Student Number:
Teacher:

BOOKLET 1

St George Girls High School

Mathematics Advanced

2026 Trial HSC Examination

General Instructions

- Reading time – 10 minutes
- Working time – 3 hours
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided
- For questions in **Section I**, use the Multiple-Choice answer sheet provided
  For questions in **Section II:**
  - Answer the questions in the space provided
  - Show relevant mathematical reasoning and/or calculations
  - Marks may not be awarded for incomplete or poorly presented solutions, or where multiple solutions are provided

Total marks: 100

Section I – 10 marks (pages 2 – 8)

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section

Section II – 90 marks (pages 9 – 37)

- Attempt Questions 11 – 33
- Allow about 2 hours and 45 minutes for this section

Q1 – Q10	/10
Q11 – Q17	/19
Q18 – Q22	/17
Q23 – Q26	/17
Q27 – Q29	/14
Q30 – Q31	/12
Q32 – Q33	/11
Total	**/100**
	%

Section I

10 marks
Attempt Questions 1 – 10
Allow about 15 minutes for this section

Use the Multiple-Choice Answer Sheet provided for Questions 1 to 10.

1. There are 42 members in a dancing school. 10 members do neither Hip Hop nor Jazz, 15 do both Hip Hop and Jazz, and there are 20 members who do Jazz altogether.

   Given that a member chosen at random does Hip Hop, what is the probability that this member does not do Jazz?

   (A) $\frac{4}{9}$

   (B) $\frac{5}{9}$

   (C) $\frac{2}{7}$

   (D) $\frac{9}{14}$

2. What is the correct solution to $\int e^2 + e^{4x}\, dx$?

   (A) $e^2x + 4e^{4x} + C$

   (B) $e^2x + \frac{e^{4x}}{4} + C$

   (C) $2e^2 + 4e^{4x} + C$

   (D) $\frac{e^2}{2} + \frac{e^{4x}}{4} + C$

3. Which interval gives the range of the function $y = 5 - 2\sin 3x$?

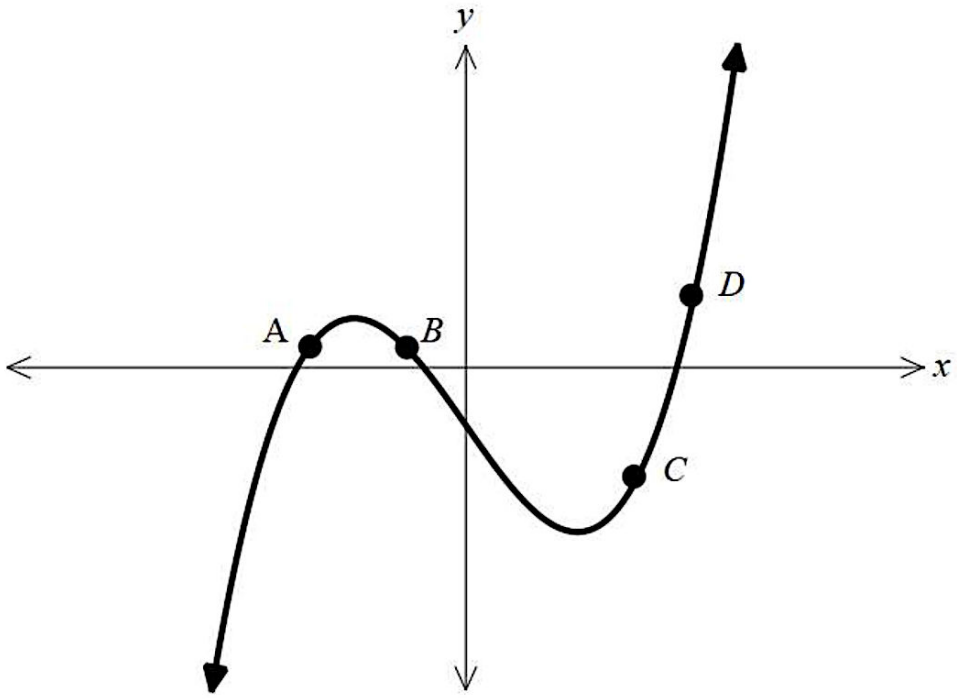
   (A) $[-2, 2]$

   (B) $[3, 7]$

   (C) $[2, 8]$

   (D) $[2, 7]$

4. Which point on the following diagram has $y > 0$, $\frac{dy}{dx} > 0$ and $\frac{d^2y}{dx^2} < 0$?



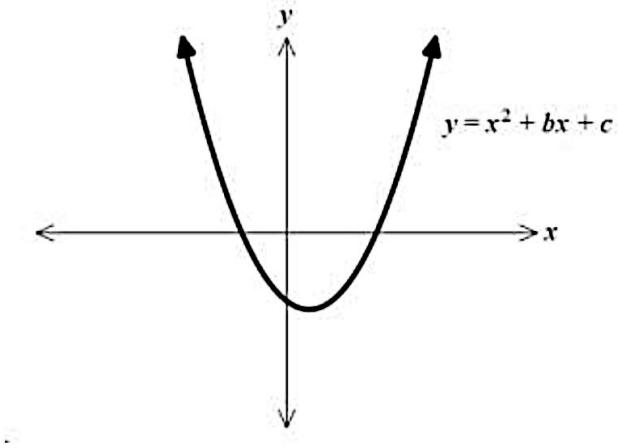
   (A) A

   (B) B

   (C) C

   (D) D

5. The diagram below shows the graph of $y = x^2 + bx + c$, where b and c are constants.



Which of the following are true?

