North Sydney Girls High School

Student Number

2026 **HSC TRIAL EXAMINATION**

Mathematics Advanced

General Instructions

- Reading Time – 10 minutes
- Working Time – 3 hours
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided
- For questions in Section II, show relevant mathematical reasoning and/or calculations

Total marks: 100

Section I – 10 marks (pages 2 – 6)

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section

Section II – **90 marks** (pages 9 – 48)

- Attempt Questions 11 – 32
- Allow about 2 hours and 45 minutes for this section

Question	1–10	11–18	19-25	26-32	Total
Mark	/10	/30	/29	/31	/100

Section I

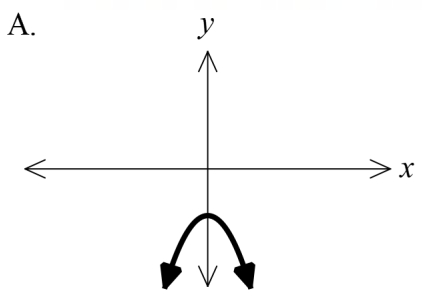
10 marks
Attempt Questions 1-10
Allow about 15 minutes for this section.

Use the multiple choice answer sheet for Questions 1-10.

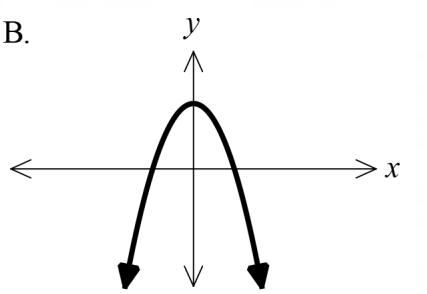
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1 Which graph best represents $y = 3 - x^2$?

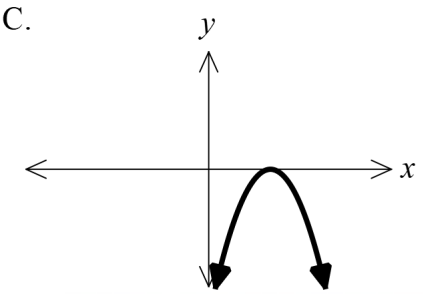
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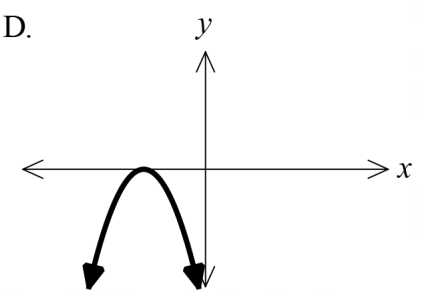
B.



C.



D.



2 What is the exact solution of the equation $3 \times 2^x = 15$?

A. $x = \log_2 5$

B. $x = \log_2 15$

C. $x = \frac{1}{3}\log_2 15$

D. $x = \log_e 5$

3 A certain recipe says the cooking time for a duck is directly proportional to its weight. The recipe says to cook a 3 kg duck for 2 hours.

How long should a 2 kg duck be cooked for?

A. 2 hours.

B. 3 hours.

C. 40 minutes.

D. 1 hour 20 minutes.

4 What is the expected value for the discrete probability distribution shown below?

x	-1	0	1	2
$p(x)$	0.2	a	0.3	0.4

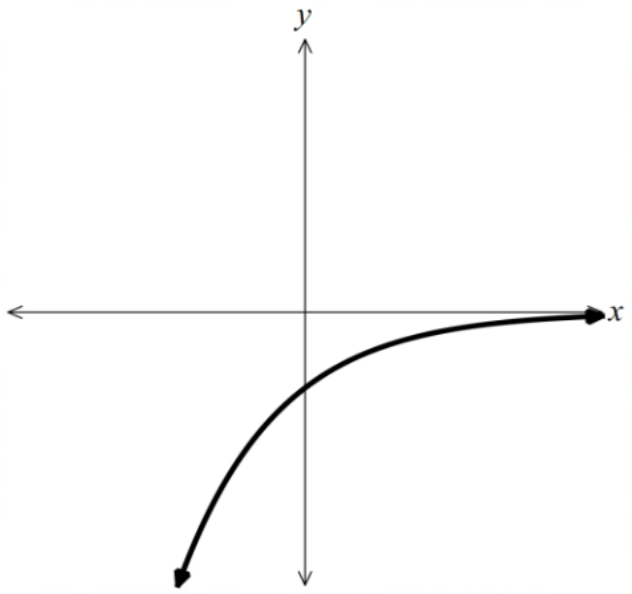
A. 0.1

B. $0.9 + a$

C. 2

D. 0.9

5 Which of the following could be the equation of the graph below?



A. $y = -\dfrac{1}{x}$

B. $y = e^{-x}$

C. $y = -e^{-x}$

D. $y = -e^{x}$

6 Which of the following is an expression for $\dfrac{d}{dx}\ln\left(3x^2\right)$?

A. $\dfrac{2}{x}$

B. $\dfrac{1}{6x}$

C. $6x$

D. $\dfrac{1}{x}$

7 Which of the following is an expression for $\int x^3\left(x^4-3\right)^5\,dx$?

A. $\dfrac{\left(x^4-3\right)^6}{24}+C$

B. $\dfrac{\left(x^4-3\right)^6}{6}+C$

C. $\dfrac{x^4\left(x^4-3\right)^6}{24}+C$

D. $\dfrac{\left(x^4-3\right)^6}{24x^4}+C$

8 If $f(2)=5$ and $f'(2)=-3$, what is the best estimate for $f(2.2)$?

A. 2.2

B. 4.4

C. 5

D. 5.6

9 What is the domain of the function $f(x) = \log_e(x+2) - \sqrt{11-x^2}$?

A. $\left[-\sqrt{11}, \sqrt{11}\right]$

B. $\left(-2, \sqrt{11}\right]$

C. $(-2, \infty)$

D. $\left(2, \sqrt{11}\right]$

10 If it is raining, the probability Bill is late to work is $\frac{3}{10}$, whereas, if it is not raining the probability of Bill being late is $\frac{1}{10}$.

The probability that it is raining is $\frac{2}{5}$.

What is the probability that it is raining given that Bill was late to work?

A. $\frac{2}{5}$

B. $\frac{9}{50}$

C. $\frac{2}{3}$

D. $\frac{3}{4}$

End of Section I

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Student Number

Mathematics Advanced
Section II Answer Booklet 1

1

Section II

90 marks
Attempt Questions 11–32
Allow about 2 hours and 45 minutes for this section

Booklet 1 – Attempt Questions 11–18 (30 marks)
Booklet 2 – Attempt Questions 19–25 (29 marks)
Booklet 3 – Attempt Questions 26–32 (31 marks)

Instructions

- Write your student number at the top of this page.
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Please turn over

Question 11 (2 marks)

Solve the equation $|2x+3|=7$. **2**

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Question 12 (5 marks)

Given the arithmetic series $27+x+y+z+11+\ldots$.

(a) Find the values of x, y and z. **2**

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(b) Find the number of terms if the series sums to 99. **3**

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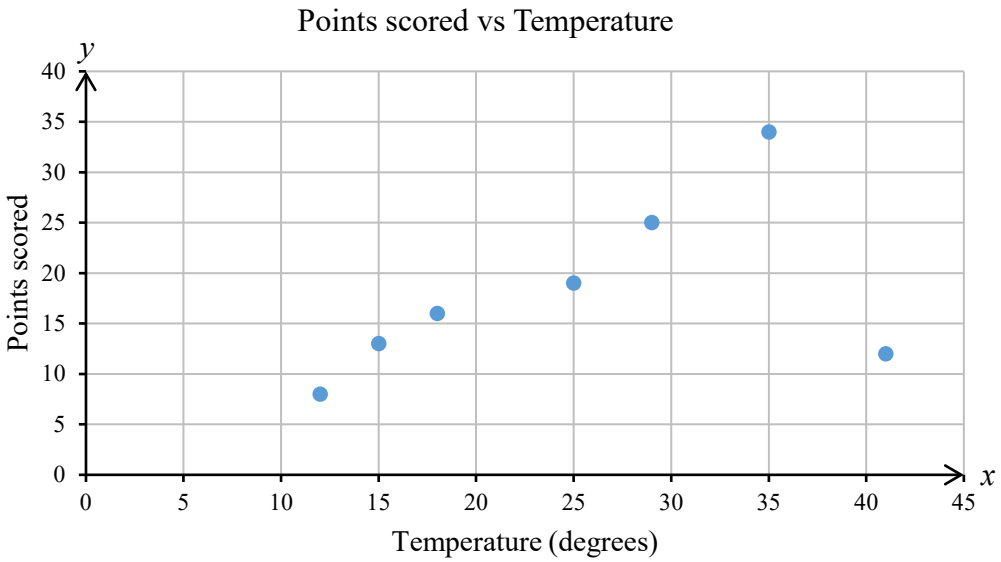
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Question 13 (3 marks)

Some basketball fans were interested in the relationship between the temperature during the game and the number of points scored.
They recorded their data and presented it in their scatter plot shown below.



The data is also presented in the table below.

