2026

Penrith High School
HIGHER SCHOOL CERTIFICATE
TRIAL EXAMINATION

Mathematics Advanced

General Instructions

- Reading time – 10 minutes
- Working time – 3 hours
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided
- In Questions 11–35 show relevant mathematical reasoning and/or calculations
- Write your NESA ID below, on the Multiple Choice Answer Sheet and the front of Booklets 1, 2, 3 & 4.

Total marks: 100

Section I – 10 marks (pages 3-10)

- Attempt Questions 1–10
- Allow about 15 minutes for this section

Section II – 90 marks (pages 11-44)

- Attempt Questions 11–35
- Allow about 2 hours and 45 minutes for this section

NESA NUMBER: | | | | | | | | | |

TEACHER: _____

	Functions	Trigonometric Functions	Calculus	Exponential & Logarithmic Functions	Statistical Analysis	Financial Mathematics
Multiple Choice:	**/3** 1, 6, 8		**/4** 2, 4, 9, 10		**/3** 3, 5, 7	

	Functions	Trigonometric Functions	Calculus	Exponential & Logarithmic Functions	Statistical Analysis	Financial Mathematics	TOTAL
Total (including Multiple Choice)	**/11**	**/22**	**/18**	**/11**	**/22**	**/18**	**/102**

Section I

10 marks
Attempt questions 1-10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1-10.

---

1 The parabola $y = x^2 + 2$ is translated down 4 units. It is then reflected about the x-axis.

What is the equation of the resulting parabola?

A. $y = -x^2 - 6$

B. $y = -x^2 - 2$

C. $y = -x^2 + 2$

D. $y = -x^2 + 6$

2 Calculate $\frac{d}{dx}\left[\int_5^x (g^4 - 2g^3 + g - 1)\, dg\right]$

A. $x^4 - 2x^3 + x$

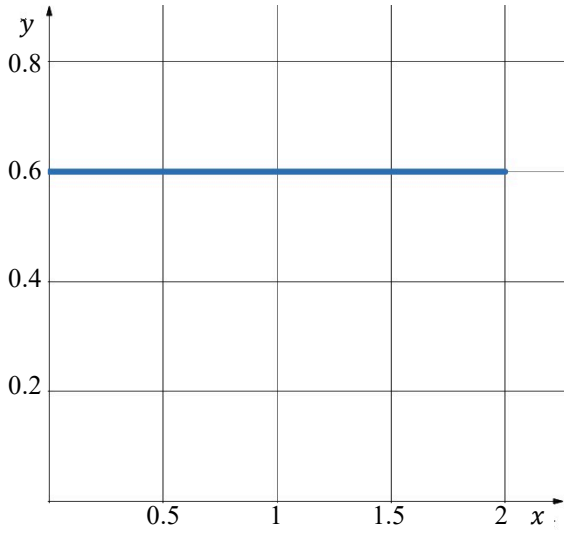
B. $x^4 - 2x^3 + x - 6$

C. $x^4 - 2x^3 + x - 1$

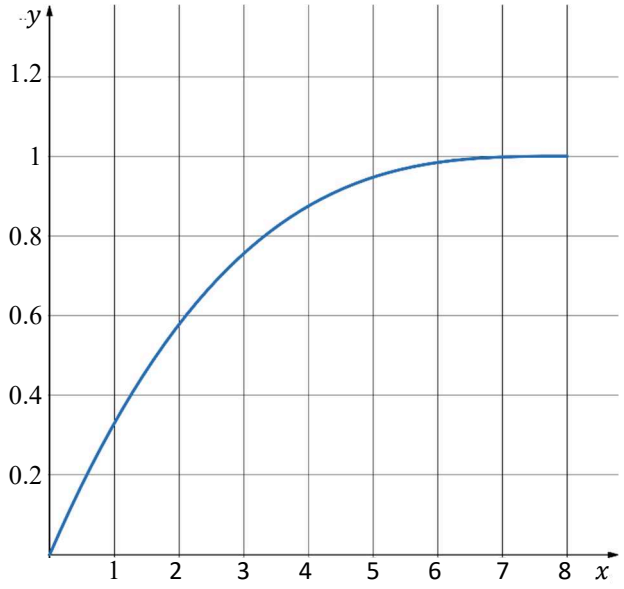
D. $x^4 - 2x^3 + x + C$

3 Which of the following could represent a probability density function for a continuous random variable?

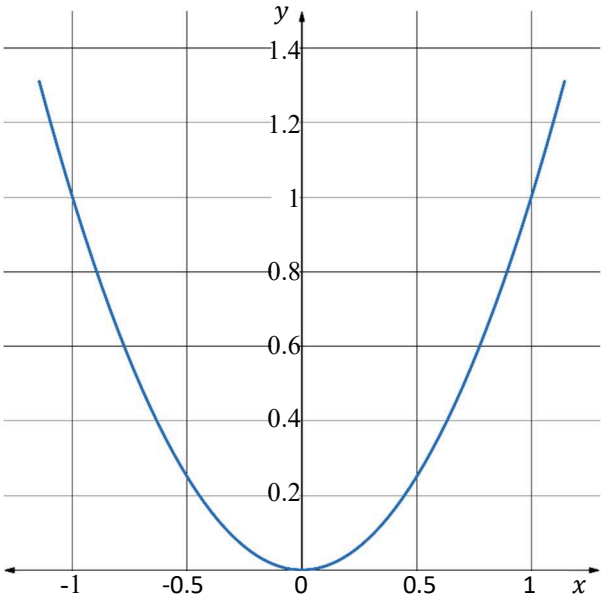
A.



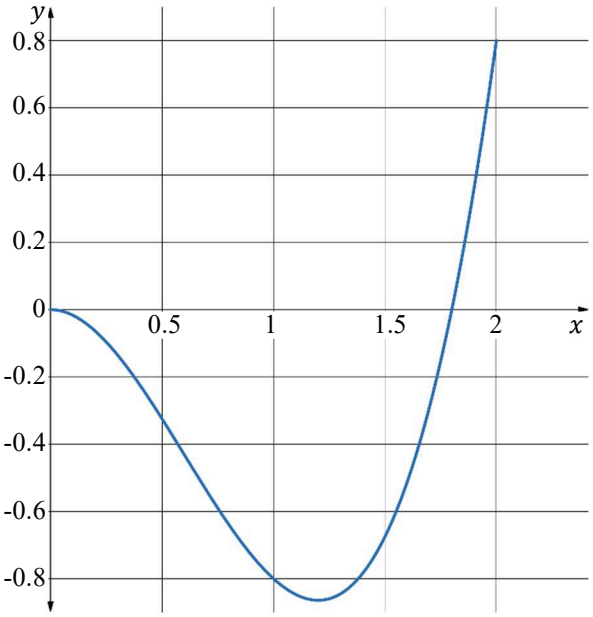
B.



C.



D.



4 Find the angle of inclination of the line $y = -8x - 5$.

A. $76°$

B. $83°$

C. $97°$

D. $104°$

5 A bus company collected data on the number of passengers travelling on a particular route each weekday during the month of June. The table shows the data collected.

Number of passengers	*Frequency*	*Cumulative Frequency*
29	2	2
30	5	7
31	6	13
32	8	21
33	1	22

The bus company calculated the mean, median, mode, range and interquartile range for this data.

One Friday in June was a public holiday and there was a festival on to celebrate. On this day there were 48 passengers on the bus route. The bus company omitted this data from the table as it was an outlier which would not occur every month.

Which of the statistics that the bus company calculated would this outlier have changed?

A. All of the statistics

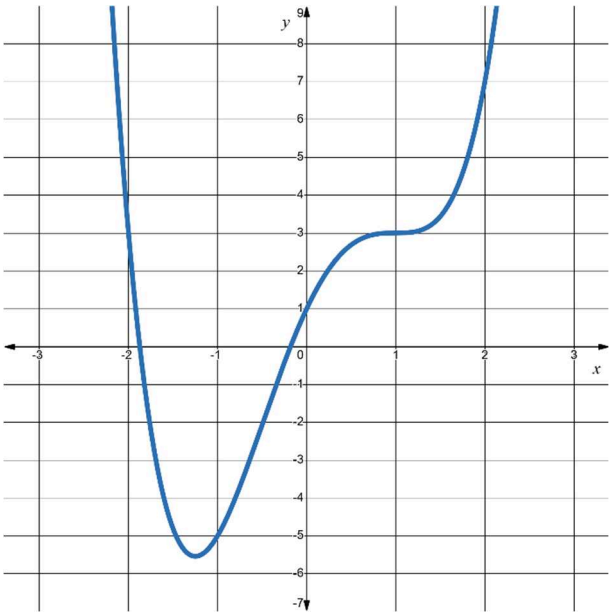
B. Mean, median and range only

C. Mean, mode and range only

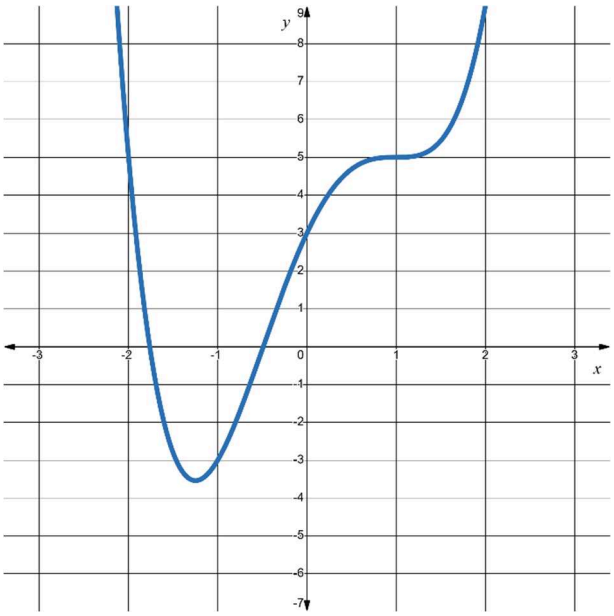
D. Mean and range only

6 Which of the following is the graph of $y = x^4 - x^3 - 3x^2 + 5x + 1$?

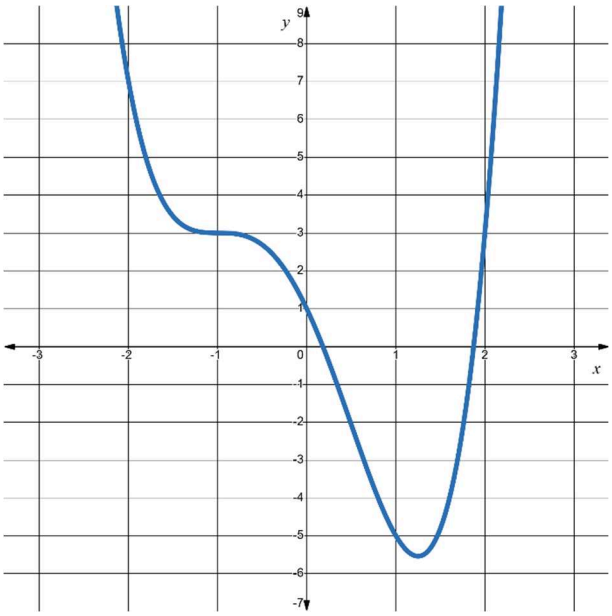
A.



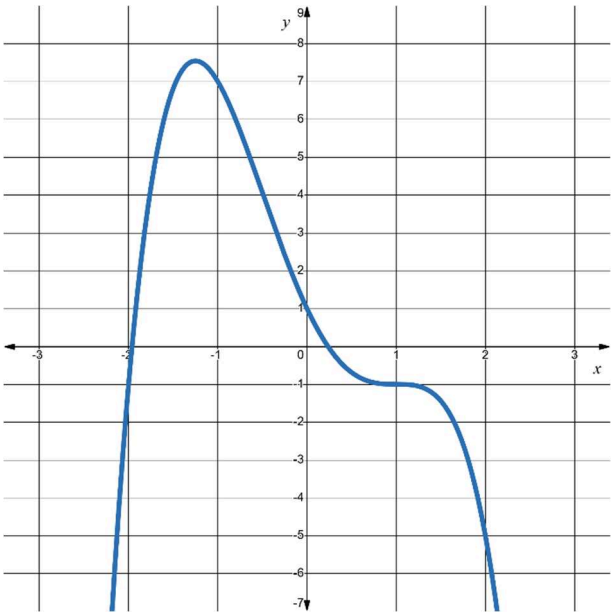
B.



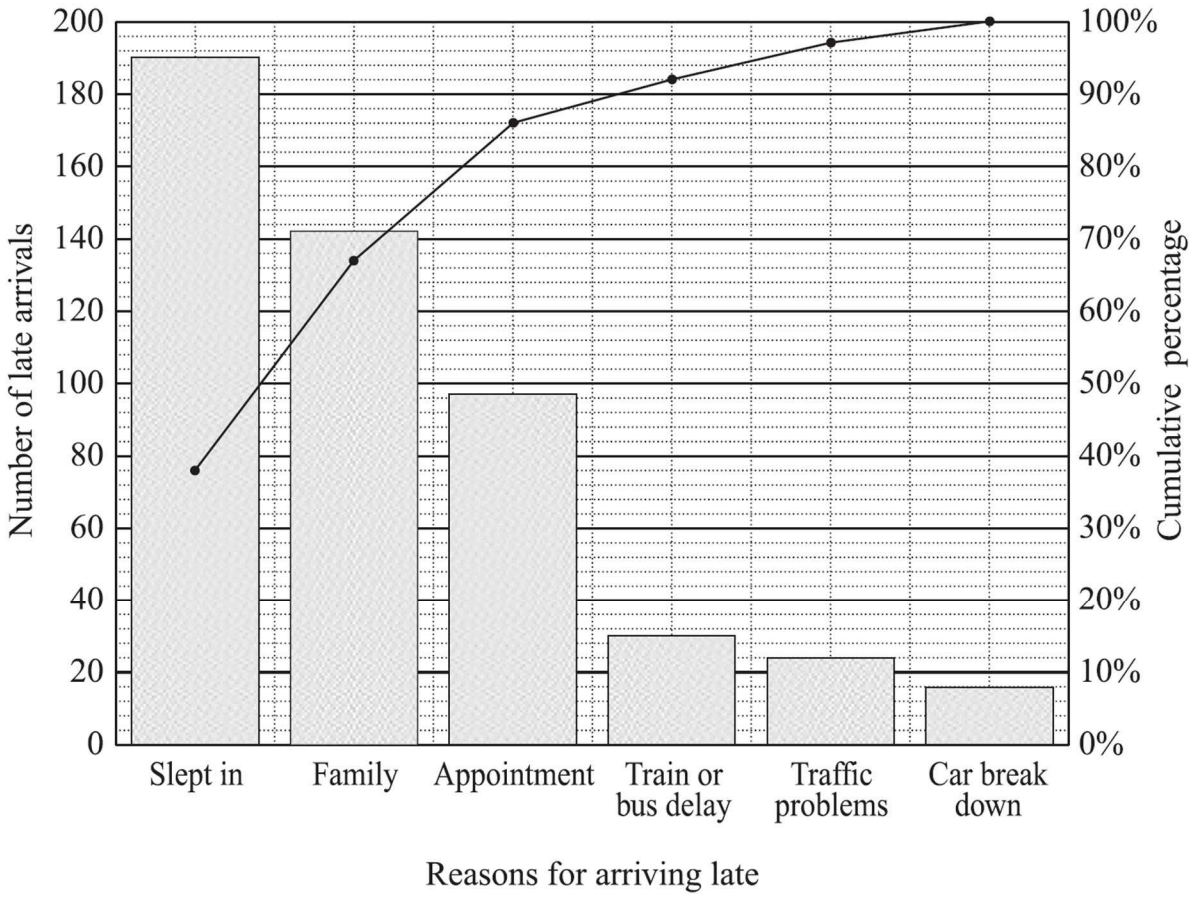
C.



D.



7 A school collected data related to the reasons given by students for arriving late. The Pareto chart shows the data collected.



What percentage of students gave the reason 'Traffic problems'?

