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Student Number

Roseville COLLEGE

**2026** HSC TRIAL EXAMINATION

# Mathematics Advanced

**General Instructions**

- Reading time – 10 minutes
- Working time – 3 hours
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided at the back of this paper
- For questions in Section II, show relevant mathematical reasoning and/or calculations
- Write your student number at the top of this page and on the multiple-choice answer sheet

**Total Marks:**

**100**

**Section I – 10 marks** (pages 2 – 6)

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section
- Answer on the Multiple Choice Sheet provided

**Section II – 90 marks** (pages 7 – 30)

- Attempt Questions 11 – 34
- Allow about 2 hours and 45 minutes for this section

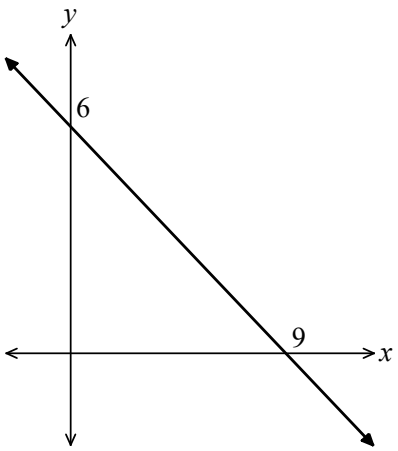
## Section I

**10 marks**
**Attempt Questions 1–10**
**Allow about 15 minutes for this section**

Use the multiple-choice answer sheet for Questions 1 – 10

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**1.** What is the equation of the line shown on the number plane below?




A. $2x - 3y - 18 = 0$

B. $2x - 3y + 6 = 0$

C. $2x + 3y - 18 = 0$

D. $2x + 3y + 6 = 0$

**2.** A six-sided die and a four-sided die are rolled together and their scores added.

The table below has been started to determine the sample space.

| Scores | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| 2 | 3 | 4 | 5 | | | |
| 3 | 4 | | | | | |
| 4 | 5 | | | | | |

Complete the table and find the probability that on a single roll of the two dice the sum of their scores is 7 or 8.

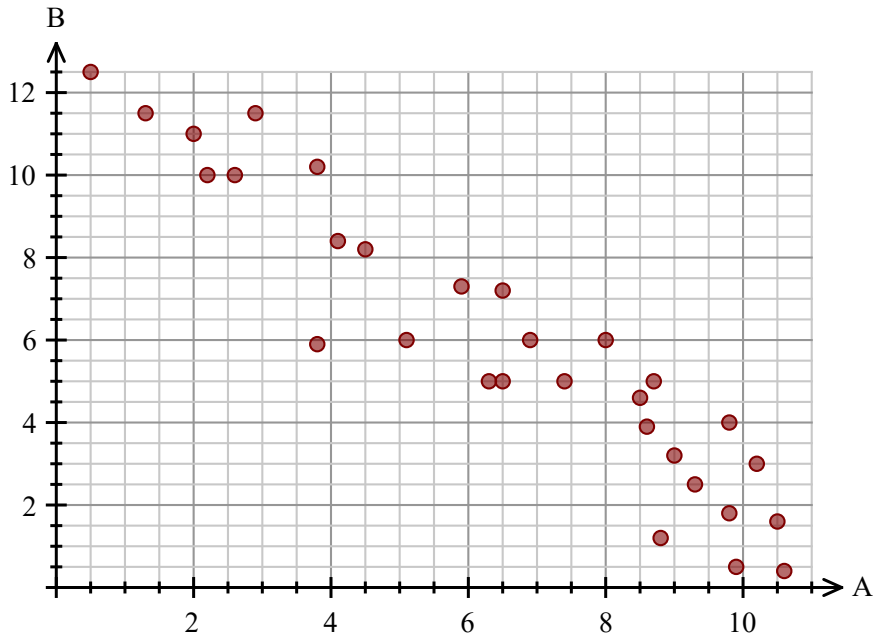
A. $\frac{5}{24}$

B. $\frac{1}{4}$

C. $\frac{7}{24}$

D. $\frac{1}{3}$

**3.** The scatterplot below compares two quantities *A* and *B*.




Which Pearson's correlation coefficient best describes the data?

A. – 0.943

B. – 0.524

C. 0.524

D. 0.943

**4.** What is the equation of the tangent to the curve $y = x^3 - 2x^2$ at the point where $x = 2$ ?

A. $y = 0$

B. $y = 2x - 4$

C. $y = 2x - 8$

D. $y = 4x - 8$

**5.** What is the value of $\int_0^1 \frac{2x\,dx}{x^2+1}$?

A. $\frac{3}{4}$

B. $\ln(\frac{3}{4})$

C. $\ln(2)$

D. 2

**6.** What are the solutions to $2\sin^2 x - \sin x - 1 = 0$ for the domain $[0, 2\pi]$?

A. $x = \frac{\pi}{6}, \frac{5\pi}{6}$ and $\frac{\pi}{2}$

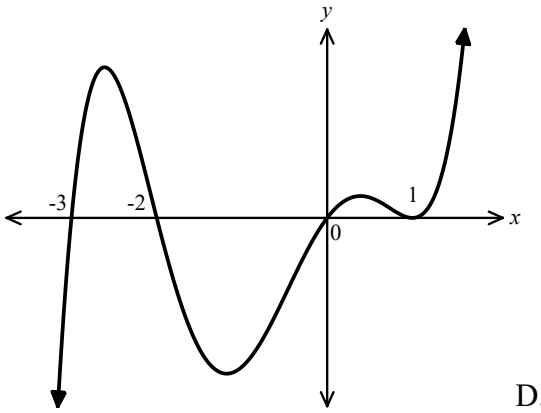
B. $x = \frac{\pi}{6}, \frac{\pi}{2}$ and $\frac{7\pi}{6}$

C. $x = \frac{5\pi}{6}, \frac{3\pi}{2}$ and $\frac{11\pi}{6}$

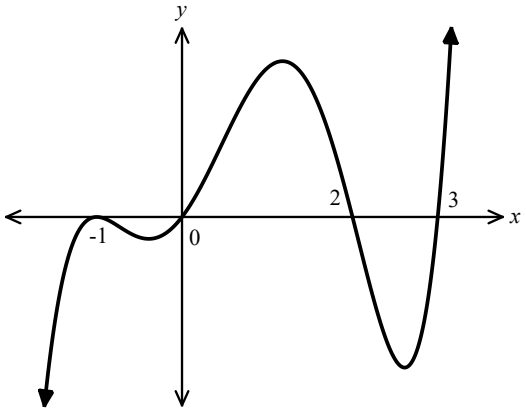
D. $x = \frac{\pi}{2}, \frac{7\pi}{6}$ and $\frac{11\pi}{6}$

**7.** Given that $P(x) = x(x+1)^2(x-2)(x-3)$, which graph accurately shows $y = P(x)$ ?

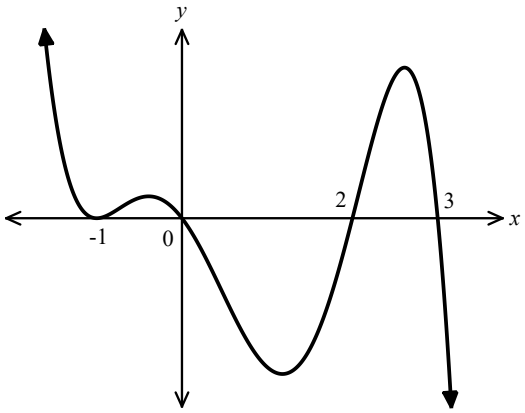
A.



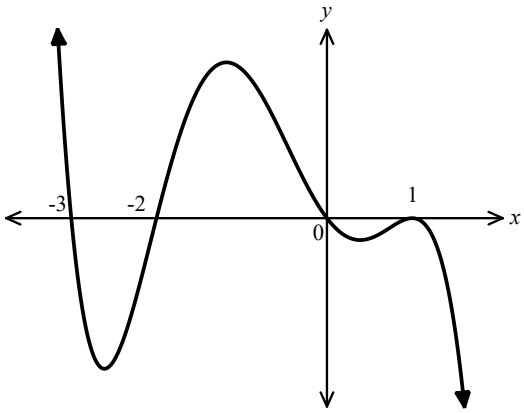

B.




C.




D.




**8.** For which domain is the curve $y = x^4 - 4x^3 + 12x$ concave down?

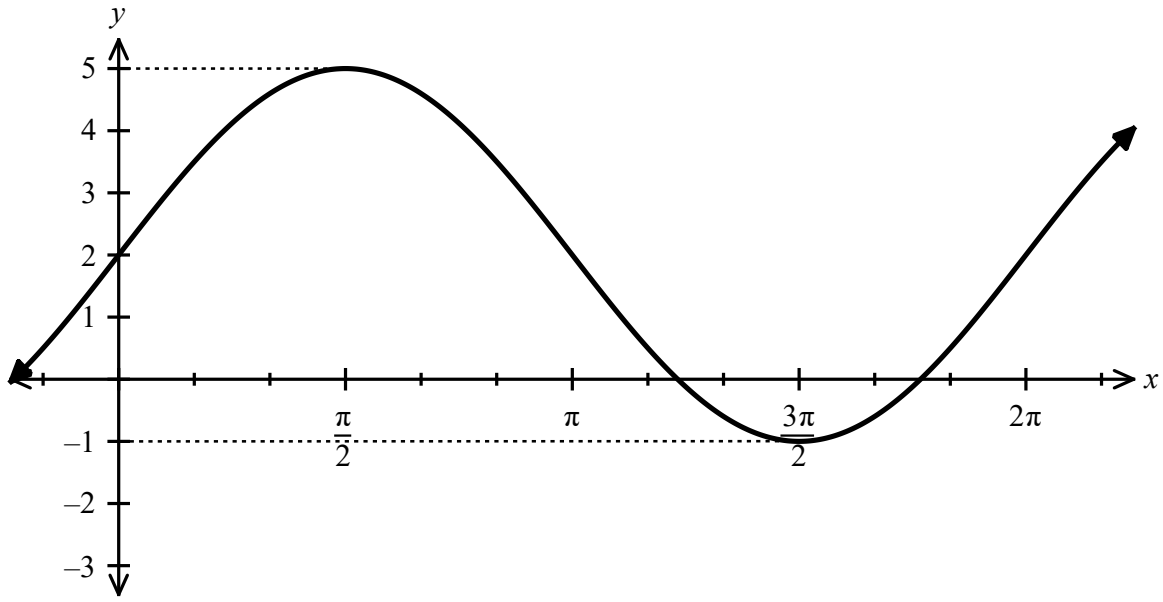
A. $(-\infty, 0)$

B. $(0, 2)$

C. $(2, \infty)$

D. $(-\infty, 0) \cup (2, \infty)$

**9.** The graph below shows a transformation of the curve $y = \cos x$.




Which equation could describe the curve above?

A. $y = 3\cos\left(x - \frac{\pi}{2}\right) + 2$

B. $y = \cos\left(3x - \frac{\pi}{2}\right) + 2$

C. $y = 2\cos\left(x + \frac{\pi}{2}\right) + 3$

D. $y = 3\cos\left(x + \frac{\pi}{2}\right) + 2$

**10.** When playing darts, on each throw, Keith has a probability of 0.88 of hitting the section of the board at which he is aiming.

When playing a particular game what is the probability that he will miss the section of the board at which he is aiming in three successive throws?

(Answer correct to 3 significant figures)

A. 0.0017

B. 0.014

C. 0.15

D. 0.68

**Section II**

**90 marks**
**Attempt Questions 11–34**
**Allow about 2 hours and 45 minutes for this section**

Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.

Your responses should include relevant mathematical reasoning and/or calculations.

Extra writing space is provided at the back of this booklet. If you use this space, clearly indicate which question you are answering.

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**Question 11** (2 marks) **2**

Find the gradient of the normal to the curve $y = 4^x$ at the point where $x = 1$.

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**Question 12** (2 marks)

Solve the equation **2**

$$x\sqrt{12} = 4\sqrt{3} - \sqrt{27}$$

giving your answer in its simplest form.