Temperature (x) in degrees Celsius	12	15	18	25	29	35	41
Points scored (y)	8	13	16	19	25	34	12

(a) Calculate the equation of the least squares regression line, giving coefficients correct to 4 significant figures. Show that the regression line predicts 21 points when the temperature is 33 degrees. **2**

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(b) Give a reason why the predicted points scored when the temperature is 33 degrees is very different to the actual data points when the temperature is 29 or 35 degrees. Suggest how the prediction could be improved. **1**

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Question 14 (2 marks)

Find $\int_0^{\frac{\pi}{2}} 3\sin 2x\,dx$. **2**

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Question 15 (6 marks)

(a) Find the equation of the tangent to the curve $y = 4\sqrt{x} - \frac{2}{x} - 5$ at $x = 1$. **3**

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(b) Find the exact value of c if the line found in part (a) is also a tangent to $y = c + \ln x$. **3**

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Question 16 (3 marks)

Solve $\log_2(x-2)+\log_2(x+1)=2$. **3**

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Question 17 (4 marks)

(a) Show that $(1-\sin^2 2x)\sec 2x = \cos 2x$. **2**

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(b) Hence or otherwise solve $2(1-\sin^2 2x)\sec 2x = 1$ for $0 \le x \le \pi$. **2**

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Question 18 (5 marks)

The acceleration a ms^{-2} of a particle after t seconds is given by $a = -6t + 18$.

(a) The particle is initially at rest and 2 metres to the right of the origin. Find the equations for the velocity and displacement of the particle. **3**

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(b) Determine the maximum displacement of the particle. **2**

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Proceed to Booklet 2 for Questions 19-25

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Student Number

Mathematics Advanced
Section II Answer Booklet 2

2

Booklet 2 – Attempt Questions 19–25 (29 marks)

Instructions

- Write your student number at the top of this page.
- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
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Please turn over

Question 19 (2 marks)

Find the derivative of $y = \sin 5x \tan\left(e^x\right)$. **2**

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Question 20 (7 marks)

The waiting time in minutes for commuters at a train station can be modelled by the continuous probability distribution with probability density function, $f(x) = \begin{cases} ke^{-x}, 0 \le x \le 4 \\ 0, \quad \text{otherwise} \end{cases}$.

(a) Find the value of k. **3**

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(b) Find the cumulative distribution function as a piecewise function. **2**

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(c) Find the median waiting time. **2**

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Question 21 (6 marks)

The rate at which the depth of snow at a ski resort is changing in a particular year can be modelled by the function $\frac{dh}{dt} = -6t^2 + 78t - 180$ where h is the depth of snow in centimetres and t is the number of months since the beginning of the year and $0 \le t \le 12$.

(a) For which values of t is the depth of snow increasing? **3**

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(b) If the depth of snow is 300 cm at the start of the year, find the maximum depth of snow during the year. **3**

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Question 22 (3 marks)

(a) Find $\frac{d}{dx}(x \ln x)$. **1**

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(b) Hence find $\int \ln x \, dx$. **2**

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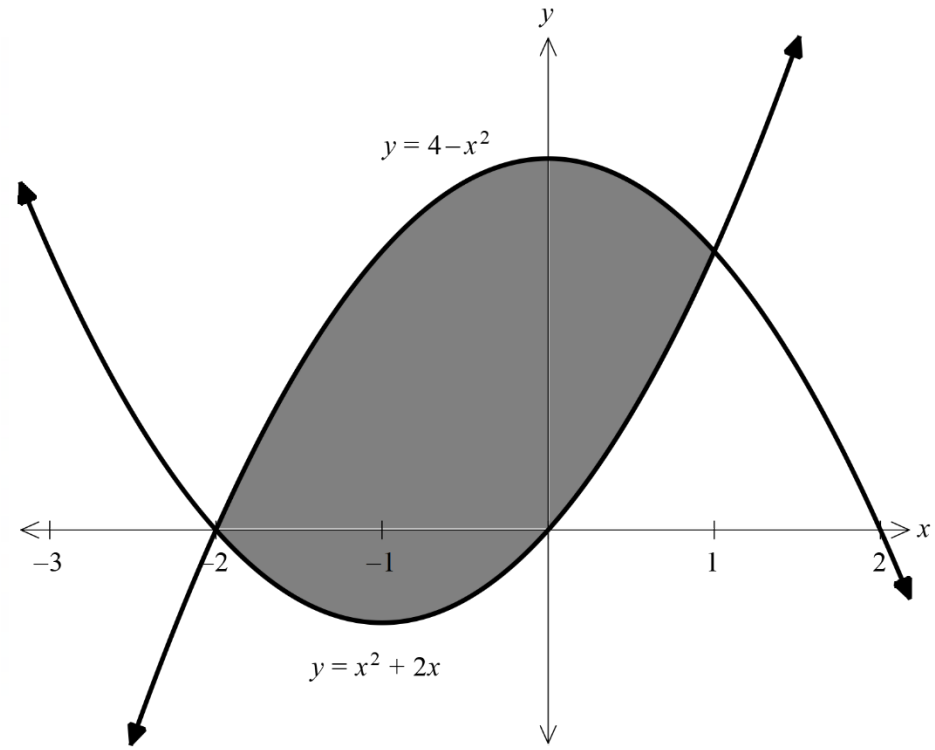
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Question 23 (3 marks)

The curves $y = x^2 + 2x$ and $y = 4 - x^2$ are shown below. **3**
Their intersection points are $(-2, 0)$ and $(1, 3)$.
Find the area of the shaded region.



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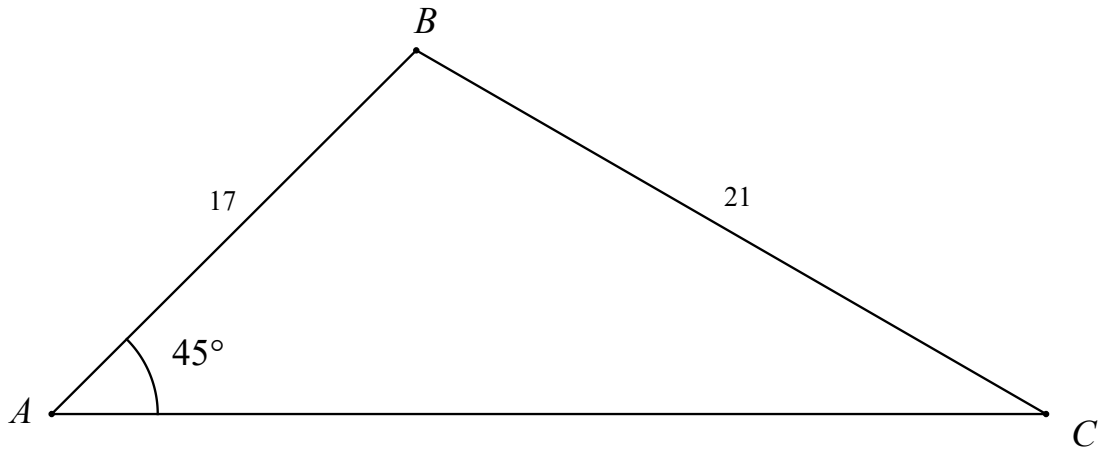
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Question 24 (4 marks)

Triangle ABC is shown below with dimensions as marked.



(a) Determine the possible value(s) of angle C. **2**

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(b) Hence, or otherwise, find the possible length(s) of AC. **2**

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Question 25 (4 marks)

Consider the line $y = mx$ and the circle $(x+1)^2 + (y-7)^2 = 10$.

(a) Show that the x - coordinates of their point(s) of intersection satisfy **1**
$(m^2+1)x^2 + (2-14m)x + 40 = 0$

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(b) Hence, or otherwise, find the values of m such that the line intersects the circle twice. **3**

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Proceed to Booklet 3 for Questions 26-32

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Student Number

Mathematics Advanced
Section II Answer Booklet 3

3

Booklet 3 – Attempt Questions 26–32 (31 marks)

Instructions

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Please turn over

Question 26 (2 marks)

Use two applications of the trapezoidal rule to estimate $\int_1^3 \frac{\ln x}{x^2} dx$ **2**

leaving your answer in exact form.

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Question 27 (5 marks)

Consider the curve with equation $y = \frac{x^5}{5} + \frac{x^4}{3} - \frac{7x}{3}$.

(a) Verify that there is a minimum turning point when $x = 1$. **2**

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(b) Find the coordinates of any points of inflection. **3**

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Question 28 (4 marks)

A problem gambler gets good tips so he is able to make a profit during the day, but then he keeps betting and makes a loss each night.
He is trying to stop gambling and reduce the amount he bets by a certain percentage compared to the previous day/night session. This means his profit/loss each session follows a geometric series.
So if his profit/loss is x dollars the first day, y dollars the first night and z dollars the next day then $x + y + z + ...$ forms a geometric series.

