point F is observed on a bearing of $055°T$ from A and a bearing of $315°T$ from B.

(a) Sketch a diagram showing the above information. **1**

(b) Find the distance from Station A to the flare. **2**

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Question 16 **2**

Differentiate $f(x) = x^2 \cos x$

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Question 17 **3**

Given $f(x) = \sqrt{x+1}$ and $g(x) = 2x - 3$.

Find the domain and range of $f\big(g(x)\big)$.

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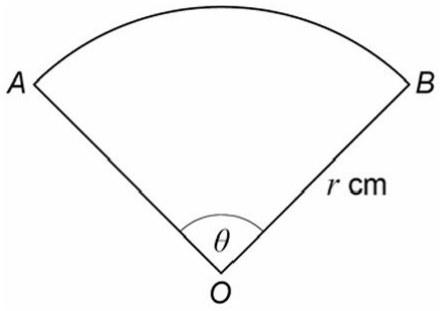
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Question 18

The diagram below shows a sector AOB of a circle with centre O and radius r **4**



The angle AOB is θ radians

The sector has an area of $9\ \text{cm}^2$ and perimeter 15cm.

Find the value of θ and r

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Question 19

Adrian picks two notes at random from five \$5, three \$10 and two \$20 notes.

Find the probability that Adrian has picked a total of at least \$20. **3**

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Question 20

Evaluate $\int_0^1 \frac{4x+2}{x^2+x+1}\,dx$. **2**

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Question 21

(a) Sketch the curve $y = \ln(x + 1)$, showing all important features. **2**

(b) Using the trapezoidal rule with 4 subintervals, estimate the value of $\int_1^3 \ln(x + 1)\, dx$ **2**

correct to 3 decimal places

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Question 22

The amount of money gained by an entrepreneur doubles every day. On day 1, she earns $\$9$ and on day 2 she earns $\$18$. **3**

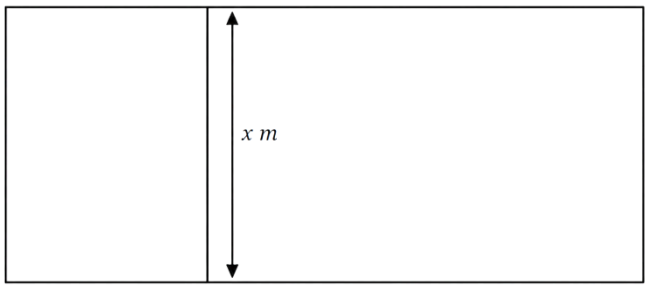
On which day will her cumulative earnings first exceed $\$1{,}000{,}000$?

Question 23

Prove that $\dfrac{\sin x}{1+\cos x}+\dfrac{1+\cos x}{\sin x}=2\operatorname{cosec} x$ **3**

Question 24

A farmer has a large patch of land, on which he needs to build two fenced-in, rectangular areas as shown in the diagram.



(a) Given the farmer has 260m of fencing, show that the total area of the two rectangles is given by **1**

$$A = 130x - \frac{3x^2}{2}$$

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(b) Hence or otherwise, find the maximum area of the paddock, showing that it is a maximum area. **3**

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Question 25

In a survey of 200 students.

- 120 play soccer
- 90 play basketball
- 170 play at least one of the two sports.

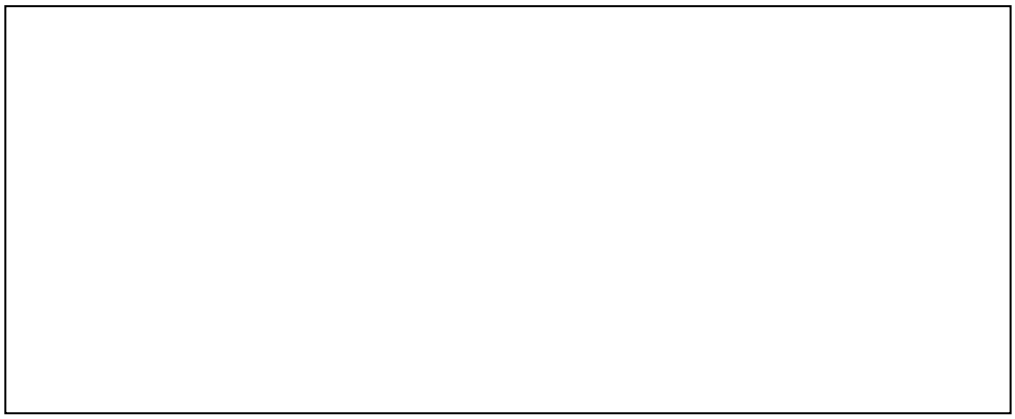
(a) By finding the number of students that play both sports. Draw a Venn diagram showing all the relevant information. **2**

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(b) Two students are randomly selected without replacement. Find the probability that at least one of them plays exactly one of the two sports. **2**

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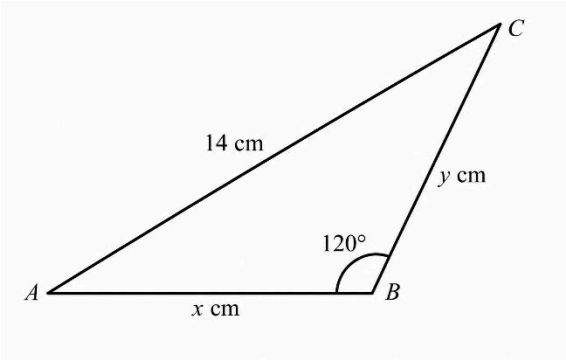
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Question 26

In the triangle ABC below, $AB = x$ cm, $BC = y$ cm. $AC = 14$ cm and angle $ABC = 120°$



(a) Show that $x^2 + y^2 + xy = 196$ **1**

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(b) Given that the perimeter of triangle ABC is 30 cm, find the exact area of triangle ABC **3**

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Question 27

Let $f(x) = \dfrac{1}{e^x + 1}$ and $g(x) = f(x) - \dfrac{1}{2}$ **3**

Prove that $g(x)$ is an **odd** function

Question 28

Solve $\ln(6e^x + 1) + x = 0$, writing your answer in logarithmic form **3**

Question 29

A particle, P, travels in a straight line from O such that its velocity, v m/s, at time t seconds after passing O is given by $v = 24\left(e^{-\frac{t}{6}} - \frac{1}{e}\right)$

(a) Find the value of t at which the particle P is instantaneously at rest. **2**

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(b) Find the total distance travelled by the particle P for the first 9 seconds to the nearest metre. **3**

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Question 30

(a) Show that $\frac{d}{dx}\left(\frac{\sin x - \cos x}{\sin x + \cos x}\right) = \frac{2}{(\sin x + \cos x)^2}$ **2**

(b) Hence evaluate $\int_0^{\frac{\pi}{4}} \frac{\sin x - \cos x + 2}{(\sin x + \cos x)^2}\, dx$ **3**

Question 31

Given the function $f(x) = \ln(x^2 + 1) - x$

(a) Find the stationary point(s) and determine their nature. **3**

(b) Find the point(s) of inflection. **2**

(c) Sketch the graph of $y = f(x)$, $\text{for} -2 \leq x \leq 2$ showing all important features. **2**

Question 32

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The temperature T °C inside a greenhouse is modelled by

$$T = 20 - 8\cos\left(\frac{\pi}{12}(t-4)\right)$$

where t is the number of hours after midnight.