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**Question 13** (4 marks)

(a) Find $\int 10x^4 - 3\sqrt{x}\ \ dx$. **2**

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(b) Evaluate $\int_2^3 e^{2x-4}\ \ dx$. **2**

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**Question 14** (3 marks)

Solve $\log_3(x^2 - 4x + 4) = 1 + \log_3(x - 2)$ **3**

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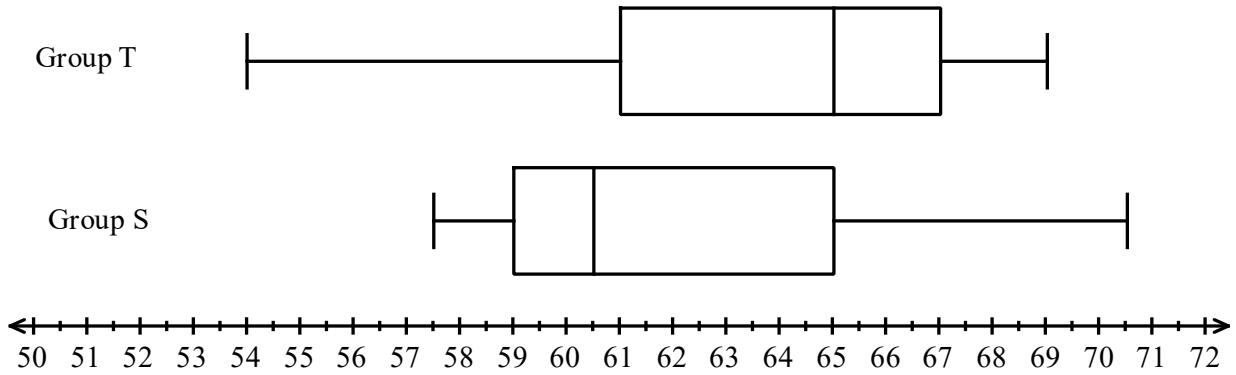
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**Question 15** (4 marks)

The parallel box plots below compare the scores of two groups (S and T) on a task.




(a) What is the median for Group S? **1**

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(b) Compare the skew of the two sets of data. **1**

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(c) Compare the spread of the two sets of data, mentioning both range and interquartile range. **2**

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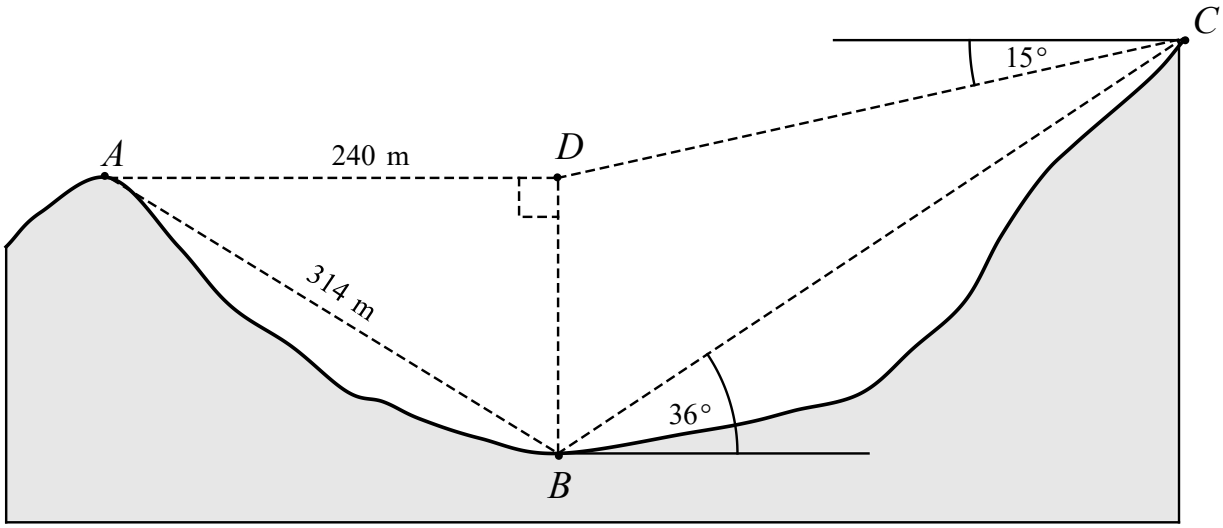
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**Question 16** (3 marks)

The diagram below shows the cross-section of a valley.




*A* and *C* are the points at the top of each side of the valley, and *B* is the point at the lowest point in the valley.

*D* is the point vertically above *B* and horizontally level with *A*.

From *B* the angle of elevation of *C* is 36º and from *C* the angle of depression of *D* is 15º.

The distance *AB* = 314 m and *AD* = 240 m.

(a) Calculate the distance *BD* correct to the nearest metre. **1**

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(b) Calculate the distance *CD* correct to the nearest metre. **2**

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**Question 17** (6 marks)

(a) Find $\frac{d}{dx}(x^4 \sin x)$. **2**

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(b) If $y = \frac{x^2 - 4}{e^x}$, find y′. **2**

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(c) When $f(x) = \ln(\cos x + 1)$, evaluate $f'\left(\frac{\pi}{2}\right)$. **2**

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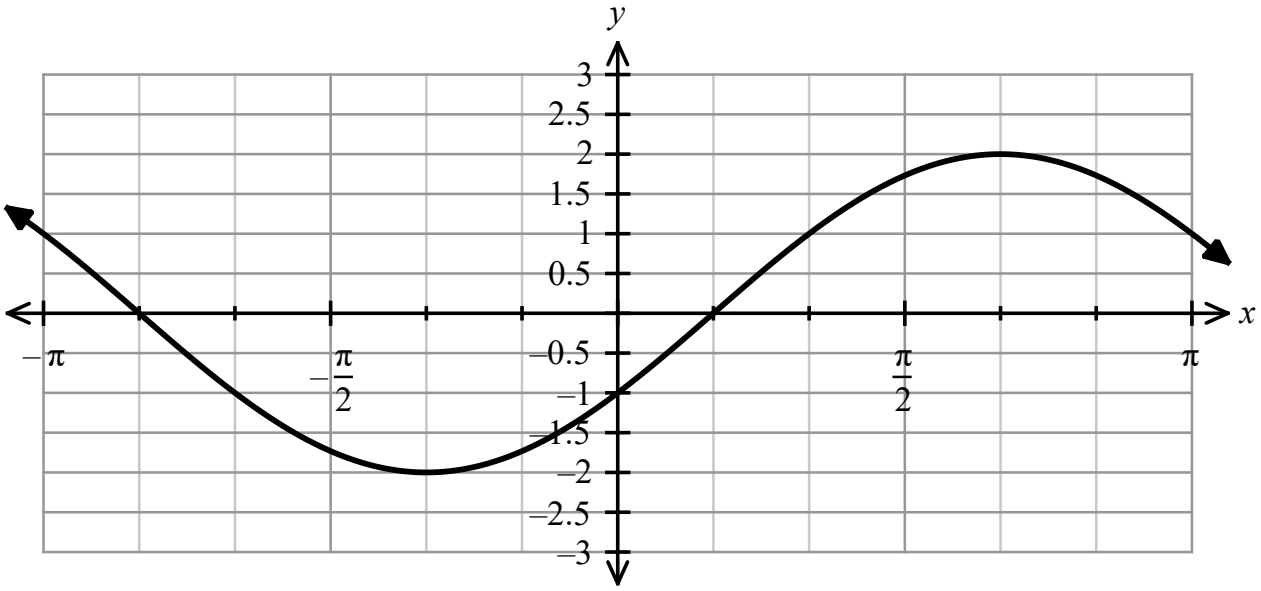
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**Question 18** (2 marks)

The graph of $y = 2\sin\left(x - \frac{\pi}{6}\right)$ is shown below for $[-\pi, \pi]$.




(a) Use the graph to solve $2\sin\left(x - \frac{\pi}{6}\right) = 0$. **1**

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(b) Use the graph to solve $2\sin\left(x - \frac{\pi}{6}\right) + 1 = 0$. **1**

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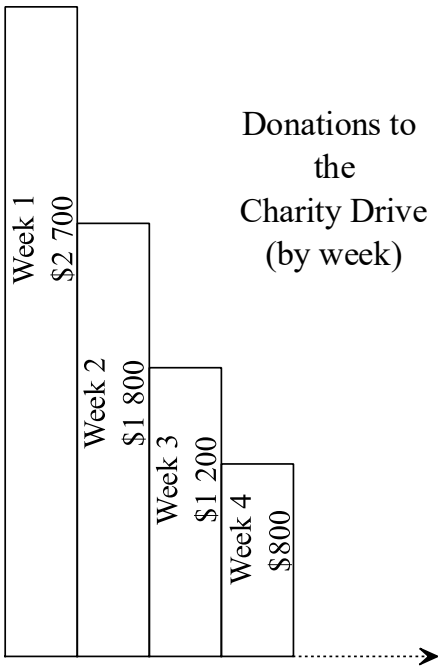
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**Question 19** (5 marks)

A charity holds its regular fundraising drive, and records the donations received each week for the first four weeks after the drive is launched.

The graphic on the right shows the results.




The director of the charity notices that the amount of donations received each week follows a geometric series.

She makes some predictions on the future donations, by assuming that the same pattern will be followed.

(a) What is the common ratio of the geometric series? **1**

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(b) How much would she predict to be received in Week 8 (to the nearest dollar)? **1**

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(c) What total amount would be raised in the first ten weeks? **1**

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(d) She claims that the drive will never raise more than \$8 100 in total. **2**

Explain how she can make this prediction and why she decides to launch a new drive after thirteen weeks.

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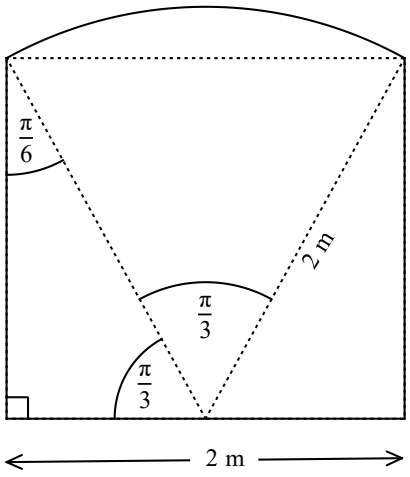
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**Question 20** (3 marks)

A two-metre-wide arched door in a restaurant is in the shape of a rectangle with a segment of a circle above it.




The top of the door is the arc of a circle with radius 2 metres and its centre is in the middle of the base of the door. The angles shown are measured in radians.

(a) Find the exact length of the arc at the top of the door. **1**

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(b) Calculate the exact area of the door. **2**

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**Question 21** (5 marks)

A curve is defined by the equation $y = (x + 2)^2(x - 4)$.

(a) Find the turning points on the curve and their nature. **2**

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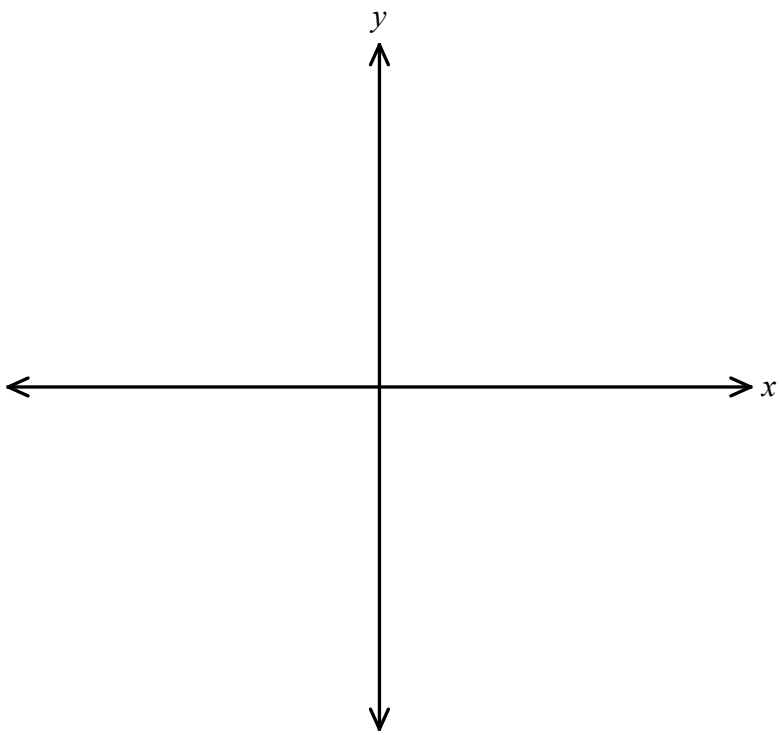
(b) Find the co-ordinates of any inflection point(s). **1**

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(c) Draw a sketch of the curve in the space below, showing all turning and inflection points and intercepts on the axes. **2**



**Question 22** (3 marks)

For two events $A$ and $B$ it is known that $P(A) = 0.4$, $P(B) = 0.3$ and $P(B|A) = 0.5$.