(a) Given the profit on the third day was \$27 and the loss on the fourth night was \$8, show that the ratio between successive profit/loss is $-\frac{2}{3}$ and find the profit on the first day. **2**

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(b) How much profit or loss will the gambler eventually make? **2**

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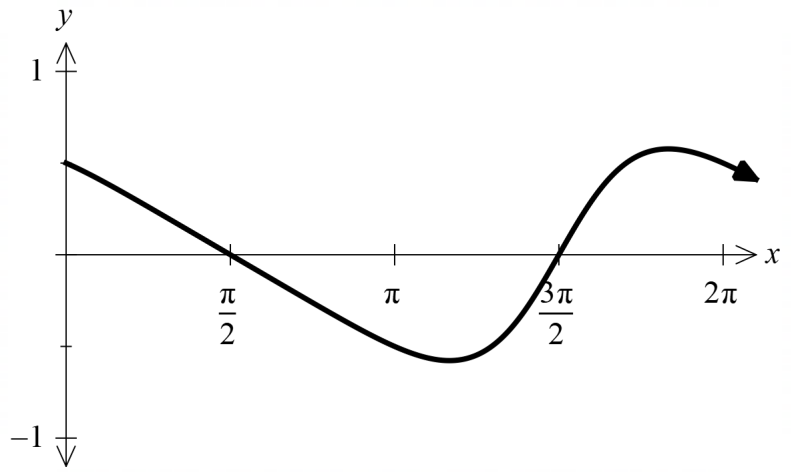
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Question 29 (4 marks)

Below is a sketch of $f(x) = \dfrac{\cos x}{2 + \sin x}$.



(a) Find the area of the region between the curve and the x-axis for $0 \le x \le 2\pi$. **3**

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(b) What is the area of the region between the curve and the x-axis for $0 \le x \le 2k\pi$ where k is a positive integer? **1**

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Question 30 (6 marks)

A population of bears follows a yearly cycle. The population peaks at 120 at the end of August. The lowest the population of bears gets to is 80. The population, P, can be modelled by the equation:

$$P = a\cos\left(\frac{\pi}{6}(t-8)\right)+c$$

where t is the number of months since the start of the year.

(a) Find the values of a and c. **2**

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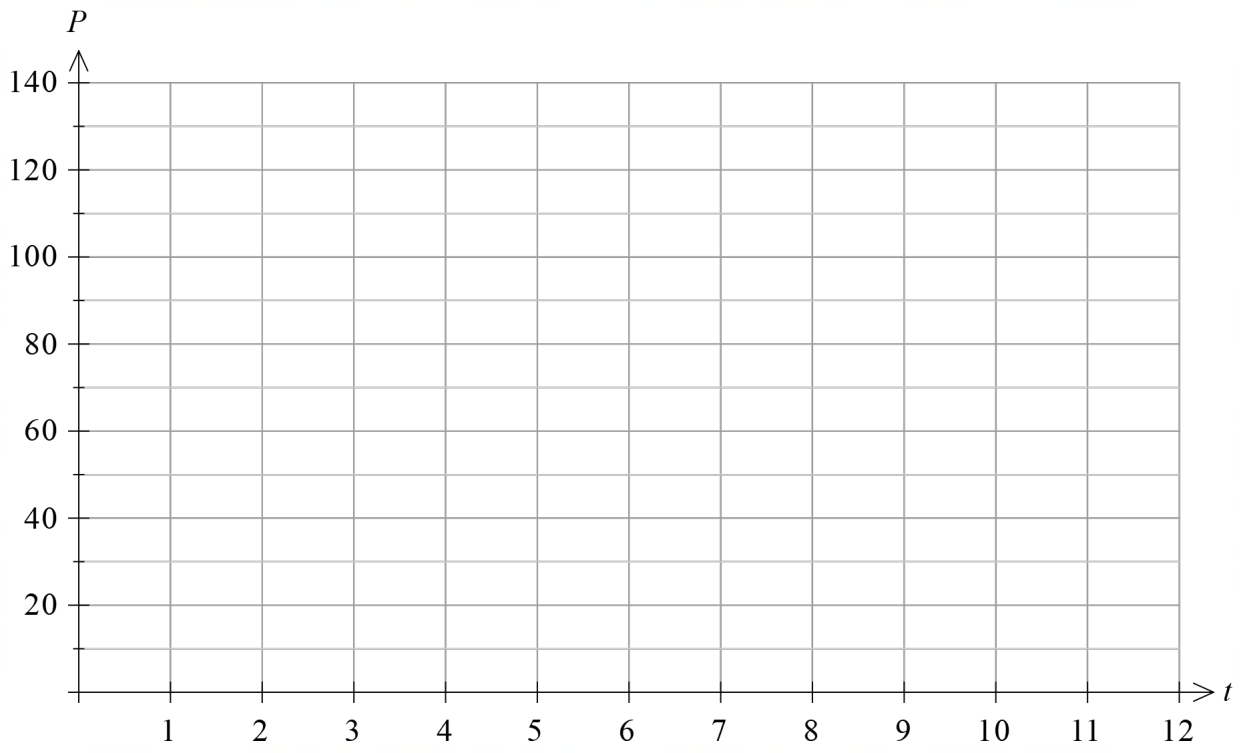
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(b) Sketch the graph of the population on the axes below for $0 \le t \le 12$. **2**



Question 30 continues on next page

(c) When the population of bears is above 110 it becomes unsafe for tourists to visit the bears' habitat. During which months should tourists stay away? **2**

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End of Question 30

Question 31 (3 marks)

Let a quadratic function be given by $f(x) = (x+1)(x-5)$. **3**

The graph of $f(x)$ is translated vertically by a units and horizontally by b units.

The transformed function is $g(x)$ whose graph passes through the points $(-2,-20)$ and $(0,-20)$.

Find the values of a and b.

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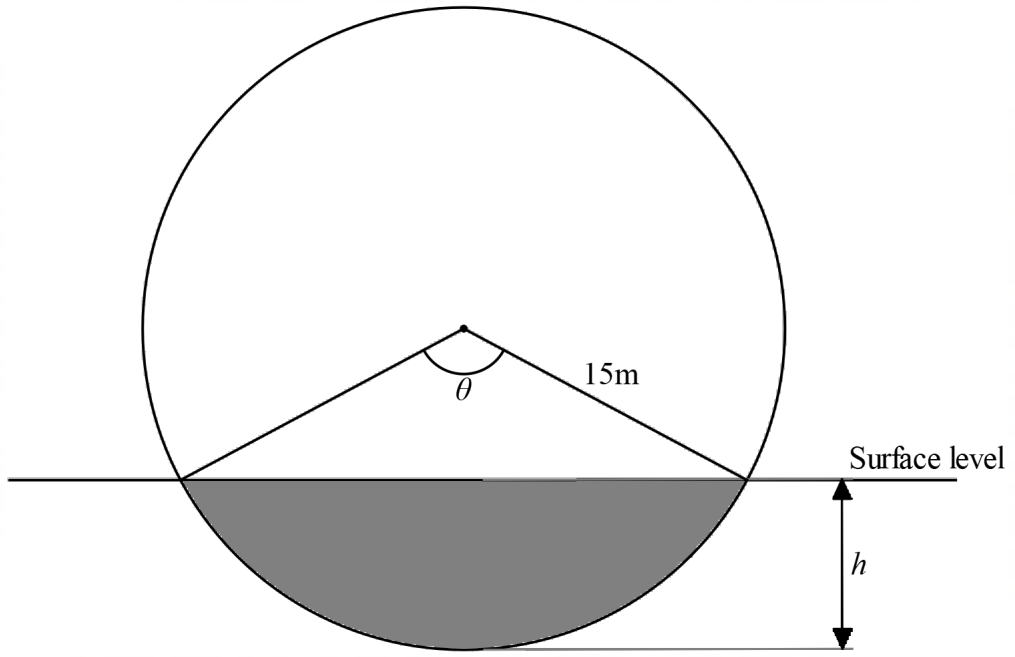
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Question 32 (7 marks)

At a mining site a circular digger with a radius of 15 metres digs out ore as shown below.
The revenue is proportional to the area of the segment dug up, which is shown shaded in the diagram below with a central angle θ.
Maintenance costs are proportional to the depth, h, that the digger is digging.



(a) Initially the mining company plans to dig at a depth of 3 m. **3**
They estimate they will earn $\$25 / m^2$ of ore dug out and maintenance is $\$18 / m$ of depth. Calculate the estimated profit or loss.

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Question 32 continues on the next page

(b) The mining company does some more study and find that the profit, P, is given by: **4**

$$P = \frac{1125}{2}(\theta - \sin\theta) - 600\left(1 - \cos\frac{\theta}{2}\right)$$

Part of the graph of $y = 15\cos x + 8\sin\frac{x}{2}$ is shown below. Use this to find the depth they should dig to maximise profits given that the deepest they can dig is $7.5\,\text{m}$.



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Mark	/10	/30	/29	/31	/100

Section I

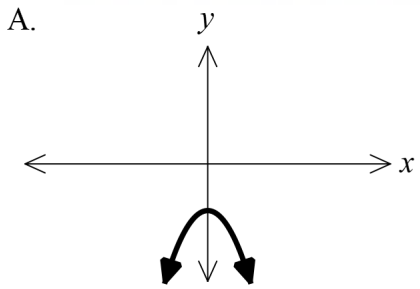
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Allow about 15 minutes for this section.