An automatic shade cloth is deployed whenever the temperature exceeds 26°C.

Determine the total amount of time each day that the shade cloth is deployed.

Question 33

(a) Sketch the graphs of $y = -2\cos 3x + 1$ and $y = -\frac{2}{\pi}x + 1$ on the same axes, showing all important features, for $0 \leq x \leq \pi$. **3**

(b) Explain how the number of solutions for $2\cos 3x = \frac{2}{\pi}x$ can be found using the graph drawn in part (a) **1**

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Question 34 **4**

Let g be the function obtained from f by a sequence of transformations such that

$$g(x) = af(b - x) + c.$$

The point $(2, 5)$ on the graph of $y = f(x)$ corresponds to the point $(7, -11)$ on the graph of $y = g(x)$.

The point $(4, 1)$ on the graph of $y = f(x)$ corresponds to the point $(5, -3)$ on the graph of $y = g(x)$.

Find the values of a, b and c.

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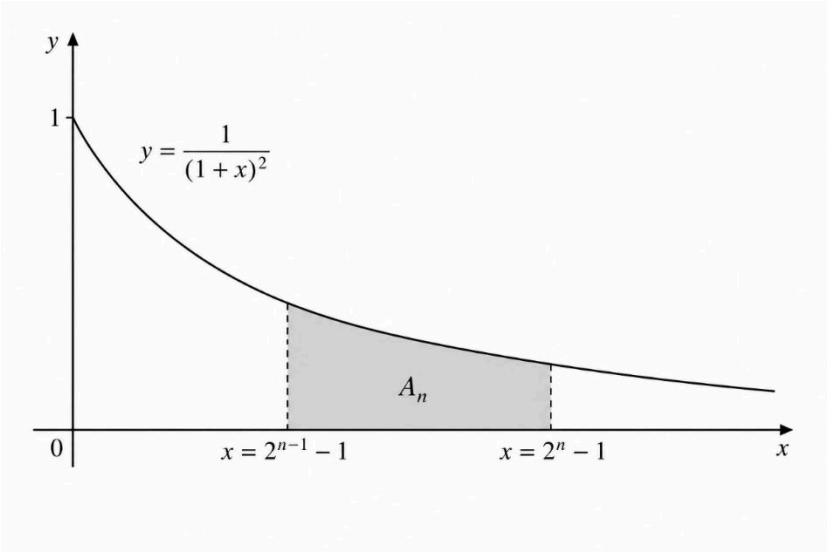
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Please turn the page for Question 35.

Question 35

Let $f(x) = \dfrac{1}{(1+x)^2}, x \geq 0.$

The shaded region $A_n$is bounded by the curve $y = f(x)$, the x-axis, and the vertical lines $x = 2^{n-1} - 1$ and $x = 2^n - 1$ as shown below.



(a) Show that $A_n = \frac{1}{2^n}$ **2**

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(b) Show that $A_n = A_{n+1} + A_{n+2} + A_{n+3} + \cdots$ **2**

(c) Find the value of S given that $S = A_1 + 2A_2 + 3A_3 + 4A_4 + \cdots$. **3**

End of Examination

Extra Working Space

Extra Working Space

Extra Working Space

Extra Working Space

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Section I

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Allow about 15 minutes for this section

Use the multiple-choice sheet for Questions 1-10

1 What is the 20th term of the following arithmetic sequence

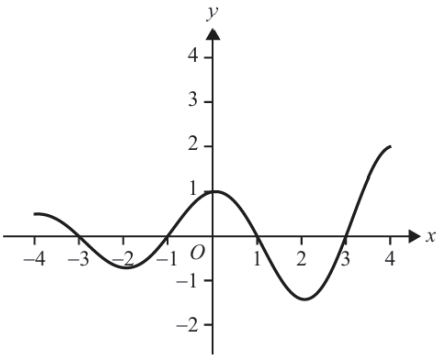
$$-7, -3, 1, 5, \ldots$$

$a = -7$
$d = 4$

$T_{20} = a + (n-1)d$
$= -7 + 19 \times 4$
$= 69$

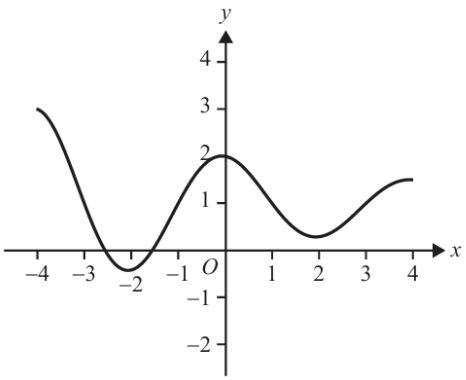
A. 65
B. 69
C. 73
D. 77

2 The graph of $y = f(x)$ is shown below.

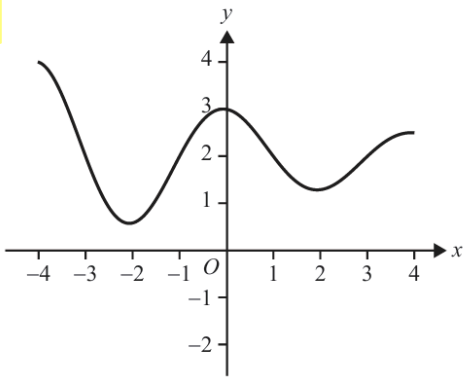


Which of the following options best represents the graph of $y = f(-x) + 2$?

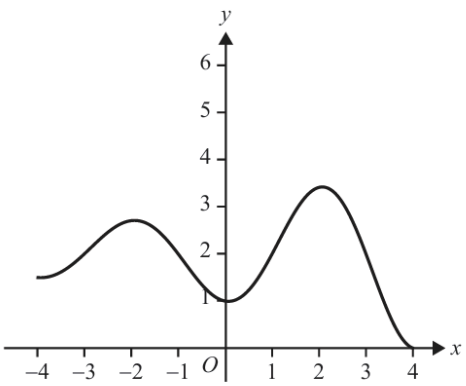
A.



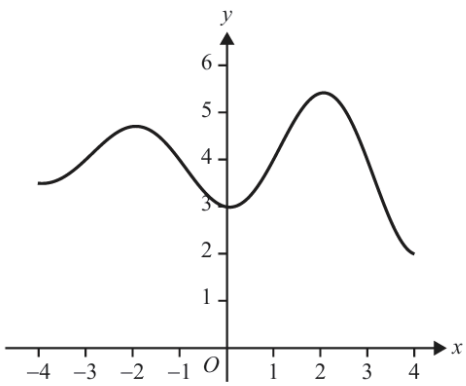
B.



C.



D.



3 The equation $x^2 + y^2 - 2x + 6y + 6 = 0$ represents a circle with centre C and radius r

Which of the following is correct?