(a) Calculate $P(A \cap B)$. **1**

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(b) State why events $A$ and $B$ are not independent. **1**

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(c) Calculate $P(A|B)$. **1**

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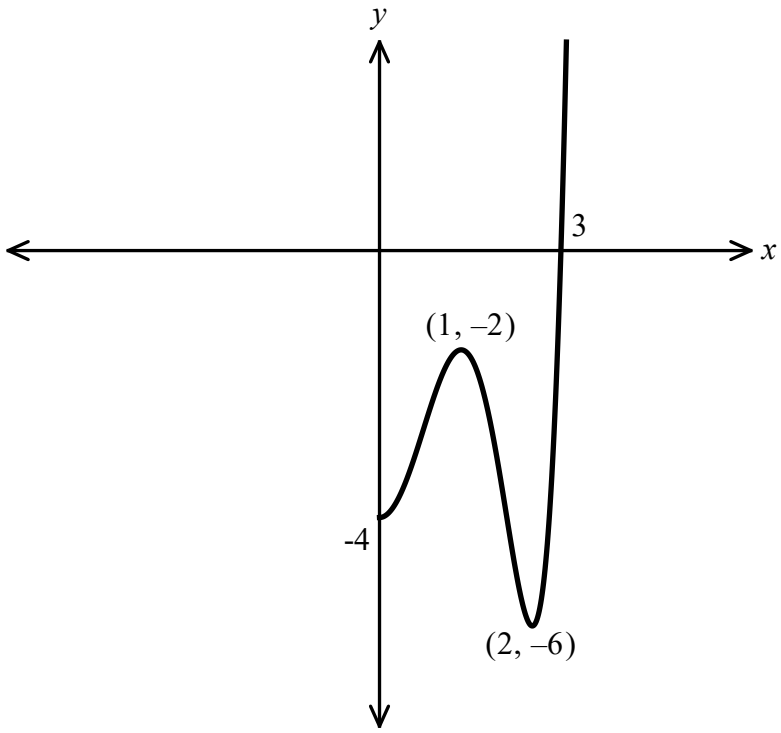
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**Question 23** (4 marks)

(a) The graph of $y = g(x)$ is shown for $[0, \infty)$, and it is known that $g(x)$ is an even function. **2**



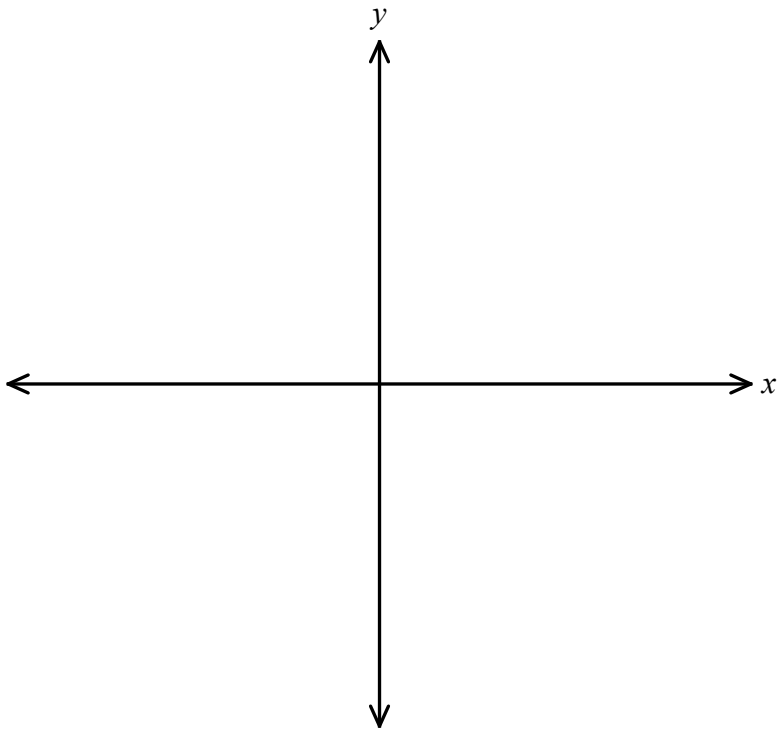

Draw and label the section of $y = g(x)$ for $(-\infty, 0)$.

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(b) It is known that $y = h(x)$ is an odd function of degree 5 which has a double root at $x = 2$ and which has a leading coefficient of 2. **2**

Draw a sketch of $y = h(x)$ on the axes below.



**Question 24** (3 marks)

The table below show the future value of an annuity of $1 for a variety of periods and interest rates. The contributions are made at the end of each year. **3**

**Future Value of an Annuity of $1.00.**

| Years | Interest Rate Per Annum | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | 5% | 6% | 7% | 8% | 9% | 10% | 11% | 12% |
| 6 | 6.802 | 6.975 | 7.153 | 7.336 | 7.523 | 7.716 | 7.913 | 8.115 |
| 7 | 8.142 | 8.394 | 8.654 | 8.923 | 9.200 | 9.487 | 9.783 | 10.089 |
| 8 | 9.549 | 9.897 | 10.260 | 10.637 | 11.028 | 11.436 | 11.859 | 12.300 |
| 9 | 11.027 | 11.491 | 11.978 | 12.488 | 13.021 | 13.579 | 14.164 | 14.776 |
| 10 | 12.578 | 13.181 | 13.816 | 14.487 | 15.193 | 15.937 | 16.722 | 17.549 |
| 11 | 14.207 | 14.972 | 15.784 | 16.645 | 17.560 | 18.531 | 19.561 | 20.655 |
| 12 | 15.917 | 16.870 | 17.888 | 18.977 | 20.141 | 21.384 | 22.713 | 24.133 |

Penny invested a sum of $3 500 at the end of each year into an annuity that paid 9% pa and continued to do so for 12 years.

After 5 years had passed, she decided that she needed to invest in a second annuity so that the total value of the two investments when the first annuity matured would be $100 000. The best interest rate that she could obtain for the second annuity was 6% pa.

What was the total yearly amount that Penny paid into the two annuities from the sixth year until the twelfth year?

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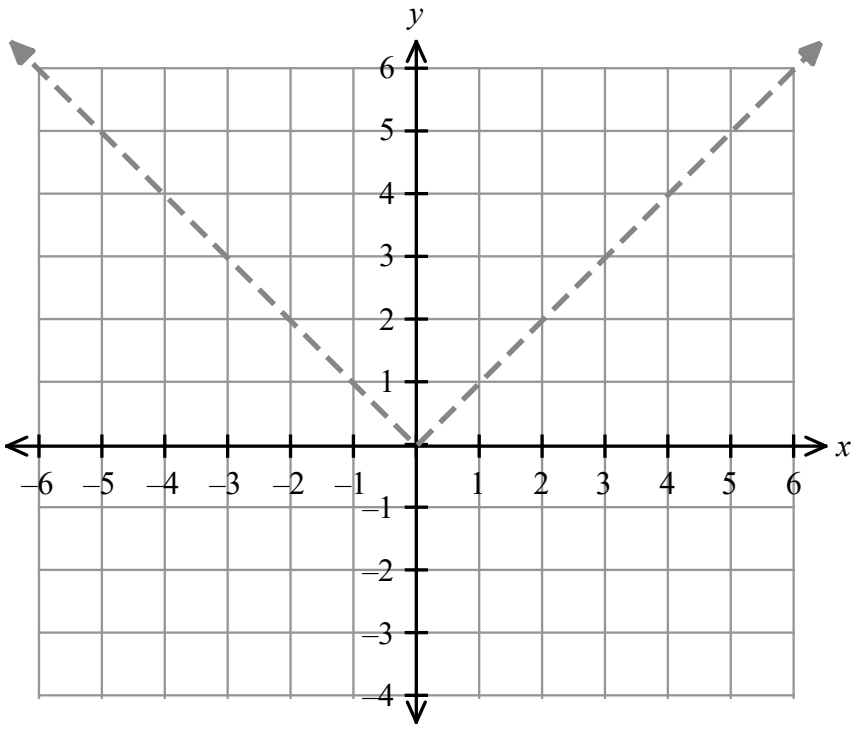
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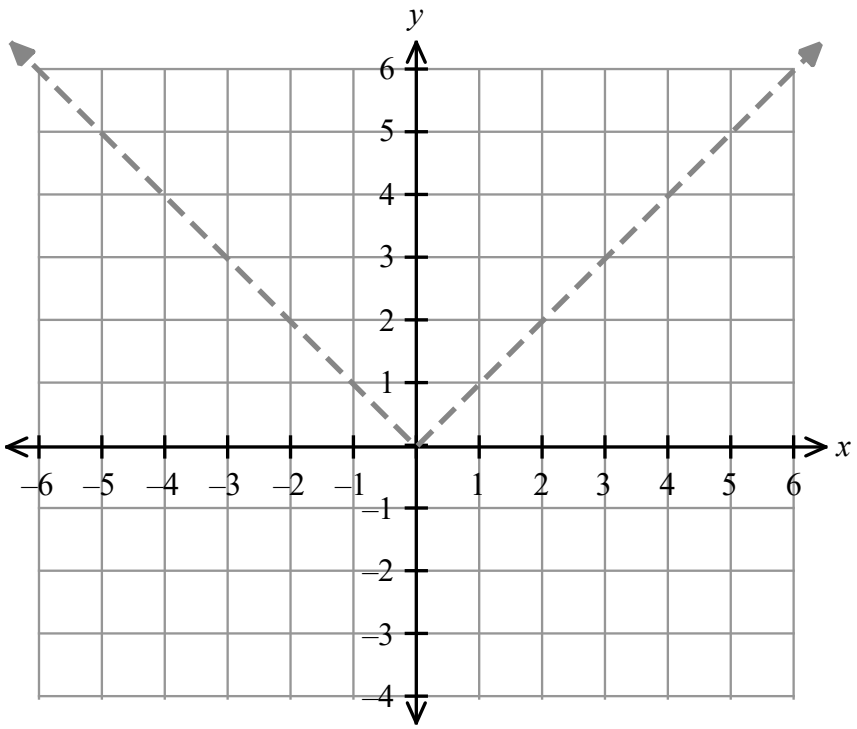
**Question 25** (3 marks)

On each of the number planes below, the broken line represents the curve $y = |x|$.

(a) On the graph below, draw the function $y = |x - 3|$ ? **1**




(b) On this graph, draw the function $y = |2x + 4| - 1$? **2**



**Question 26** (3 marks)

A particle is moving in a straight line on a number line. **3**

It is initially at rest and its velocity in m/s at a time *t* seconds is given by:

$$v = 12t^3 - 96t^2 + 180t$$

Given that when $t = 1,\ x = 60$, find its initial acceleration and initial displacement.

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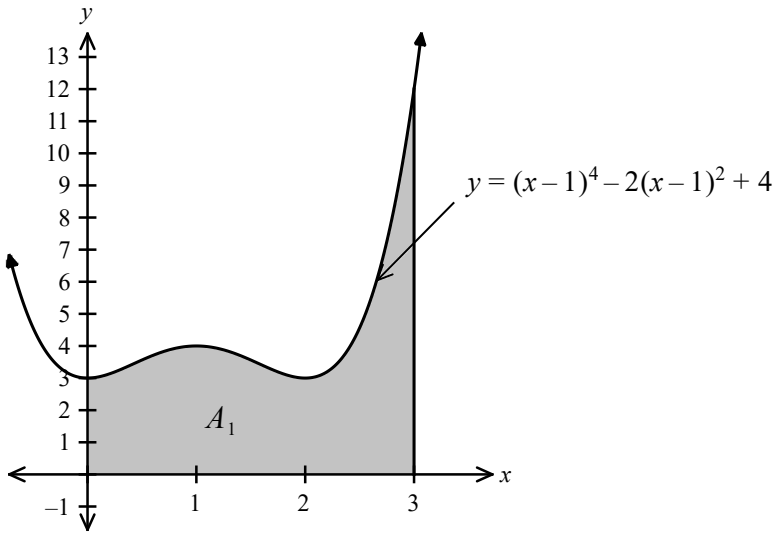
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**Question 27** (6 marks)

The diagram below shows the curve $y = (x-1)^4 - 2(x-1)^2 + 4$.




The shaded area $A_1$ is the area enclosed between the $x$-axis, the curve and the lines $x = 0$ and $x = 3$.

(a) Use three applications of the trapezoidal rule to estimate the area $A_1$. **2**

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(b) Explain why $\int_0^3 (x-1)^4 - 2(x-1)^2 + 4 \; dx = \int_{-1}^2 x^4 - 2x^2 + 4 \; dx$. **1**

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(c) Hence or otherwise find the exact value of $A_1$. **2**

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(d) Explain why the approximation you found for $A_1$ is greater than the exact value of $A_1$. **1**
You may wish to draw on the diagram above to illustrate your answer.

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**Question 28** (2 marks)

An arithmetic series has a common difference of 6 and its $20^{th}$ term is 130. **2**

Find the sum of the first twenty terms of the series.