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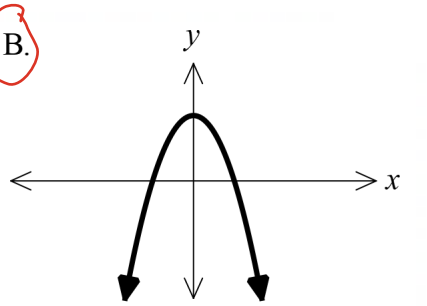
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1 Which graph best represents $y = 3 - x^2$?

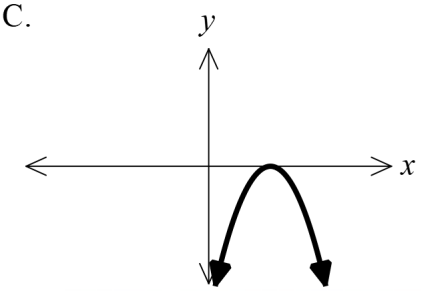
A.



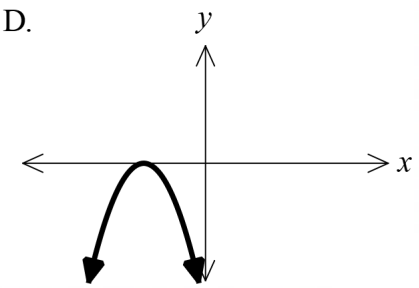
B.



C.



D.



2 What is the exact solution of the equation $3 \times 2^x = 15$?

A. $x = \log_2 5$

B. $x = \log_2 15$

C. $x = \frac{1}{3}\log_2 15$

D. $x = \log_e 5$

3 A certain recipe says the cooking time for a duck is directly proportional to its weight. The recipe says to cook a 3 kg duck for 2 hours.

How long should a 2 kg duck be cooked for?

A. 2 hours.

B. 3 hours.

C. 40 minutes.

D. 1 hour 20 minutes.

$\frac{2}{3} \times 2$ hours

$\frac{4}{3}$ hours

4 What is the expected value for the discrete probability distribution shown below?

x	-1	0	1	2
$p(x)$	0.2	a	0.3	0.4

A. 0.1

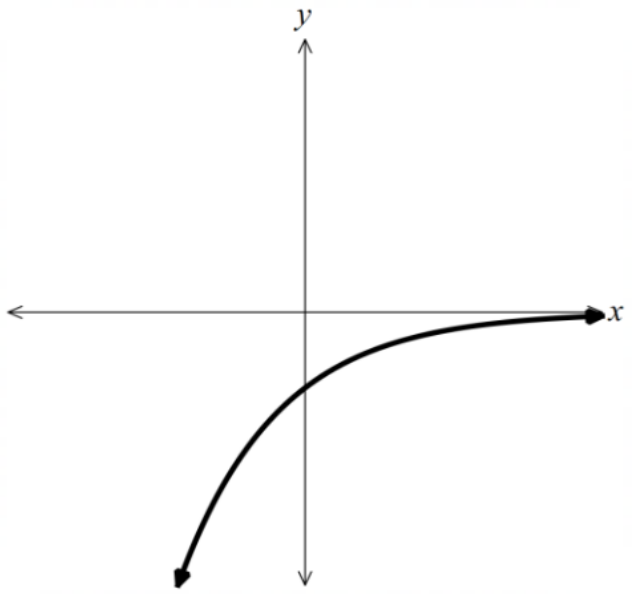
B. $0.9 + a$

C. 2

D. 0.9

$-0.2 + 0.3 + 0.8$

5 Which of the following could be the equation of the graph below?



A. $y = -\frac{1}{x}$

B. $y = e^{-x}$

C. $y = -e^{-x}$

D. $y = -e^{x}$

6 Which of the following is an expression for $\frac{d}{dx}\ln\left(3x^2\right)$?

A. $\frac{2}{x}$

B. $\frac{1}{6x}$

C. $6x$

D. $\frac{1}{x}$

$\frac{d}{dx}(\ln 3 + 2\ln x)$

7 Which of the following is an expression for $\int x^3\left(x^4-3\right)^5\,dx$?

A. $\dfrac{\left(x^4-3\right)^6}{24}+C$

B. $\dfrac{\left(x^4-3\right)^6}{6}+C$

C. $\dfrac{x^4\left(x^4-3\right)^6}{24}+C$

D. $\dfrac{\left(x^4-3\right)^6}{24x^4}+C$

$\frac{1}{4}\int 4x^3(x^4-3)^5\,dx$

$\frac{1}{4}\times\frac{(x^4-3)^6}{6}+C$

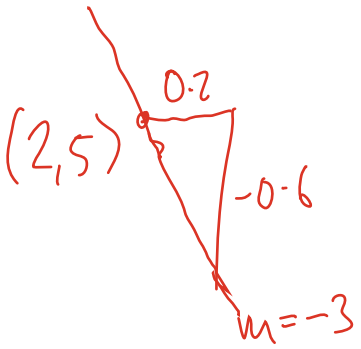
8 If $f(2)=5$ and $f'(2)=-3$, what is the best estimate for $f(2.2)$?

A. 2.2

B. 4.4

C. 5

D. 5.6



9 What is the domain of the function $f(x) = \log_e(x+2) - \sqrt{11-x^2}$?

A. $\left[-\sqrt{11}, \sqrt{11}\right]$

B. $\left(-2, \sqrt{11}\right]$

C. $(-2, \infty)$

D. $\left(2, \sqrt{11}\right]$

10 If it is raining, the probability Bill is late to work is $\frac{3}{10}$, whereas, if it is not raining the probability of Bill being late is $\frac{1}{10}$.

The probability that it is raining is $\frac{2}{5}$.

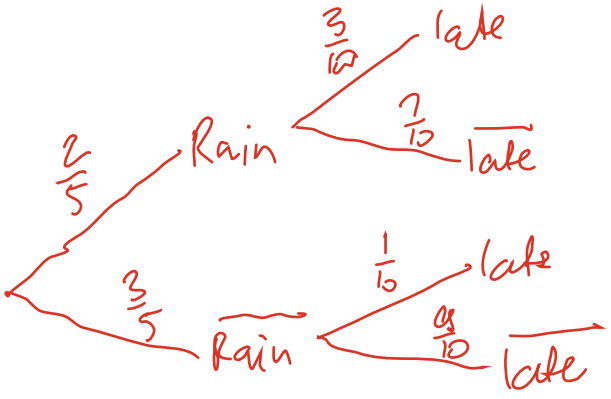
What is the probability that it is raining given that Bill was late to work?

A. $\frac{2}{5}$

B. $\frac{9}{50}$

C. $\frac{2}{3}$

D. $\frac{3}{4}$



$$P(\text{rain} \mid \text{late}) = \frac{P(\text{rain and late})}{P(\text{late})}$$

$$= \frac{\frac{2}{5} \times \frac{3}{10}}{\frac{2}{5} \times \frac{3}{10} + \frac{3}{5} \times \frac{1}{10}} = \frac{6}{6+3}$$

End of Section I

Student Number

Mathematics Advanced
Section II Answer Booklet 1

1

Section II

90 marks
Attempt Questions 11–32
Allow about 2 hours and 45 minutes for this section

Booklet 1 – Attempt Questions 11–18 (30 marks)
Booklet 2 – Attempt Questions 19–25 (29 marks)
Booklet 3 – Attempt Questions 26–32 (31 marks)

Instructions

- Write your student number at the top of this page.
- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
- Your responses should include relevant mathematical reasoning and/or calculations.
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Please turn over

Question 11 (2 marks)

Solve the equation $|2x+3|=7$. **2**

$2x+3 = \pm 7$

$2x = 4$ or $2x = -10$

$x = 2$ or $x = -5$

Question 12 (5 marks)

Given the arithmetic series $27+x+y+z+11+\ldots$.

(a) Find the values of x, y and z. **2**

$a = 27$, $T_5 = 11$

$\therefore 11 = 27 + 4d$

$-16 = 4d$

$d = -4$

$\therefore x = 23$

$y = 19$

$z = 15$

(b) Find the number of terms if the series sums to 99. **3**

$S_n = \frac{n}{2}(2a + (n-1)d)$

$99 = \frac{n}{2}(54 - 4(n-1))$

$198 = n(58 - 4n)$

$4n^2 - 58n + 198 = 0$

$2n^2 - 29n + 99 = 0$

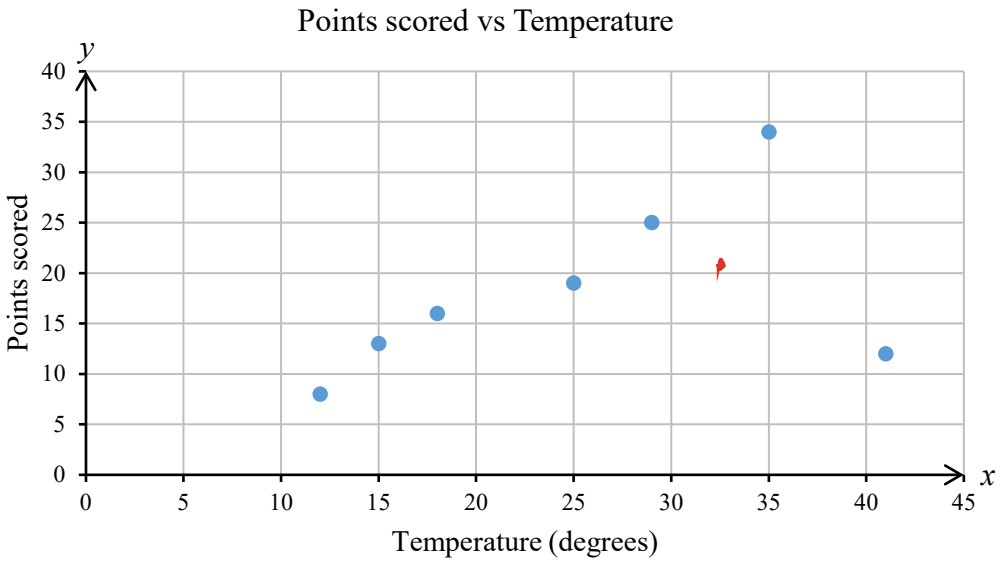
$(2n - 11)(n - 9) = 0$

$\therefore n = \frac{11}{2}$ or $n = 9$ however n is an integer

$\therefore n = 9$

Question 13 (3 marks)

Some basketball fans were interested in the relationship between the temperature during the game and the number of points scored.
They recorded their data and presented it in their scatter plot shown below.