$(x-1)^2 + (y-3)^2 = -6+1+9$
$= 4$

A. $C = (1, -3)$ and $r = 2$

B. $C = (1, -3)$ and $r = 4$

C. $C = (-1, 3)$ and $r = 2$

D. $C = (-1, 3)$ and $r = 4$

4 Which of the following is equivalent to $-\log_a\left(\frac{x}{y}\right) - \log_a y$?

$\log_a\left(\frac{y}{x}\right) - \log y$
$= \log_a\left(\frac{1}{x}\right)$

A. $\log_a\left(\frac{1}{x}\right)$

B $\log_a\left(\frac{1}{y}\right)$

C. $\log_a\left(\frac{x}{y^2}\right)$

D. $\log_a\left(\frac{y}{x}\right)$

5 A fair spinner numbered 1 to 12 is spun once.

Let A be the event that the number spun is odd, and let B be the event that the number spun is greater than 6.

Which statement is true?

$A = \{1, 3, 5, 7, 9, 11\}$
$P(A) = \frac{6}{12} = \frac{1}{2}$
$B = \{7, 8, 9, 10, 11, 12\}$
$P(B) = \frac{6}{12} = \frac{1}{2}$
$A \cap B = \{7, 9, 11\}$
$P(A \cap B) = \frac{1}{4}$
$P(A) \times P(B) = \frac{1}{4}$

A. A and B are independent.

B. A and B are mutually exclusive.

C. A and B are neither independent nor mutually exclusive.

D. A and B are both independent and mutually exclusive.

6 A camera uses the exposure value $E = \log_2\left(\frac{1}{t}\right)$ where t is the shutter time.

If the shutter time is halved, E increases by

A. 0.5

B. 1

C. 2

D. $\log_2(2t)$

7 What is the number of solutions to the equation $\sin 2x\,(\sec x - 1) = 0$ for $0 \leq x \leq 2\pi$

A. 2

B. 3

C. 5

D. 7

$\sin 2x = 0$

$2x = 0, \pi, 2\pi, 3\pi, 4\pi$

$x = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi$

$\sec x = 1$

$\frac{1}{\cos x} = 1$

$\therefore x = 0, 2\pi$

but $x \neq \frac{\pi}{2}, \frac{3\pi}{2}$

$\therefore x = 0, \pi, 2\pi$

8 One day, at a particular school, m students walked to school and the remaining n students travelled to school using a different form of transport.

Of the m students who walked, 20% took at at least 30 minutes to get to school.
Of the n students who used a different form of transport, 40% took at least 30 minutes to get to school.

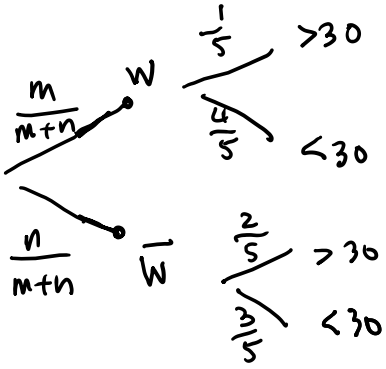
Given that a randomly selected student took at least 30 minutes to get to school, the probability they walked to school is given by

A. $\frac{m}{m+n}$

B. $\frac{2n}{m+n}$

C. $\frac{m}{5(m+n)}$

D. $\frac{m}{m+2n}$



$P(W \mid >30) = \frac{P(W \cap >30)}{P(>30)}$

$= \frac{\frac{m}{5(m+n)}}{\frac{m}{5(m+n)} + \frac{2n}{5(m+n)}}$

$= \frac{m}{m+2n}$

9 Let $f(x) = ae^{kx}$, where $a > 0$ and $k > 0$

Suppose that $f(p) = 3$ and $f(q) = 12$.

What is the value of $f\left(\frac{p+q}{2}\right)$?

A. 6

B. 7.5

C. 6.5

D. Cannot be determined with the given information.

$f(p) = 3 \Rightarrow ae^{pk} = 3 \therefore e^{kp} = \frac{3}{a}$

$f(q) = 12 \Rightarrow ae^{qk} = 12 \therefore e^{qk} = \frac{12}{a}$

Now $f\left(\frac{p+q}{2}\right) = ae^{k\left(\frac{p+q}{2}\right)}$

$= ae^{\frac{kp}{2}} \cdot e^{\frac{kq}{2}}$

$= a\left(e^{kp}\right)^{\frac{1}{2}} \cdot \left(e^{kq}\right)^{\frac{1}{2}}$

$= a \cdot \sqrt{\frac{3}{a}} \cdot \sqrt{\frac{12}{a}}$

$= \sqrt{36}$

10 What is the value of $\int_0^3 f(x)\,dx$ given that $f(x) + f(3-x) = 10$, for all $x \in [0,3]$?

A. 10

B. 15

C. 20

D. 30

$f(3-x) \Rightarrow f(x)$ translated to the left by 3 units and flipped around y axis.

This is equivalent to flipping around the line $x = \frac{3}{2}$

$\therefore \int_0^3 f(3-x)\,dx = \int_0^3 f(x)$

$\therefore \int_0^3 f(x) + f(3-x)\,dx = 2\int_0^3 f(x)\,dx$

Now

$\int_0^3 f(x) + f(3-x)\,dx$

$= \int_0^3 10\,dx$

$= 30$

$\therefore \int_0^3 f(x)\,dx = 15$

SECTION II

90 Marks
Attempt Questions 11-34
Allow about 2 hours and 45 minutes for this section

Question 11

Find the sum of the following arithmetic series **2**

$$2 + 5 + 8 + \ldots + 53$$

$a = 2, \; d = 3$

$\therefore \quad 53 = 2 + (n-1)3$

$51 = 3(n-1)$

$n - 1 = 17$

$n = 18$

$\therefore \quad S_{18} = \frac{18}{2}(2 + 53)$

$= 9 \times 55$

$= 495$

Question 12

Solve $|2x - 5| = 3$ **2**

$2x - 5 = \pm 3$

$2x = 5 \pm 3$

$\therefore \; 2x = 8 \quad \text{or} \quad 2x = 2$

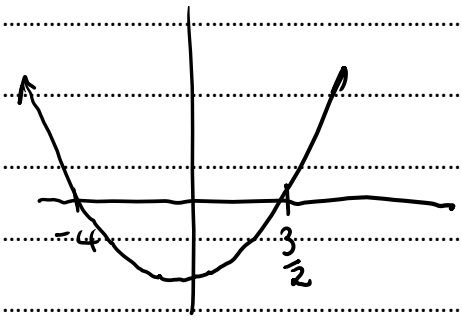
$x = 4 \quad \text{or} \quad x = 1$

Question 13 **2**

Solve $2x^2 + 5x - 12 > 0$

$2x^2+5x-12 > 0$

$(2x-3)(x+4) > 0$



$\therefore \quad x < -4 \quad \text{or} \quad x > \frac{3}{2}$

Question 14

Find the equation of the tangent to the curve $f(x) = 6\sqrt{x+1} + 5$ at the point where $x = 8$ **3**