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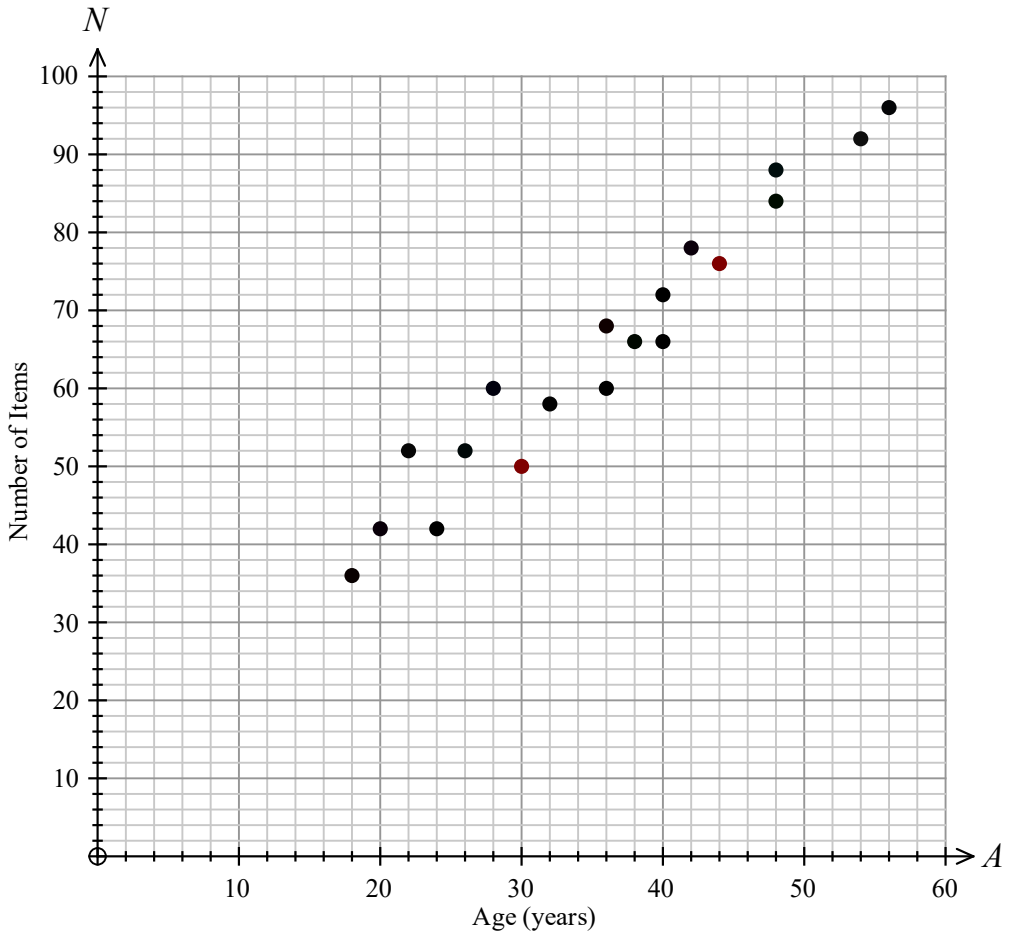
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**Question 29** (4 marks)

At a Heroes and Villains Comic convention a group of attendees record their age and the number of items of memorabilia in their collection. The scatterplot below records the data collected.




(a) Draw a line of best fit on the scatterplot. **1**

No calculations are necessary, draw the line by eye.

(b) Determine the equation of your line of best fit. **2**

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(c) Estimate the number of items in the collection of a fifty-year-old collector. **1**

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**Question 30** (5 marks)

A discrete random variable $X$ has a probability distribution as shown in the table.

| $x$ | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| $P(X = x)$ | 0.10 | 0.30 | 0.45 | 0.10 | $k$ |

(a) Find $k$. **1**

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(b) Find $P(X > 2)$. **1**

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(c) Calculate $E(X)$ **1**

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(d) Calculate the standard deviation of $X$. **2**

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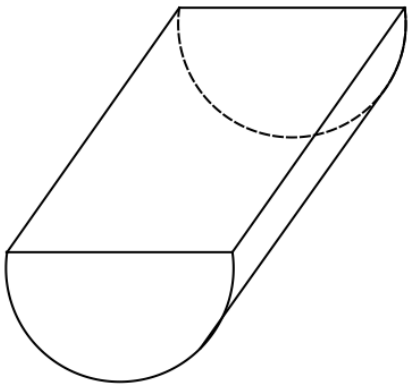
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**Question 31** (6 marks)

The diagram below shows half a closed cylinder with radius $r$ and height $h$.

The total surface area of the shape is $\sqrt[3]{27\pi}$.



(a) Show that the volume, $V$ can be expressed as **3**

$$V = \frac{\pi}{2(\pi+2)}\left[\sqrt[3]{27\pi}r - \pi r^3\right]$$

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**Question 31 continues on page 26**

**Question 31 (continued)**

(b) Hence, find the maximum volume. **3**

**Question 32** (4 marks)

(a) Show that $\sin\theta - \cos\theta\sin\theta\left(\sin\left(\frac{\pi}{2} - \theta\right)\right) = \sin^3\theta$ for $\left[0, \frac{\pi}{2}\right]$. **2**

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(b) Hence or otherwise, solve $2\sqrt{2}\sin\theta - 2\sqrt{2}\cos\theta\sin\theta\left(\sin\left(\frac{\pi}{2} - \theta\right)\right) = 1$ for $\left[0, \frac{\pi}{2}\right]$. **2**

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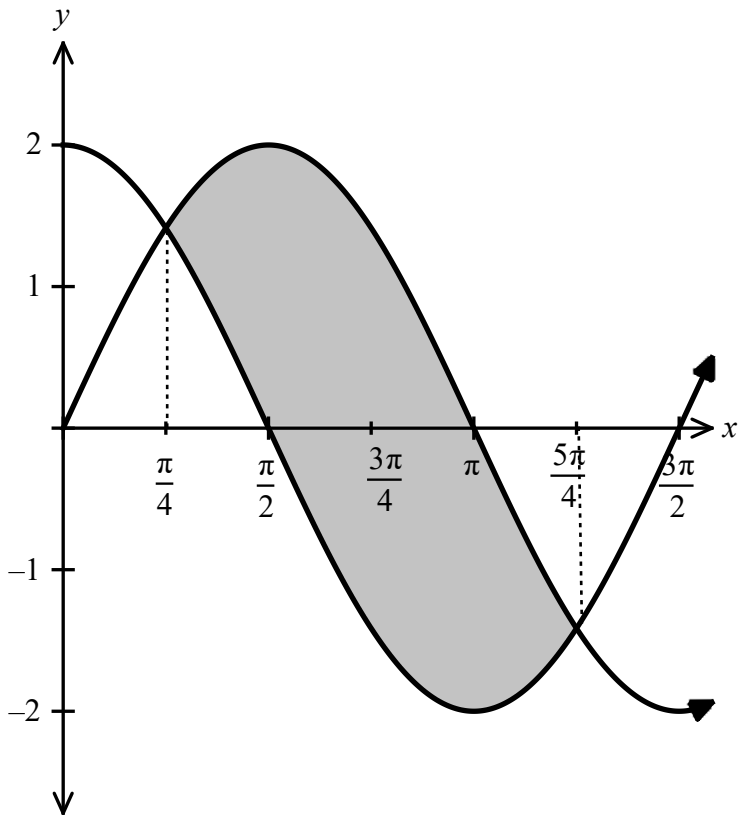
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**Question 33** (3 marks)

The diagram shows the shaded area enclosed between the curves $y = 2 \sin x$ and $y = 2 \cos x$. **3**




Calculate the exact value of the shaded area.

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**Question 34** (5 marks)

Mitch opened an investment account with an initial deposit of \$20 000 on 1st January 2023.

The account earned interest at a rate of 8% per annum, compounding quarterly.

He withdrew \$1 000 on the 1st April 2023 and then on 1st July 2023 and continued to do so on the first day of each successive quarter.

His withdrawals occurred after the quarterly interest was added.

Let $\$A_n$ be the amount remaining in the account after $n$ withdrawals have been made.

(a) Show that the amount remaining after the July 1st withdrawal is given by **1**

$A_2 = (20000 \times 1.02 - 1000) \times 1.02 - 1000.$

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(b) Show that $A_n = 50000 - 30000 \times 1.02^n$. **2**

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**Question 34 continues on page 30**

**Question 34 (continued)**

(c) What is the total number of withdrawals that Mitch could make before the account runs out of funds. **2**

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## End of Paper

Section II Extra writing space

If you use this space, clearly indicate which question you are answering.

Roseville College

2026

HSC TRIAL
EXAMINATION

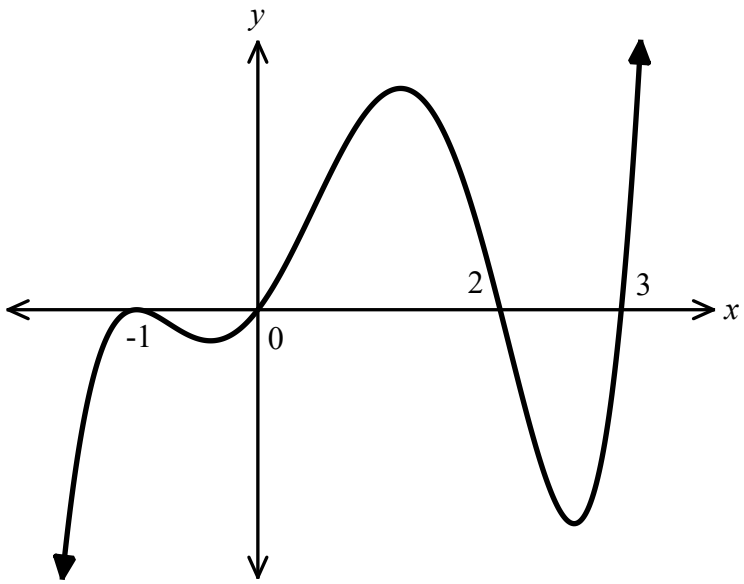
# Advanced Mathematics
# Solutions

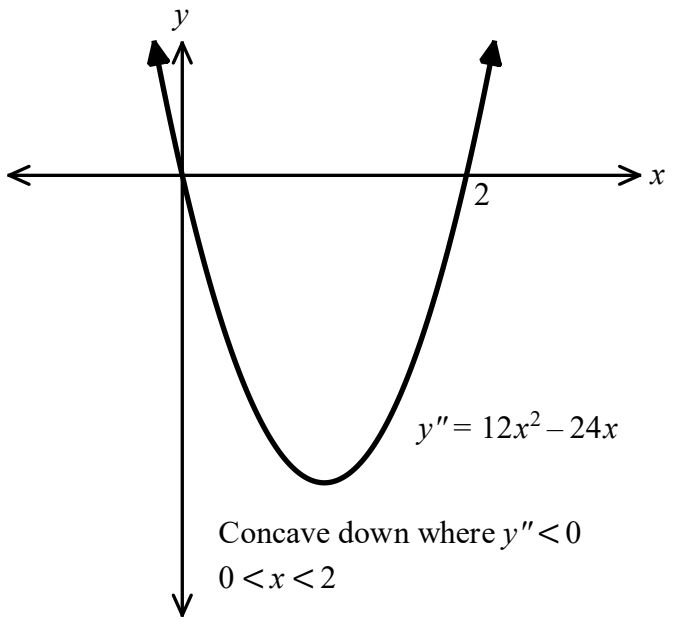
Section I

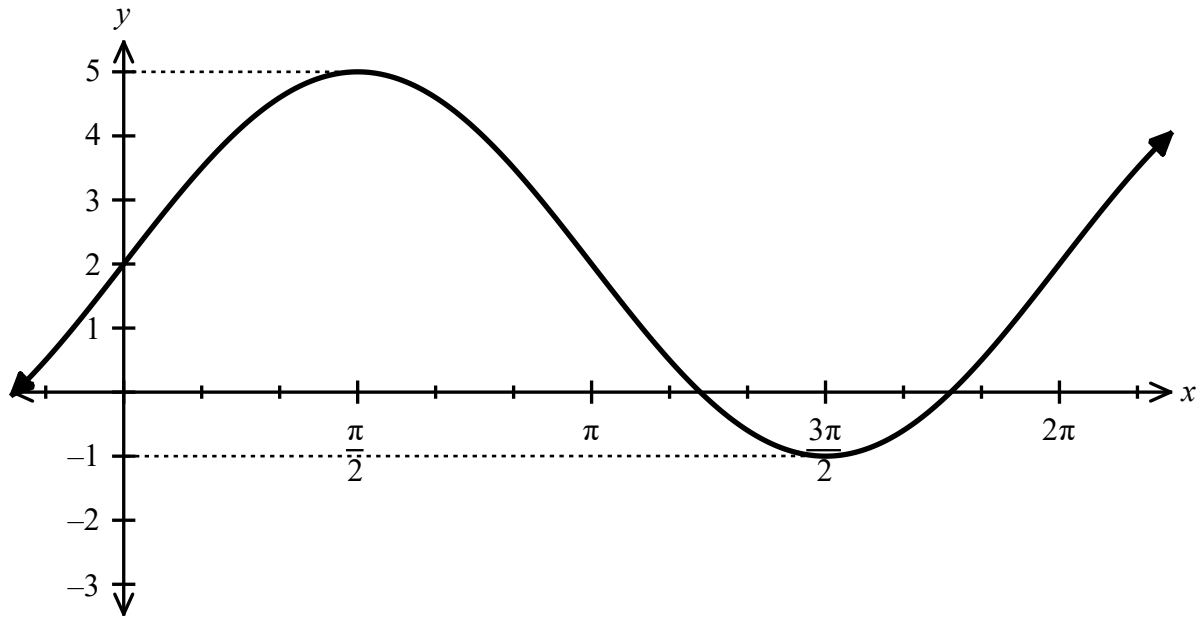
<table>
<tr><th>No</th><th>Working</th><th>Answer</th></tr>
<tr><td>1.</td><td>Gradient $m = -\frac{6}{9} = -\frac{2}{3}$<br>$\mathrm{y} - \text{intercept } b = 6$<br>Equation $y = -\frac{2}{3}\,x + 6$<br>$3y = -2x + 18$<br>$2x + 3y - 18 = 0$</td><td>C</td></tr>
<tr><td>2.</td><td>Completed table<br><table><tr><td>Scores</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td></tr><tr><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td></tr><tr><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td></tr><tr><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td></tr><tr><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td><td>10</td></tr></table>$P(7 \text{ or } 8) = \frac{7}{24}$</td><td>C</td></tr>
<tr><td>3.</td><td>By calculation, the correlation coefficient is – 0.9425 but by inspection, the grouping of the scores is close to a straight line and the gradient is a negative slope, so – 0.943 is the only possible choice,</td><td>A</td></tr>
</table>