A. 2.5%

B. 5%

C. 12%

D. 97%

8 Which of the following describes the graph of $y^2 - 6y + x^2 - 16 = 0$?

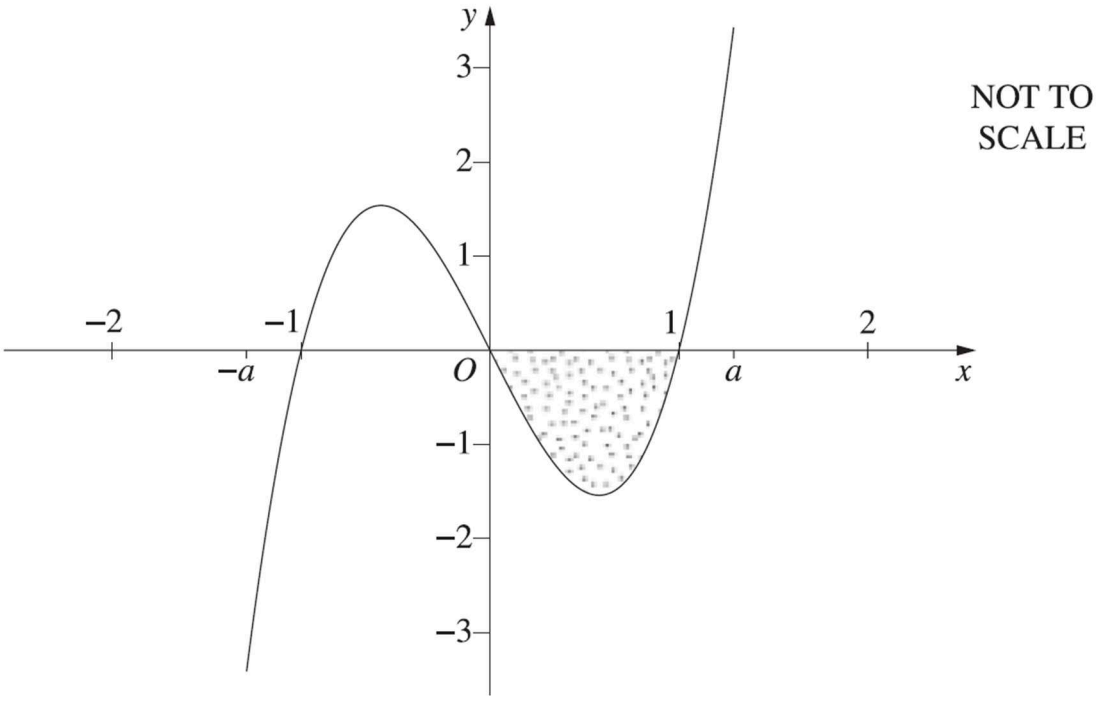
A. A circle with centre at $(0, -3)$ and radius 5 units.

B. A circle with centre at $(0, 3)$ and radius 5 units.

C. A circle with centre at $(0, 3)$ and radius 4 units.

D. A circle with centre at $(3, 0)$ and radius $\sqrt{7}$ units.

9 The diagram shows the graph $y = f(x)$, where $f(x)$ is an odd function.
The shaded area is 1 square unit.



A new function, $g(x)$, is found by taking the graphing of $y = f(x)$ and translating it by 3 units upwards.

Find $\int_{-1}^{1} g(x)\, dx$.

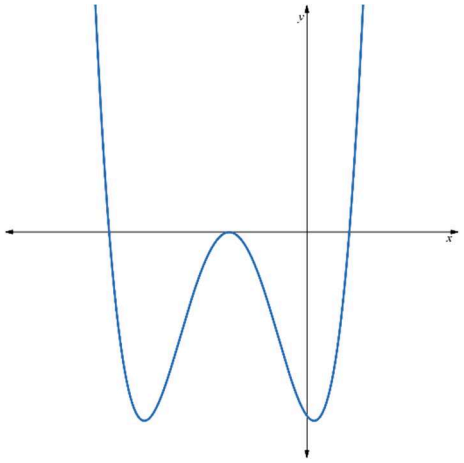
A. 2

B. 3

C. 4

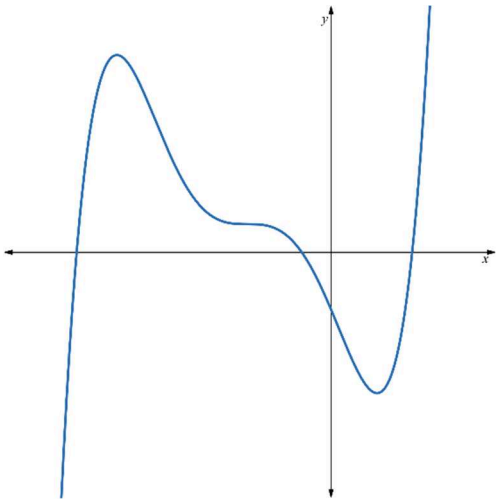
D. 6

10 The graph of a function $y = f'(x)$ is shown below.

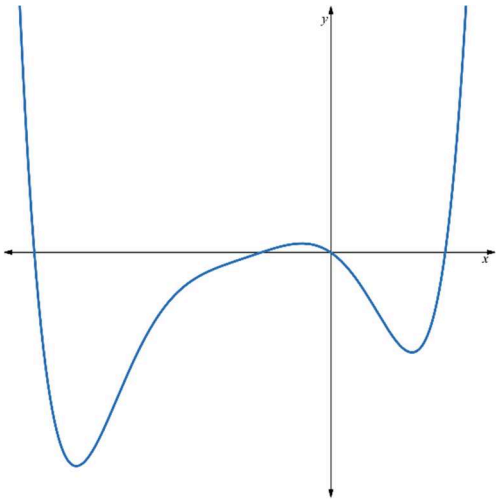


Which of the following is the graph of $y = f(x)$?

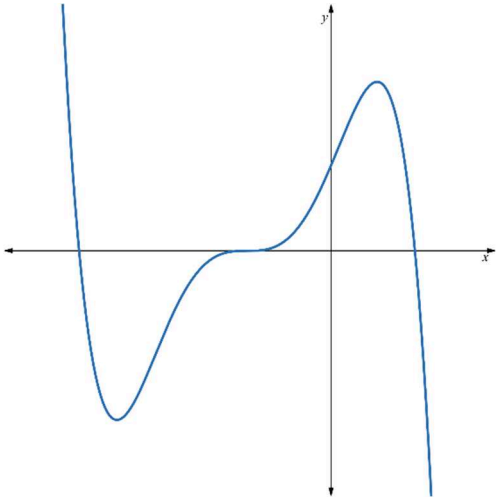
A.



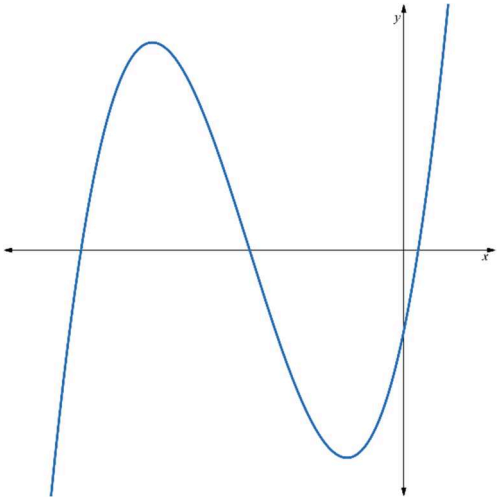
B.



C.



D.



NESA Number: | | | | | | | | | |

2026

Penrith High School
HIGHER SCHOOL CERTIFICATE
TRIAL EXAMINATION

Mathematics Advanced
Section II Answer Booklet 1

Section II

90 marks
Attempt Questions 11–32
Allow about 2 hours and 45 minutes for this section

Booklet 1 – Attempt Questions 11 – 18 (23 marks)
Booklet 2 – Attempt Questions 19 – 24 (25 marks)
Booklet 3 – Attempt Questions 25 – 30 (22 marks)
Booklet 4 – Attempt Questions 32 – 35 (22 marks)

Instructions

- Answer the questions in the spaces provided. Sufficient spaces are provided for typical responses
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided at the back of this booklet. If you use this space, clearly indicate which question you are answering.
- Write your NESA ID above

	Functions	Trigonometric Functions	Calculus	Exponential & Logarithmic Functions	Statistical Analysis	Financial Mathematics
Booklet 1	**/4** 16	**/6** 13, 18	**/4** 12, 15		**/9** 11, 14, 17	

Please turn over

Question 11 (2 marks)

A survey of 230 students found that 180 caught a train, 75 caught a bus and 40 caught neither on the way to school. **2**

What is the probability that a student caught a bus, given that they also caught a train?

Question 12 (2 marks)

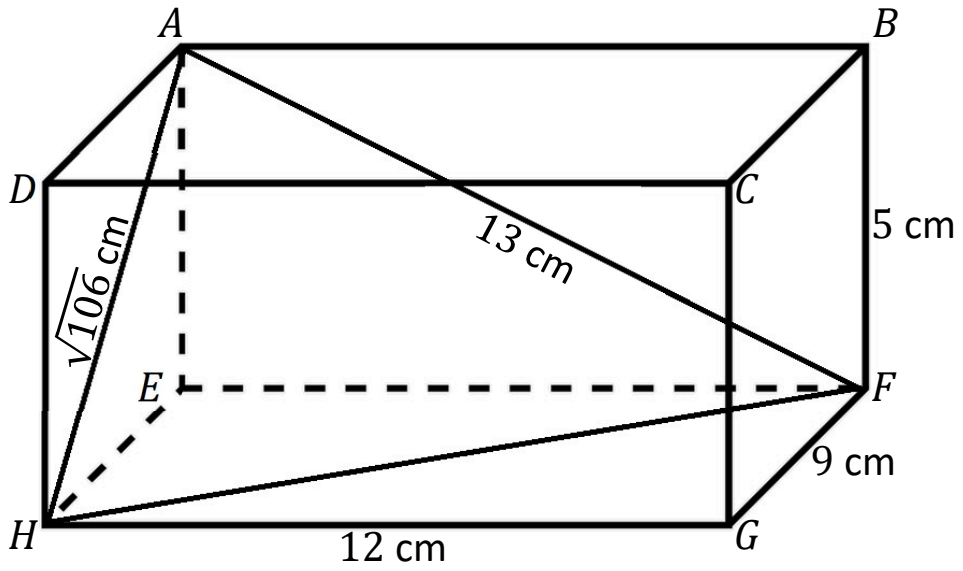
Isaac is training for a marathon. His watch saves his running velocity, $v(t)$, in metres per minute every 20 minutes. The table gives the velocity, $v(t)$ at time t minutes. **2**

t	0	20	40	60	80
$v(t)$	160	175	178	166	150

Use the trapezoidal rule and the velocity at each of the five time values to find the approximate distance in kilometres that Isaac has run in his 80 minute training session.

Question 13 (3 marks)

The rectangular prism below has $BF = 5$ cm, $FG = 9$ cm, $GH = 12$ cm, $AH = \sqrt{106}$ cm and $AF = 13$ cm, as shown in the diagram below. **3**



NOT TO SCALE

Find the size of $\angle HAF$, correct to the nearest degree.

Question 14 (4 marks)

(a) A discrete random variable X has the probability distribution table shown below. **2**

$X = x$	3	5	a	12
$P(x)$	0.1	m	0.2	0.4

Find the value of a and m given an expected value of 8.4.

(b) Hence find the variance of X. **2**

Question 15 (2 marks)

Differentiate $f(x) = x^2 - 3x + 2$ using the first principles formula $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$. **2**

Question 16 (4 marks)

For which values of k does $y = kx^2 + (k-2)x + 1$ have real roots? Provide your answer in simplified exact form. **4**

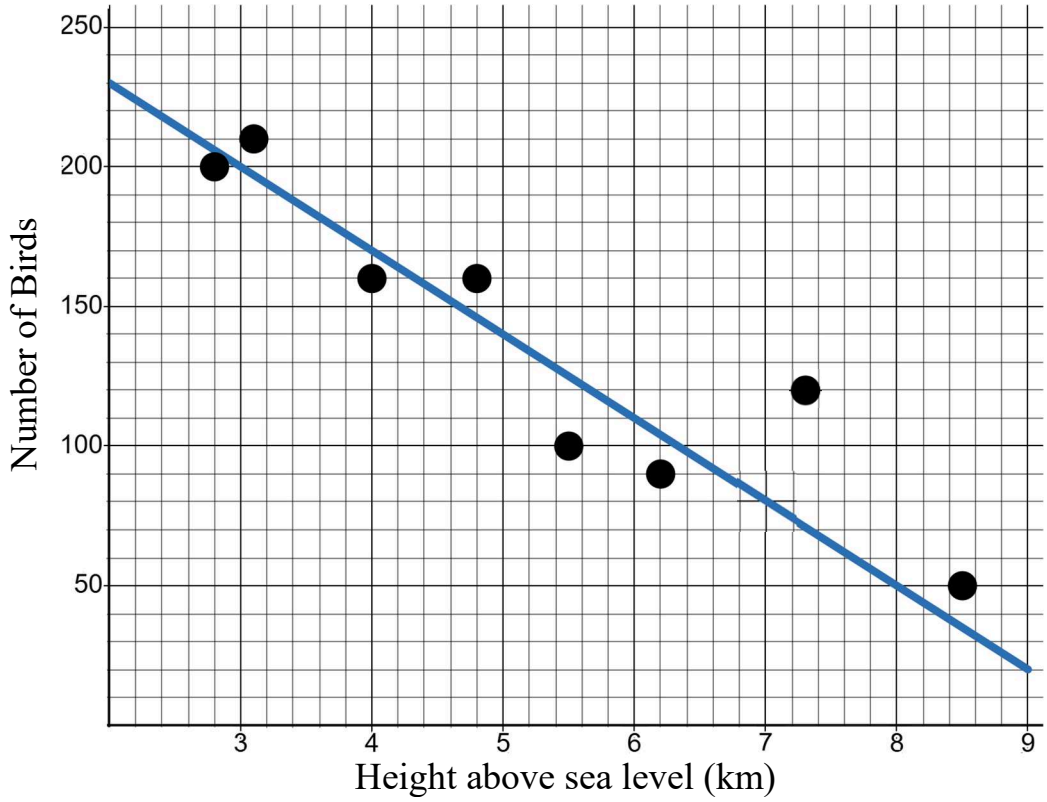
Question 17 (3 marks)

Rosanne studies the number of birds B flying above a National Park. She compares the number of birds at different heights h, in kilometres, between 2 and 9 km above sea level. She finds that the data she gathers follows a linear pattern which can be modelled by a least squares regression line with equation **3**

$$B = -30h + c$$

where c is a constant.

Some of the data points that Rosanne collected along with the least squares regression line are shown in the scatterplot below.



Find the value of c, interpret its meaning in the given context and comment on the accuracy of this value as an estimate.

Question 18 (3 marks)

Solve $5\cos(2x + \frac{\pi}{3}) + 2 = 0$ for $0 \leq x \leq \pi$. **3**

Section II extra writing space

If you use this space, clearly indicate which question you are answering.

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TRIAL EXAMINATION

Mathematics Advanced
Section II Answer Booklet 2

Booklet 2 – Attempt Questions 19 – 24 (25 marks)

Instructions

- Answer the questions in the spaces provided. Sufficient spaces are provided for typical responses
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided at the back of this booklet. If you use this space, clearly indicate which question you are answering.
- Write your NESA ID above

	Functions	Trigonometric Functions	Calculus	Exponential & Logarithmic Functions	Statistical Analysis	Financial Mathematics
Booklet 2	**/2** 21		**/2** 22	**/7** 23	**/2** 19, 24	**/12** 20, 24

Please turn over

Question 19 (2 marks)

Charlie is playing basketball. He has a 13% chance of scoring a goal from beyond the halfway line. How many shots must he take to ensure at least a 97% chance of scoring at least one goal? **2**

Question 20 (2 marks)

A table of future value interest factors for an annuity of $1 is shown. **2**

Rate (r) / Period (n)	*0.005*	*0.01*	*0.015*	*0.02*	*0.03*	*0.06*
7	7.10588	7.21354	7.32300	7.43428	7.66246	8.39384
28	29.97452	32.12910	34.48148	37.05121	42.93092	68.52811
56	64.44140	74.58098	86.79754	101.55826	141.15377	418.82235
84	104.07393	130.67227	166.17264	213.86661	365.88054	2209.41674

Owais deposits $2 000 into a savings account every three months for 7 years at an interest rate of 6% per annum, compounding quarterly.

Find the amount of money in Owais' savings account at the end of the 7 years.

Question 21 (2 marks)

At the point $(3, -4)$ on the curve $y = f(x)$, the tangent has equation $y = 2x - 10$. Determine the equation of the tangent to the curve $y = f(-x)$ at the point $(-3, -4)$. **2**

Question 22 (2 marks)

Find $\int \frac{3x - 4}{3x^2 - 8x} \, dx$. **2**

Question 23 (7 marks)

(a) Use a logarithm law to write $\ln\frac{3x-1}{x+2}$ in the form $\ln a - \ln b$, where a and b are algebraic expressions with no fractions. **1**

(b) Hence, show that $y = \ln\frac{3x-1}{x+2}$ has a gradient of 1 at $x = \frac{-5\pm\sqrt{133}}{6}$. **3**

Question 23 continues on the next page.