$f'(x) = \frac{3}{\sqrt{x+1}}$

$f'(8) = \frac{3}{3} = 1$

$f(8) = 6\sqrt{9} + 5$

$= 23$

$\therefore$ equation of tangent: $y - 23 = 1(x - 8)$

$y = x - 8 + 23$

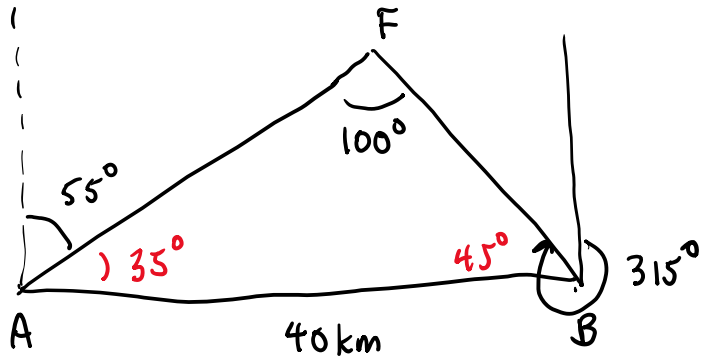
$= x + 15$

Question 15

Stations A and B are 40km apart where station B is due east of A.

A flare at point F is observed on a bearing of $055°T$ from A and a bearing of $315°T$ from B.

(a) Sketch a diagram showing the above information. **1**



(b) Find the distance from Station A to the flare. **2**

$$\frac{AF}{\sin 45^\circ} = \frac{40}{\sin 100^\circ}$$

$$AF = \frac{40 \sin 45^\circ}{\sin 100^\circ}$$

$$\approx 28.7 \text{ km}$$

Question 16 **2**

Differentiate $f(x) = x^2 \cos x$

$u = x^2$ $\quad v = \cos x$

$u' = 2x$ $\quad v' = -\sin x$

$\therefore f'(x) = 2x\cos x - x^2 \sin x$

Question 17 **3**

Given $f(x) = \sqrt{x+1}$ and $g(x) = 2x - 3$.

Find the domain and range of $f(g(x))$.

$f(g(x)) = \sqrt{(2x-3)+1}$

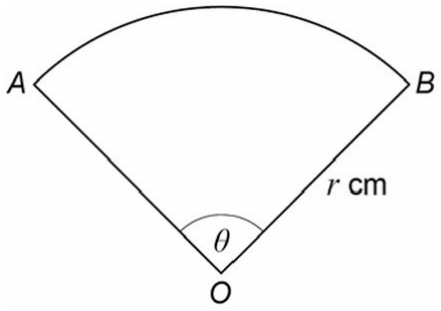
$= \sqrt{2x-2}$

$\therefore 2x - 2 \geq 0$

$\therefore D: x \geq 1$ $\quad R: y \geq 0$

Question 18

The diagram below shows a sector AOB of a circle with centre O and radius r **4**



The angle AOB is θ radians

The sector has an area of $9\ \text{cm}^2$ and perimeter 15cm.

Find the value of θ and r

$A = \frac{1}{2}r^2\theta \Rightarrow \frac{1}{2}r^2\theta = 9$ ①

$P = 2r + r\theta \Rightarrow 2r + r\theta = 15$

$\therefore r(2+\theta) = 15$ ②

From ① $\theta = \frac{18}{r^2}$

$\therefore r\left(2 + \frac{18}{r^2}\right) = 15$

$2r + \frac{18}{r} = 15$

$2r^2 + 18 = 15r$

$2r^2 - 15r + 18 = 0$

$(2r - 3)(r - 6) = 0$

$\therefore r = \frac{3}{2}$ or $r = 6$

$\therefore \theta = 8$ or $\theta = \frac{1}{2}$

as $8 > 2\pi$ reject

$\therefore r = 6\text{cm}$, $\theta = \frac{1}{2}$

Question 19

Adrian picks two notes at random from five \$5, three \$10 and two \$20 notes.

Find the probability that Adrian has picked a total of at least \$20. **3**

Possible outcomes: \$5 + \$20, \$10 + \$10, \$10 + \$20, \$20 + \$20

$P(5,20) = \left(\frac{5}{10} \times \frac{2}{9}\right) \times 2 = \frac{20}{90}$

$P(10,10) = \frac{3}{10} \times \frac{2}{9} = \frac{6}{90}$

$P(10,20) = \left(\frac{3}{10} \times \frac{2}{9}\right) \times 2 = \frac{12}{90}$

$P(20,20) = \frac{2}{10} \times \frac{1}{9} = \frac{2}{90}$

$\therefore \quad P = \frac{20 + 6 + 12 + 2}{90}$

$= \frac{4}{9}$

Question 20

Evaluate $\int_0^1 \frac{4x+2}{x^2+x+1}\,dx$. **2**

$I = \int_0^1 \frac{2(2x+1)}{x^2+x+1}\,dx$

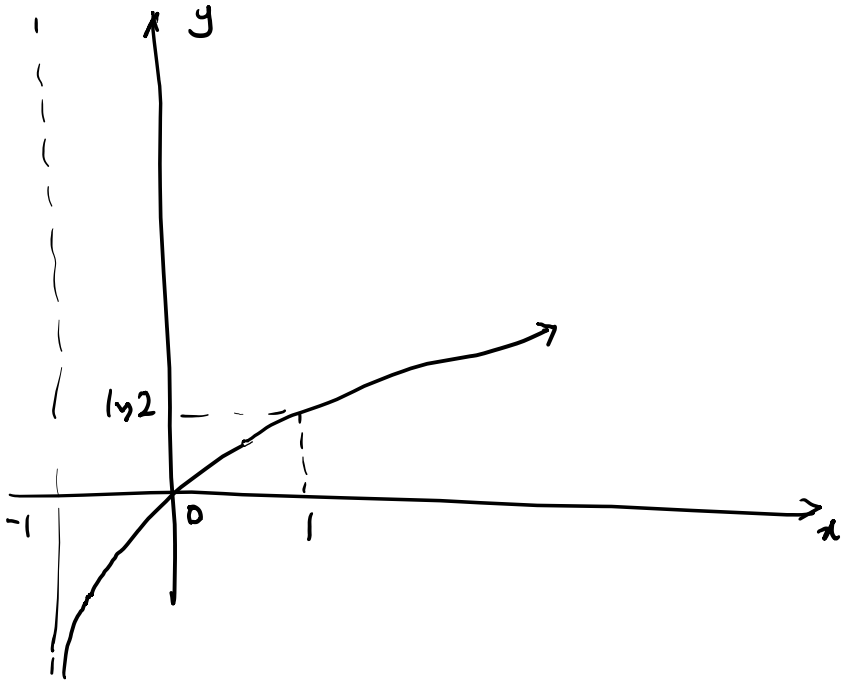
$= 2\left[\ln|x^2+x+1|\right]_0^1$

$= 2[\ln 3 - \ln 1]$

$= 2\ln 3$

Question 21

(a) Sketch the curve $y = \ln(x + 1)$, showing all important features. **2**



(b) Using the trapezoidal rule with 4 subintervals, estimate the value of $\int_1^3 \ln(x + 1)\, dx$ correct to 3 decimal places **2**

x	1	1.5	2	2.5	3
y	$\ln 2$	$\ln 2.5$	$\ln 3$	$\ln 3.5$	$\ln 4$

$$\therefore \int_1^3 \ln(x+1)\,dx \approx \frac{0.5}{2}\left[\ln 2 + \ln 4 + 2(\ln 2.5 + \ln 3 + \ln 3.5)\right]$$

$$\approx 2.154 \text{ (to 3 d.p)}$$

Question 22

The amount of money gained by an entrepreneur doubles every day. On day 1, she earns \$9 and on day 2 she earns \$18. **3**

On which day will her cumulative earnings first exceed \$1,000,000?