(A) $b > 0$ and $c > 0$

(B) $b > 0$ and $c < 0$

(C) $b < 0$ and $c > 0$

(D) $b < 0$ and $c < 0$

6. Given that the graph of $y = f(x)$ has a local maximum at $(-4, 3)$, what will the graph of $y = -2f\left(\frac{1}{2}x\right)$ have?

(A) A local minimum at $(-8, -6)$

(B) A local minimum at $(-2, -6)$

(C) A local maximum at $(-8, 6)$

(D) A local maximum at $(-2, 6)$

7. A vessel initially contains 100 litres of water. It is being emptied at a rate of $\frac{dV}{dt} = -2 - \frac{20}{t+1}$ where V is the volume in litres after t minutes.

   Which of the following gives the exact volume of water (in litres) remaining in the vessel after five minutes?

   (A) $10 + 20 \ln 6$

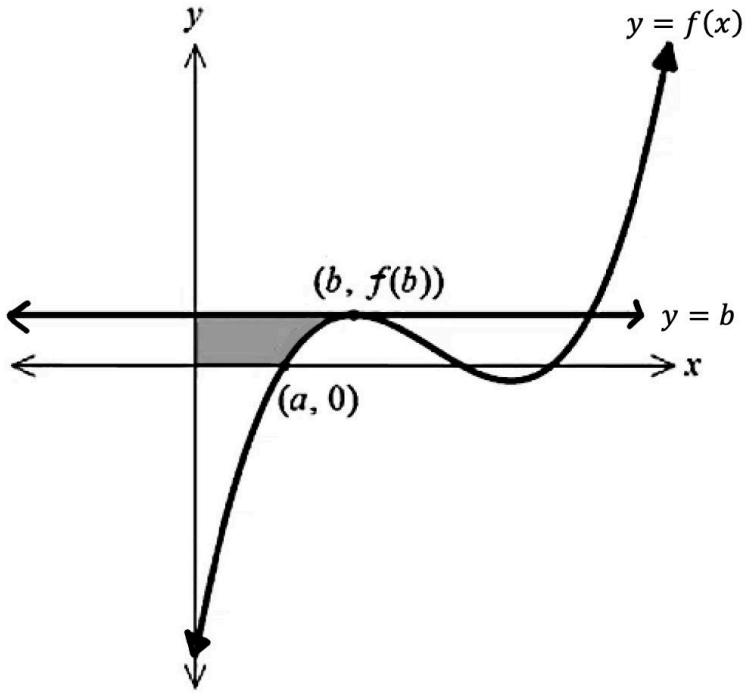
   (B) $90 - 20 \ln 6$

   (C) $90 + 20 \ln 6$

   (D) $110 + 20 \ln 6$

8. Part of the graph of $y = f(x)$ is shown below.

   The horizontal line, $y = b$, is a tangent to the local maximum at $\big(b, f(b)\big)$. The shaded area is bounded by the graph of $y = f(x)$, the coordinate axes, and the horizontal line.



   Which is the correct expression for the area of the shaded region?

   (A) $af(b) - \int_a^b f(x)\ dx$

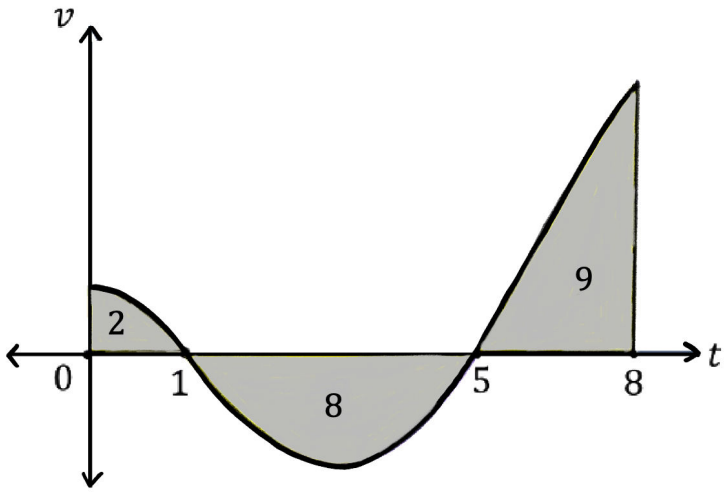
   (B) $af(b) + \int_a^b f(x)\ dx$

   (C) $bf(b) - \int_a^b f(x)\ dx$

   (D) $\int_0^b f(x)\ dx - bf(b)$

9. The graph below shows the velocity, v m/s, of a particle moving along a straight line for the first 8 seconds.

   The areas between the velocity curve and the time (t) axis during this time interval are as shown below.



   Given that the displacement of the particle at $t = 0$ is 4 m, find the displacement of the particle at $t = 8$ seconds.

   (A) $x = 2$ m

   (B) $x = 3$ m

   (C) $x = 7$ m

   (D) $x = 24$ m

10. The functions $h(x)$ and $g(x)$ are defined for all real x, and $h(x) = [g(x)]^4$.

    Given that $h'(-2) = 96$, $g'(-2) = -3$, what is the value of $h(-2)$?

    (A) -16

    (B) -2

    (C) 2

    (D) 16

End of Section I

Section II – Booklet 1

Attempt Questions 11 – 26 (53 marks)
Allow about 95 minutes for Section II – Booklet 1

- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided on pages 22-23 of Booklet 1. If you use this space, clearly indicate which question you are answering.