The data is also presented in the table below.

Temperature (x) in degrees Celsius	12	15	18	25	29	35	41
Points scored (y)	8	13	16	19	25	34	12

(a) Calculate the equation of the least squares regression line, giving coefficients correct to 4 significant figures. Show that the regression line predicts 21 points when the temperature is 33 degrees. **2**

From calculator $A = 7.781$, $B = 0.4145$

$\therefore y = 0.4145x + 7.781$

when $x = 33$

$y = 0.4145 \times 33 + 7.781$

$= 21$

(b) Give a reason why the predicted points scored when the temperature is 33 degrees is very different to the actual data points when the temperature is 29 or 35 degrees. Suggest how the prediction could be improved. **1**

The point (41,12) is an outlier which drags the regression line down.
The prediction could be improved by recalculating the regression line excluding this point or directly interpolating between (29,25) and (35,34).

Question 14 (2 marks)

Find $\int_0^{\frac{\pi}{2}} 3\sin 2x\,dx$. **2**

$\left[-\frac{3}{2}\cos 2x\right]_0^{\frac{\pi}{2}} = -\frac{3}{2}(-1) - \left(-\frac{3}{2}\right)(1)$

$= 3$

Question 15 (6 marks)

(a) Find the equation of the tangent to the curve $y = 4\sqrt{x} - \frac{2}{x} - 5$ at $x = 1$. **3**

$y' = \frac{2}{\sqrt{x}} + \frac{2}{x^2}$

at $x=1$, $y' = 2+2 = 4$ and $y = 4 - 2 - 5 = -3$

$\therefore$ tangent is $y + 3 = 4(x - 1)$

$y = 4x - 7$

(b) Find the exact value of c if the line found in part (a) is also a tangent to $y = c + \ln x$. **3**

$y' = \frac{1}{x}$

for $y = 4x - 7$ to be tangent need

$\frac{1}{x} = 4$, $\therefore x = \frac{1}{4}$

$\therefore$ point of contact $y = 4\left(\frac{1}{4}\right) - 7 = -6$

$\therefore \left(\frac{1}{4}, -6\right)$ on $y = c + \ln x$

$\therefore -6 = c + \ln\left(\frac{1}{4}\right)$

$-6 = c - \ln 4$

$c = -6 + \ln 4$

Question 16 (3 marks)

Solve $\log_2(x-2)+\log_2(x+1)=2$. **3**

$\log_2[(x-2)(x+1)]=2$

$x^2-x-2=4$

$x^2-x-6=0$

$(x-3)(x+2)=0$

$\therefore x=3$ or $x=-2$

however need $x>2$ and $x>-1$

$\therefore x=3$ only solution

Question 17 (4 marks)

(a) Show that $(1-\sin^2 2x)\sec 2x = \cos 2x$. **2**

LHS: $(1-\sin^2 2x)\sec 2x$

$= \cos^2 2x \left(\frac{1}{\cos 2x}\right)$

$= \cos 2x$

$=$ RHS

(b) Hence or otherwise solve $2(1-\sin^2 2x)\sec 2x = 1$ for $0 \le x \le \pi$. **2**

$2\cos 2x = 1$

$\therefore \cos 2x = \frac{1}{2}$ $\quad 0 \le x \le \pi$

$0 \le 2x \le 2\pi$



$\therefore 2x = \frac{\pi}{3}, \frac{5\pi}{3}$

$x = \frac{\pi}{6}, \frac{5\pi}{6}$

Question 18 (5 marks)

The acceleration a ms^{-2} of a particle after t seconds is given by $a = -6t + 18$.

(a) The particle is initially at rest and 2 metres to the right of the origin. Find the equations for the velocity and displacement of the particle. **3**

$v = -3t^2 + 18t + C_1$

$t = 0, v = 0 \therefore C_1 = 0$

$\therefore v = -3t^2 + 18t$

$\therefore x = -t^3 + 9t^2 + C_2$

$t = 0, x = 2 \therefore C_2 = 2$

$\therefore x = -t^3 + 9t^2 + 2$

(b) Determine the maximum displacement of the particle. **2**

stationary points $v = 0$

$-3t + 18t = 0$

$-3t(t - 6) = 0$

$\therefore t = 0$ or $t = 6$

at $t = 0$, $a = 18$ $\therefore$ minimum point

$t = 6$, $a = -18$ $\therefore$ maximum point

$\therefore$ maximum displacement $= -6^3 + 9 \times 6^2 + 2$

$= 110$ m

Proceed to Booklet 2 for Questions 19-25

Student Number

Mathematics Advanced
Section II Answer Booklet 2

2

Booklet 2 – Attempt Questions 19–25 (29 marks)

Instructions

- Write your student number at the top of this page.
- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided on pages at the end of each booklet. If you use this space, clearly indicate which question you are answering.

Please turn over

Question 19 (2 marks)

Find the derivative of $y = \sin 5x \tan\left(e^x\right)$. **2**

let $u = \sin 5x$ $\quad v = \tan(e^x)$

$u' = 5\cos 5x$ $\quad v' = e^x \sec^2(e^x)$

$\therefore \frac{dy}{dx} = e^x \sin 5x \sec^2(e^x) + 5\cos 5x \tan(e^x)$

Question 20 (7 marks)

The waiting time in minutes for commuters at a train station can be modelled by the continuous probability distribution with probability density function, $f(x)=\begin{cases} ke^{-x}, 0 \le x \le 4 \\ 0, \quad \text{otherwise} \end{cases}$.

(a) Find the value of k. **3**

$\int_{-\infty}^{\infty} f(x)\,dx = 1$

$\therefore \int_0^4 ke^{-x}\,dx = 1$

$\left[-ke^{-x}\right]_0^4 = 1$

$-ke^{-4} + k = 1$

$k\left(1-e^{-4}\right) = 1$

$k = \dfrac{1}{1-e^{-4}} = \dfrac{e^4}{e^4-1}$

(b) Find the cumulative distribution function as a piecewise function. **2**

$\int_0^x ke^{-t}\,dt = \left[-ke^{-t}\right]_0^x$

$= -ke^{-x} + k$

$= k\left(1-e^{-x}\right)$

$\therefore F(x) = \begin{cases} 0, & x < 0 \\ \dfrac{1-e^{-x}}{1-e^{-4}}, & 0 \le x \le 4 \\ 1, & x > 4 \end{cases}$

(c) Find the median waiting time. **2**

for median $F(x) = \frac{1}{2}$

$\therefore \dfrac{1-e^{-x}}{1-e^{-4}} = \frac{1}{2}$

$2 - 2e^{-x} = 1 - e^{-4}$

$e^{-x} = \frac{1}{2}\left(1 + e^{-4}\right)$

$x = -\ln\left[\frac{1}{2}\left(1+e^{-4}\right)\right]$

$= 0.675$ minutes

$= 40$ seconds

Question 21 (6 marks)

The rate at which the depth of snow at a ski resort is changing in a particular year can be modelled by the function $\frac{dh}{dt} = -6t^2 + 78t - 180$ where h is the depth of snow in centimetres and t is the number of months since the beginning of the year and $0 \le t \le 12$.