(c) (i) Anika thought there was a fault in the answer to Question 23b. She said the answer should be $x = \frac{-5+\sqrt{133}}{6}$ only. However, she is incorrect, there is no error in the answer provided in Question 23b. Anika made an error, as she was solving the question: **1**

"Hence, show that $y = \ln a - \ln b$ has a gradient of 1 at $x = \frac{-5\pm\sqrt{133}}{6}$,,

Instead of the question:

"Hence, show that $y = \ln\frac{3x-1}{x+2}$ has a gradient of 1 at $x = \frac{-5\pm\sqrt{133}}{6}$,,

Provide the restriction that Anika used that lead to her incorrect answer.

(c) (ii) Substitute $x = \frac{-5-\sqrt{133}}{6}$ into a and b, provide answers correct to 2 decimal places, and hence explain why $x = \frac{-5-\sqrt{133}}{6}$ is not a solution to Question 23b if you start with the equation $y = \ln a - \ln b$ but it is a solution if you start with the equation $y = \ln\frac{3x-1}{x+2}$. **2**

Question 24 (10 marks)

One morning a farmer's crops were attacked by 2 000 locusts (insects). During daylight each day, including the first, the locusts population increases by 15%. Unfortunately, the farmer doesn't realise the severity of the locusts for the first two weeks.

(a) Show that there are 14 151 locusts at the farm when the farmer realises the severity of the locust plague, to the nearest whole. **1**

(b) The farmer does not wish to spray his farm with pesticide, so he decides to try a locust trap instead. At the start of the 15th day the farmer installs the locust trap, which traps the locust at nighttime. Find the minimum number of locusts per day that the trap must catch to reduce the number of locust from the 15th day onwards. **1**

(c) Once the trap is in use, the number of locusts at the end of the nth day is given by A_n. The locust trap is guaranteed to successfully catch exactly 4 000 locusts per day. **2**

(i) Show that $A_2 = 14\,151(1.15)^2 - 4000(1.15) - 4000$.

Question 24 continues on the next page.

(ii) Show that $A_n = 14\,151(1.15)^n - 4000\left(\frac{1.15^n - 1}{0.15}\right)$. **3**

(iii) How many days since the farmer installs the trap will it take to catch all of the locusts on their farm? **3**

Section II extra writing space

If you use this space, clearly indicate which question you are answering.

NESA Number: | | | | | | | | | |

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TRIAL EXAMINATION

Mathematics Advanced
Section II Answer Booklet 3

Booklet 3 – Attempt Questions 25 – 30 (22 marks)

Instructions

- Answer the questions in the spaces provided. Sufficient spaces are provided for typical responses
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided at the back of this booklet. If you use this space, clearly indicate which question you are answering.
- Write your NESA ID above

	Functions	Trigonometric Functions	Calculus	Exponential & Logarithmic Functions	Statistical Analysis	Financial Mathematics
Booklet 3	**/2**	**/6**	**/4**		**/8**	**/2**
	27	25, 30	28		29	26

Please turn over

Question 25 (2 marks)

Find x, given that $\sin(x + 10)° = \cos(3x + 20)°$ and $0° \leq x \leq 90°$. **2**

Question 26 (2 marks)

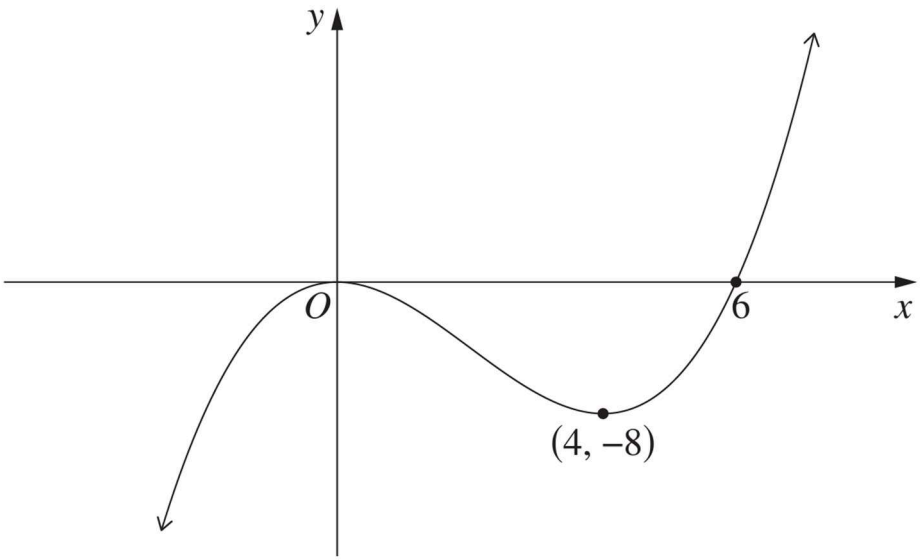
A computer program is designed to select any positive integer x. The probability distribution shown in the table below, where m is a constant, shows the probabilities of selecting the first four positive integers only. **2**

x	1	2	3	4	...
$P(x)$	$3m$	$6m^2$	$12m^3$	$24m^4$	...

Find the value of m.

Question 27 (2 marks)

Consider the graph of $y = f(x)$ as shown. **2**



NOT TO SCALE

Sketch the graph of $y = f(2x + 4)$, labelling the x-intercepts and the coordinates of the turning points.

Question 28 (4 marks)

The noise level $N(t)$, in decibels during a 90 minute football match can be modelled by the function **4**

$$N(t) = \frac{1}{20000}(0.25t^4 - 35t^3 + 1225t^2 + 1\,800\,000)$$

where t is the time in minutes that have been played in the match and $0 \leq t \leq 90$.

Find the loudest noise level during the football match.

Question 29 (8 marks)

Elena, the owner of Elena's Escape Rooms, records the time taken in minutes for each group to complete her escape room. She recognises this data as a continuous random variable, X, which can be modelled by the probability density function

$$f(x) = \begin{cases} -\frac{1}{4500}(x-10)(x-40) & 10 \leq x \leq 40 \\ 0 & \text{elsewhere} \end{cases}$$

(a) Find the mode time taken to complete the escape room at Elena's Escape Rooms. **2**

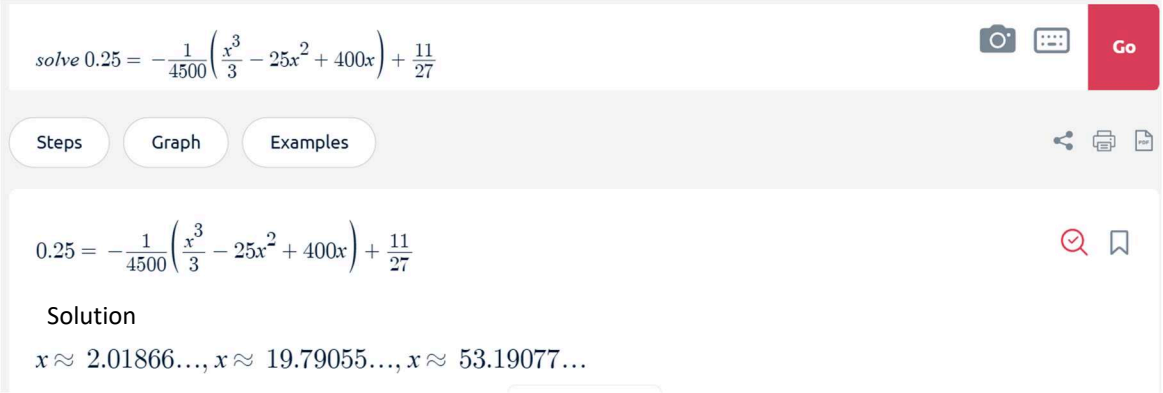
(b) (i) Show that the cumulative density function for the time taken to complete the escape room at Elena's Escape Rooms is given by **2**

$$F(x) = -\frac{1}{4500}\left(\frac{x^3}{3} - 25x^2 + 400x\right) + \frac{11}{27}$$

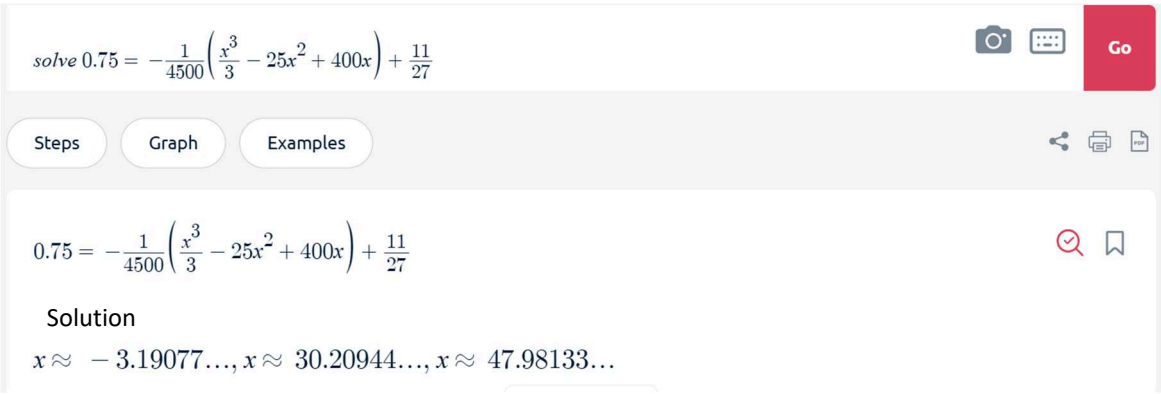
Questions 29 continues on the next page.

(b) (ii) Elena used the cumulative density function found above and an online calculator to find the 25th and 75th percentiles for her escape room. Two screenshots of the calculations made by the online calculator are shown below. **1**

Screenshot 1:



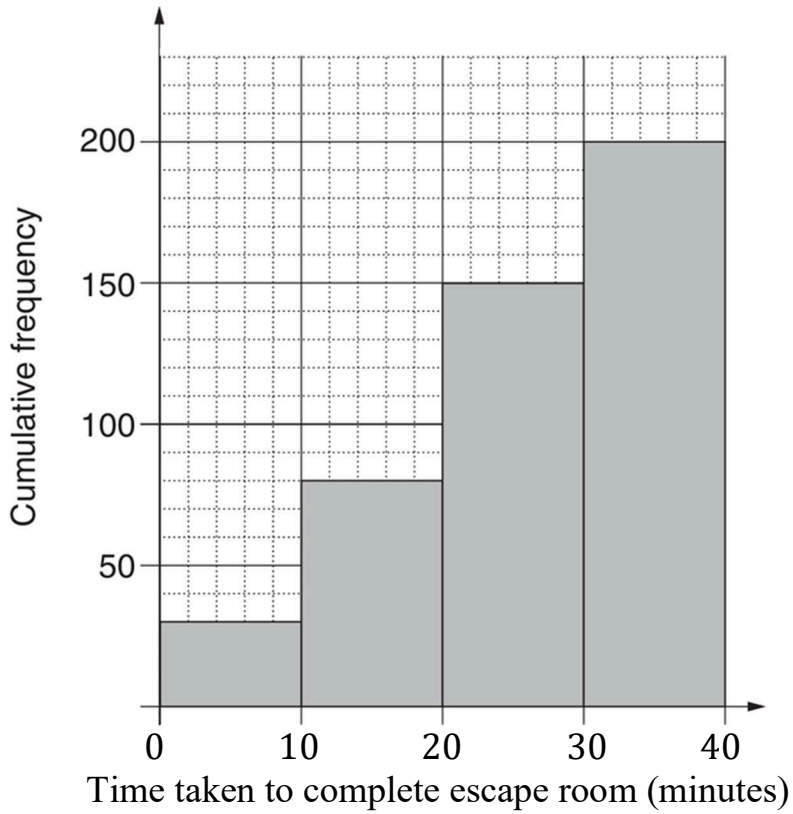
Screenshot 2:



Using the screenshots above, calculate the interquartile range for the time taken to complete Elena's Escape Room.

Questions 29 continues on the next page.

(c) (i) A different escape room company, Escape From Axel, also gathers data on the time its customers take to complete their escape room. Axel displays the data in the cumulative frequency histogram below. **2**



Calculate the interquartile range for the time taken to complete the escape room at Escape From Axel.

(c) (ii) Find the modal time group for the time taken to complete the escape room at Escape From Axel. **1**

Question 30 (4 marks)

Identify the period, amplitude, phase shift and vertical shift of the graph formed by the equation $y = 12\cos(\frac{\pi}{3}(x-2)) + 4$. **4**

Section II extra writing space

If you use this space, clearly indicate which question you are answering.

NESA Number: | | | | | | | | | |

2026

Penrith High School
HIGHER SCHOOL CERTIFICATE
TRIAL EXAMINATION

Mathematics Advanced
Section II Answer Booklet 4

Booklet 4 – Attempt Questions 31 – 35 (22 marks)

Instructions

- Answer the questions in the spaces provided. Sufficient spaces are provided for typical responses
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided at the back of this booklet. If you use this space, clearly indicate which question you are answering.
- Write your NESA ID above

	Functions	Trigonometric Functions	Calculus	Exponential & Logarithmic Functions	Statistical Analysis	Financial Mathematics
Booklet 4		**/10** 32, 35	**/4** 33	**/4** 31		**/4** 34

Please turn over

Question 31 (4 marks)

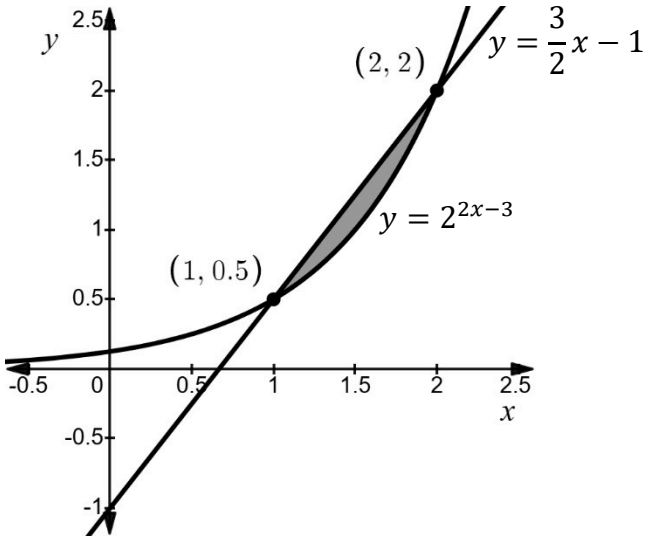
Given $f(x) = 16^x$ and $g(x) = \frac{1}{2}x + \frac{3}{4}$ show that the derivative of $y = f\big(g(x)\big)$ is $\ln 2\,(2^{2x+4})$. **4**

Question 32 (3 marks)

Find $\int \frac{\tan^4 x}{1 - \sec^2 x}\ dx$. **3**

Question 33 (4 marks)

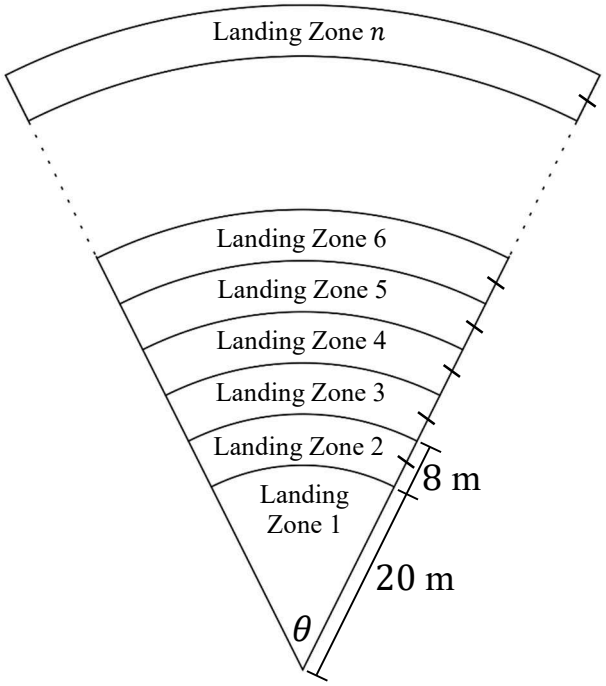
The curve $y = 2^{2x-3}$ meets the line $y = \frac{3}{2}x - 1$ at $(1, 0.5)$ and $(2, 2)$, as shown in the diagram. **4**



Show that the shaded region has an area of $\frac{5\ln 2 - 3}{4\ln 2}$ units^2.