Question 11 (1 mark)

The first three terms of an arithmetic sequence are 5, 9, and 13. **1**

Find the 19^{th} term.

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Question 12 (2 marks)

Find the equation of the tangent to the curve $y = 1 + 2x^2 - \frac{4}{x^2}$ at the point $(2, 8)$. **2**

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Question 13 (4 marks)

The probability distribution of a discrete random variable X is given in the table below.

x	0	2	3	5
$\mathrm{P}(X = x)$	0.3	$2k$	$3k$	0.1

(a) Find the value of k. **1**

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(b) Calculate the expected value, $E(X)$. **1**

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(c) Hence, find the standard deviation. **1**

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(d) Find $P(X \text{ is odd})$. **1**

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Question 14 (2 marks)

Evaluate $\int_0^1 \frac{x}{\sqrt{x^2+2}}\,dx$ correct to 4 decimal places. **2**

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Question 15 (3 marks)

Solve the equation $\sin^2 x - \sqrt{3}\sin x \cos x = 0$ for $0 \leq x \leq 2\pi$. **3**

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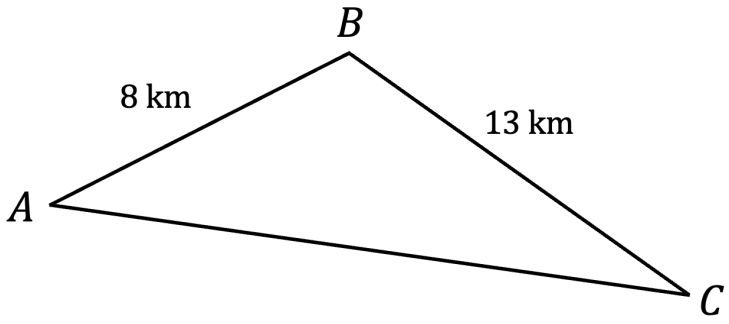
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Question 16 (4 marks)

In the diagram below, a car travels 8 km on a bearing of 080° from Town A to Town B.

It then travels 13 km on a bearing of 132° from Town B to Town C.



(a) Find the size of $\angle ABC$. **1**

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(b) Find the bearing of Town C from Town A. **3**

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Question 17 (3 marks)

The graph of $y = f(x)$ passes through the points $(2, 5)$ and $(3, a)$. **3**

If its gradient function is given by $f'(x) = 3x^2 - 4x$, find the value of a.

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Question 18 (3 marks)

Prove that $\sec x + \tan x = \dfrac{\cos x}{1 - \sin x}$. **3**

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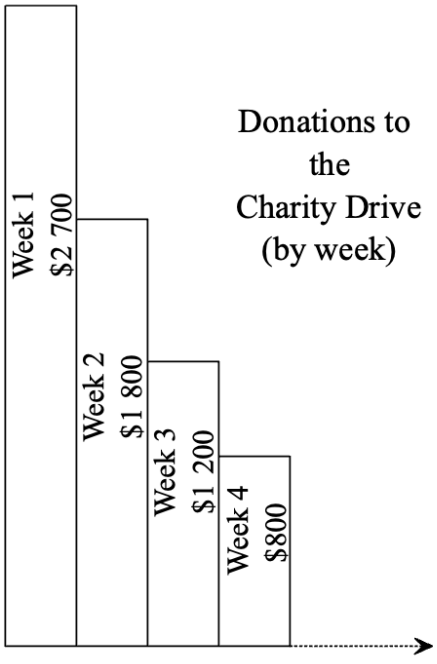
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Question 19 (5 marks)

A charity starts a fundraising drive, and records the donations received each week for the first four weeks after the drive is launched.

The graph on the right shows the results.



The director of the charity notices that the amount of donations received each week follows a geometric sequence.

She makes some predictions of the future donations, by assuming that the same pattern will be followed.

(a) Find the common ratio. **1**

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(b) How much would she predict to be received in Week 8 (to the nearest dollar)? **1**

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(c) What total amount would be raised in the first thirteen weeks? **1**

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(d) She claims that the drive will never raise more than \$8100 in total. **2**

Explain how she can make this prediction, and why she decides to launch a new fundraising drive after Week 13.

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Question 20 (3 marks)

The population of Victoria Street in Kogarah can be modelled by the function $V = 1500e^{-0.06t}$, where t is the number of years since the 1st of January 2026.

(a) Find the population of Victoria Street at the end of September 2026. **1**

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(b) Find the rate at which the population of Victoria Street is changing on the 1st of January 2031. **2**

Give your answer correct to the nearest whole number.

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Question 21 (3 marks)

Solve the following simultaneous equations: **3**

$$\log_3 x + \log_3 y = 8$$
$$\log_x y = 3$$

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Question 22 (3 marks)

The area enclosed by the curve $y = x^2$, the y-axis and the line $y = a$, where $a > 0$, in the first quadrant is exactly 10 units^2. Find the value of a correct to 2 decimal places. **3**

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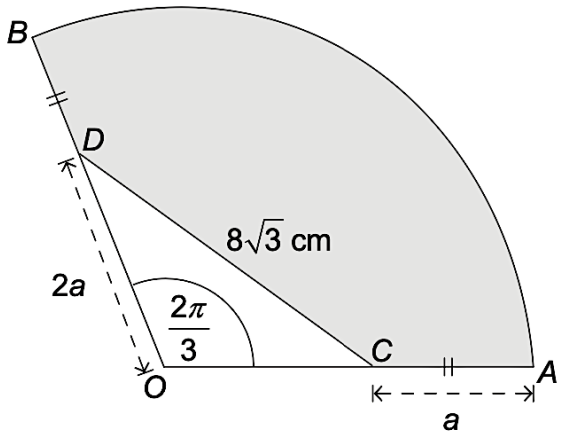
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Question 23 (3 marks)

The triangle ODC is inside a sector that has a central angle of $\frac{2\pi}{3}$, as shown in the diagram below. **3**



The lengths of the sector and triangle are as follows:

- $DC = 8\sqrt{3}$ cm
- $AC = BD$
- $OC = OD = 2AC$

Find the exact area of the shaded region.