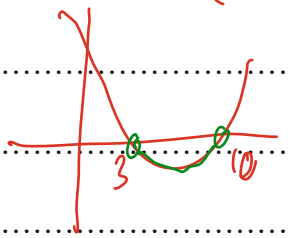
(a) For which values of t is the depth of snow increasing? **3**

Increasing when $\frac{dh}{dt} > 0$

$\therefore -6t^2 + 78t - 180 > 0$

$t^2 - 13t + 30 < 0$

$(t-10)(t-3) < 0$



$\therefore 3 < t < 10$

(b) If the depth of snow is 300 cm at the start of the year, find the maximum depth of snow during the year. **3**

$h = -2t^3 + 39t^2 - 180t + C$

$t = 0, h = 300 \therefore C = 300$

$\therefore h = -2t^3 + 39t^2 - 180t + 300$

max point at $t = 10$ as increasing then decreasing at this point from (a)

at $t = 10$ $h = -2 \times 10^3 + 39 \times 10^2 - 180 \times 10 + 300$

$= 400$ m

check $t = 12$: $h = -2 \times 12^3 + 39 \times 12^2 - 180 \times 12 + 300$

$= 300$ m

$\therefore$ max depth during year is 400m

Question 22 (3 marks)

(a) Find $\frac{d}{dx}(x \ln x)$. **1**

$\frac{d}{dx}(x \ln x) = x\left(\frac{1}{x}\right) + \ln(x) \times 1$

$= 1 + \ln x$

(b) Hence find $\int \ln x \, dx$. **2**

By (a) and fundamental theorem of calculus

$\int 1 + \ln x \, dx = x \ln x + C$

$\therefore \int dx + \int \ln x \, dx = x \ln x + C$

$x + \int \ln x \, dx = x \ln x + C$

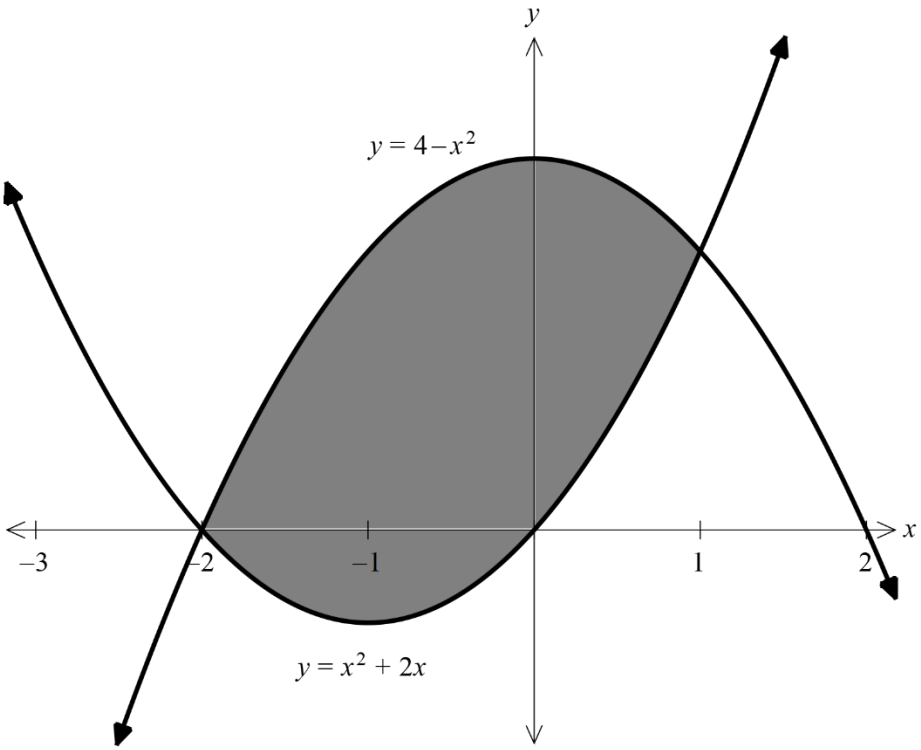
$\therefore \int \ln x \, dx = x \ln x - x + C$

Question 23 (3 marks)

The curves $y = x^2 + 2x$ and $y = 4 - x^2$ are shown below. **3**

Their intersection points are $(-2, 0)$ and $(1, 3)$.

Find the area of the shaded region.



$$\text{Area} = \int_{-2}^{1} 4 - x^2 - (x^2 + 2x)\,dx$$

$$= \int_{-2}^{1} 4 - 2x - 2x^2\,dx$$

$$= \left[4x - x^2 - \frac{2x^3}{3}\right]_{-2}^{1}$$

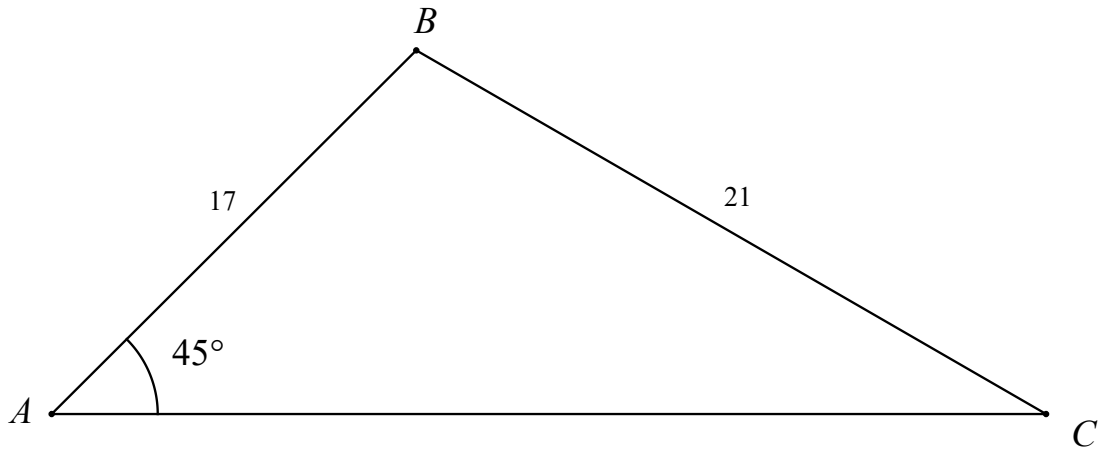
$$= 3 - \frac{2}{3} - \left(-8 - 4 + \frac{16}{3}\right)$$

$$= 15 - \frac{18}{3}$$

$$= 9 \text{ units}^2$$

Question 24 (5 marks)

Triangle ABC is shown below with dimensions as marked.



(a) Determine the possible value(s) of angle C. **2**

$\frac{\sin \angle C}{17} = \frac{\sin 45}{21}$

$\sin \angle C = \frac{17}{21\sqrt{2}}$

$= 0.572$

$\angle C = 34.9^\circ$ or $180 - 34.9 = 145.1$

but $145.1 + 45 = 190.1 > 180^\circ$

$\therefore \angle C = 34.9^\circ$

(b) Hence, or otherwise, find the possible length(s) of AC. **2**

$\angle B = 180 - 45 - 34.9$

$= 100.1^\circ$

$\therefore \frac{AC}{\sin 100.1} = \frac{21}{\sin 45^\circ}$

$AC = 21\sqrt{2}\sin(100.1)$

$= 29.2$

Question 25 (4 marks)

Consider the line $y = mx$ and the circle $(x+1)^2 + (y-7)^2 = 10$.

(a) Show that the x - coordinates of their point(s) of intersection satisfy $(m^2+1)x^2 + (2-14m)x + 40 = 0$ **1**

Substituting $y = mx$ into circle equation:

$(x+1)^2 + (mx-7)^2 = 10$

$x^2 + 2x + 1 + m^2x^2 - 14mx + 49 = 10$

$(m^2+1)x^2 + (2-14m)x + 40 = 0$

(b) Hence, or otherwise, find the values of m such that the line intersects the circle twice. **3**

for two intersection points $\Delta > 0$

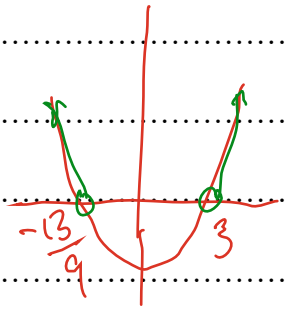
$\therefore (2-14m)^2 - 4(m^2+1)(40) > 0$

$4 - 56m + 196m^2 - 160m^2 - 160 > 0$

$36m^2 - 56m - 156 > 0$

$9m^2 - 14m - 39 > 0$

$(9m+13)(m-3) > 0$



$\therefore m < -\frac{13}{9}$ or $m > 3$

Proceed to Booklet 3 for Questions 26-32

Student Number

Mathematics Advanced
Section II Answer Booklet 3

3

Booklet 3 – Attempt Questions 26–32 (31 marks)

Instructions

- Write your student number at the top of this page.
- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
- Your responses should include relevant mathematical reasoning and/or calculations.
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Please turn over

Question 26 (2 marks)

Use two applications of the trapezoidal rule to estimate $\int_1^3 \frac{\ln x}{x^2}\,dx$ **2**

leaving your answer in exact form.

$$\int_1^3 \frac{\ln x}{x^2} = \frac{1}{2}\left[\frac{\ln(1)}{1^2} + \frac{\ln(3)}{3^2} + 2\frac{\ln(2)}{2^2}\right]$$
$$= \frac{1}{2}\left(\frac{\ln 3}{9} + \frac{\ln 2}{2}\right)$$
$$= \frac{1}{36}(2\ln 3 + 9\ln 2)$$
$$= \frac{1}{36}\ln(2^9 \times 3^2)$$

Question 27 (5 marks)

Consider the curve with equation $y = \frac{x^5}{5} + \frac{x^4}{3} - \frac{7x}{3}$.