Question 34 (4 marks)

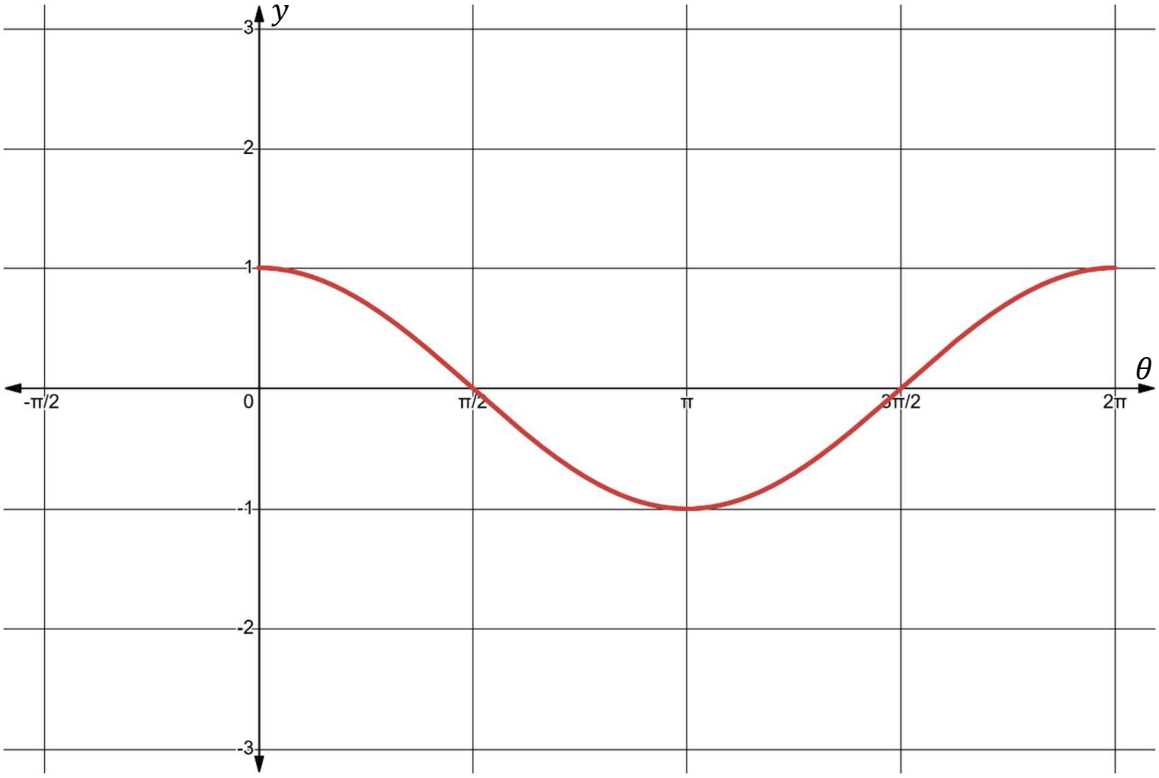
Mrs Palmer is setting up a javelin landing zone. The landing zone takes the form of a sector with angle θ in radians. The first arc is 20 metres from the vertex of the sector and consecutive arcs are 8 metres apart. This is shown in the diagram below. **4**



The perimeter and internal arcs separating each landing zone must be marked on the grass with black paint, as shown in the diagram. Show that the total length of the lines to be marked with paint can be given by $L_n = 4(n^2\theta + 4n\theta + 4n + 6)$, where n is the number of landing zones.

Question 35 (7 marks)

(a) The graph of $y = \cos\theta$ for $0 \leq \theta \leq 2\pi$ is shown below. On the same Cartesian Plane, sketch the graph of $y = -\sin\theta + 1$ for $0 < \theta < 2\pi$. **3**



(b) Hence or otherwise, find any point(s) of intersection of $y = \cos\theta$ and $y = -\sin\theta + 1$ for $0 < \theta < 2\pi$. **2**

Question 35 continues on the next page

(c) Hence or otherwise, solve $\sin^2\theta + \sin\theta\cos\theta - \sin\theta = 0$ for $0 < \theta < 2\pi$. **2**

Section II extra writing space

If you use this space, clearly indicate which question you are answering.

2026

Penrith High School
HIGHER SCHOOL CERTIFICATE
TRIAL EXAMINATION

Mathematics Advanced

General Instructions

- Reading time – 10 minutes
- Working time – 3 hours
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided
- In Questions 11–35 show relevant mathematical reasoning and/or calculations
- Write your NESA ID below, on the Multiple Choice Answer Sheet and the front of Booklets 1, 2, 3 & 4.

Total marks: 100

Section I – 10 marks (pages 3-10)

- Attempt Questions 1–10
- Allow about 15 minutes for this section

Section II – 90 marks (pages 11-44)

- Attempt Questions 11–35
- Allow about 2 hours and 45 minutes for this section

NESA NUMBER: | | | | | | | | | |

TEACHER: Solutions

	Functions	Trigonometric Functions	Calculus	Exponential & Logarithmic Functions	Statistical Analysis	Financial Mathematics
Multiple Choice:	/3 1, 6, 8		/4 2, 4, 9, 10		/3 3, 5, 7	

	Functions	Trigonometric Functions	Calculus	Exponential & Logarithmic Functions	Statistical Analysis	Financial Mathematics	TOTAL
Total (including Multiple Choice)	/11	/22	/18	/9	/22	/18	/100

Section I

10 marks
Attempt questions 1-10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1-10.

---

1 The parabola $y = x^2 + 2$ is translated down 4 units. It is then reflected about the x-axis.

What is the equation of the resulting parabola?

A. $y = -x^2 - 6$

B. $y = -x^2 - 2$

C. $y = -x^2 + 2$

D. $y = -x^2 + 6$

$y = (x^2 + 2) - 4$
$= x^2 - 2$
$y = -(x^2 - 2)$
$= -x^2 + 2$

2 Calculate $\frac{d}{dx}\left[\int_5^x (g^4 - 2g^3 + g - 1)\, dg\right]$

A. $x^4 - 2x^3 + x$

B. $x^4 - 2x^3 + x - 6$

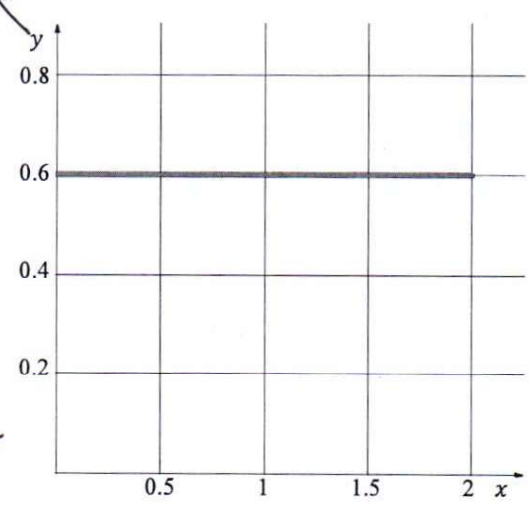
C. $x^4 - 2x^3 + x - 1$

D. $x^4 - 2x^3 + x + C$

By the fundamental theorem of calculus.

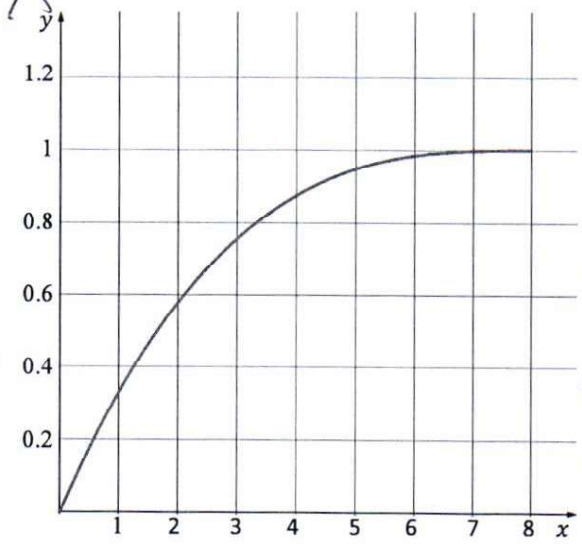
3 Which of the following could represent a probability density function for a continuous random variable?

A.



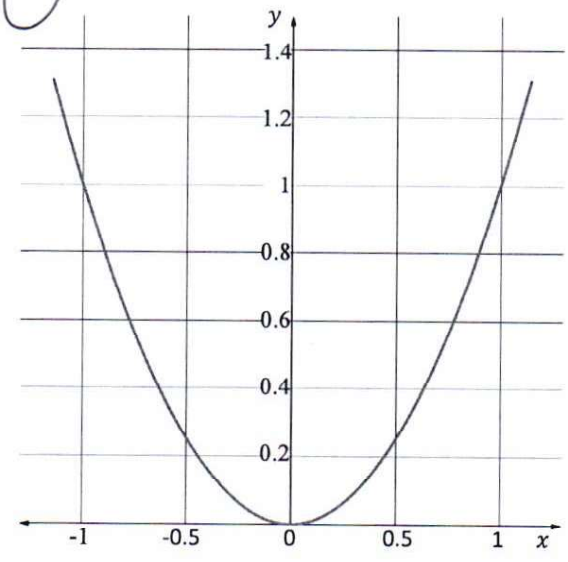
Area → $= 0.6 \times 2 = 1.2 \neq 1$

B.



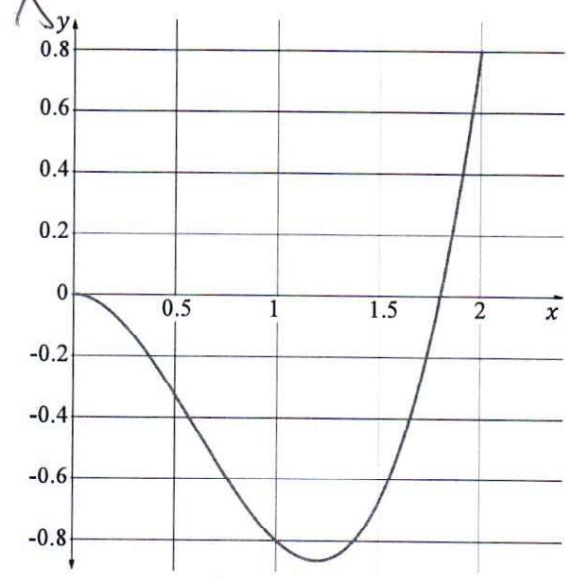
Area is clearly greater than 1

C.



By process of elimination this is the answer. Note x can be negative in a PDF.

D.



$y \geq 0$ in a PDF

4 Find the angle of inclination of the line $y = -8x - 5$.

A. 76°

B. 83°

C. 97°

D. 104°

$m = \tan\theta$

$-8 = \tan\theta$

$\tan^{-1}(-8) = \theta$

$\theta = -83$

$180 - 83 = 97$

5 A bus company collected data on the number of passengers travelling on a particular route each weekday during the month of June. The table shows the data collected.

Number of passengers	*Frequency*	*Cumulative Frequency*
29	2	2
30	5	7
31	6	13
32	8	21
33	1	22

The bus company calculated the mean, median, mode, range and interquartile range for this data.

One Friday in June was a public holiday and there was a festival on to celebrate. On this day there were 48 passengers on the bus route. The bus company omitted this data from the table as it was an outlier which would not occur every month.

Which of the statistics that the bus company calculated would this outlier have changed?

A. All of the statistics

B. Mean, median and range only

C. Mean, mode and range only

D. Mean and range only

An outlier will not change mode. So eliminate A and C.

Now B and C differ only by median. So will it change the median?

At the moment there are 22 scores, so median is the average of 11th and 12th scores (31)

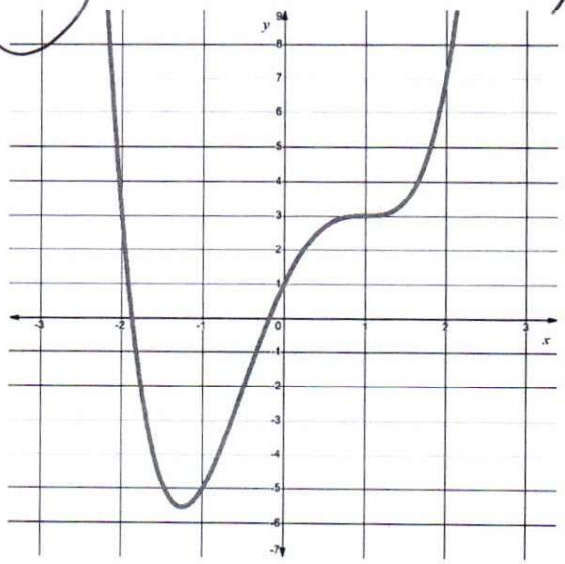
If another score was added there will be 23 scores, so the median will be the 12th score. Which is also 31.

So median won't change.

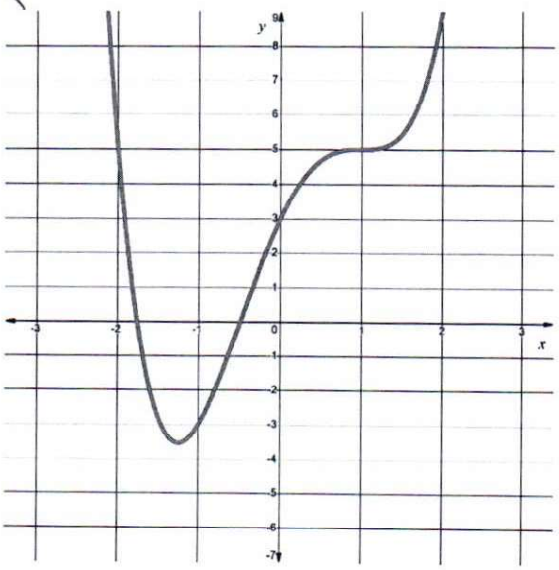
6 Which of the following is the graph of $y = x^4 - x^3 - 3x^2 + 5x + 1$?

y-intercept should be 1 so eliminate B.

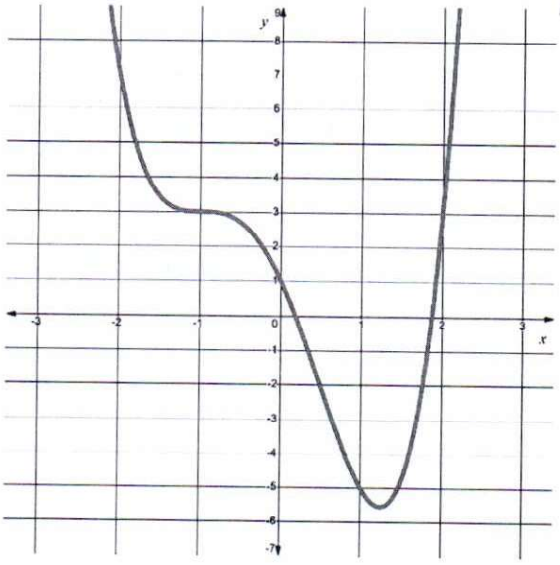
A.



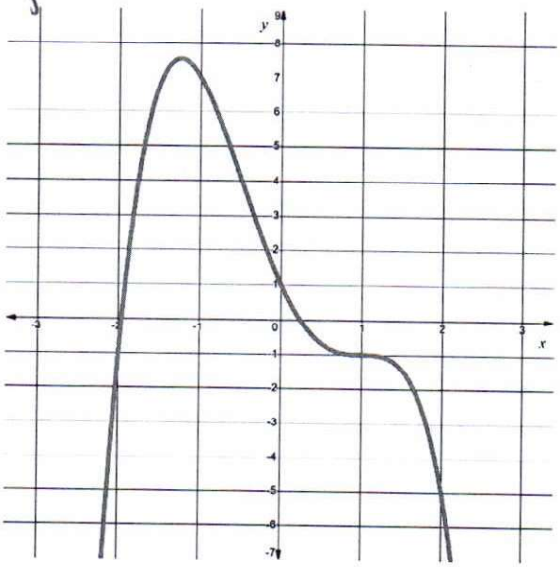
B.



C.



D.



To choose between A or C we can simply substitute an x-value in.