$T_n = 9(2^{n-1})$

$\therefore$ Cumulative earning: $S_n = \frac{9(2^n - 1)}{2-1}$

$= 9(2^n - 1)$

$\therefore \quad 9(2^n - 1) > 1{,}000{,}000$

$2^n > \frac{1{,}000{,}000}{9} + 1$

$n > \log_2(111112.111\ldots)$

$\doteq 16.76$

By day 17

Question 23

Prove that $\dfrac{\sin x}{1+\cos x} + \dfrac{1+\cos x}{\sin x} = 2\operatorname{cosec} x$ **3**

$LHS = \frac{\sin^2 x + (1+\cos x)^2}{\sin x(1+\cos x)}$

$= \frac{\sin^2 x + 1 + 2\cos x + \cos^2 x}{\sin x(1+\cos x)}$

$= \frac{2 + 2\cos x}{\sin x(1+\cos x)}$

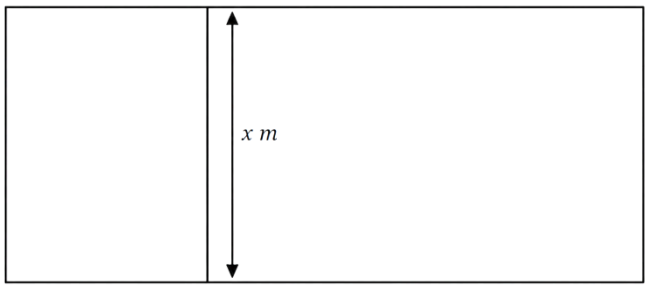
$= \frac{2(1+\cos x)}{\sin x(1+\cos x)}$

$= \frac{2}{\sin x}$

$= RHS$

Question 24

A farmer has a large patch of land, on which he needs to build two fenced-in, rectangular areas as shown in the diagram.



(a) Given the farmer has 260m of fencing, show that the total area of the two rectangles is given by **1**

$$A = 130x - \frac{3x^2}{2}$$

Let the horizontal length be L

$\therefore \quad 2L + 3x = 260$

$L = 130 - \frac{3x}{2}$

Now $A = xL$

$= x\left(130 - \frac{3x}{2}\right)$

$= 130x - \frac{3x^2}{2}$

(b) Hence or otherwise, find the maximum area of the paddock, showing that it is a maximum area. **3**

$A' = 130 - 3x$

Stationary when $A' = 0$

$\therefore 3x = 130$

$x = \frac{130}{3}$

testing for maximum:

$A'' = -3 < 0$

$\therefore$ always concave down

$\therefore x = \frac{130}{3}$ gives maximum area

$\therefore$ max area $= 130\left(\frac{130}{3}\right) - \frac{3\left(\frac{130}{3}\right)^2}{2}$

$= \frac{8450}{3} m^2$

$\fallingdotseq 2816.7 m^2$

Question 25

In a survey of 200 students.

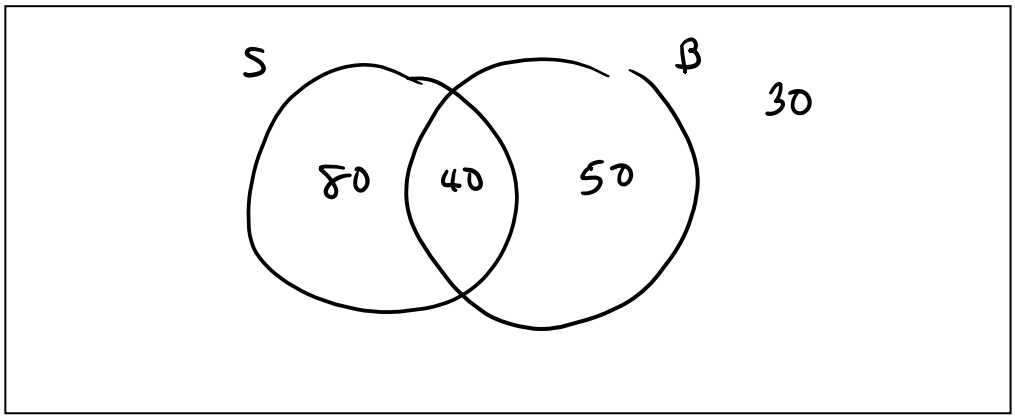
- 120 play soccer
- 90 play basketball
- 170 play at least one of the two sports.

(a) By finding the number of students that play both sports. Draw a Venn diagram showing all the relevant information. **2**

$n(S \cup B) = n(S) + n(B) - n(S \cap B)$

$\therefore \quad 170 = 120 + 90 - n(S \cap B)$

$\therefore \quad n(S \cap B) = 40$



(b) Two students are randomly selected without replacement. Find the probability that at least one of them plays exactly one of the two sports. **2**

There are 130 students that does exactly one sport.

$\therefore$ n(not exactly one sport) $= 200 - 130 = 70$

Probability at least two selected play exactly one sport:

$= 1 - P$(neither selected plays exactly one sport)

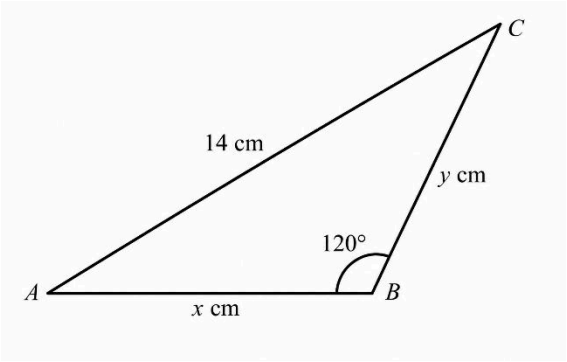
$= 1 - \frac{70}{200} \cdot \frac{69}{199}$

$= 1 - \frac{4830}{39800}$

$= \frac{3497}{3980}$

Question 26

In the triangle ABC below, $AB = x$ cm, $BC = y$ cm. $AC = 14$ cm and angle $ABC = 120°$



(a) Show that $x^2 + y^2 + xy = 196$ **1**

Using the cosine rule

$AC^2 = AB^2 + BC^2 - 2(AB)(BC)\cos\angle ABC$

$14^2 = x^2 + y^2 - 2xy\cos 120°$

Since $\cos 120° = -\frac{1}{2}$

$196 = x^2 + y^2 + xy$

i.e. $x^2 + y^2 + xy = 196$

(b) Given that the perimeter of triangle ABC is 30 cm, find the exact area of triangle ABC **3**