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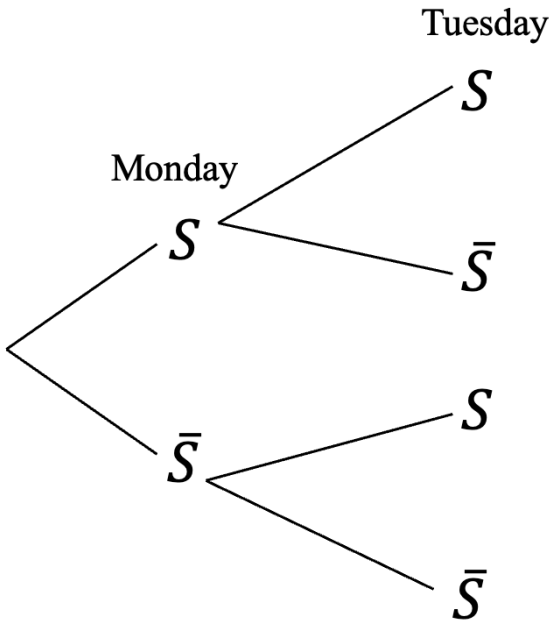
Question 24 (6 marks)

In a particular city, the probability that it will snow on Monday is x.

If it does snow on Monday, the probability that it will snow on Tuesday is $\frac{1}{4}x$.

If it does not snow on Monday, the probability that it will snow on Tuesday is x.

(a) Let S be the probability that it snows on any given day. Complete the probability tree diagram below. **2**



(b) Find an expression, in terms of x, that gives the probability that it will snow on Tuesday. **2**

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(c) Find the greatest probability of snow on Tuesday by first finding the value of x. **2**

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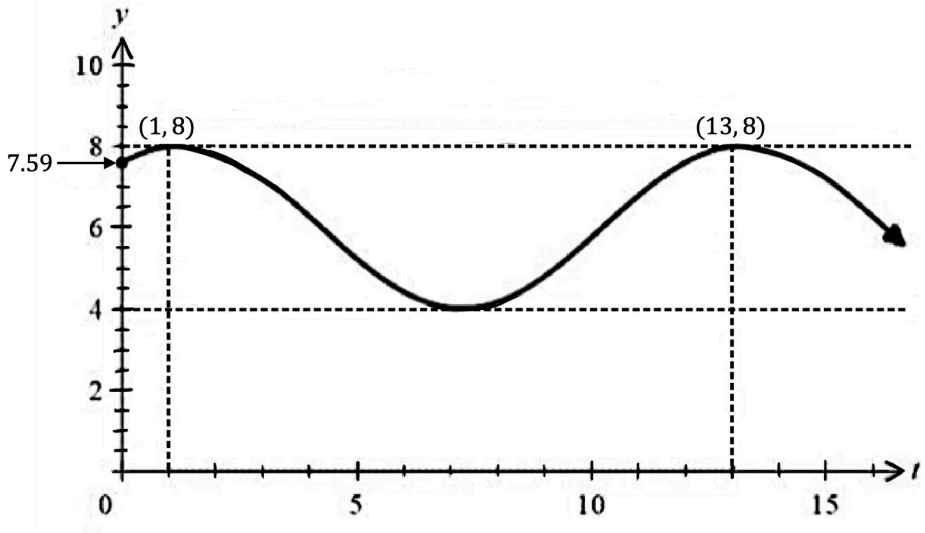
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Question 25 (2 marks)

The trigonometric function below is defined by the equation $y = a\sin[b(t + c)] + d$. **2**



Find the values of a, b, c and d correct to 2 decimal places where required.

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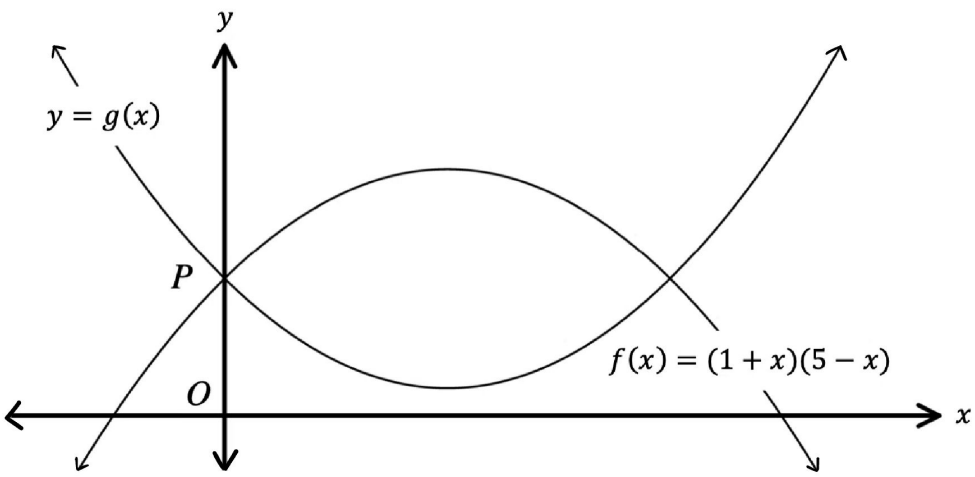
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Question 26 (6 marks)

The figure above shows the curve with equation $f(x) = (1 + x)(5 - x)$, and the curve with equation $y = g(x)$.