Eg. $x = 1$

$y = 1^4 - 1^3 - 3(1)^2 + 5(1) + 1$

$= 3$

$\therefore$ A

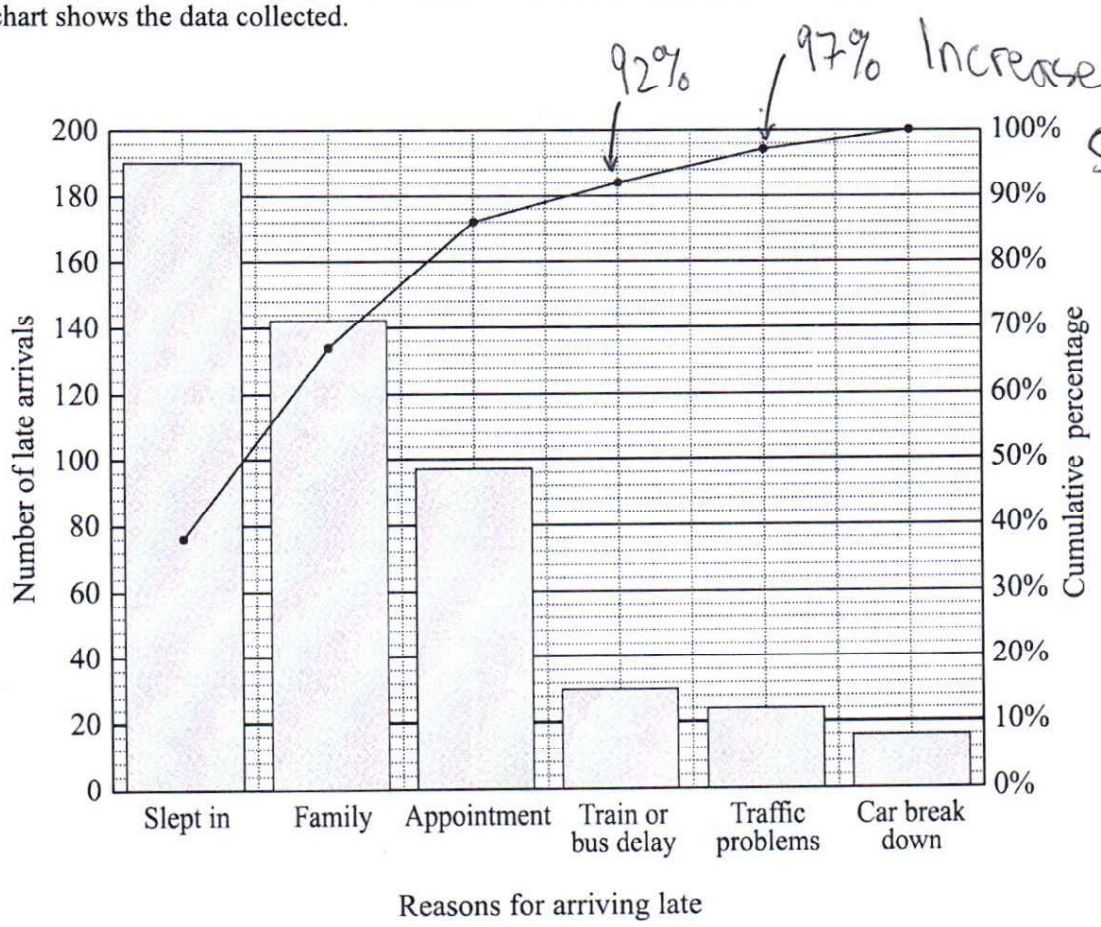
$a = 1$

> 0

So edges should be up

so eliminate D.

7 A school collected data related to the reasons given by students for arriving late. The Pareto chart shows the data collected.



What percentage of students gave the reason 'Traffic problems'?

A. 2.5%

B. 5%

C. 12%

D. 97%

8 Which of the following describes the graph of $y^2 - 6y + x^2 - 16 = 0$?

A. A circle with centre at $(0, -3)$ and radius 5 units.

B. A circle with centre at $(0, 3)$ and radius 5 units.

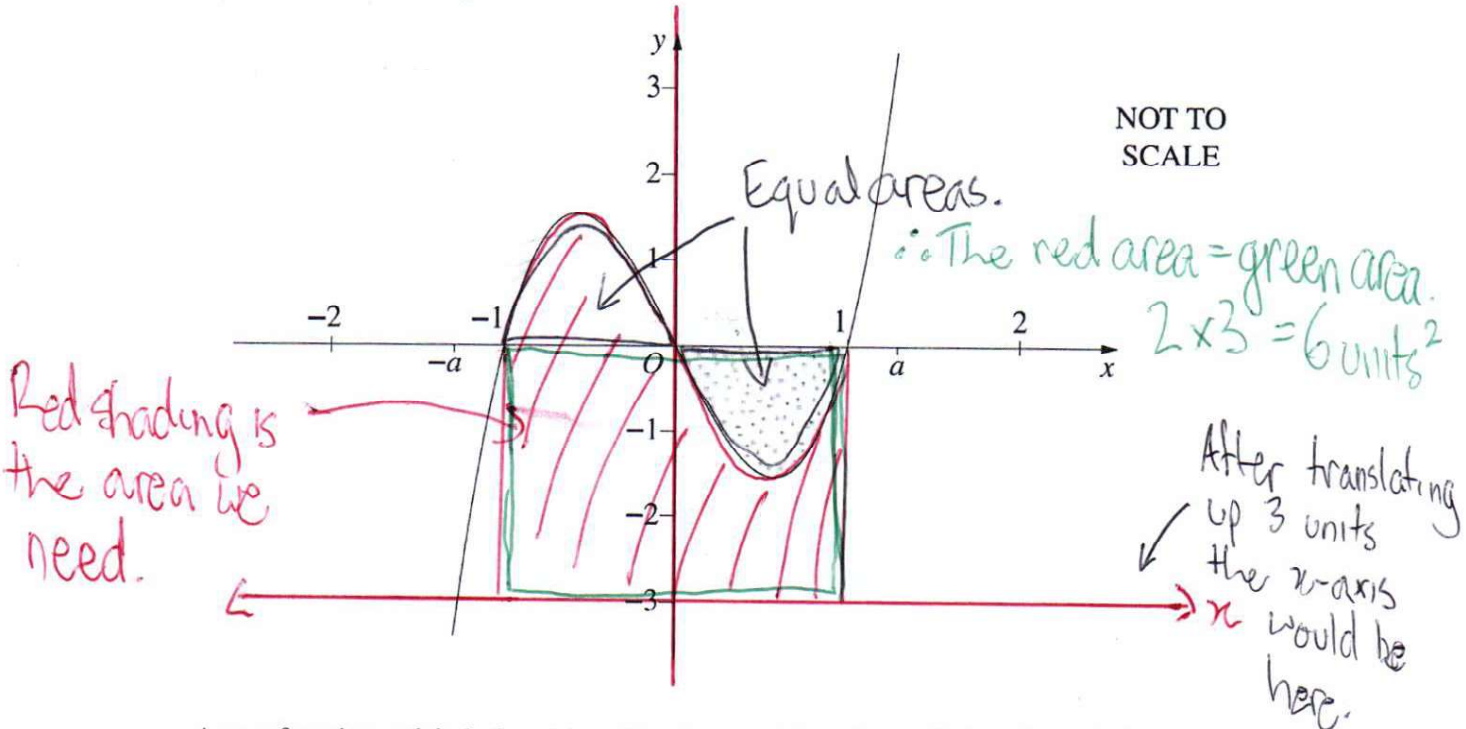
C. A circle with centre at $(0, 3)$ and radius 4 units.

D. A circle with centre at $(3, 0)$ and radius $\sqrt{7}$ units.

$y^2 - 6y + \left(\frac{6}{2}\right)^2 + x^2 = 16 + \left(\frac{6}{2}\right)^2$

$(y-3)^2 + x^2 = 25$

9 The diagram shows the graph $y = f(x)$, where $f(x)$ is an odd function.
The shaded area is 1 square unit.



A new function, $g(x)$, is found by taking the graphing of $y = f(x)$ and translating it by 3 units upwards.

Find $\int_{-1}^{1} g(x)\,dx$.

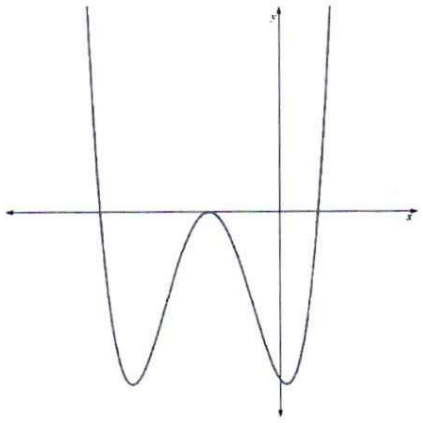
A. 2

B. 3

C. 4

D. 6

10 The graph of a function $y = f'(x)$ is shown below.



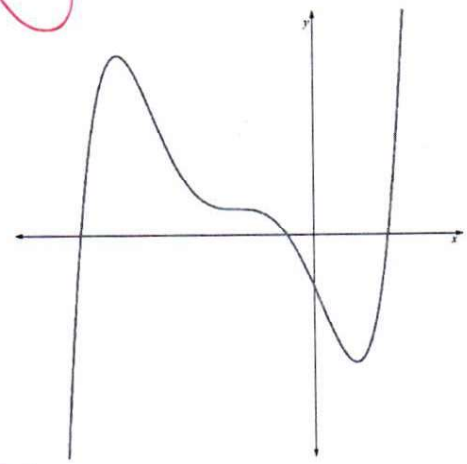
y-intercepts turn into stationary points. So eliminate B and D

On the edges the y values of $y=f'(x)$ are positive, so the gradients of $y=f(x)$ should be positive (uphill) on the edges.

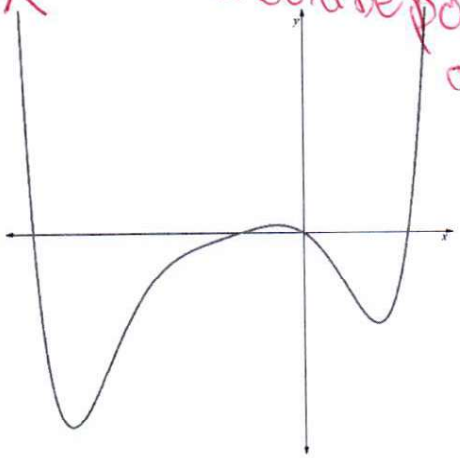
∴ A.

Which of the following is the graph of $y = f(x)$?

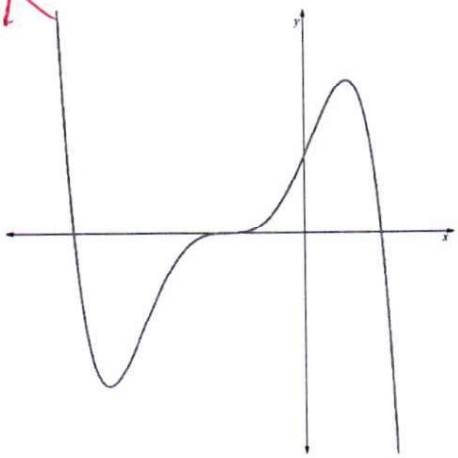
A.



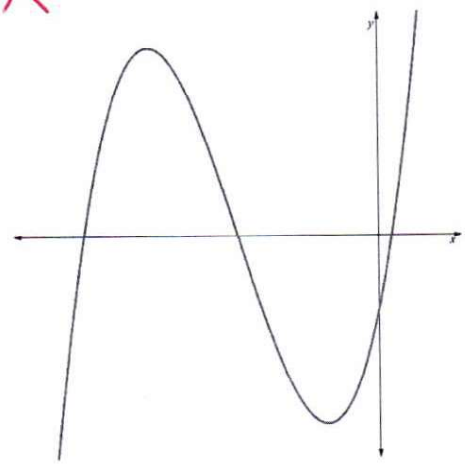
B.



C.



D.



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2026

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TRIAL EXAMINATION

Mathematics Advanced
Section II Answer Booklet 1

Section II

90 marks
Attempt Questions 11–32
Allow about 2 hours and 45 minutes for this section

Booklet 1 – Attempt Questions 11 – 18 (23 marks)
Booklet 2 – Attempt Questions 19 – 24 (23 marks)
Booklet 3 – Attempt Questions 25 – 30 (22 marks)
Booklet 4 – Attempt Questions 32 – 35 (22 marks)

Instructions

- Answer the questions in the spaces provided. Sufficient spaces are provided for typical responses
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided at the back of this booklet. If you use this space, clearly indicate which question you are answering.
- Write your NESA ID above

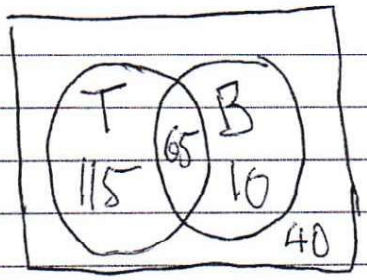
	Functions	Trigonometric Functions	Calculus	Exponential & Logarithmic Functions	Statistical Analysis	Financial Mathematics
Booklet 1	/4 16	/6 13, 18	/4 12, 15		/9 11, 14, 17	

Please turn over

Question 11 (2 marks)

A survey of 230 students found that 180 caught a train, 75 caught a bus and 40 caught neither on the way to school. **2**

What is the probability that a student caught a bus, given that they also caught a train?



230 − 40 = 190 left for the circles.

180 + 75 = 255

255 − 190 = 65 double counted

∴ 65 in intersection.

$P(B|T) = \frac{65}{180}$

comments: Many students did the question correctly.

- Few Mistakes include incorrect numbers in the venn diagram and hence in working
- did not include 40 in any calculations.
- Overall, okay.

Question 12 (2 marks)

Isaac is training for a marathon. His watch saves his running velocity, $v(t)$, in metres per minute every 20 minutes. The table gives the velocity, $v(t)$ at time t minutes. **2**

t	0	20	40	60	80
$v(t)$	160	175	178	166	150

Use the trapezoidal rule and the velocity at each of the five time values to find the approximate distance in kilometres that Isaac has run in his 80 minute training session.

$$\frac{b-a}{2n}\{f(a)+f(b)+2[f(x_1)+f(x_2)+f(x_3)]\}$$

$$\frac{80-0}{2\times 4}\{160+150+2[175+178+166]\}$$

$$= 13480\text{ m} = 13.482\text{ km}$$

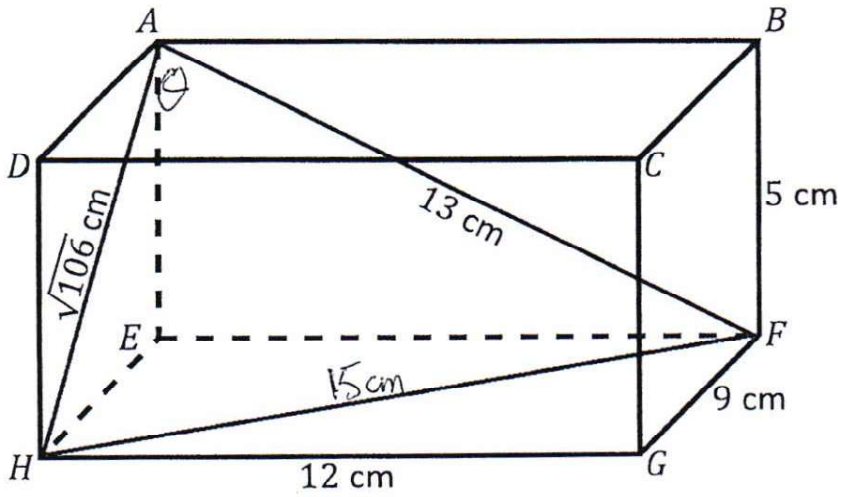
comments: Most students used the formula, but very few with incorrect numbers.

- Many students used n as 5 (should have been 4)
- Some students wrote 13480 in km (deducted 1 mark)

I ignored the conversion from km to m.

Question 13 (3 marks)

The rectangular prism below has $BF = 5$ cm, $FG = 9$ cm, $GH = 12$ cm, $AH = \sqrt{106}$ cm and $AF = 13$ cm, as shown in the diagram below. **3**



NOT TO SCALE

Find the size of $\angle HAF$, correct to the nearest degree.

$HF^2 = 12^2 + 9^2$

$= 225$

$HF = \pm\sqrt{225}$

$= 15 \quad (HF > 0)$

$\cos\theta = \dfrac{\sqrt{106}^2 + 13^2 - 15^2}{2 \times \sqrt{106} \times 13}$

$= 0.1867...$

$\theta = \cos^{-1}(0.1867...)$

$= 79^\circ$

Comments: Most of the students did the question correctly. However, some students used different numbers for the triangle and made the question easier.

- Many students used the cosine rule correctly, however, very few swapped the values around.
- Overall well done.