(a) Verify that there is a minimum turning point when $x = 1$. **2**

$\frac{dy}{dx} = x^4 + \frac{4x^3}{3} - \frac{7}{3}$

$\frac{d^2y}{dx^2} = 4x^3 + 4x^2$

at $x = 1$, $\frac{dy}{dx} = 1 + \frac{4}{3} - \frac{7}{3} = 0$

$\frac{d^2y}{dx^2} = 4 + 4 = 8 > 0$

$\therefore$ minimum turning point

(b) Find the coordinates of any points of inflection. **3**

Possible points of inflection $\frac{d^2y}{dx^2} = 0$

$4x^3 + 4x^2 = 0$

$4x^2(x + 1) = 0$

$x = 0$ or -1

x	-2	-1	$-\frac{1}{2}$	0	1
$\frac{d^2y}{dx^2}$	-48	0	$\frac{1}{2}$	0	8
	$\cap$	$\cdot$	$\cup$	$\cdot$	$\cup$

Concavity only changes at $x = -1$

$y = -\frac{1}{5} + \frac{1}{3} + \frac{7}{3} = -\frac{1}{5} + \frac{8}{3}$

$= \frac{37}{15}$

Point of inflection at $\left(-1, \frac{37}{15}\right)$

Question 28 (4 marks)

A problem gambler gets good tips so he is able to make a profit during the day, but then he keeps betting and makes a loss each night.
He is trying to stop gambling and reduce the amount he bets by a certain percentage compared to the previous day/night session. This means his profit/loss each session follows a geometric series.
So if his profit/loss is x dollars the first day, y dollars the first night and z dollars the next day then $x+y+z+...$ forms a geometric series.

(a) Given the profit on the third day was \$27 and the loss on the fourth night was \$8, show that the ratio between successive profit/loss is $-\frac{2}{3}$ and find the profit on the first day. **2**

D1 N1 D2 N2 D3 N3 D4 N4

\$27 −\$8

for geometric seris $T_5 = 27$ and $T_8 = -8$

$\therefore ar^4 = 27$ — (1)

$ar^7 = -8$ — (2)

(2) ÷ (1) $r^3 = \frac{-8}{27}$

$\therefore r = \frac{-2}{3}$

$\therefore a\left(\frac{-2}{3}\right)^4 = 27$, $a = \frac{3^7}{2^4} = \$136.69$

(b) How much profit or loss will the gambler eventually make? **2**

$|r| < 1$ $\therefore$ limiting sum exists

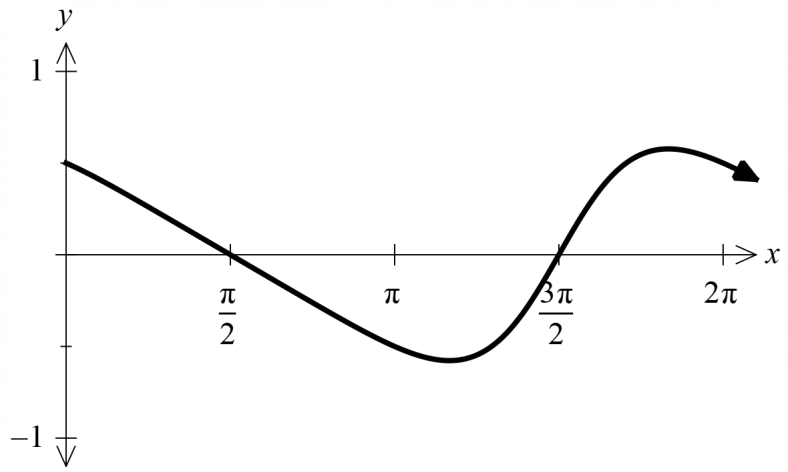
$S_\infty = \frac{\frac{3^7}{2^4}}{1+\frac{2}{3}}$

$= \frac{3^7}{2^4} \times \frac{3}{5}$

$\therefore$ gambler eventually makes \$82.01

Question 29 (4 marks)

Below is a sketch of $f(x) = \dfrac{\cos x}{2 + \sin x}$.



(a) Find the area of the region between the curve and the x-axis for $0 \le x \le 2\pi$. **3**

$$\text{Area} = \int_0^{\frac{\pi}{2}} \frac{\cos x}{2+\sin x}\,dx - \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \frac{\cos x}{2+\sin x}\,dx + \int_{\frac{3\pi}{2}}^{2\pi} \frac{\cos x}{2+\sin x}\,dx$$

$$= \left[\ln(2+\sin x)\right]_0^{\frac{\pi}{2}} - \left[\ln(2+\sin x)\right]_{\frac{\pi}{2}}^{\frac{3\pi}{2}} + \left[\ln(2+\sin x)\right]_{\frac{3\pi}{2}}^{2\pi}$$

$$= \ln 3 - \ln 2 - (\ln 1 - \ln 3) + \ln 2 - \ln 1$$

$$= 2\ln 3$$

(b) What is the area of the region between the curve and the x-axis for $0 \le x \le 2k\pi$ where k is a positive integer? **1**

$2k\ln 3$

Question 30 (6 marks)

A population of bears follows a yearly cycle. The population peaks at 120 at the end of August. The lowest the population of bears gets to is 80. The population, P, can be modelled by the equation:

$$P = a\cos\left(\frac{\pi}{6}(t-8)\right)+c$$

where t is the number of months since the start of the year.

(a) Find the values of a and c. **2**

$c = \frac{80+120}{2} = 100$

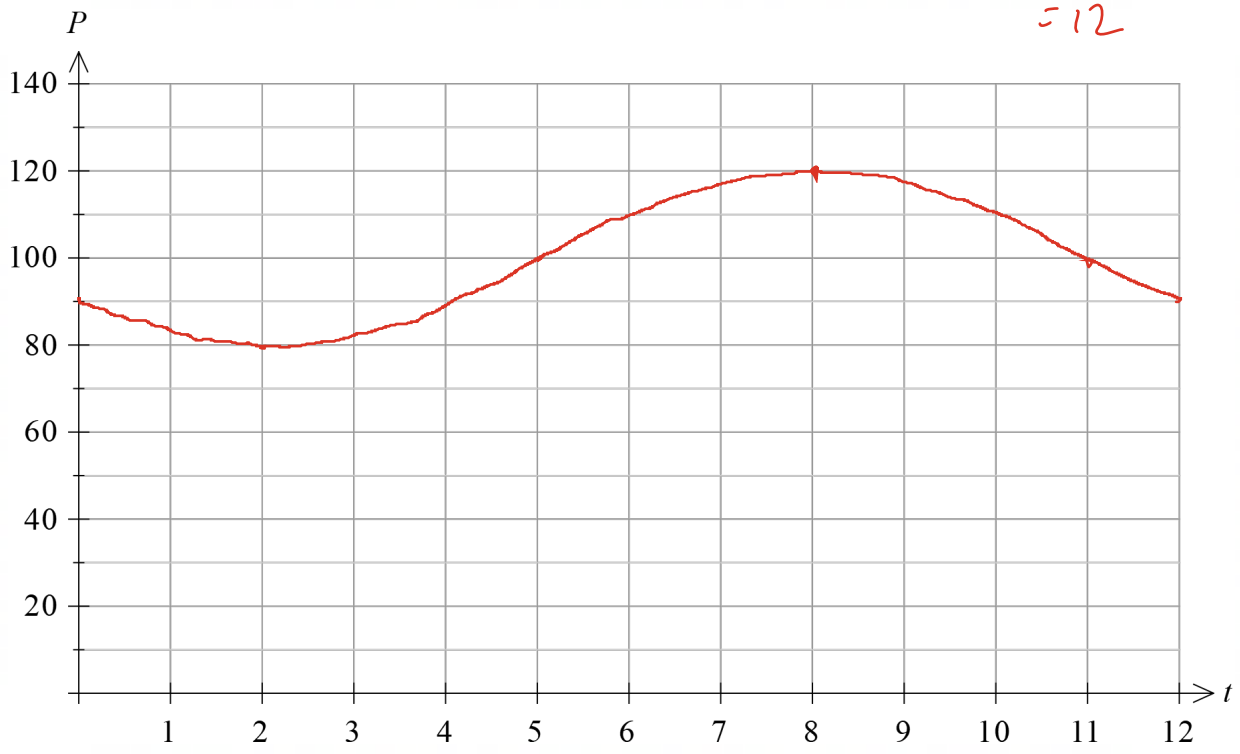
$a = \frac{120-80}{2} = 20$

$t=0: P = 20\cos\left(-\frac{4\pi}{3}\right)+100$

$= 90$

(b) Sketch the graph of the population on the axes below for $0 \le t \le 12$. **2**

period $= \frac{2\pi}{\frac{\pi}{6}} = 12$



Question 30 continues on next page

Question 30 (continued)

(c) When the population of bears is above 110 it becomes unsafe for tourists to visit the bears' habitat. During which months should tourists stay away? **2**

$110 = 20\cos\left(\frac{\pi}{6}(t-8)\right) + 100$

$\cos\left(\frac{\pi}{6}(t-8)\right) = \frac{1}{2}$

$\frac{\pi}{6}(t-8) = \frac{\pi}{3}, \frac{5\pi}{3}, -\frac{\pi}{3}$

$t-8 = 2, 10, -2$

$t = 10, \cancel{14}, 6$

$\therefore 6 \leq t \leq 10$

∴ tourists should stay away

July, August, September, October

End of Question 30

Question 31 (3 marks)

Let a quadratic function be given by $f(x) = (x+1)(x-5)$. **3**

The graph of $f(x)$ is translated vertically by a units and horizontally by b units.