Both curves meet the y-axis at the point P.



The curve with equation $y = g(x)$ can be obtained by a series of transformations of the curve with equation $y = f(x)$.

(a) List the sequence of transformations applied to $f(x)$ to give $g(x)$, and hence **find the equation** of the curve $y = g(x)$. **3**

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(b) Calculate the area bounded by the two curves $y = f(x)$ and $y = g(x)$. **3**

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Section II

Student Number:

Teacher:

Section II – Booklet 2

Attempt Question 27 – 33 (37 marks)
Allow about 70 minutes for
Section II – Booklet 2

BOOKLET 2

Instructions

- Write your Teacher's Name and Student Number at the top of this page.
- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided on pages 38-39 of Booklet 2. If you use this space, clearly indicate which question you are answering.

Q27 – Q29	/14
Q30 – Q31	/12
Q32 – Q33	/11

Please turn over

Question 27 (3 marks)

(a) Differentiate $2x \cos x$. **1**

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(b) Hence, find $\int x \sin x \, dx$. **2**

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Question 28 (5 marks)

The number of hours, h, of daylight in a certain city can be modelled by the function of the form $h(t) = 12 + \frac{5}{2}\cos\left[\frac{\pi}{183}(t - 22)\right]$ for $1 \leq t \leq 366$, where t is the number of days from the 1st of January in a leap year.

For example, $t = 1$ corresponds to the 1st of January and $t = 366$ corresponds to 31st of December.

(a) Find the maximum number of hours of daylight. **1**

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(b) On what date does this occur for the first time? **1**

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(c) On how many days of the year is the number of hours of daylight at least 10 hours and 45 minutes? **3**

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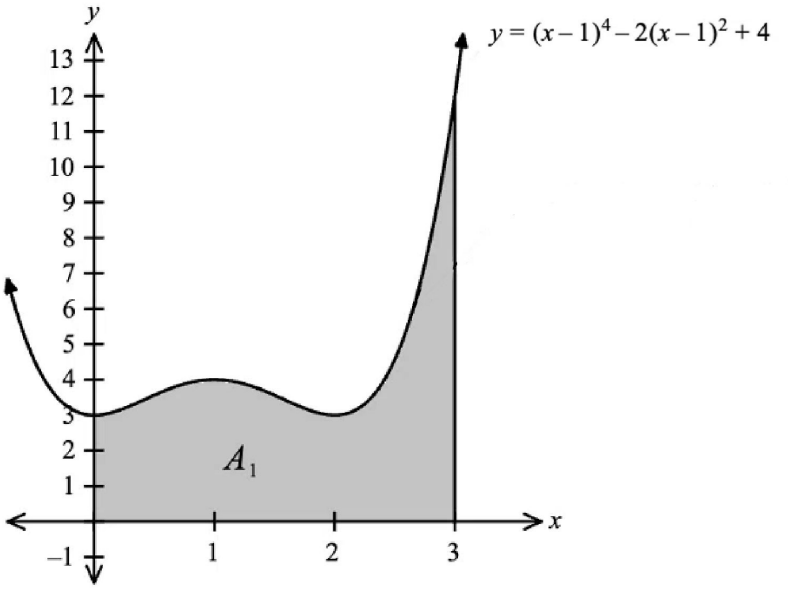
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Question 29 (6 marks)

The diagram below shows the graph of $y = (x-1)^4 - 2(x-1)^2 + 4$.



The shaded area, A_1, is the area enclosed between the x-axis, the curve and the lines $x = 0$ and $x = 3$.

(a) Use three applications of the Trapezoidal rule to estimate the area A_1. **2**

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(b) Explain why $\int_0^3 (x-1)^4 - 2(x-1)^2 + 4\,dx = \int_{-1}^2 x^4 - 2x^2 + 4\,dx$. **1**

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(c) Hence, or otherwise, find the exact value of A_1. **2**

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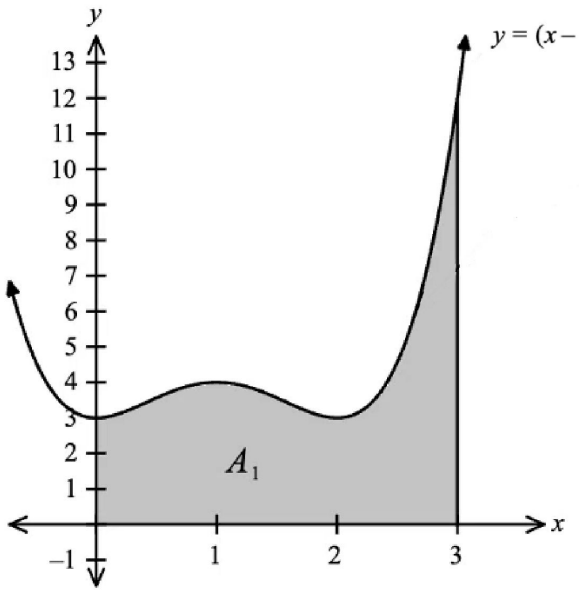
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(d) Explain why the approximation found for A_1 in part (a) is greater than the exact value of A_1 found in part (c). **1**

You may wish to draw on the diagram below to illustrate your answer.