Question 14 (4 marks)

(a) A discrete random variable X has the probability distribution table shown below. **2**

$X = x$	3	5	a	12
$P(x)$	0.1	m	0.2	0.4

Find the value of a and m given an expected value of 8.4.

$0.1 + m + 0.2 + 0.4 = 1$

$m = 0.3$

well done.

$3 \times 0.1 + 5 \times 0.3 + a \times 0.2 + 12 \times 0.4 = 8.4$

$0.2a + 6.6 = 8.4$

$0.2a = 1.8$

$a = 9$

well done.

(b) Hence find the variance of X. **2**

$E(x^2) - \mu^2$

$[3^2 \times 0.1 + 5^2 \times 0.3 + 9^2 \times 0.2 + 12^2 \times 0.4] - 8.4^2$

$= 11.64$

comments (b)

- Majority of students did very well, except, few students input incorrect numbers, hence lost a mark.

- Overall well done

Question 15 (2 marks)

Differentiate $f(x) = x^2 - 3x + 2$ using the first principles formula $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$. 2

$$\lim_{h \to 0} \frac{[(x+h)^2 - 3(x+h) + 2] - [x^2 - 3x + 2]}{h}$$

$$\lim_{h \to 0} \frac{x^2 + 2xh + h^2 - 3x - 3h + 2 - x^2 + 3x - 2}{h}$$

$$\lim_{h \to 0} \frac{2xh + h^2 - 3h}{h} = 2x + 0 - 3$$

$$= 2x - 3$$

$$\lim_{h \to 0} 2x + h - 3$$

comments: Very well done, except very few forgot to write h^2 while expanding.

Question 16 (4 marks)

For which values of k does $y = kx^2 + (k-2)x + 1$ have real roots? Provide your answer in simplified exact form. 4

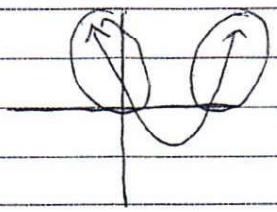
$\Delta \geq 0$

$b^2 - 4ac \geq 0$

$(k-2)^2 - 4(k)(1) \geq 0$

$k^2 - 4k + 4 - 4k \geq 0$

$k^2 - 8k + 4 \geq 0$



$k^2 - 8k + 4 = 0$

$$k = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(1)(4)}}{2(1)}$$

$$= \frac{8 \pm \sqrt{48}}{2} = \frac{8 \pm 4\sqrt{3}}{2} = 4 \pm 2\sqrt{3}$$

$k \leq 4 - 2\sqrt{3}$, $k \geq 4 + 2\sqrt{3}$

comments:
Many students did it well.
common mistakes include
i) $\Delta > 0$
ii) incorrect discriminant calculations
iii) choice of incorrect interval.
iv) overall okay

Question 17 (3 marks)

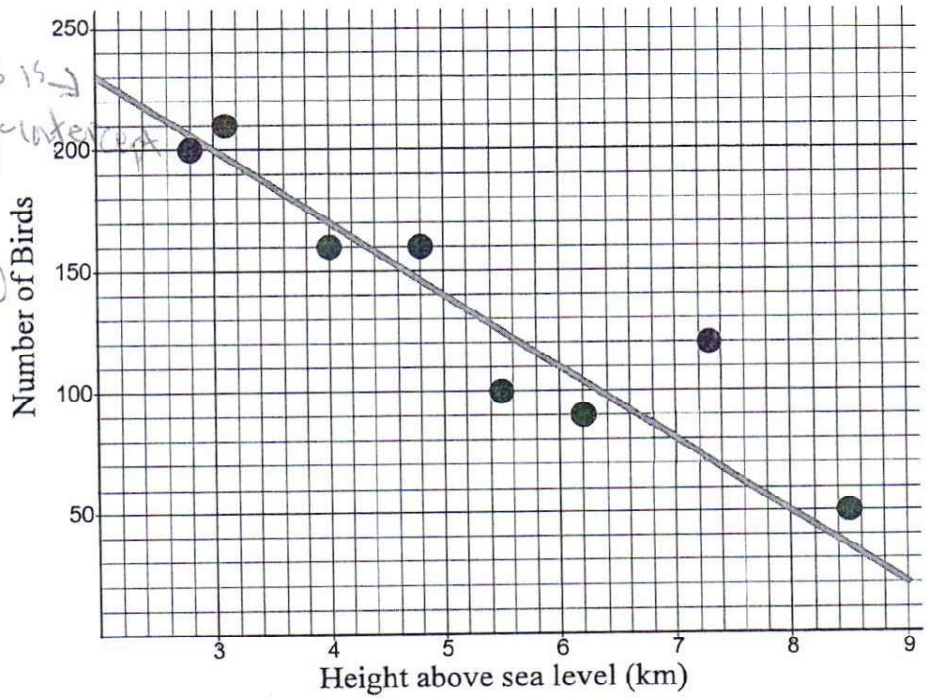
Rosanne studies the number of birds B flying above a National Park. She compares the number of birds at different heights h, in kilometres, between 2 and 9 km above sea level. She finds that the data she gathers follows a linear pattern which can be modelled by a least squares regression line with equation **3**

$$B = -30h + c$$

where c is a constant.

Some of the data points that Rosanne collected along with the least squares regression line are shown in the scatterplot below.

NOTE: This is not the y-intercept as $x=2$ NOT $x=0$ here.



Find the value of c, interpret its meaning in the given context and comment on the accuracy of this value as an estimate.

sub any point on the line.
sub (3, 200)
$200 = -30(3) + C$
$C = 290$

This means at sea level there would be 290 birds. However this is extrapolation (beyond the data range) so we can't be sure of its accuracy.

Comments: Some students wrote 230 as value of C, which is incorrect.
- Very few did not give the context or accuracy, hence lost a mark. Overall, okay.

Question 18 (3 marks)

Solve $5\cos(2x + \frac{\pi}{3}) + 2 = 0$ for $0 \leq x \leq \pi$. 3

$\cos\left(2x + \frac{\pi}{3}\right) = -\frac{2}{5}$

$0 \leq x \leq \pi$

$0 \leq 2x \leq 2\pi$

$\frac{\pi}{3} \leq 2x + \frac{\pi}{3} \leq \frac{7\pi}{3}$

Ref $\angle = \cos^{-1}\left(\frac{2}{5}\right)$

$= 1.159\ldots$

S	A
T	C

$2x + \pi/3 = \pi - 1.159,\ \pi + 1.159$

$2x = \frac{2\pi}{3} - 1.159,\ \frac{2\pi}{3} + 1.159$

$x = \frac{\pi}{3} - \frac{1.159}{2},\ \frac{\pi}{3} + \frac{1.159}{2}$

$= 0.4677,\ 1.6267$

Comments: Some students showed a correct working of this question. However, common mistakes include

- using degrees and radians in the same working
- using $2/5$ instead of $-2/5$ and making the question easy.

NESA Number: | | | | | | | | | |

2026

Penrith High School
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TRIAL EXAMINATION

Solution

Mathematics Advanced
Section II Answer Booklet 2

Booklet 2 – Attempt Questions 19 – 24 (23 marks)

Instructions

- Answer the questions in the spaces provided. Sufficient spaces are provided for typical responses
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided at the back of this booklet. If you use this space, clearly indicate which question you are answering.
- Write your NESA ID above

	Functions	Trigonometric Functions	Calculus	Exponential & Logarithmic Functions	Statistical Analysis	Financial Mathematics
Booklet 2	/2 21		/2 22	/5 23	/2 19, 24	/12 20, 24

Please turn over

Question 19 (2 marks)

Charlie is playing basketball. He has a 13% chance of scoring a goal from beyond the halfway line. How many shots must he take to ensure at least a 97% chance of scoring at least one goal? **2**

$1 - P(\text{no goal})^n = 0.97$

$1 - 0.87^n = 0.97$

$0.03 = 0.87^n$

$\log 0.03 = n \log 0.87$

$n = \dfrac{\log 0.03}{\log 0.87}$

$n = 25.18$

$\therefore$ 26 shots

1 mark - Some progress towards solution

2 marks - Correct solution

* Some students found it difficult to setup the equation at the start.

Question 20 (2 marks)

A table of future value interest factors for an annuity of \$1 is shown. **2**

Rate (r) / *Period (n)*	*0.005*	*0.01*	*0.015*	*0.02*	*0.03*	*0.06*
7	7.10588	7.21354	7.32300	7.43428	7.66246	8.39384
28	29.97452	32.12910	34.48148	37.05121	42.93092	68.52811
56	64.44140	74.58098	86.79754	101.55826	141.15377	418.82235
84	104.07393	130.67227	166.17264	213.86661	365.88054	2209.41674

Owais deposits \$2 000 into a savings account every three months for 7 years at an interest rate of 6% per annum, compounding quarterly.

Find the amount of money in Owais' savings account at the end of the 7 years.

$r = 0.06 \div 4 = 0.015$ $\quad n = 7 \times 4 = 28$

$\$2000 \times 34.48148 = \$68962.96$

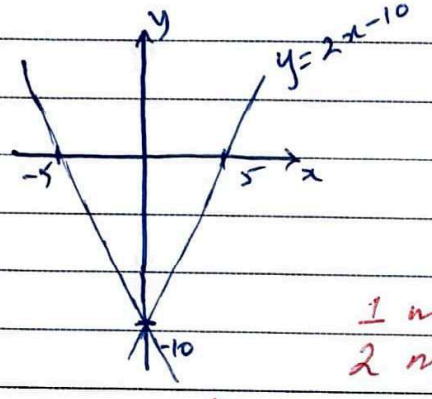
1 mark - finding the correct value of n and r.

2 marks - correct solution

* Some students used $r = 0.06$ and $n = 7$

Question 21 (2 marks)

At the point $(3, -4)$ on the curve $y = f(x)$, the tangent has equation $y = 2x - 10$. Determine the equation of the tangent to the curve $y = f(-x)$ at the point $(-3, -4)$. **2**



The tangent is reflected across the y-axis too.

$\therefore\ y = -2x - 10$

1 mark – some correct steps
2 marks – correct solution

* Many students failed to realise that the tangent is reflected in the y-axis.

Question 22 (2 marks)

Find $\int \frac{3x-4}{3x^2-8x}\ dx$. **2**

$f(x) = 3x^2 - 8x$

$f'(x) = 6x - 8$

$$\int \frac{3x-4}{3x^2-8x}\ dx = \frac{1}{2}\int \frac{2(3x-4)}{3x^2-8x}\ dx$$

$$= \frac{1}{2}\ln\left|3x^2-8x\right| + C$$

1 mark – solution had some error such as not putting the absolute value sign.
2 marks – correct solution

This question was generally done well.

Question 23 (7 marks)

(a) Use a logarithm law to write $\ln\frac{3x-1}{x+2}$ in the form $\ln a - \ln b$, where a and b are algebraic expressions with no fractions. **1**

$\ln(3x-1) - \ln(x+2)$

(b) Hence, show that $y = \ln\frac{3x-1}{x+2}$ has a gradient of 1 at $x = \frac{-5\pm\sqrt{133}}{6}$. **3**

$y = \ln(3x-1) - \ln(x+2)$

$y' = \frac{3}{3x-1} - \frac{1}{x+2}$

$1 = \frac{3}{3x-1} - \frac{1}{x+2}$

$= \frac{3(x+2) - 1(3x-1)}{(3x-1)(x+2)}$

$(3x-1)(x+2) = 3(x+2) - (3x-1)$

$3x^2+5x-2 = 3x+6-3x+1$

$= 7$

$3x^2+5x-9 = 0$

$x = \frac{-5\pm\sqrt{5^2-4\times3\times-9}}{2\times3}$

$= \frac{-5\pm\sqrt{133}}{6}$

Question 23 continues on the next page.

(c) (i) Anika thought there was a fault in the answer to Question 23b. She said the answer should be $x = \frac{-5+\sqrt{133}}{6}$ only. However, she is incorrect, there is no error in the answer provided in Question 23b. Anika made an error, as she was solving the question: **1**

"Hence, show that $y = \ln a - \ln b$ has a gradient of 1 at $x = \frac{-5\pm\sqrt{133}}{6}$,,

Instead of the question:

"Hence, show that $y = \ln\frac{3x-1}{x+2}$ has a gradient of 1 at $x = \frac{-5\pm\sqrt{133}}{6}$,,

Provide the restriction that Anika used that lead to her incorrect answer.

$3x-1>0$ $\quad x+2>0$

$x>\frac{1}{3}$ $\quad x>-2$

$\boxed{x>\frac{1}{3}}$

NOTE: $x>0$ is not the correct restriction.

(c) (ii) Substitute $x = \frac{-5-\sqrt{133}}{6}$ into a and b, provide answers correct to 2 decimal places, and hence explain why $x = \frac{-5-\sqrt{133}}{6}$ is not a solution to Question 23b if you start with the equation $y = \ln a - \ln b$ but it is a solution if you start with the equation $y = \ln\frac{3x-1}{x+2}$. **2**

$a = 3\left(\frac{-5-\sqrt{133}}{6}\right)-1$ $\quad b = \frac{-5-\sqrt{133}}{6}+2$

$= -9.27$ $\quad = -0.76$

This is not a solution using $\ln(3x-1)-\ln(x+2)$ as the arguments are both less than $\frac{1}{3}$ which is the restriction, or we can say the argument is negative but you can only take the log of a positive number.

However, it is a solution using $\ln\left(\frac{3x-1}{x+2}\right)$ as the numerator and denominator are both negative, which simplifies to a positive argument.

Question 24 (10 marks)

One morning a farmer's crops were attacked by 2 000 locusts (insects). During daylight each day, including the first, the locusts population increases by 15%. Unfortunately, the farmer doesn't realise the severity of the locusts for the first two weeks.

(a) Show that there are 14 151 locusts at the farm when the farmer realises the severity of the locust plague, to the nearest whole. **1**

$A = P(1+r)^n$

$= 2000(1+0.15)^{14}$

$= 14151.41$

$= 14151$

generally done well.

(b) The farmer does not wish to spray his farm with pesticide, so he decides to try a locust trap instead. At the start of the 15th day the farmer installs the locust trap, which traps the locust at nighttime. Find the minimum number of locusts per day that the trap must catch to reduce the number of locust from the 15th day onwards. **1**

On the 15th day, there are $14151 \times 0.15 = 2122.65$ new locusts born.

∴ The trap must catch at least 2123 locusts.

generally done well. Few students made calculation errors.

(c) Once the trap is in use, the number of locusts at the end of the nth day is given by A_n. The locust trap is guaranteed to successfully catch exactly 4 000 locusts per day. **2**

(i) Show that $A_2 = 14\,151(1.15)^2 - 4000(1.15) - 4000$.

$A_0 = 14151$

$A_1 = 14151(1.15) - 4000$

$A_2 = [14151(1.15) - 4000](1.15) - 4000$

$= 14151(1.15)^2 - 4000(1.15) - 4000$

1 mark – Some progress.

2 marks – correct solution.

This question was generally done well.

Question 24 continues on the next page.

(ii) Show that $A_n = 14\,151(1.15)^n - 4000\left(\frac{1.15^n - 1}{0.15}\right)$. **3**

$A_3 = \left[14151(1.15)^2 - 4000(1.15) - 4000\right](1.15) - 4000$

(1) $= 14151(1.15)^3 - 4000(1.15)^2 - 4000(1.15) - 4000$

$A_n = 14151(1.15)^n - 4000(1.15)^{n-1} - \cdots - 4000(1.15) - 4000$

$= 14151(1.15)^n - 4000\left[1 + (1.15) + (1.15)^2 + \cdots + (1.15)^{n-1}\right]$

(1) $1 + (1.15) + (1.15)^2 + \cdots + (1.15)^{n-1}$ is a G.P. $(a = 1 \quad r = 1.15)$

$A_n = 14151(1.15)^n - 4000\left(\frac{1.15^n - 1}{1.15 - 1}\right)$

(1) $= 14151(1.15)^n - 4000\left(\frac{1.15^n - 1}{0.15}\right)$

* Some students had difficulty identifying the correct series.

(iii) How many days since the farmer installs the trap will it take to catch all of the locusts on their farm? **3**

(1) $14151(1.15)^n - 4000\left(\frac{1.15^n - 1}{0.15}\right) = 0$

$14151(1.15)^n - \frac{80,000}{3}(1.15)^n + \frac{80000}{3} = 0$.