| No | Working | Answer |
|---|---|---|
| 4. | $y = x^3 - 2x^2$<br>$\frac{dy}{dx} = 3x^2 - 4x$<br>When $x = 2$<br>$y = (2)^3 - 2(2)^2 = 8 - 8 = 0$<br>$\frac{dy}{dx} = 3(2)^2 - 4(2) = 12 - 8 = 4$<br><br>So $m = 4$ and point is $(2, 0)$<br>$y - 0 = 4(x - 2)$<br>$y = 4x - 8$ | **D** |
| 5. | $\int_0^1 \frac{2x\,dx}{x^2+1} = \left[\ln(x^2+1)\right]_0^1$<br>$= \ln(2) - \ln(1)$<br>$= \ln(2)$ | **C** |

| No | Working | Answer |
|---|---|---|
| 6. | $2\sin^2 x - \sin x - 1 = 0$<br>Letting $\sin x = u$<br>$2u^2 - u - 1 = 0$<br>$2u^2 - 2u + u - 1 = 0$<br>$2u(u-1) + (u-1) = 0$<br>$(2u+1)(u-1) = 0$<br>$2u + 1 = 0$ and $u - 1 = 0$<br>$2u = -1$ and $u = 1$<br>$u = -\frac{1}{2}$ and $u = 1$<br>$\sin x = -\frac{1}{2}$ and $\sin x = 1$<br>$x = \frac{\pi}{2}, \frac{7\pi}{6}$ and $\frac{11\pi}{6}$ for $[0, 2\pi]$. | **D** |

| No | Working | Answer |
|---|---|---|
| 7. | $P(x) = x(x+1)^2(x-2)(x-3)$ will have zeros at $x = 0,\ 2,\ 3$, and a double zero at $x = -1$.<br><br>It's degree is 5, and it has a leading coefficient of 1, so will have a positive slope beyond the turning points.<br><br>This describes graph B.<br><br><br> | **B** |

| No | Working | | Answer |
|---|---|---|---|
| 8. | $y = x^4 - 4x^3 + 12x$<br>$y' = 4x^3 - 12x^2 + 12$<br>$y'' = 12x^2 - 24x$<br>$y'' = 12x(x - 2)$<br>$y'' = 0$ when $x = 0$ or $x = 2$<br><br>Test concavity at<br>$x = -1,\ y'' = 36$<br>$x = 1,\ y'' = -12$<br>and $x = 3,\ y'' = 36$<br>Concave down when $y'' < 0$,<br><br>so between $x = 0$ and $x = 2$<br><br>Solution: $0 < x < 2$, so domain is (0, 2). | Alternative to testing concavity<br><br>Sketch $y''$<br><br><br><br>Concave down where $y'' < 0$<br>$0 < x < 2$<br><br>Solution: $0 < x < 2$, so domain is (0, 2). | **B** |

| No | Working | Answer |
|---|---|---|
| 9. | <br>The difference between maximum and minimum is 6 units, so amplitude is 3 units, so cos function is multiplied by three.<br>The period is $2\pi$ units, (unchanged) so the $x$ term has not been multiplied by a factor (other than 1)<br>$y$ = cos $x$ normally has a maximum turning pt at $x = 0$, but here it is at $x = \frac{\pi}{2}$ so graph has been translated by $\frac{\pi}{2}$ to the right, so the argument becomes $(x - \frac{\pi}{2})$<br>The entire graph has been translated 2 units upward, so the equation is $y = 3\cos\left(x - \frac{\pi}{2}\right) + 2$ | **A** |

| No | Working | Answer |
|---|---|---|
| 10. | Prob of missing with one throw $= 1 - 0.88 = 0.12$<br>Probability of missing with three successive throws $= 0.12^3$<br>$= 0.001728$<br>$= 0.0017$ 2 sig fig | **A** |

Roseville College

# 2026 Trial Higher School Certificate Examination

Mathematics Advanced

Name _____ Teacher _____

## Section I – Multiple Choice Answer Sheet

Allow about 25 minutes for this section

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

**Sample:** 2 + 4 = (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ⊗ C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word **correct** and drawing an arrow as follows.




| | | | | |
|---|---|---|---|---|
| 1. | A ○ | B ○ | C ● | D ○ |
| 2. | A ○ | B ○ | C ● | D ○ |
| 3. | A ● | B ○ | C ○ | D ○ |
| 4. | A ○ | B ○ | C ○ | D ● |
| 5. | A ○ | B ○ | C ● | D ○ |
| 6. | A ○ | B ○ | C ○ | D ● |
| 7. | A ○ | B ● | C ○ | D ○ |
| 8. | A ○ | B ● | C ○ | D ○ |
| 9. | A ● | B ○ | C ○ | D ○ |
| 10. | A ● | B ○ | C ○ | D ○ |

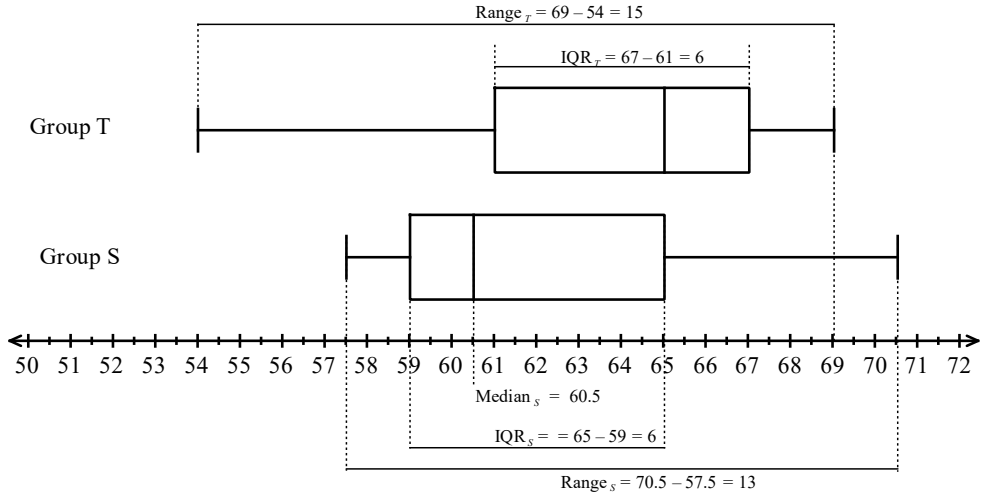
# Roseville College
# Advanced Mathematics Trial HSC

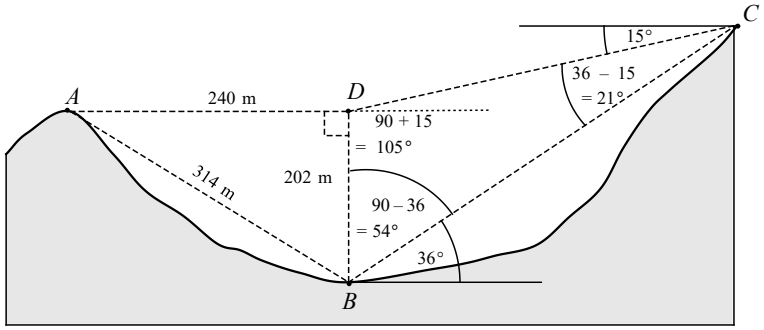
## Solutions 2026

## Section II

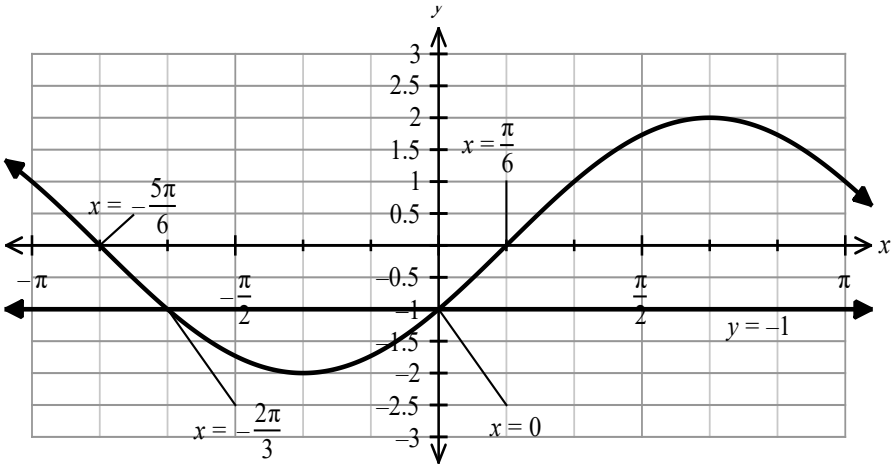
| Question | | Working and answer | Marks | Mark Allocation |
|---|---|---|---|---|
| 11. | | $y = 4^x$<br>$\frac{dy}{dx} = (\ln 4).\ 4^x$<br>When $x = 1,\ \frac{dy}{dx} = 4(\ln 4)$<br>Gradient of tangent $= 4(\ln 4)$<br>Gradient of normal $= -\frac{1}{4(\ln 4)} = -\frac{1}{\ln(256)}$ | **2** | 2 marks for correct answer in any exact form<br><br>1 mark for correct use of rule to obtain $\frac{dy}{dx} = (\ln 4).\ 4^x$ oem |
| 12. | | $x\sqrt{12} = 4\sqrt{3} - \sqrt{27}$<br>$2x\sqrt{3} = 4\sqrt{3} - 3\sqrt{3}$<br>$2x\sqrt{3} = \sqrt{3}$<br>$\therefore x = \frac{1}{2}$ | **2** | 2 marks provides correct solution<br><br>1 mark Simplifies one surd correctly, or equivalent merit |

<table>
<tr><th>Question</th><th>Working and answer</th><th>Marks</th><th>Mark Allocation</th></tr>
<tr><td>13.</td><td>a)</td><td>$\int 10x^4 - 3\sqrt{x}\ dx = \int 10x^4 - 3x^{\frac{1}{2}}\ dx$<br>$= \frac{10x^5}{5} - 3.\ \frac{2}{3}\ x^{\frac{3}{2}} + C$<br>$= 2x^5 - 2\sqrt{x^3}\ + C$</td><td>2</td><td>2 marks for correct solution<br><br>1 mark if only one of the components of the integral is found correctly</td></tr>
<tr><td>13.</td><td>b)</td><td>$\int_2^3 e^{2x-4}\ dx = \left[\frac{1}{2}\ e^{2x-4}\right]_2^3$<br>$= \left(\frac{1}{2}\ e^{2\times3-4}\right) - \left(\frac{1}{2}\ e^{2\times2-4}\right)$<br>$= \frac{1}{2}\ e^2 - \frac{1}{2}\ e^0$<br>$= \frac{e^2-1}{2}$</td><td>2</td><td>2 marks for correct solution<br><br>1 mark if either the integral or substitution steps are completed incorrectly oem</td></tr>
<tr><td>14.</td><td></td><td>$\log_3(x^2 - 4x + 4) = 1 + \log_3(x-2)$<br>$\log_3(x-2)^2 = 1 + \log_3(x-2)$<br>$2\log_3(x-2) = 1 + \log_3(x-2)$<br>$\log_3(x-2) = 1$<br>$x - 2 = 3$<br>$x = 5$</td><td>3</td><td><table><tr><th>Criteria</th><th>Marks</th></tr><tr><td>• Correct solution</td><td>3</td></tr><tr><td>• Uses a log law to simplify</td><td>2</td></tr><tr><td>• Factors the quadratic</td><td>1</td></tr></table></td></tr>
</table>

| Question | | Working and answer | Marks | Mark Allocation |
|---|---|---|---|---|
| 15. | |  | | |
| 15. | a) | The median of group S is 60.5. | **1** | 1 mark for correct answer |
| 15. | b) | Both sets are skewed, Group T is negatively skewed, and Group S is positively skewed. | **1** | 1 mark for correct answer |
| 15. | c) | Both Groups have the same interquartile range of 6, while Group T has a range (15) which is 2 greater than Group S (13), the difference being in the extremities and the skew. | **2** | 2 marks for correct range and interquartile range and comparison of quantities<br><br>1 mark for finding the range and interquartile range with a minor error and comparison of quantities |

| Question | | Working and answer | Marks | Mark Allocation |
|---|---|---|---|---|
| 16. | a) | $BD^2 = 314^2 - 240^2$<br>$= 40996$<br>$BD = \sqrt{40996}$<br>$= 202.47468$<br>$= 202$ m correct to the nearest metre | **1** | 1 mark for correct answer |
| 16. | b) | <br><br>Calculate angles in triangle *CBD* as shown.<br>In $\Delta\, CBD$<br>$\frac{CD}{\sin B} = \frac{BD}{\sin C}$<br>$\frac{CD}{\sin 54°} = \frac{202.474}{\sin 21°}$<br>$CD = \frac{202.474 \sin 54°}{\sin 21°}$<br>$= 457.08735$<br>$= 457$ m (nearest metre) | **2** | 2 marks for correct answer<br><br>1 mark for correct angle s and working with an error in calculation or algebra |

| Question | | Working and answer | Marks | Mark Allocation |
|---|---|---|---|---|
| 17. | a) | $\frac{d}{dx}(x^4 \sin x) = x^4.\cos x + 4x^3.\sin x$ | **2** | 2 marks for correct answer<br>1 mark for correct use of rule with an algebraic error |
| 17. | b) | $y = \frac{x^2 - 4}{e^x}$<br>$y' = \frac{e^x.\ 2x - (x^2 - 4).e^x}{e^{2x}}$<br>$= \frac{e^x(\ 2x - x^2 + 4)}{e^{2x}}$<br>$= \frac{2x - x^2 + 4}{e^x}$ | **2** | 2 marks for correct answer<br>1 mark for correct use of rule with an algebraic error |
| 17. | c) | $f(x) = \ln(\cos x + 1)$<br>$f'(x) = \frac{-\sin x}{\cos x + 1}$<br>$f'\left(\frac{\pi}{2}\right) = \frac{-\sin\frac{\pi}{2}}{\cos\frac{\pi}{2} + 1} = \frac{-1}{0 + 1} = -1$ | **2** | 2 marks for correct answer<br>1 mark for correct use of rule with an algebraic error |