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Question 30 (7 marks)

Consider the function with equation $y = x^3e^x$.

This will have the following derivatives (do NOT prove this).

$$\frac{dy}{dx} = x^2e^x(x+3)$$

$$\frac{d^2y}{dx^2} = xe^x(x^2+6x+6)$$

(a) Find any stationary points and determine their nature. **2**

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(b) Describe what happens to the function as $x \to \pm\infty$. **2**

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(c) Find the global minimum value of $y = x^3e^x$. **1**

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(d) You are given that $(-4.7, -0.9)$ and $(-1.3, -0.6)$ are points of inflection. **2**

Sketch the graph of $y = x^3e^x$.

Question 31 (5 marks)

A particle starts to move from the origin along the x-axis.

Its velocity at time, t seconds, is given by $v = 8 + \dfrac{6}{2t+1}$ metres per second.

(a) Find the acceleration of the particle after 2 seconds. **2**

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(b) Explain why the particle is never at rest. **1**

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(c) Find the exact distance travelled by the particle between $t = 0$ and $t = 4$. **2**

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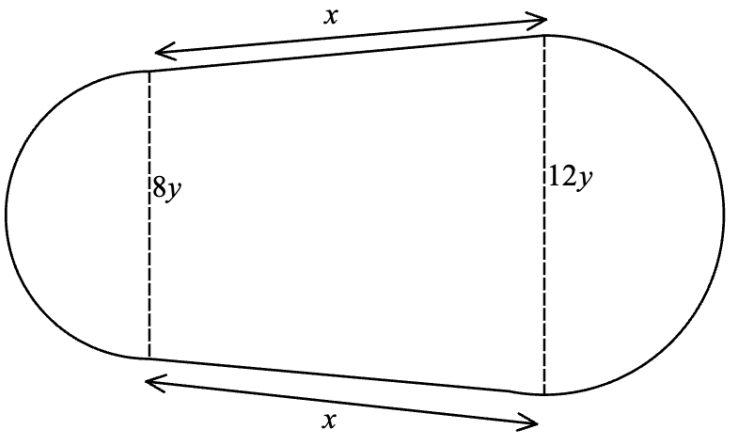
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Question 32 (5 marks)

A plan for a cycling track is shown below.

It is comprised of two equal straight sections which are x km long, and two semicircular sections with diameters $8y$ km and $12y$ km respectively.



The average speed of a cyclist on the track is given by $S = 10\pi y(x^2 + 6x + 5)$ km/h.

(a) Show that the total distance around the track is given by $D = 10\pi y + 2x$. **1**

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(b) Given that the total length of the track is to be 6 km, show that $y = \dfrac{3 - x}{5\pi}$. **1**

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(c) Hence, find the value of x which gives the maximum average speed around the track. **3**

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Question 33 (6 marks)

In a laboratory environment, a certain bacteria culture grows such that its population size in the n^{th} hour is given by $P_n = b \times 10^{n-1}$, where $b > 0$.

Let $B_n = \log_{10}(P_n)$.

(a) Given that $B_1 + B_2 + B_3 + \cdots + B_{10} = 25$, show that $b = 0.01$. **3**

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(b) Find the smallest value of n such that $B_1 + B_2 + B_3 + \cdots + B_n > 50$. **3**

END OF EXAMINATION

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If you use this space, clearly indicate which question you are answering.

Section I

10 marks
Attempt Questions 1 – 10
Allow about 15 minutes for this section

Use the Multiple-Choice Answer Sheet provided for Questions 1 to 10.

1. There are 42 members in a dancing school. 10 members do neither Hip Hop nor Jazz, 15 do both Hip Hop and Jazz, and there are 20 members who do Jazz altogether.

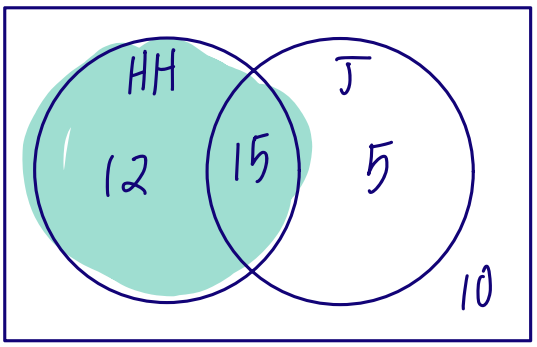
   Given that a member chosen at random does Hip Hop, what is the probability that this member does not do Jazz?