(1) $-\frac{37547}{3}(1.15)^n = -\frac{80,000}{3}$

$37547(1.15)^n = 80,000$

$(1.15)^n = \frac{80,000}{37547}$

(1) $n\log(1.15) = \log\left(\frac{80,000}{37547}\right)$

$n = \frac{\log\left(\frac{80,000}{37547}\right)}{\log 1.15}$

$= 5.41$

$\therefore$ 6 days

* Generally done well.

* Some students made calculation errors

* marks were not deducted if the answer wasn't rounded to 6 days.

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2026

Penrith High School
HIGHER SCHOOL CERTIFICATE
TRIAL EXAMINATION

Mathematics Advanced
Section II Answer Booklet 3

Booklet 3 – Attempt Questions 25 – 30 (22 marks)

Instructions

- Answer the questions in the spaces provided. Sufficient spaces are provided for typical responses
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided at the back of this booklet. If you use this space, clearly indicate which question you are answering.
- Write your NESA ID above

	Functions	Trigonometric Functions	Calculus	Exponential & Logarithmic Functions	Statistical Analysis	Financial Mathematics
Booklet 3	**/2**	**/6**	**/4**		**/8**	**/2**
	27	25, 30	28		29	26

Please turn over

Question 25 (2 marks)

Find x, given that $\sin(x + 10)° = \cos(3x + 20)°$ and $0° \leq x \leq 90°$. **2**

$\cos(90 - (x+10)) = \cos(3x+20)$ (1)

$90 - x - 10 = 3x + 20$

$4x = 60$

$x = 15$ (1)

Alternative :

$(x+10) + (3x + 20) = 90$

$4x + 30 = 90$

$x = 15$

* $\sin\theta \neq \cos(180 - \theta)$

$\sin\theta \neq \cos(90 + \theta)$

* Penalised for guessing and checking.

Question 26 (2 marks)

A computer program is designed to select any positive integer x. The probability distribution shown in the table below, where m is a constant, shows the probabilities of selecting the first four positive integers only. **2**

x	1	2	3	4	...
$P(x)$	$3m$	$6m^2$	$12m^3$	$24m^4$	...

× 2m × 2m × 2m

Find the value of m.

G.P. with $a = 3m$, $r = 2m$

For $\Sigma P(x) = 1$, $|r| < 1$ & $r \neq 0$.

$S_\infty = \frac{a}{1-r}$

$1 = \frac{3m}{1-2m}$ (1)

$1 - 2m = 3m$

$5m = 1$

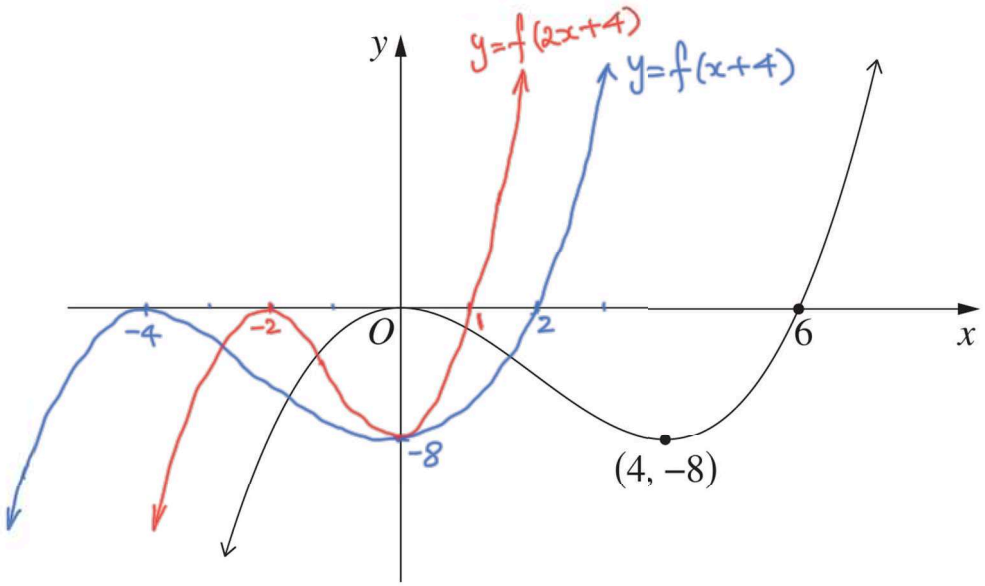
$\therefore m = \frac{1}{5}$ (1)

* No marks for :
- $m = 0$
- S_∞ is not found
- Guessing and checking

Done well

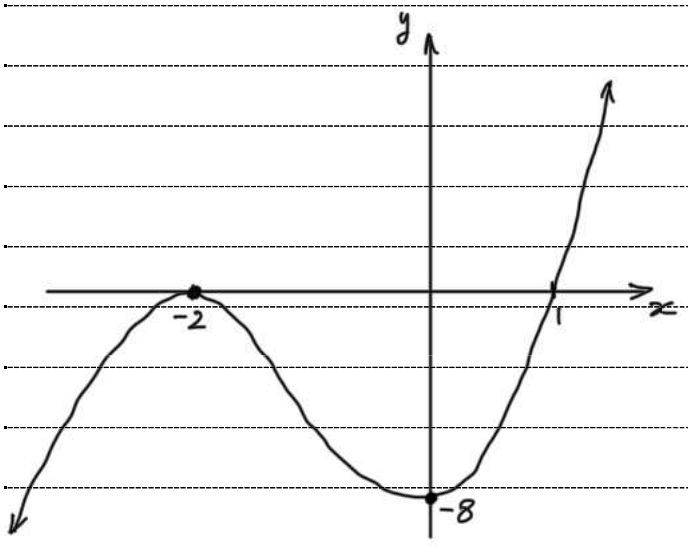
Question 27 (2 marks)

Consider the graph of $y = f(x)$ as shown. **2**



NOT TO SCALE

Sketch the graph of $y = f(2x + 4)$, labelling the x-intercepts and the coordinates of the turning points.



Note:

$f(x)$ → $f(x+4)$ → $f(2x+4)$

Left 4 units

Horizontal dilation by a factor of $\frac{1}{2}$

Question 28 (4 marks)

The noise level $N(t)$, in decibels during a 90 minute football match can be modelled by the function **4**

$$N(t) = \frac{1}{20000}(0.25t^4 - 35t^3 + 1225t^2 + 1\,800\,000)$$

where t is the time in minutes that have been played in the match and $0 \leq t \leq 90$.

Find the loudest noise level during the football match.

$N'(t) = \frac{1}{20000}(t^3 - 105t^2 + 2450t)$

Let $N'(t) = 0$ for stationary points,

$0 = \frac{1}{20000}(t^3 - 105t^2 + 2450t)$

$0 = t(t^2 - 105t + 2450)$

$0 = t(t-70)(t-35)$

$\therefore t = 0, 70, 35$ (1)

$N = 90, 90, 108.76$ (1)

* Must check the values of the endpoints, $N(0)$ and $N(90)$.
* No marks for guessing and checking.
* Most common mistake:
  Max. = 108.8
  Without checking $N(90)$.

Can be skipped & just compare n values.

$N''(t) = \frac{1}{20000}(3t^2 - 210t + 2450)$

$N''(0) = \frac{1}{20000}(2450) = \frac{49}{400} > 0$ $\therefore$ Min. turning point at $t=0$.

$N''(35) = \frac{1}{20000}(3(35)^2 - 210(35) + 2450) = -\frac{49}{800} < 0$ $\therefore$ Max. turning point at $x=35$.

$N''(70) = \frac{1}{20000}(3(70)^2 - 210(70) + 2450) = \frac{49}{400} > 0$ $\therefore$ Min. turning point at $x = 70$.

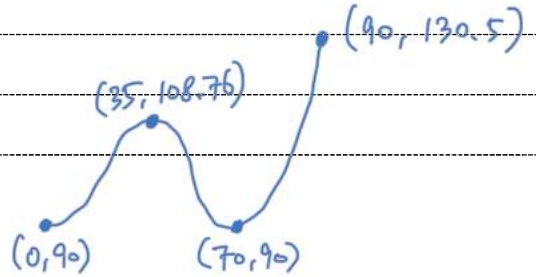
Check end points,

$N(0) = 90$ from above

$N(90) = \frac{1}{20000}(0.25(90)^4 - 35(90)^3 + 1225(90)^2 + 1800000)$

$= 130.5$ (1)

$\therefore$ The loudest noise level is 130.5 decibels. (1)



Question 29 (8 marks)

Elena, the owner of Elena's Escape Rooms, records the time taken in minutes for each group to complete her escape room. She recognises this data as a continuous random variable, X, which can be modelled by the probability density function

$$f(x) = \begin{cases} -\frac{1}{4500}(x-10)(x-40) & 10 \leq x \leq 40 \\ 0 & \text{elsewhere} \end{cases}$$

Alternative
Sub $f(x)=0$ for x-intercepts,
$0 = -\frac{1}{4500}(x-10)(x-40)$
$x = 10, 40$
$f(x)$ is concaving down parabola since it is quadratic & $a<0$.

Done well

(a) Find the mode time taken to complete the escape room at Elena's Escape Rooms. **2**

Max. $f(x)$ occurs at $x = \frac{10+40}{2} = 25$
$\therefore$ Mode $= 25$

$f(x) = -\frac{1}{4500}(x^2 - 50x + 400)$

$f'(x) = -\frac{1}{4500}(2x - 50)$

Let $f'(x) = 0$ for stationary point,

$0 = -\frac{1}{4500}(2x-50)$

$x = 25$

Since $a = -\frac{1}{4500} < 0$ $f(x)$ is concave down.

$\therefore f(x)$ is max. at $x = 25$.

$\therefore$ Mode $= 25$.

* Mode is the x value when $f(x)$ is maximum. Mode $\neq f(x)$.

* No marks if no steps shown.

(b) (i) Show that the cumulative density function for the time taken to complete the escape room at Elena's Escape Rooms is given by **2**

Done well

$$F(x) = -\frac{1}{4500}\left(\frac{x^3}{3} - 25x^2 + 400x\right) + \frac{11}{27}$$

$F(x) = -\frac{1}{4500}\int_{10}^{x}(x^2 - 50x + 400)\,dx$ (1)

$= -\frac{1}{4500}\left[\frac{x^3}{3} - 25x^2 + 400x\right]_{10}^{x}$

$= -\frac{1}{4500}\left(\frac{x^3}{3} - 25x^2 + 400x - \left(\frac{10^3}{3} - 25(10)^2 + 400\right)\right)$

$= -\frac{1}{4500}\left(\frac{x^3}{3} - 25x^2 + 400x - \frac{5500}{3}\right)$ (1)

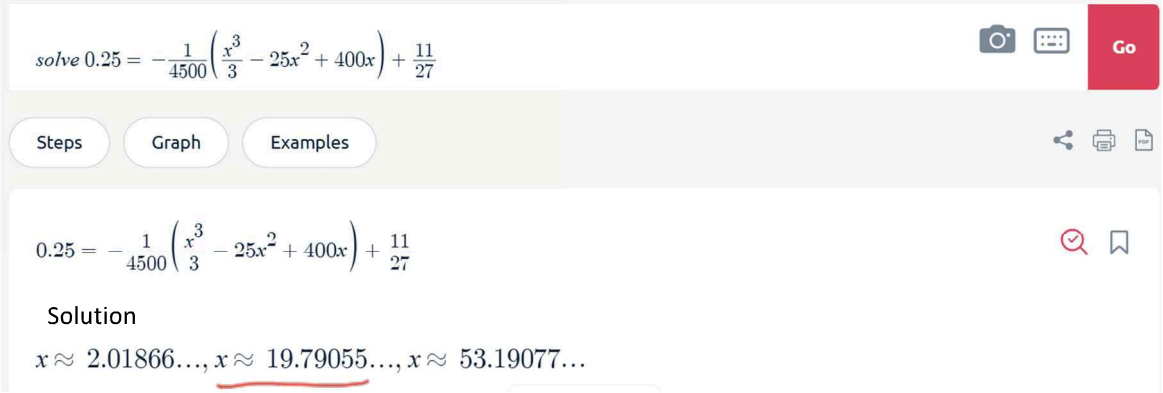
$= -\frac{1}{4500}\left(\frac{x^3}{3} - 25x^2 + 400x\right) + \frac{11}{27}$

Questions 29 continues on the next page.

Done well

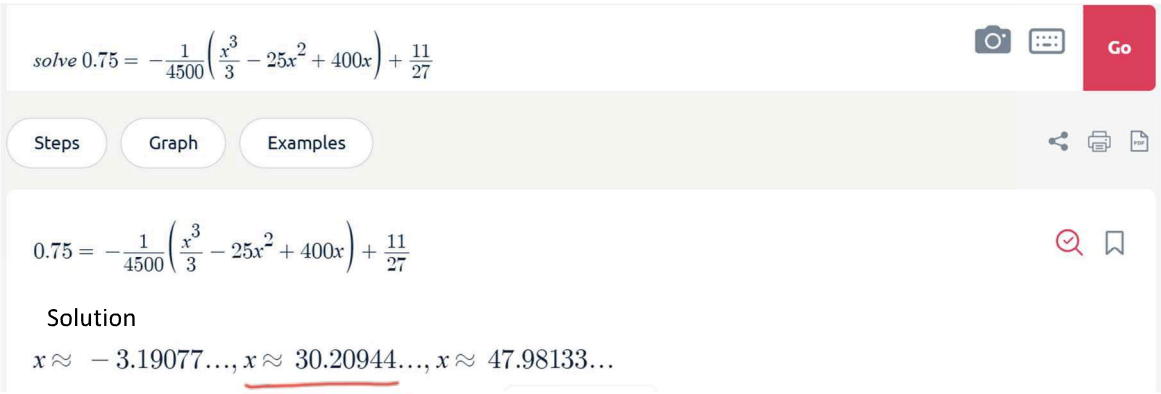
(b) (ii) Elena used the cumulative density function found above and an online calculator to find the 25th and 75th percentiles for her escape room. Two screenshots of the calculations made by the online calculator are shown below. **1**

Screenshot 1:



Since $10 \leq x \leq 40$

Screenshot 2:



Since $10 \leq x \leq 40$

Using the screenshots above, calculate the interquartile range for the time taken to complete Elena's Escape Room.

$IQR = Q_3 - Q_1$

$= 30.20944 - 19.79055$ (since $10 \leq x \leq 40$)

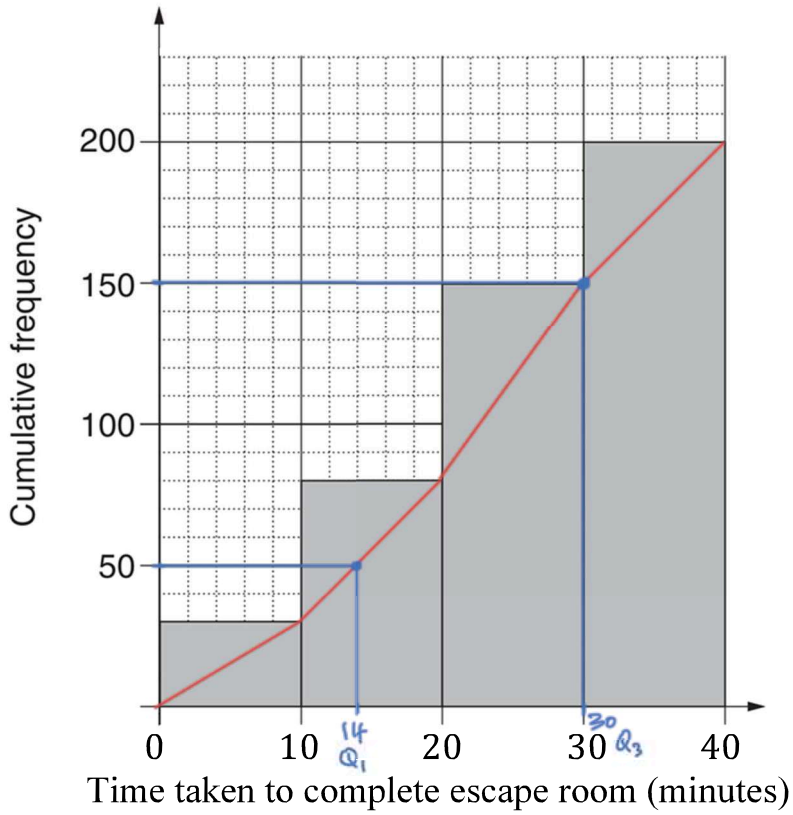
$= 10.41889$

$= 10.42$ min (2 d.p.)

Questions 29 continues on the next page.

(c) (i) A different escape room company, Escape From Axel, also gathers data on the time its customers take to complete their escape room. Axel displays the data in the cumulative frequency histogram below. **2**

Poorly done



Calculate the interquartile range for the time taken to complete the escape room at Escape From Axel.