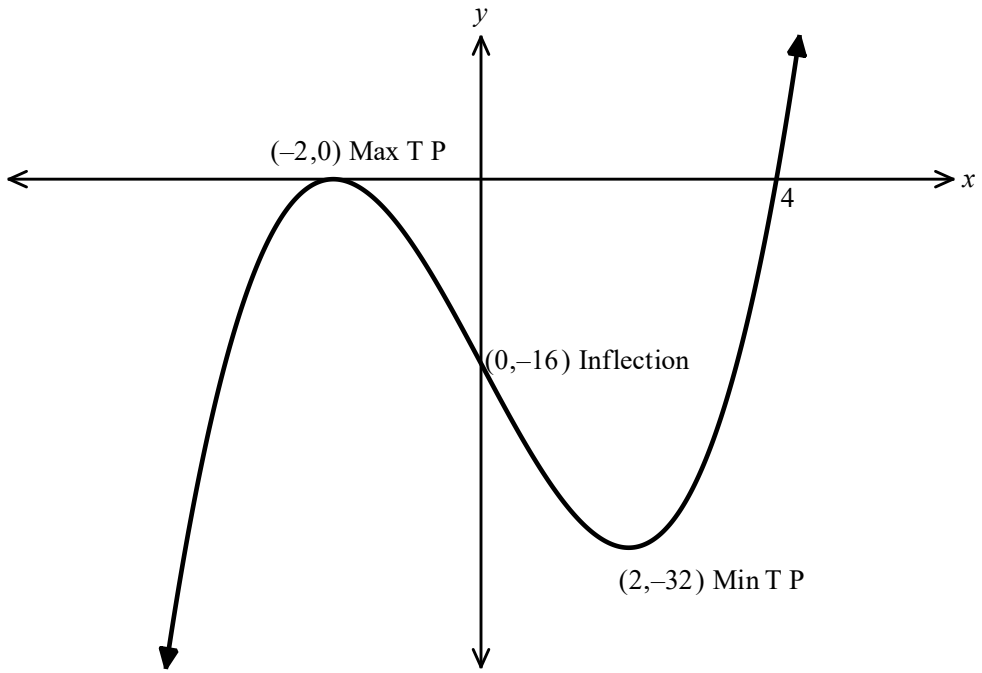
| Question | | Working and answer | Marks | Mark Allocation |
|---|---|---|---|---|
| 18. | a) | $2\sin\left(x-\frac{\pi}{6}\right)=0$<br>The solutions are where the curve crosses the $x$-axis, so<br>$x=-\frac{5\pi}{6}$ and $\frac{\pi}{6}$<br><br> | **1** | 1 mark for correct answer |
| 18. | b) | $2\sin\left(x-\frac{\pi}{6}\right)+1=0$<br>$2\sin\left(x-\frac{\pi}{6}\right)=-1$<br>The solutions are where the curve crosses the line $y=-1$ so<br>$x=-\frac{2\pi}{3}$ and 0 | **1** | 1 mark for correct answer |

| Question | | Working and answer | Marks | Mark Allocation |
|---|---|---|---|---|
| 19. | a) | Ratio $r = \frac{1800}{2700} = \frac{2}{3}$ | **1** | 1 mark for correct answer |
| 19. | b) | $T_n = ar^{n-1}$<br>$T_8 = 2700 \times \left(\frac{2}{3}\right)^7 = 158.0246$<br>$= \$158$ (nearest dollar) | **1** | 1 mark for correct answer |
| 19. | c) | $S_n = \frac{a(1-r^n)}{1-r}$<br>$S_{10} = \frac{2700\left(1-\left(\frac{2}{3}\right)^{10}\right)}{1-\left(\frac{2}{3}\right)}$<br>$= 7959.533$<br>$= \$7960$ (nearest dollar) | **1** | 1 mark for correct answer |

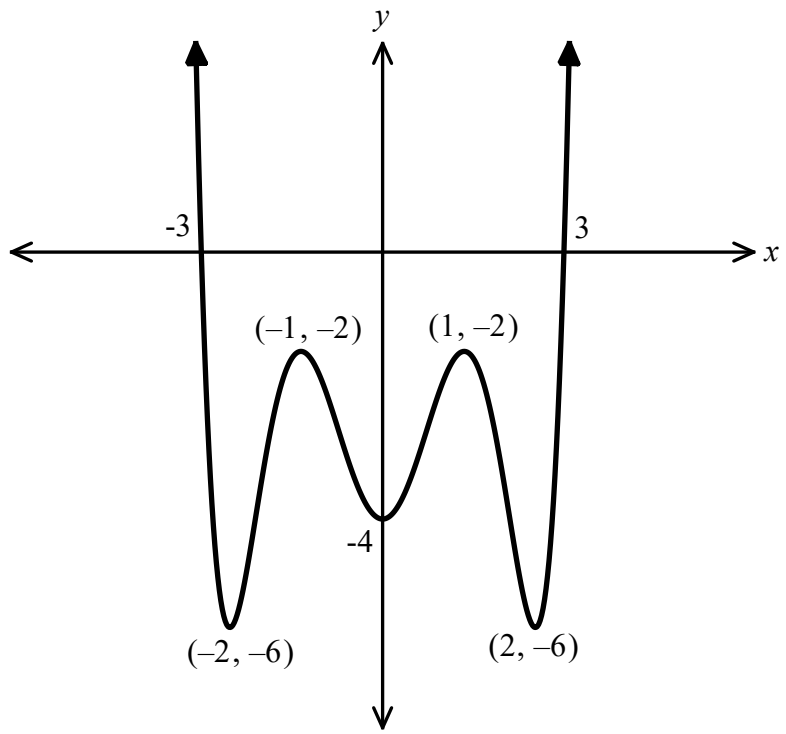
| Question | | Working and answer | Marks | Mark Allocation |
|---|---|---|---|---|
| 19. | d) | Since $\|r\| < 1$, there is a limiting sum, given by<br>$S = \frac{a}{1-r}$<br>$S = \frac{2700}{1-\frac{2}{3}}$<br>$= \$8\,100$<br>$S_{13} = \frac{2700\left(1-\left(\frac{2}{3}\right)^{13}\right)}{1-\left(\frac{2}{3}\right)}$<br>$= \$8058$ (nearest dollar)<br>Which is only \$42 less than the theoretical maximum it can ever reach<br>So it would make sense to relaunch the drive at this point. | **2** | 2 marks for comparing $S_{13}$ with the limiting sum and possibly also finding $T_{13}$ =\$13.87 and making a relevant comment.<br><br>1 mark for calculating $S_{13}$ only, or of calculating $T_{13}$ =\$13.87 only and not finding the limiting sum oem |

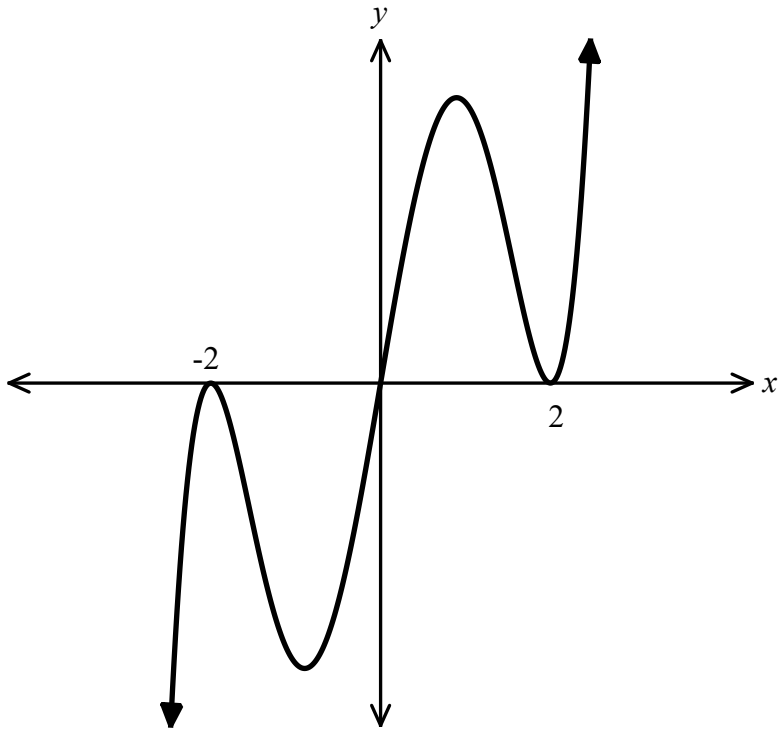
| Question | | Working and answer | Marks | Mark Allocation |
|---|---|---|---|---|
| 20. | a) | $l = r\theta$<br>$= 2 \times \frac{\pi}{3}$<br>$= \frac{2\pi}{3}$ metres | **1** | 1 mark for correct answer |
| 20. | b) | Area of sector $= \frac{1}{2} r^2 \theta$<br>$= \frac{1}{2} \times 2^2 \times \frac{\pi}{3}$<br>$= \frac{2\pi}{3}$ m$^2$<br>Area of right triangle $= \frac{1}{2} a b \sin\theta$<br>$= \frac{1}{2} \times 1 \times 2 \times \sin\frac{\pi}{3}$<br>$= 1 \times \frac{\sqrt{3}}{2}$<br>$= \frac{\sqrt{3}}{2}$ m$^2$<br>Area door $= \frac{2\pi}{3} + 2 \times \frac{\sqrt{3}}{2}$<br>$= \frac{2\pi}{3} + \sqrt{3}$ m$^2$<br>Alternative:<br>Find the height ($\sqrt{3}$) with Pythagoras Thm and use<br>A= $\frac{1}{2}$ base × height | **2** | 2 marks for correct answer<br>1 mark for correct area of sector or correct area of triangle oem |

| Question | | Working and answer | Marks | Mark Allocation |
|---|---|---|---|---|
| 21. | a) | $y = (x+2)^2(x-4)$<br>$x-$ intercepts are $x = -2$ (double)and $x = 4$<br>$y = (x-4)(x^2+4x+4)$<br>$= x^3+4x^2+4x-4x^2-16x-16$<br>$y = x^3-12x-16$<br>$\frac{dy}{dx} = 3x^2-12 = 3(x^2-4)$<br>$= 3(x+2)(x-2)$<br>Turning points at $x = 2$ and $x = -2$<br>Points are $(2,-32)$ and $(-2,0)$<br>$\frac{d^2y}{dx^2} = 6x$<br>Test turning points<br>$x = 2, \quad \frac{d^2y}{dx^2} = 6(2) = 12$ so concave up (min)<br>$x = -2, \quad \frac{d^2y}{dx^2} = 6(-2) = -12$ so concave down (max) | **2** | 2 marks for correct derivative and correct turning points<br><br>1 mark for turning points found correctly from incorrect derivative oem |
| 21. | b) | $\frac{d^2y}{dx^2} = 6x$<br>Possible inflection when $6x = 0,\ x = 0,\ y = -16$ (also $y$ intercept)<br>Since change of concavity (from above) $(0,-16)$ is an inflection | **1** | 1 mark for correct inflection point |