   (A) $\frac{4}{9}$

   (B) $\frac{5}{9}$

   (C) $\frac{2}{7}$

   (D) $\frac{9}{14}$



$\frac{12}{27} = \frac{4}{9}$

2. What is the correct solution to $\int e^2 + e^{4x}\, dx$?

   (A) $e^2x + 4e^{4x} + C$

   (B) $e^2x + \frac{e^{4x}}{4} + C$

   (C) $2e^2 + 4e^{4x} + C$

   (D) $\frac{e^2}{2} + \frac{e^{4x}}{4} + C$

3. Which interval gives the range of the function $y = 5 - 2\sin 3x$?

amplitude = 2
centre = 5
∴ max. value = 5+2
= 7
min. value = 5−2
= 3

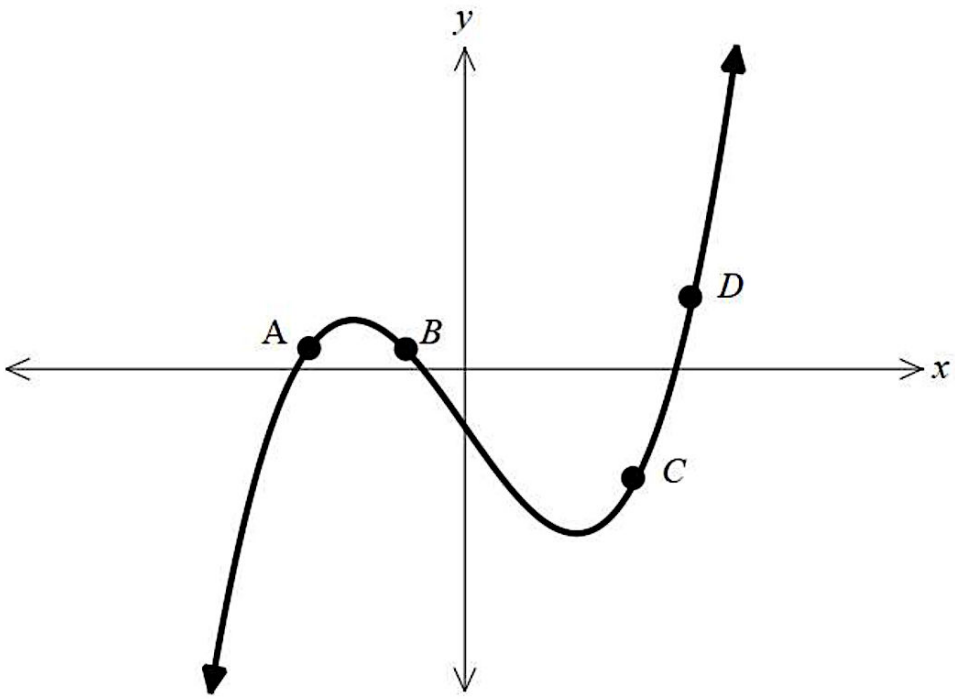
(A) $[-2, 2]$

(B) $[3, 7]$

(C) $[2, 8]$

(D) $[2, 7]$

4. Which point on the following diagram has $y > 0$, $\frac{dy}{dx} > 0$ and $\frac{d^2y}{dx^2} < 0$?



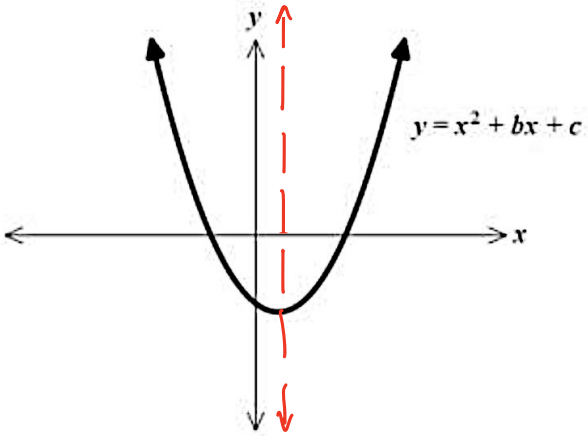
(A) A

(B) B

(C) C

(D) D

5. The diagram below shows the graph of $y = x^2 + bx + c$, where b and c are constants.



Which of the following are true?

y-int < 0, $\therefore c < 0$

axis of sym > 0

$\therefore \frac{-b}{2a} > 0$

since $a > 0$ (concave up)

$b < 0$

(A) $b > 0$ and $c > 0$

(B) $b > 0$ and $c < 0$

(C) $b < 0$ and $c > 0$

(D) $b < 0$ and $c < 0$

6. Given that the graph of $y = f(x)$ has a local maximum at $(-4, 3)$, what will the graph of $y = -2f\left(\frac{1}{2}x\right)$ have?

- reflection in x-axis
  $\therefore$ local max. becomes a local min.
  $(-4, 3) \rightarrow (-4, -3)$
- V.D. s.f. 2
  $(-4, -3) \rightarrow (-4, -6)$
- H.D. s.f. 2
  $(-4, -6) \rightarrow (-8, -6)$

(A) A local minimum at $(-8, -6)$

(B) A local minimum at $(-2, -6)$

(C) A local maximum at $(-8, 6)$

(D) A local maximum at $(-2, 6)$

7. A vessel initially contains 100 litres of water. It is being emptied at a rate of $\frac{dV}{dt} = -2 - \frac{20}{t+1}$ where V is the volume in litres after t minutes.

   Which of the following gives the exact volume of water (in litres) remaining in the vessel after five minutes?