$\because$ Perimeter $= 30$cm $\Rightarrow$ $x + y + 14 = 30$

$\therefore x + y = 16$

$\therefore x^2 + 2xy + y^2 = 256$

Subtract expression from part (a)

$xy = 60$

$\therefore A = \frac{1}{2}xy\sin 120°$

$= \frac{1}{2} \times 60 \times \frac{\sqrt{3}}{2}$

$= 15\sqrt{3}$ cm^2

Question 27

Let $f(x) = \frac{1}{e^x + 1}$ and $g(x) = f(x) - \frac{1}{2}$ **3**

Prove that $g(x)$ is an **odd** function

$$g(-x) = f(-x) - \frac{1}{2}$$

$$= \frac{1}{e^{-x}+1} - \frac{1}{2}$$

$$= \frac{e^x}{1+e^x} - \frac{1}{2}$$

$$= \frac{2e^x - (1+e^x)}{2(1+e^x)}$$

$$= \frac{e^x - 1}{2(1+e^x)}$$

Now $-g(x) = \frac{1}{2} - f(x)$

$$= \frac{1}{2} - \frac{1}{e^x+1}$$

$$= \frac{e^x + 1 - 2}{2(e^x+1)}$$

$$= \frac{e^x - 1}{2(e^x+1)}$$

$$= g(-x)$$

Question 28

Solve $\ln(6e^x + 1) + x = 0$, writing your answer in logarithmic form **3**

$\ln(6e^x+1) = -x$

$\therefore \quad 6e^x + 1 = e^{-x}$

$6e^{2x} + e^x = 1$

$6e^{2x} + e^x - 1 = 0$

let $u = e^x$

$\therefore \quad 6u^2 + u - 1 = 0$

$(3u-1)(2u+1) = 0$

$\therefore u = \frac{1}{3}$ or $u = -\frac{1}{2}$

$e^x = \frac{1}{3}$ or $e^x = -\frac{1}{2}$

$x = \ln\left(\frac{1}{3}\right)$ $\quad \therefore x$ has no solutions

$= -\ln 3$

Examination continues on next page

Question 29

A particle, P, travels in a straight line from O such that its velocity, v m/s, at time t seconds after passing O is given by $v = 24\left(e^{-\frac{t}{6}} - \frac{1}{e}\right)$

(a) Find the value of t at which the particle P is instantaneously at rest. **2**

at rest: $v = 0 \Rightarrow 24\left(e^{-\frac{t}{6}} - \frac{1}{e}\right) = 0$

$e^{-\frac{t}{6}} - \frac{1}{e} = 0$

$e^{-\frac{t}{6}} = \frac{1}{e}$

$-\frac{t}{6} = \ln\left(\frac{1}{e}\right)$

$= -1$

$t = 6$

(b) Find the total distance travelled by the particle P for the first 9 seconds to the nearest metre. **3**

$x = 24\int e^{-\frac{t}{6}} - \frac{1}{e}\, dt$

$= 24\left(-6 \cdot e^{-\frac{t}{6}} - \frac{1}{e}t\right) + C$

$= -144e^{-\frac{t}{6}} - \frac{24}{e}t + C$

when $t = 0 \quad x = -144 + C$

when $t = 6 \quad x = -144e^{-1} - \frac{144}{e} + C$

$= -\frac{288}{e} + C$

when $t = 9 \quad x = -144e^{-\frac{3}{2}} - \frac{216}{e} + C$

$\therefore$ distance travelled from $t = 0$ to $t = 6$: $144 - \frac{288}{e}$

distance from $t = 6$ to 9 : $144e^{-\frac{3}{2}} - \frac{72}{e}$

$\therefore$ total distance: $144 - \frac{360}{e} + 144e^{-\frac{3}{2}}$

≈ 44 m (to nearest metre)

Question 30

(a) Show that $\frac{d}{dx}\left(\frac{\sin x - \cos x}{\sin x + \cos x}\right) = \frac{2}{(\sin x + \cos x)^2}$ **2**

$u = \sin x - \cos x \qquad v = \sin x + \cos x$

$u' = \cos x + \sin x \qquad v' = \cos x - \sin x$

$\therefore \frac{d}{dx}\left(\frac{\sin x - \cos x}{\sin x + \cos x}\right) = \frac{(\sin x + \cos x)^2 + (\sin x - \cos x)^2}{(\cos x + \sin x)^2}$

$= \frac{\sin^2 x + 2\cos x \sin x + \cos^2 x + \sin^2 x - 2\sin x \cos x + \cos^2 x}{(\cos x + \sin x)^2}$

$= \frac{2(\sin^2 x + \cos^2 x)}{(\cos x + \sin x)^2}$

$= \frac{2}{(\cos x + \sin x)^2}$

(b) Hence evaluate $\int_0^{\frac{\pi}{4}} \frac{\sin x - \cos x + 2}{(\sin x + \cos x)^2}\, dx$ **3**

$I = \int_0^{\frac{\pi}{4}} \frac{\sin x - \cos x}{(\sin x + \cos x)^2} + \frac{2}{(\sin x + \cos x)^2}\, dx$

$= -\int_0^{\frac{\pi}{4}} \frac{\cos x - \sin x}{(\sin x + \cos x)^2}\, dx + \int_0^{\frac{\pi}{4}} \frac{2}{(\sin x + \cos x)^2}\, dx$

$= \left[\frac{1}{\sin x + \cos x}\right]_0^{\frac{\pi}{4}} + \left[\frac{\sin x - \cos x}{\sin x + \cos x}\right]_0^{\frac{\pi}{4}}$

$= \left[\left(\frac{1}{\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}}\right) - \left(\frac{1}{1}\right)\right] + \left[\left(\frac{\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}}{0 + 1}\right) - \left(\frac{-1}{1}\right)\right]$