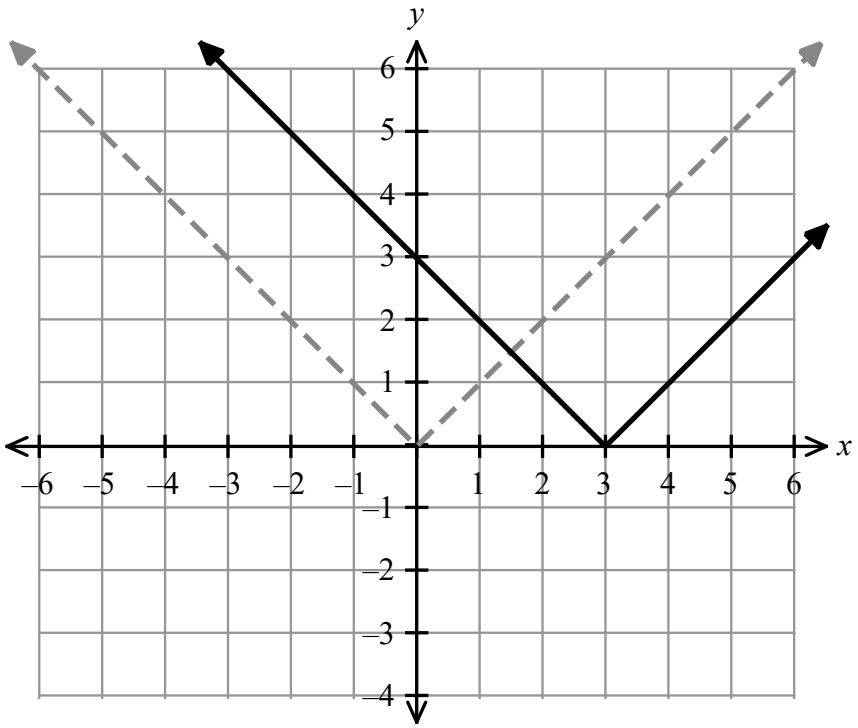
| Question | | Working and answer | Marks | Mark Allocation |
|---|---|---|---|---|
| 21. | c) |  | **2** | 2 marks for all points shown and curve drawn correctly (using the results of a) and b))<br><br>1 mark for graph with incorrect points shown (from previous parts) or curve drawn incorrectly oem |

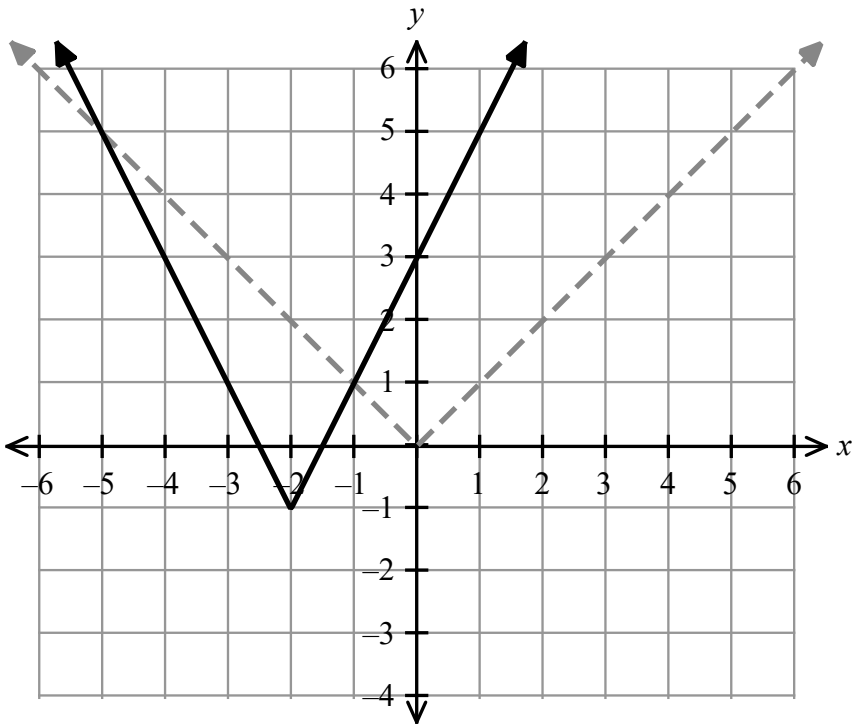
| Question | | Working and answer | Marks | Mark Allocation |
|---|---|---|---|---|
| 22. | a) | $P(A \cap B) = P(A)P(B\|A) = 0.4 \times 0.5 = 0.2$ | **1** | 1 mark for correct answer |
| 22. | b) | $P(A \cap B) \neq P(A)P(B)$<br><br>OR<br><br>$P(B\|A) \neq P(B)$ | **1** | 1 mark for correct answer |
| 22. | c) | $P(A\|B) = \frac{0.2}{0.3} = \frac{2}{3}$ | 1 | 1 mark for correct answer |

| Question | | Working and answer | Marks | Mark Allocation |
|---|---|---|---|---|
| 23. | a) | As it is an even function, it is symmetric about the $y$-axis. Draw the reflection in $x = 0$.<br><br> | **2** | 2 marks for correct graph with labels.<br><br>1 mark for a graph with a minor error in shape or with no labels oem |

| Question | | Working and answer | Marks | Mark Allocation |
|---|---|---|---|---|
| 23. | b) | The curve must pass through the origin, to be an odd function, and has a double root at x – 2, so has $x(x-2)^2$ as factors. [Touches $x$ axis at $x-2$]. As it is degree five the remaining factors are $(x+2)^{2.}$<br><br>A positive leading coefficient means it passes through the origin with a positive gradient. Draw the information above in 1st quadrant and then draw a point symmetric reflection of this in the 3rd quadrant.<br><br><br> | **2** | 2 marks for correct graph with labels.<br><br>1 mark for a graph with a minor error in shape or with no labels oem |

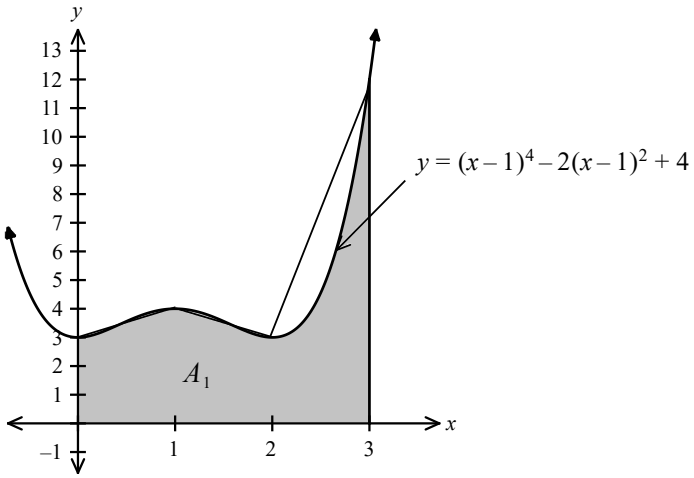
| Question | Working and answer | Marks | Mark Allocation |
|---|---|---|---|
| 24. | Interest factor for 12 years at 9% pa $= 20.141$<br>Value of first annuity after 12 years $= 20.141 \times 3500$<br>$= \$70493.50$<br>Additional amount required from 2nd annuity<br>$= \$100\,000 - \$70\,493.50$<br>$= \$29\,506.50$<br>2nd annuity has 7 years to run at 6% pa. with annual contributions of $\$C$.<br>Interest factor for 7 years at 6% $pa = 8.394$<br>$29506.5 = 8.394 \times C$<br>$C = \frac{29506.5}{8.394} = \$3515.19$ to the nearest cent.<br>Total annual amount paid for years 6 – 12 = $\$3500 + 3515.19$<br>$= \$7\,015.19$ | **3** | 3 marks for correct answer<br><br>2 marks for correct total value of first annuity and amount needed from the 2nd annuity or equivalent merit.<br><br>1 mark for some correct and relevant working |

| Question | | Working and answer | Marks | Mark Allocation |
|---|---|---|---|---|
| 25. | a) |  | **1** | 1 mark for correct graph. |

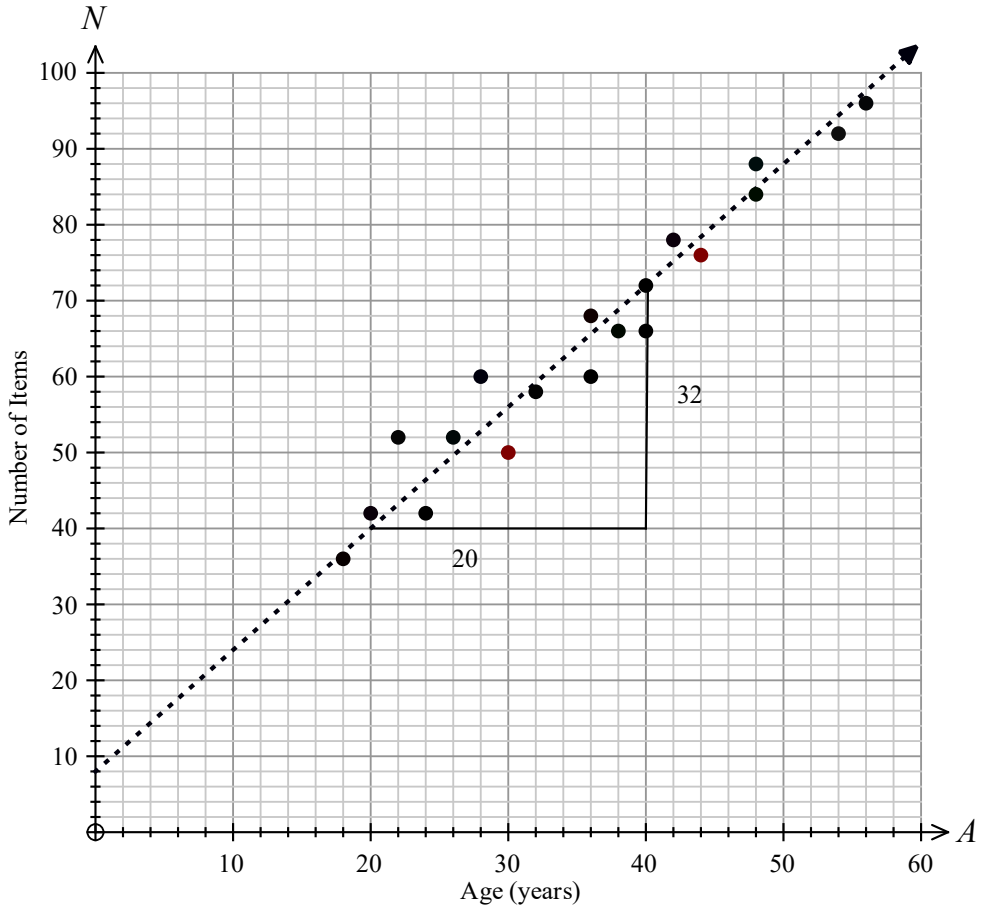
| Question | | Working and answer | Marks | Mark Allocation |
|---|---|---|---|---|
| 25. | b) |  | **2** | 2 marks for correct graph.<br>1mark for correct change in slope or correct translation of vertex oem |

| Question | | Working and answer | Marks | Mark Allocation |
|---|---|---|---|---|
| 26. | | $v = \dot{x} = 12t^3 - 96t^2 + 180t$<br>$\ddot{x} = 36t^2 - 192t + 180$<br>When $t = 0\ a = \ddot{x} = 180$<br>$x = \int 12t^3 - 96t^2 + 180t\ dt$<br>$= \frac{12t^4}{4} - \frac{96t^3}{3} + \frac{180t}{2} + C$<br>$x = 3t^4 - 32t^3 + 90t^2 + C$<br>When $t = 1,\ x = 60$<br>$60 = 61 + C$<br>$C = -1$<br>$x = 3t^4 - 32t^3 + 90t^2 - 1$<br>When $t = 0\ x = 3(0)^4 - 32(0)^3 + 90(0)^2 - 1 = -1$<br>The particle is initially 1 unit to the left of the origin and accelerating at 180 m/s$^2$ toward the right. | **3** | 3 marks for correct initial acceleration and displacement<br><br>2 marks for correct initial displacement oem<br><br>1 mark for one of correct initial acceleration oem |

| Question | | Working and answer | Marks | Mark Allocation |
|---|---|---|---|---|
| 27. | a) | $f(0) = (-1)^4 - 2\times(-1)^2 + 4 = 3 \quad f(1) = (0)^4 - 2\times(0)^2 + 4 = 4$<br>$f(2) = (1)^4 - 2\times(1)^2 + 4 = 3 \quad f(3) = (2)^4 - 2\times(2)^2 + 4 = 12$<br>$A_1 \approx \frac{3-0}{2\times 3}\big(f(0) + f(3) + 2\big(f(1) + f(2)\big)\big)$<br>$A_1 \approx \frac{1}{2}\big(3 + 12 + 2(4+3)\big) = \frac{29}{2} = 14.5$ | **2** | 2 marks for correct answer<br>1 mark for correctly substituting into the function values oem |
| 27. | b) | Substituting (x – 1) in place of $x$ in $x^4 - 2x^2 + 4$ causes a translation of the curve to the right, one unit, giving the graph shown. So, the bounds would be one unit to the left for the original function so -1 and 2.<br>Or can simply state that substituting 3 into (x-1) gives 2 and substituting 0 into (x-1) gives (-1), the bounds of the other integral | **1** | 1 mark for any correct explanation |
| 27. | c) | $\int_{-1}^{2} x^4 - 2x^2 + 4\ dx = \left[\frac{x^5}{5} - \frac{2x^3}{3} + 4x\right]_{-1}^{2}$<br>$= \left(\frac{2^5}{5} - \frac{2\times 2^3}{3} + 4\times 2\right) - \left(\frac{-1^5}{5} - \frac{2\times -1^3}{3} + 4\times -1\right)$<br>$= \frac{96 - 80 + 120}{15} - \frac{-3 + 10 - 60}{15}$<br>$= \frac{136 - (-53)}{15} = \frac{63}{5}$<br>$= 12.6$ | **2** | 2 marks for correct value<br>1 mark correct integration, using either method with an error in |