The transformed function is $g(x)$ whose graph passes through the points $(-2,-20)$ and $(0,-20)$.

Find the values of a and b.

by symmetry $g(x)$ vertex when $x = -1$

$g(x)$ music as translations don't change coefficient of x^2

$\therefore g(x) = (x+1)^2 + k$

$y(0) = -20$

$\therefore -20 = 1 + k$

$k = -21$

vertex $f(x)$: $x = \frac{-1+5}{2} = 2$

$f(2) = 3(-3) = -9$

vertex $f(x)$ is $(2, -9)$

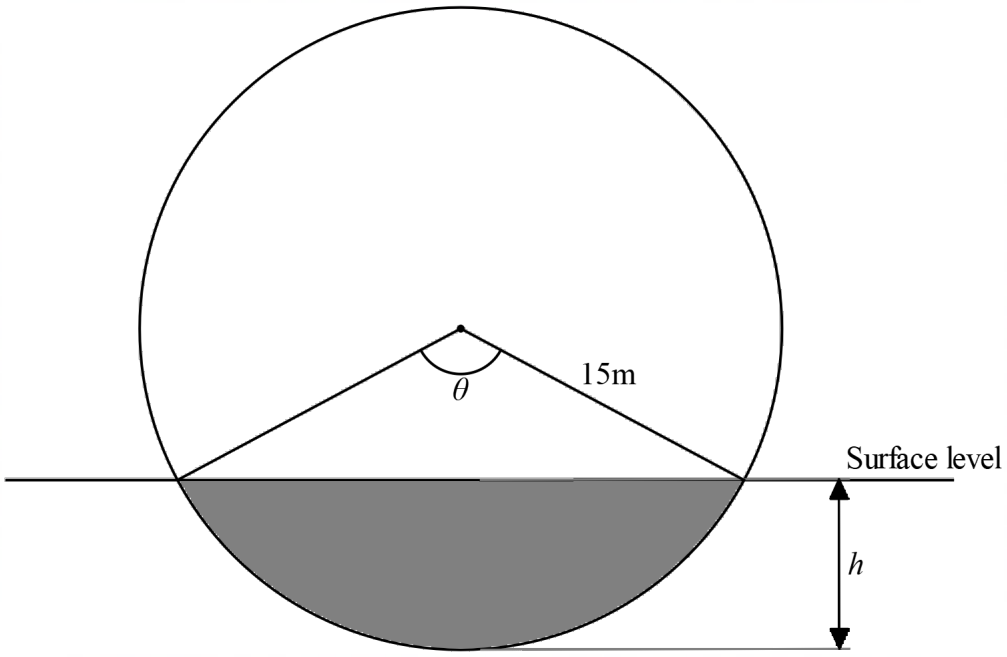
vertex $g(x)$ is $(-1, -21)$

$\therefore$ shifted left 3 units, down 12 units

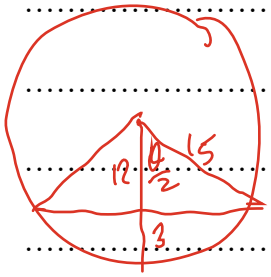
$\therefore a = -12$ and $b = -3$

Question 32 (6 marks)

At a mining site a circular digger with a radius of 15 metres digs out ore as shown below.
The revenue is proportional to the area of the segment dug up, which is shown shaded in the diagram below with a central angle θ.
Maintenance costs are proportional to the depth, h, that the digger is digging.



(a) Initially the mining company plans to dig at a depth of 3 m. **3**
They estimate they will earn \$25 / m^2 of ore dug out and maintenance is \$18 / m of depth. Calculate the estimated profit or loss.



$\cos\frac{\theta}{2} = \frac{12}{15}$

$\frac{\theta}{2} = 0.6435$ radians

$\theta = 1.287$ radians

$\therefore$ Area segment $= \frac{1}{2} \times 15^2 (1.287 - \sin(1.287))$

$= 36.7877 \text{ m}^2$

$\therefore$ Profit $= 25 \times 36.7877 - 18 \times 3$

$= \$865.69$

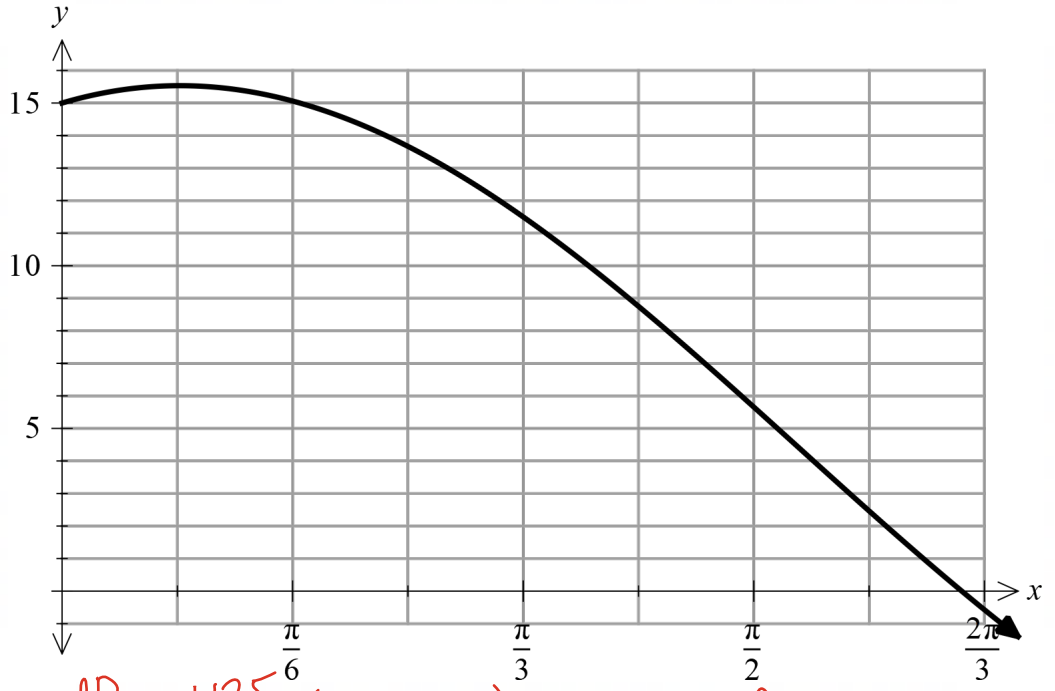
Question 32 continues on the next page

Question 32 (continued)

(b) The mining company does some more study and find that the profit, P, is given by: **4**

$$P = \frac{1125}{2}(\theta - \sin\theta) - 600\left(1 - \cos\frac{\theta}{2}\right)$$

Part of the graph of $y = 15\cos x + 8\sin\frac{x}{2}$ is shown below. Use this to find the depth they should dig to maximise profits given that the deepest they can dig is 7.5 m.



$\frac{dP}{d\theta} = \frac{1125}{2}(1-\cos\theta) - 300\sin\left(\frac{\theta}{2}\right)$

stationary points $\frac{dP}{d\theta} = 0$

$0 = \frac{1125}{2}(1-\cos\theta) - 300\sin\left(\frac{\theta}{2}\right)$

$\frac{1125}{2}\cos\theta + 300\sin\left(\frac{\theta}{2}\right) = \frac{1125}{2}$

$45\cos\theta + 24\sin\left(\frac{\theta}{2}\right) = 45$

$15\cos\theta + 8\sin\left(\frac{\theta}{2}\right) = 15$

$\therefore$ from graph $\theta = 0$ or $\theta \approx \frac{\pi}{6}$

checking nature $\frac{dP}{d\theta} = \frac{75}{2}\left(15 - 15\cos\theta - 8\sin\left(\frac{\theta}{2}\right)\right)$

to left of $\theta \approx \frac{\pi}{6}$ $15\cos\theta + 8\sin\frac{\theta}{2} > 15$ from graph

$\therefore \frac{dP}{d\theta} = \frac{75}{2}\left(15 - \left(15\cos\theta + 8\sin\frac{\theta}{2}\right)\right) < 0$

to right of $\theta \approx \frac{\pi}{6}$ $15\cos\theta + 8\sin\frac{\theta}{2} < 15$ from graph

$\therefore \frac{dP}{d\theta} > 0$

End of paper

Section II extra writing space

If you use this space, clearly indicate which question you are answering.

$\theta \approx \frac{\pi}{6}$ $\therefore$ minimum point at $\theta = \frac{\pi}{6}$

$\therefore$ maximum at endpoints of domain

$\theta = 0$, $P = \frac{1125}{2}(0 - \sin 0) - 600(1 - \cos 0)$

$= 0$

max depth 7.5 m

$\therefore$ max $\theta = \frac{2\pi}{3}$

$P = \frac{1125}{2}\left(\frac{2\pi}{3} - \sin\left(\frac{2\pi}{3}\right)\right) - 600\left(1 - \cos\left(\frac{2\pi}{3}\right)\right)$

$\approx \$390.96$

$\therefore$ maximum profit of \$390.96 when they dig 7.5m deep.