$= \left[\frac{\sqrt{2}}{2} - 1\right] + (1)$

$= \frac{\sqrt{2}}{2}$

Question 31

Given the function $f(x) = \ln(x^2 + 1) - x$

(a) Find the stationary point(s) and determine their nature. **3**

$f'(x) = \frac{2x}{x^2+1} - 1$

Stationary point when $f'(x) = 0$

$\therefore \frac{2x}{x^2+1} - 1 = 0$

$\therefore \frac{2x}{x^2+1} = 1$

$\therefore x^2 + 1 = 2x$

$x^2 - 2x + 1 = 0$

$\therefore (x-1)^2 = 0$

$x = 1$

when $x = 1$ $y = \ln 2 - 1$

Testing the nature

x	0	1	2
y'	-1	0	$-\frac{1}{5}$
	\	—	/

$\therefore (1, \ln 2 - 1)$ is a horizontal point of inflection

(b) Find the point(s) of inflection. **2**

$$f''(x) = \frac{(x^2+1)\cdot 2 - 2x\cdot 2x}{(x^2+1)^2}$$

$$= \frac{2x^2+2-4x^2}{(x^2+1)^2}$$

$$= \frac{2-2x^2}{(x^2+1)^2}$$

$$= \frac{2(1-x)(1+x)}{(x^2+1)^2}$$

∴ possible point of inflection when $f''(x)=0$

$\Rightarrow$ $x=1$ or $x=-1$

∴ $y=\ln 2-1$ or $y=\ln 2+1$

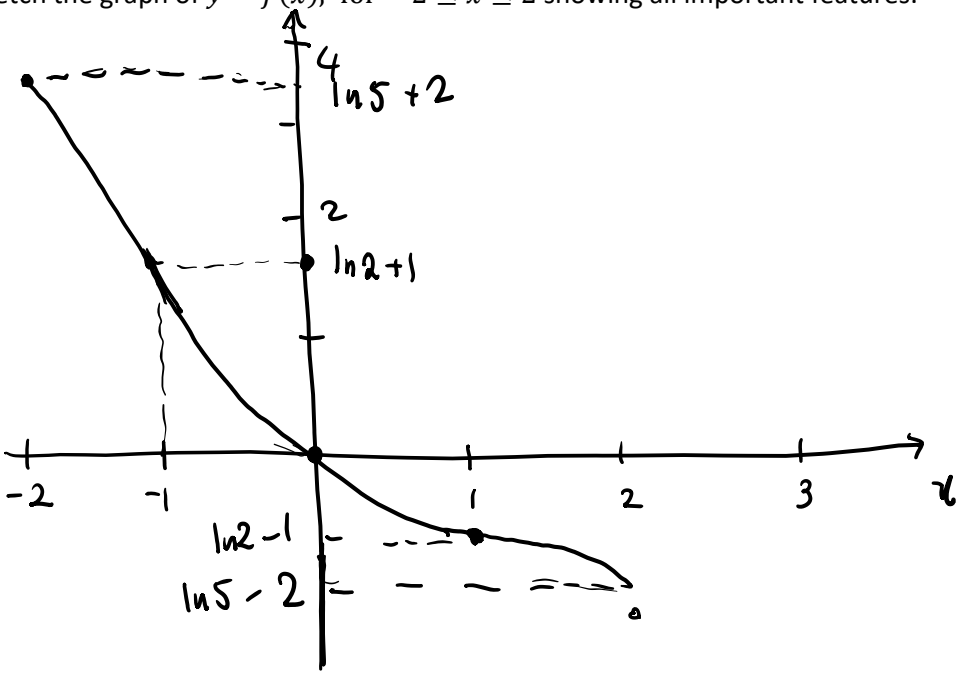
From (a) $(1, \ln 2-1)$ is a horizontal point of inflection

Testing $x=-1$

x	-2		-1		0
$f''(x)$	$-\frac{6}{25}$		0		2
	$\cap$	·		$\cup$	

∴ $(-1, \ln 2+1)$ is a point of inflection

(c) Sketch the graph of $y = f(x)$, for $-2 \le x \le 2$ showing all important features. **2**



Question 32

4

The temperature T °C inside a greenhouse is modelled by

$$T = 20 - 8\cos\left(\frac{\pi}{12}(t-4)\right)$$

where t is the number of hours after midnight.

An automatic shade cloth is deployed whenever the temperature exceeds 26°C.

Determine the total amount of time each day that the shade cloth is deployed.

$T = \frac{2\pi}{n} = \frac{2\pi}{\frac{\pi}{12}}$

$= 24$ hours

$\therefore$ so domain: $0 \leq t \leq 24$

$-\frac{\pi}{3} \leq \frac{\pi}{12}(t-4) \leq \frac{5\pi}{3}$

solve for equality:

$20 - 8\cos\left(\frac{\pi}{12}(t-4)\right) = 26$

$\cos\left(\frac{\pi}{12}(t-4)\right) = -\frac{3}{4}$

$\therefore$ in quadrants 2 and 3

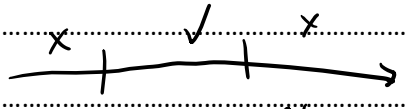
$\therefore \frac{\pi}{12}(t-4) = \pi - \cos^{-1}\left(\frac{3}{4}\right)$ or $\pi + \cos^{-1}\left(\frac{3}{4}\right)$

$t - 4 = 12 - \frac{12\cos^{-1}\left(\frac{3}{4}\right)}{\pi}$ or $12 + \frac{12\cos^{-1}\left(\frac{3}{4}\right)}{\pi}$

$t = 16 - \frac{12\cos^{-1}\left(\frac{3}{4}\right)}{\pi}$ or $16 + \frac{12\cos^{-1}\left(\frac{3}{4}\right)}{\pi}$

≈ 13.239 or 18.761

Testing region:



Test $t = 14$

$T = 26.9$ which satisfies the inequation

$\therefore$ total time $= 18.761 - 13.239$

≈ 5.52 hours

Question 33

3

(a) Sketch the graphs of $y = -2\cos 3x + 1$ and $y = -\frac{2}{\pi}x + 1$ on the same axes, for $0 \leq x \leq \pi$

$y = -\frac{2}{\pi}x + 1$

$\Rightarrow$ x int:

$\frac{2x}{\pi} = 1$

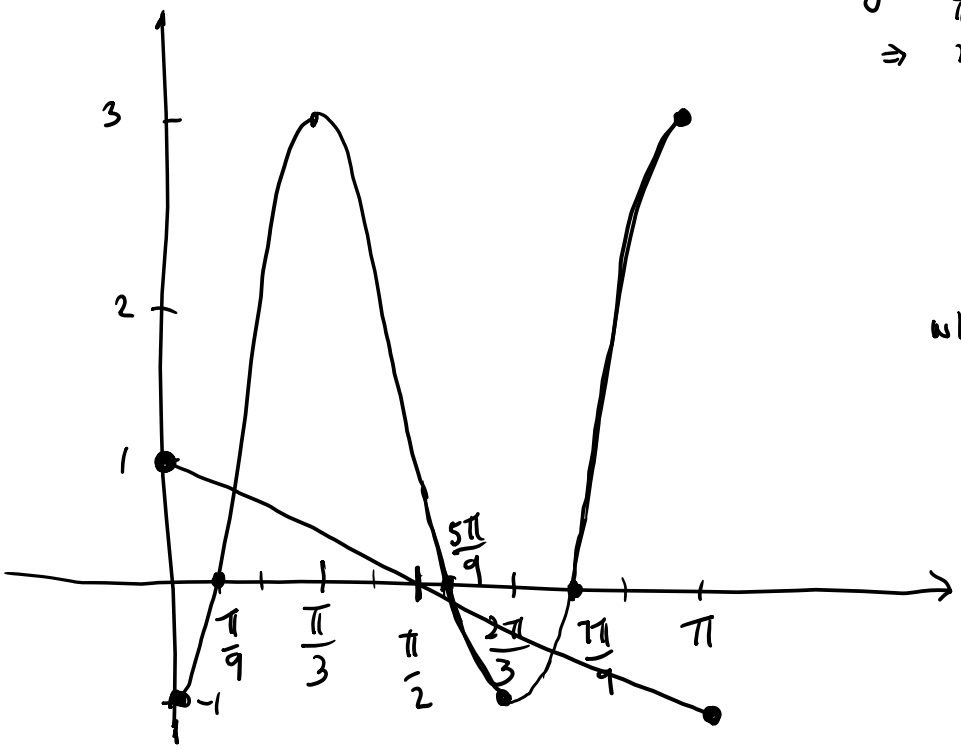
$2x = \pi$

$x = \frac{\pi}{2}$

when $x = \pi$

$y = -2 + 1$

$= -1$



$\frac{5\pi}{9} \approx 1.75$

$\frac{\pi}{2} \approx 1.57$

(b) Explain how the number of solutions for $2\cos 3x = \frac{2}{\pi}x$ can be found using the graph drawn in part (a)