$IQR = Q_3 - Q_1$

$= 30 - 14$

$= 16$ minutes

* 1 mark for either Q_1 or Q_3
* No marks if both Q_1 and Q_3 are incorrect.
* Time is a continuous variable. Use the ogive for Q_1 and Q_3.

Done well

(c) (ii) Find the modal time group for the time taken to complete the escape room at Escape From Axel. **1**

20 – 30 minutes

* Most common mistake: Modal group = 30.

Done well

Question 30 (4 marks)

Identify the period, amplitude, phase shift and vertical shift of the graph formed by the equation $y = 12\cos(\frac{\pi}{3}(x-2)) + 4$. **4**

Amplitude = 12

Vertical shift = 4 units up

Period = $\frac{2\pi}{\frac{\pi}{3}}$

$= 6$

Phase shift = 2 units to the right.

* Penalised if 'phase shift' is not stated.

"Horizontal shift by 2 units to the right" is not sufficient, as it does not demonstrate whether you understand the meaning of phase shift or simply describing every translation.

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2026

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TRIAL EXAMINATION

Mathematics Advanced
Section II Answer Booklet 4

Booklet 4 – Attempt Questions 31 – 35 (22 marks)

Instructions

- Answer the questions in the spaces provided. Sufficient spaces are provided for typical responses
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided at the back of this booklet. If you use this space, clearly indicate which question you are answering.
- Write your NESA ID above

	Functions	Trigonometric Functions	Calculus	Exponential & Logarithmic Functions	Statistical Analysis	Financial Mathematics
Booklet 4		/10 32, 35	/4 33	/4 31		/4 34

Please turn over

Question 31 (4 marks)

Given $f(x) = 16^x$ and $g(x) = \frac{1}{2}x + \frac{3}{4}$ show that the derivative of $y = f(g(x))$ is $\ln 2\,(2^{2x+4})$. **4**

$f(g(x)) = 16^{\frac{1}{2}x + \frac{3}{4}}$

$f'(g(x)) = \frac{1}{2} \times 16^{\frac{1}{2}x+\frac{3}{4}} \times \ln 16$

$= \frac{1}{2} \times (2^4)^{\frac{1}{2}x+\frac{3}{4}} \times \ln 2^4$

$= \frac{1}{2} \times 2^{2x+3} \times 4 \ln 2$

$= 2^{2x+3} \times 2 \ln 2$

$= 2^{2x+3} \times 2^1 \times \ln 2$

$= 2^{2x+3+1} \times \ln 2$

$= \ln 2\,(2^{2x+4})$

ALTERNATE METHOD

$f(g(x)) = 16^{\frac{1}{2}x+\frac{3}{4}}$

$= (2^4)^{\frac{1}{2}x+\frac{3}{4}}$

$= 2^{2x+3}$

$f'(g(x)) = 2 \times 2^{2x+3} \times \ln 2$

$= 2^1 \times 2^{2x+3} \times \ln 2$

$= 2^{2x+3+1} \ln 2$

$= \ln 2\,(2^{2x+4})$

Question 32 (3 marks)

Find $\int \frac{\tan^4 x}{1-\sec^2 x}\,dx$.

3

Writing this led to fewer errors. Many made the mistake of: $\tan^2 x = 1 - \sec^2 x$.

$1 + \tan^2 x = \sec^2 x$
$\tan^2 x = \sec^2 x - 1$
$-\tan^2 x = 1 - \sec^2 x$

$= \int \frac{\tan^4 x}{-\tan^2 x}\,dx$

$= \int -\tan^2 x\,dx$

$= \int 1 - \sec^2 x\,dx$

$= x - \tan x + C$

NOTE: Many students made errors with negatives. Some made two errors, which cancelled each other out and lead to the correct answer. However, two errors were made so marks were lost despite the correct answer.

ALTERNATE METHODS:

$\int -\tan^2 x\,dx$

$= -\int \sec^2 x - 1\,dx$

$= -(\tan x - x) + C$

$= x - \tan x + C$

$\int \frac{(\tan^2 x)^2}{1-\sec^2 x}\,dx$

$= \int \frac{(-\tan^2 x)^2}{1-\sec^2 x}\,dx$

$= \int \frac{(1-\sec^2 x)^2}{1-\sec^2 x}\,dx$

$= \int 1 - \sec^2 x\,dx$

$= x - \tan x + C$

$\int \frac{(\tan^2 x)^2}{1-\sec^2 x}\,dx$

$= \int \frac{(\sec^2 x - 1)^2}{1-\sec^2 x}\,dx$

$= -\int \frac{(\sec^2 x - 1)^2}{\sec^2 x - 1}\,dx$

$= -\int \sec^2 x - 1\,dx$

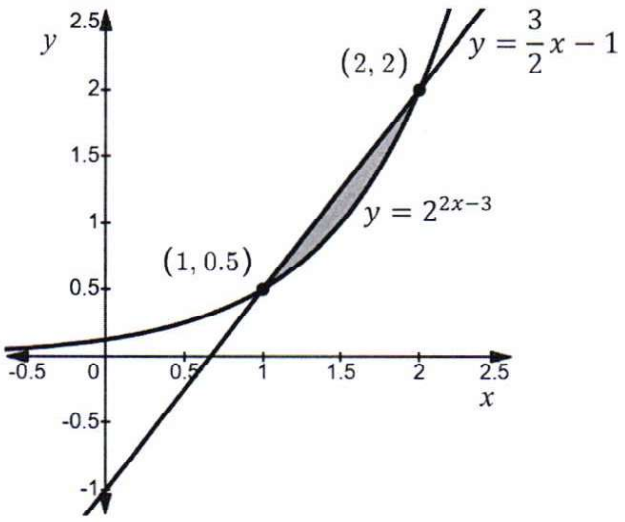
$= -(\tan x - x) + C$

$= x - \tan x + C$

NOTE: $\tan^4 x \neq 1 - \sec^4 x$.
These formulas are for powers of 2 not 4.

Question 33 (4 marks)

The curve $y = 2^{2x-3}$ meets the line $y = \frac{3}{2}x - 1$ at $(1, 0.5)$ and $(2, 2)$, as shown in the diagram. **4**



Show that the shaded region has an area of $\frac{5\ln 2 - 3}{4\ln 2}$ units^2.

$$\int_1^2 \frac{3}{2}x - 1\,dx - \int_1^2 2^{2x-3}\,dx$$

$$\left[\frac{3}{4}x^2 - x\right]_1^2 - \frac{1}{2}\int_1^2 2\times 2^{2x-3}\,dx$$

$$\left(\frac{3}{4}(2)^2 - (2)\right) - \left(\frac{3}{4}(1)^2 - (1)\right) - \frac{1}{2}\left[\frac{2^{2x-3}}{\ln 2}\right]_1^2$$

$$5/4 - \frac{1}{2}\left(\frac{2^{2(2)-3}}{\ln 2} - \frac{2^{2(1)-3}}{\ln 2}\right)$$

$$5/4 - \frac{1}{2}\left(\frac{2}{\ln 2} - \frac{2^{-1}}{\ln 2}\right)$$

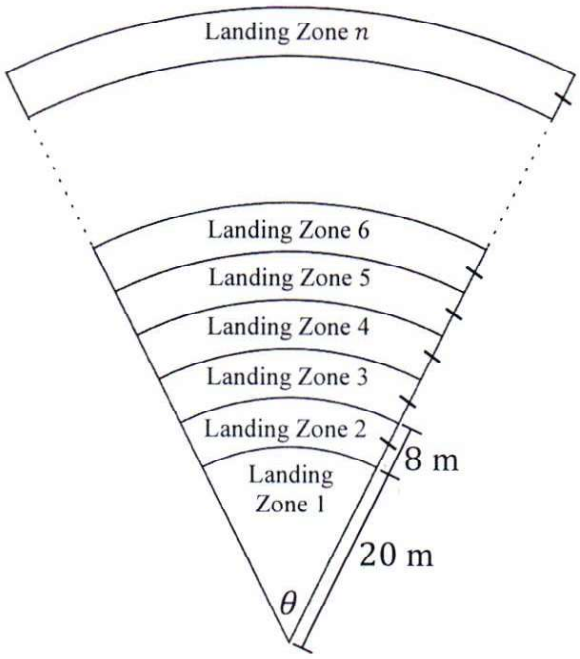
$$5/4 - \frac{2}{2\ln 2} + \frac{1}{4\ln 2}$$

$$\frac{5\ln 2}{4\ln 2} - \frac{4}{4\ln 2} + \frac{1}{4\ln 2} = \frac{5\ln 2 - 3}{4\ln 2}$$

NOTE: Less errors were made when the integrals were kept separate rather than subtracting the functions first then integrating.

Question 34 (4 marks)

Mrs Palmer is setting up a javelin landing zone. The landing zone takes the form of a sector with angle θ in radians. The first arc is 20 metres from the vertex of the sector and consecutive arcs are 8 metres apart. This is shown in the diagram below. **4**



The perimeter and internal arcs separating each landing zone must be marked on the grass with black paint, as shown in the diagram. Show that the total length of the lines to be marked with paint can be given by $L_n = 4(n^2\theta + 4n\theta + 4n + 6)$, where n is the number of landing zones.

Straight lines → $= [20 + (n-1)8] \times 2$

$= [20 + 8n - 8] \times 2$

$= [12 + 8n] \times 2$

$= 24 + 16n$

Arcs → $20\theta + 28\theta + 36\theta + \ldots [20 + (n-1)8]\theta$

$= \theta(20 + 28 + 36 + \ldots (20 + (n-1)8))$

sum of AP, $a = 20$, $d = 8$

$= \theta\left(\frac{n}{2}(2 \times 20 + (n-1)8)\right)$

$= \theta\left(\frac{n}{2}(40 + 8n - 8)\right)$

$= \theta\left(\frac{n}{2}(32 + 8n)\right)$

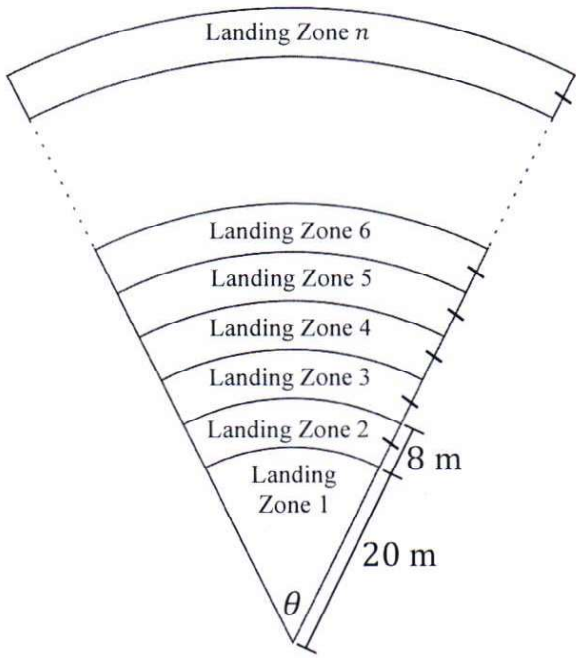
$= 16n\theta + 4n^2\theta$

Total $= 24 + 16n + 16n\theta + 4n^2\theta$

$= 4(n^2\theta + 4n\theta + 4n + 6)$

Question 34 (4 marks)

Mrs Palmer is setting up a javelin landing zone. The landing zone takes the form of a sector with angle θ in radians. The first arc is 20 metres from the vertex of the sector and consecutive arcs are 8 metres apart. This is shown in the diagram below. **4**



The perimeter and internal arcs separating each landing zone must be marked on the grass with black paint, as shown in the diagram. Show that the total length of the lines to be marked with paint can be given by $L_n = 4(n^2\theta + 4n\theta + 4n + 6)$, where n is the number of landing zones.

ALTERNATE METHOD

$P_1 = 2\times 20 + 20\theta$

$= 40 + 20\theta$

$P_2 = 40 + 20\theta + 8\times 2 + (20+8)\theta$

$= 40 + 16 + 20\theta + 28\theta$

$P_3 = 40 + 16 + 20\theta + 28\theta + 8\times 2 + (20 + 8\times 2)\theta$

$= 40 + 16\times 2 + 20\theta + 28\theta + 36\theta$

$P_n = 40 + 16(n-1) + \theta(\underbrace{20 + 28 + 36 + \ldots + [20 + (n-1)8]}_{AP,\ a=20,\ l=20+(n-1)8})$

$= 40 + 16n - 16 + \theta\left(\frac{n}{2}(20 + 20 + (n-1)8)\right)$

$= 24 + 16n + \theta\left(\frac{n}{2}(40 + 8n - 8)\right)$

$= 24 + 16n + \theta\left(\frac{n}{2}(32 + 8n)\right)$

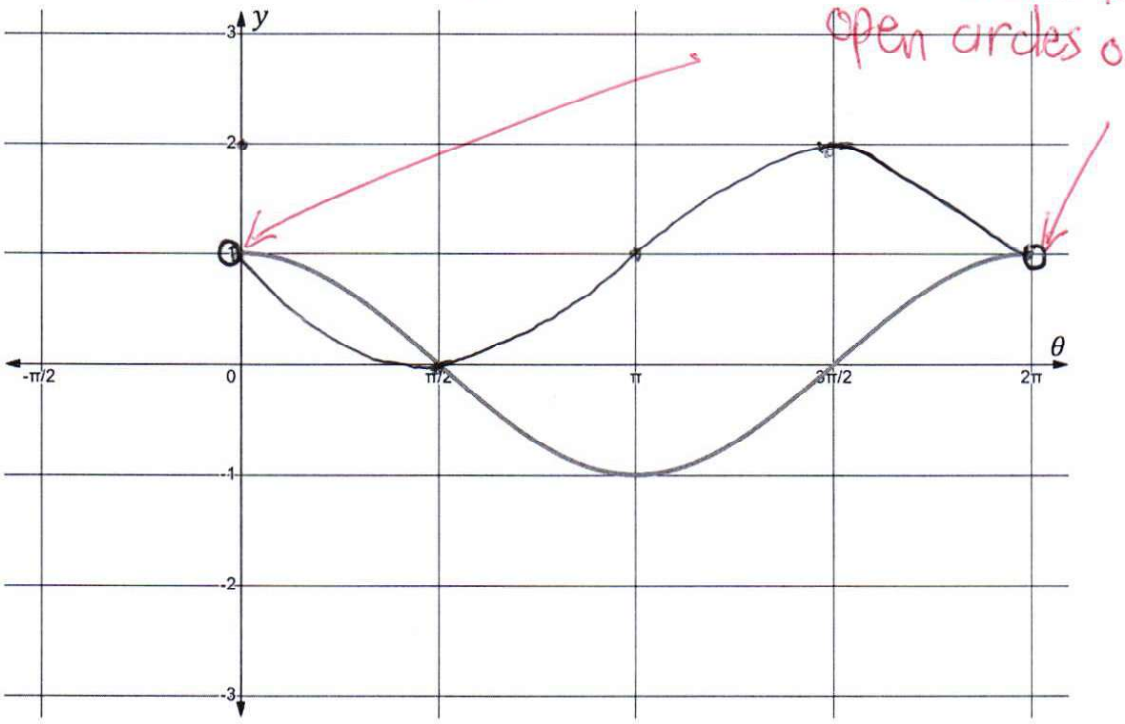
$= 24 + 16n + 16n\theta + 4n^2\theta$

$= 4(n^2\theta + 4n\theta + 4n + 6)$

Question 35 (7 marks)

(a) The graph of $y = \cos\theta$ for $0 \leq \theta \leq 2\pi$ is shown below. On the same Cartesian Plane, sketch the graph of $y = -\sin\theta + 1$ for $0 < \theta < 2\pi$. 3

NOTE: Most students forgot the open circles on the ends.



(b) Hence or otherwise, find any point(s) of intersection of $y = \cos\theta$ and $y = -\sin\theta + 1$ for $0 < \theta < 2\pi$. 2

π/2

CFE's were given if open circles were missed on the first part.

Question 35 continues on the next page

(c) Hence or otherwise, solve $\sin^2\theta + \sin\theta\cos\theta - \sin\theta = 0$ for $0 < \theta < 2\pi$. 2

$\sin\theta(\sin\theta + \cos\theta - 1) = 0$

$\sin\theta = 0$ $\quad\quad$ $\sin\theta + \cos\theta - 1 = 0$

$\theta = \pi$ $\quad\quad$ $\cos\theta = -\sin\theta + 1$

$\theta = \pi/2$ (from part ii)