| Question | | Working and answer | Marks | Mark Allocation |
|---|---|---|---|---|
| 27. | d) | The first two trapezia are close to the curve of the function, maybe slightly less, but the third is above so its area would be greater that the integral, making the approximation greater than the exact value.<br><br> | **1** | 1 mark for any correct reasoning. |

| Question | Working and answer | Marks | Mark Allocation |
|---|---|---|---|
| 28. | $T_n = a + (n-1)d$<br>$T_{20} = a + 19 \times 6 = 130$<br>$a = 130 - 114 = 16$<br>$S_n = \frac{n}{2}(a + l)$<br>$S_{20} = \frac{20}{2}(16 + 130)$<br>$= 1460$ | **2** | 2 marks for correct answer<br><br>1 mark for correct value of $a$ oem |

| Question | | Working and answer | Marks | Mark Allocation |
|---|---|---|---|---|
| 29. | a) |  | **1** | 1 mark for correct line within reasonable accuracy |

| Question | | Working and answer | Marks | Mark Allocation |
|---|---|---|---|---|
| 29. | b) | Taking two points on the line<br>e. g. (20,40)and (40,72)<br>Gradient $m = \frac{72-40}{40-20}$<br>$= \frac{32}{20} = \frac{8}{5} = 1.6$<br>$y-$ intercept $\approx 8$<br>Equation is $y = \frac{8x}{5} + 8$ or $y = 1.6x + 8$ | **2** | 2 marks for correct equation or one with gradient within 0.2 of given equation<br><br>1 mark for correct gradient but line misplaced vertically or equivalent merit |
| 29. | c) | Using equation<br>$y = 8\frac{50}{5} + 8$ $y = 1.6(50) + 8$<br>$= 88$ items $= 88$ items | **1** | 1 mark for correct answer read from answer in part (b) |
| 30. | a) | 0.10+0.30+0.45+0.10+k=1<br>k=0.05 | **1** | |
| 30. | b) | P(X>2)=0.1+0.05<br>=0.15 | **1** | |
| 30. | c) | $E(X) = 0 \times 0.10 + 1 \times 0.30 + 2 \times 0.45 + 3 \times 0.10 + 4 \times 0.05 = 1.7$ | **1** | |

<table>
<tr><th colspan="2">Question</th><th>Working and answer</th><th>Marks</th><th>Mark Allocation</th></tr>
<tr><td>30.</td><td>d)</td><td>$Var(X) = 0^2 \times 0.10 + 1^2 \times 0.30 + 2^2 \times 0.45 + 3^2 \times 0.10 + 4^2 \times 0.05 - (1.7)^2 = 0.$<br>$\sigma = \sqrt{0.91} = 0.9539\ldots$</td><td>2</td><td><table><tr><th>Criteria</th><th>Marks</th></tr><tr><td>• Correctly calculates the standard deviation</td><td>2</td></tr><tr><td>• Correctly calculates variance</td><td>1</td></tr></table></td></tr>
<tr><td>31.</td><td>a)</td><td>$A = \pi r^2 + \pi rh + 2rh$<br>$\pi r^2 + \pi rh + 2rh = \sqrt[3]{27\pi}$<br>$\pi rh + 2rh = \sqrt[3]{27\pi} - \pi r^2$<br>$rh(\pi + 2) = \sqrt[3]{27\pi} - \pi r^2$<br>$h = \frac{1}{r(\pi+2)}\left[\sqrt[3]{27\pi} - \pi r^2\right]$<br>$V = \frac{1}{2}\pi r^2 h$<br>$V = \frac{1}{2}\pi r^2\left[\frac{1}{r(\pi+2)}\left[\sqrt[3]{27\pi} - \pi r^2\right]\right]$<br>$V = \frac{\pi}{2(\pi+2)}\left[\sqrt[3]{27\pi}r - \pi r^3\right]$</td><td>3</td><td>1 mark for correct working.<br>3marks Provides correct solution<br>2 marks Finds correct expression for $h$, or equivalent merit<br>1 mark Finds an expression for Surface Area, and attempts to find $h$, or equivalent merit</td></tr>
</table>

| | | | | |
|---|---|---|---|---|
| 31. | b) | $\frac{dV}{dr}=\frac{\pi}{2(\pi+2)}\left[\sqrt[3]{27\pi}-3\pi r^2\right]$<br>$\sqrt[3]{27\pi}-3\pi r^2=0$<br>$3\pi r^2=3\pi^{\frac{1}{3}}$<br>$r^2=\pi^{-\frac{2}{3}}$<br>$r=\pm\pi^{-\frac{1}{3}}$, but $r>0$<br>$\therefore r=\pi^{-\frac{1}{3}}$<br>$\frac{d^2V}{dr^2}=\frac{\pi}{2(\pi+2)}\left[-6\pi r\right]$<br>$\frac{d^2V}{dr^2}\left(\pi^{-\frac{1}{3}}\right)<0$<br>$\therefore$ maximum<br>$V\left(\pi^{-\frac{1}{3}}\right)=\frac{\pi}{2(\pi+2)}\left[\sqrt[3]{27\pi}\left(\pi^{-\frac{1}{3}}\right)-\pi\left(\pi^{-\frac{1}{3}}\right)^3\right]$<br>$=\frac{\pi}{2(\pi+2)}\left[3\pi^{\frac{1}{3}}\times\pi^{-\frac{1}{3}}-\pi\times\pi^{-1}\right]$<br>$=\frac{\pi}{2(\pi+2)}\left[3-1\right]$<br>$=\frac{\pi}{\pi+2}$ | **3** | 3 Marks correct solution<br><br>2 marks Finds an expression for $r$, or equivalent merit<br><br>1 mark Finds $\frac{dV}{dr}$ and attempts to solve $\frac{dV}{dr}=0$, or equivalent merit |

| Question | Working and answer | Marks | Mark Allocation |
|---|---|---|---|
| 32. | a) | **2** | 2 marks for correct derivation of result<br><br>1 mark for some correct working |

$$\sin\theta - \cos\theta\sin\theta\left(\sin\left(\frac{\pi}{2} - \theta\right)\right) = \sin\theta\left(1 - \cos\theta\left(\sin\left(\frac{\pi}{2} - \theta\right)\right)\right)$$

$$= \sin\theta\ \left(1 - \cos\theta(\cos\theta)\right) \quad \text{for } \left[0, \frac{\pi}{2}\right]$$

$$= \ \sin\theta(1 - \cos^2\theta)$$

$$= \sin\theta(\sin^2\theta)$$

$$= \sin^3\theta$$

| Question | | Working and answer | Marks | Mark Allocation |
|---|---|---|---|---|
| 32. | b) | $2\sqrt{2}\sin\theta - 2\sqrt{2}\cos\theta\sin\theta\left(\sin\left(\frac{\pi}{2}-\theta\right)\right) = 1$<br>$2\sqrt{2}\sin^3\theta = 1$<br>$\sin^3\theta = \frac{1}{2\sqrt{2}}$<br>$\sin\theta = \frac{1}{\sqrt{2}}$<br>$\theta = \frac{\pi}{4}$ for $\left[0,\frac{\pi}{2}\right]$ | **2** | 2 marks for correct solution of equation<br><br>1 mark for some correct working oem |

| | | | |
|---|---|---|---|
| 33. | $A = \int_{\frac{\pi}{4}}^{\frac{5\pi}{4}} 2\sin x \ \ dx - \int_{\frac{\pi}{4}}^{\frac{5\pi}{4}} 2\cos x \ \ dx = [-2\cos x]_{\frac{\pi}{4}}^{\frac{5\pi}{4}} - [2\sin x]_{\frac{\pi}{4}}^{\frac{5\pi}{4}}$<br>$= \left(-2\cos\frac{5\pi}{4} + 2\cos\frac{\pi}{4}\right) - \left(2\sin\frac{5\pi}{4} - 2\sin\frac{\pi}{4}\right)$<br>$= \left(-2 \times -\frac{1}{\sqrt{2}} + 2 \times \frac{1}{\sqrt{2}}\right) - \left(2 \times -\frac{1}{\sqrt{2}} - 2 \times \frac{1}{\sqrt{2}}\right)$<br>$= \frac{4}{\sqrt{2}} + \frac{4}{\sqrt{2}} = \frac{8}{\sqrt{2}} = \frac{8\sqrt{2}}{2} = 4\sqrt{2}$ square units<br>OR AS A SINGLE INTEGRAL<br>$A = \int_{\frac{\pi}{4}}^{\frac{5\pi}{4}} 2\sin x - 2\cos x \ \ dx$<br>$= [-2\cos x - 2\sin x]_{\frac{\pi}{4}}^{\frac{5\pi}{4}}$<br>$= \left(-2\cos\frac{5\pi}{4} - 2\sin\frac{5\pi}{4}\right) - \left(-2\cos\frac{\pi}{4} - 2\sin\frac{\pi}{4}\right)$<br>$= \left(-2 \times -\frac{1}{\sqrt{2}} - 2 \times -\frac{1}{\sqrt{2}}\right) - \left(-2 \times \frac{1}{\sqrt{2}} - 2 \times \frac{1}{\sqrt{2}}\right)$<br>$= \left(\sqrt{2} + \sqrt{2}\right) - \left(-\sqrt{2} - \sqrt{2}\right)$<br>$= 2\sqrt{2} + 2\sqrt{2} = 4\sqrt{2}$ square units | **3** | 3 marks for correct value<br>2 marks for the correct integrations and a minor error in substitutions oem<br>1 mark for some correct working in integration or substitution |

| Question | | Working and answer | Marks | Mark Allocation |
|---|---|---|---|---|
| 34. | a) | Quarterly interest rate: $r = 0.08 \div 4 = 0.02$<br>$A_1 = 20000 \times 1.02 - 1000$<br>$A_2 = (20000 \times 1.02 - 1000) \times 1.02 - 1000$ | **1** | 1 mark for correct derivation of required result |
| 34. | b) | $A_3 = \big((20000 \times 1.02 - 1000) \times 1.02 - 1000\big) \times 1.02 - 1000$<br>$= 20000 \times 1.02^3 - 1.02^2 \times 1000 - 1.02 \times 1000 - 1000$<br>So following the pattern<br>$A_n = 20000 \times 1.02^n$<br>$- 1000(1.02^{n-1} + 1.02^{n-2} \ldots\ldots + 1.02^2 + 1.02 + 1)$<br>The section in brackets above is a geometric series with $a = 1$ and $r$<br>$= 1.02$ with $n$ terms going from 0 to $n - 1$<br>$s_n = \dfrac{1(1.02^n - 1)}{1.02 - 1}$<br>$A_n = 20000 \times 1.02^n - \dfrac{1000(1.02^n - 1)}{1.02 - 1}$<br>$= 20000 \times 1.02^n - \dfrac{1000(1.02^n - 1)}{0.02}$<br>$= 20000 \times 1.02^n - \big(50000(1.02^n - 1)\big)$<br>$= 20000 \times 1.02^n - 50000 \times 1.02^n + 50000$<br>$= 50000 - 30000 \times 1.02^n$ | **2** | 2 marks for correctly deriving the required result<br><br>1 mark for a derivation with a minor error in calculation or algebra oem |

| Question | | Working and answer | Marks | Mark Allocation |
|---|---|---|---|---|
| 34. | c) | The account runs out of funds when $A_n = 0$<br>$50000 - 30000 \times 1.02^n = 0$<br>$30000 \times 1.02^n = 50000$<br>$1.02^n = \frac{50000}{30000} = 1.\dot{6}$<br>$\ln(1.02^n) = \ln(1.\dot{6})$<br>$n\ln(1.02) = \ln(1.\dot{6})$<br>$n = \frac{\ln(1.\dot{6})}{\ln(1.02)} = 25.79585103$<br>So Mitch could make 25 withdrawals of \$1000 then another 26th partial withdrawal after that would empty the account. | **2** | 2 marks for an answer of 25 complete withdrawals or of 26 complete withdrawals with one partial withdrawal.<br>Can be found by guess and check rather than logarithms<br>1 mark for setting $50000 - 30000 \times 1.02^n = 0$ with a subsequent error or incomplete solution oem |