1

The solutions to $2\cos 3x = \frac{2}{\pi}x$ are equivalent to the solutions to $2\cos 3x + 1 = \frac{2}{\pi}x + 1$

$\therefore$ we can see the number of solutions by looking at how many intersection points on the curve drawn in part (a)

Question 34 **4**

Let g be the function obtained from f by a sequence of transformations such that

$$g(x) = af(b - x) + c.$$

The point $(2, 5)$ on the graph of $y = f(x)$ corresponds to the point $(7, -11)$ on the graph of $y = g(x)$.

The point $(4, 1)$ on the graph of $y = f(x)$ corresponds to the point $(5, -3)$ on the graph of $y = g(x)$.

Find the values of a, b and c.

$$\frac{y-c}{a} = f(-x+b)$$

g(x) is transformed in the following sequence:

- vertical dilation by factor of a
- shifted up by c units
- shifted left by b units
- flipped around y axis.

Applying transformation on (2,5) yields

$$[-(2-b), 5a+c]$$

$$= [b-2, 5a+c] \quad \therefore b-2 = 7$$

$$\therefore \boxed{b = 9}$$

and $5a+c = -11$ (1)

Applying transformation on (4,1) yields

$$(-(4-b), a+c)$$

$b-4 = 5$ and $a+c = -3$ (2)

$b = 9$

(1) - (2)

$4a = -8$

$a = -2$

$\therefore c = -1$

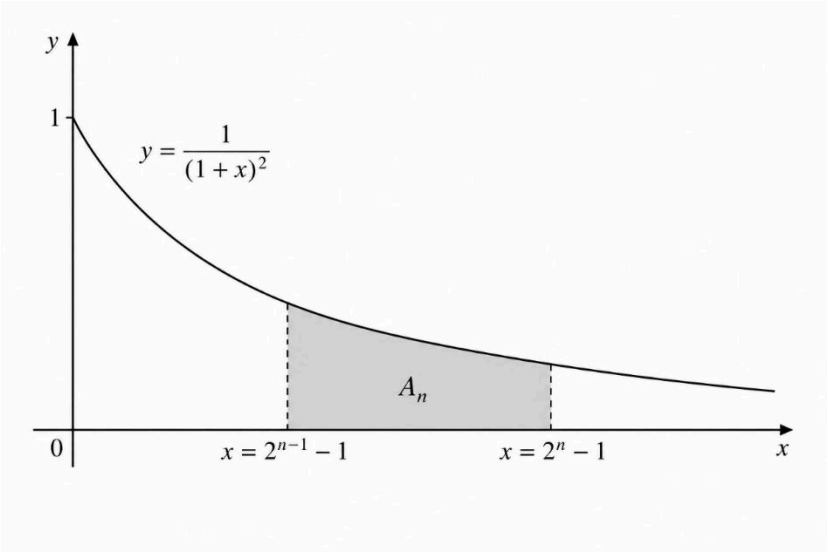
$\therefore \boxed{a = -2, \ b = 9 \text{ and } c = -1}$

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Question 35

Let $f(x) = \dfrac{1}{(1+x)^2}, x \geq 0.$

The shaded region A_n is bounded by the curve $y = f(x)$, the x-axis, and the vertical lines $x = 2^{n-1} - 1$ and $x = 2^n - 1$ as shown below.



(a) Show that $A_n = \frac{1}{2^n}$ **2**

$$A_n = \int_{2^{n-1}-1}^{2^n-1} (1+x)^{-2}\,dx$$

$$= \left[-(1+x)^{-1}\right]_{2^{n-1}-1}^{2^n-1}$$

$$= -\left[\frac{1}{1+x}\right]_{2^{n-1}-1}^{2^n-1}$$

$$= -\left[\frac{1}{1+2^n-1} - \frac{1}{1+2^{n-1}-1}\right]$$

$$= -\left(\frac{1}{2^n} - \frac{1}{2^{n-1}}\right)$$

$$= \frac{2}{2^n} - \frac{1}{2^n}$$

$$= \frac{1}{2^n}$$

(b) Show that $A_n = A_{n+1} + A_{n+2} + A_{n+3} + \cdots$ **2**

$A_{n+1} = \frac{1}{2^{n+1}}$, $A_{n+2} = \frac{1}{2^{n+2}}$, $A_{n+3} = \frac{1}{2^{n+3}}$

$\therefore RHS = \frac{1}{2^{n+1}} + \frac{1}{2^{n+2}} + \frac{1}{2^{n+3}} + \cdots$

this is a limiting sum with $a = \frac{1}{2^{n+1}}$ and $r = \frac{1}{2}$

$= \frac{\frac{1}{2^{n+1}}}{1 - \frac{1}{2}}$

$= \frac{1}{2^{n+1}} \times \frac{1}{2}$

$= \frac{1}{2^n}$

$= A_n$ (from part a)

(c) Find the value of S given that $S = A_1 + 2A_2 + 3A_3 + 4A_4 + \cdots$. **3**

$A_1 = \frac{1}{2^1}$, $A_2 = \frac{1}{2^2}$, $A_3 = \frac{1}{2^3}$ ...

$\therefore S = A_1 + 2A_2 + 3A_3 + 4A_4 + \cdots$

$= (A_1 + A_2 + A_3 + A_4 \cdots) + (A_2 + A_3 + A_4 + \cdots) + (A_3 + A_4 + A_5 \cdots) + \cdots$

$= \underbrace{\left(\frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \frac{1}{2^4} + \cdots\right)}_{\text{G.P } a = \frac{1}{2}\ r = \frac{1}{2}} + \left(\frac{1}{2^2} + \frac{1}{2^3} + \frac{1}{2^4} + \cdots\right) + \left(\frac{1}{2^3} + \frac{1}{2^4} + \frac{1}{2^5} + \cdots\right)$

$\text{G.P } a = \frac{1}{2^2}\ r = \frac{1}{2}$ etc.

$= \frac{\frac{1}{2}}{1 - \frac{1}{2}} + \frac{\frac{1}{4}}{1 - \frac{1}{2}} + \frac{\frac{1}{8}}{1 - \frac{1}{2}} + \cdots$

$= 2\left(\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots\right)$

$= 2\left(\frac{\frac{1}{2}}{1 - \frac{1}{2}}\right)$

$= 2$