   (A) $10 + 20 \ln 6$

   (B) $90 - 20 \ln 6$

   (C) $90 + 20 \ln 6$

   (D) $110 + 20 \ln 6$

$$\frac{dV}{dt} = -2 - \frac{20}{t+1}$$

$$V = -2t - 20\ln(t+1) + C$$

when $t = 0$, $V = 100$

$$100 = -2(0) - 20\ln(0+1) + C$$

$$\therefore C = 100$$

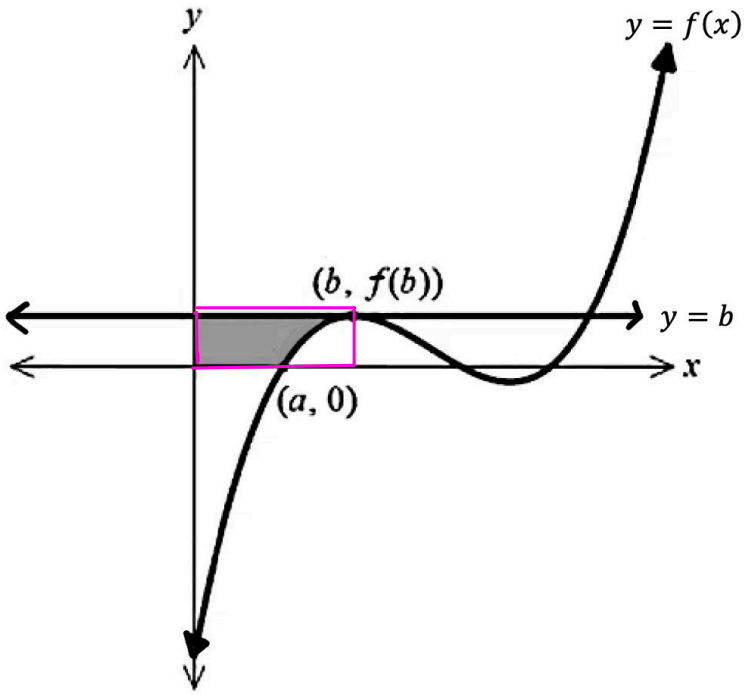
$$\therefore V = -2t - 20\ln(t+1) + 100$$

when $t = 5$,

$$V = -2(5) - 20\ln(5+1) + 100$$
$$= -10 - 20\ln 6 + 100$$
$$= 90 - 20\ln 6$$

8. Part of the graph of $y = f(x)$ is shown below.

The horizontal line, $y = b$, is a tangent to the local maximum at $\big(b, f(b)\big)$. The shaded area is bounded by the graph of $y = f(x)$, the coordinate axes, and the horizontal line.



Which is the correct expression for the area of the shaded region?

(A) $af(b) - \int_a^b f(x)\ dx$

(B) $af(b) + \int_a^b f(x)\ dx$

(C) $bf(b) - \int_a^b f(x)\ dx$

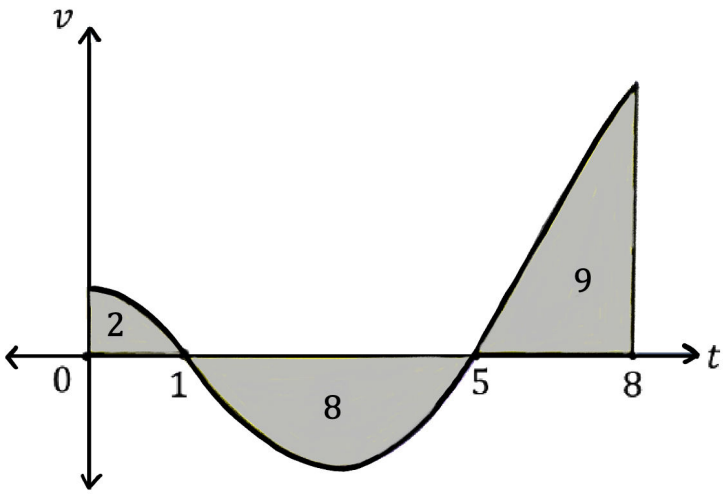
(D) $\int_0^b f(x)\ dx - bf(b)$

$A = \text{rectangle} - \int_a^b f(x)\,dx$

$= b \times f(b) - \int_a^b f(x)\,dx$

9. The graph below shows the velocity, v m/s, of a particle moving along a straight line for the first 8 seconds.

   The areas between the velocity curve and the time (t) axis during this time interval are as shown below.



   Given that the displacement of the particle at $t = 0$ is 4 m, find the displacement of the particle at $t = 8$ seconds.

(A) $x = 2$ m

(B) $x = 3$ m

(C) $x = 7$ m

(D) $x = 24$ m

$2 + 9 = 11$ m in the positive direction

8m in the negative direction

$\therefore x = 4 + 11 - 8$

$= 15 - 8$

$= 7$ m

10. The functions $h(x)$ and $g(x)$ are defined for all real x, and $h(x) = [g(x)]^4$.

Given that $h'(-2) = 96$, $g'(-2) = -3$, what is the value of $h(-2)$?

(A) -16

(B) -2

(C) 2

(D) 16

$$h'(x) = 4[g(x)]^3 \times g'(x)$$
$$h'(-2) = 4[g(-2)]^3 \times g'(-2)$$
$$96 = 4[g(-2)]^3 \times -3$$
$$= -12[g(-2)]^3$$
$$-8 = [g(-2)]^3$$
$$\therefore g(-2) = -2$$
$$\therefore h(-2) = [g(-2)]^4$$
$$= (-2)^4$$
$$= 16$$

End of Section I

12MAT HSC AT4 2026

MARKER'S COMMENTS

Q11)

$a = 5, \quad d = 4$

$T_{19} = 5 + (19-1) \times 4$

$= 5 + 18 \times 4$

$= 77$ — ① No half marks.

Q12)

$y = 1 + 2x^2 - \frac{4}{x^2}$

$= 1 + 2x^2 - 4x^{-2}$

$y' = 4x + 8x^{-3}$ — ① correct derivative

$= 4x + \frac{8}{x^3}$

when $x = 2$, $y' = 4(2) + \frac{8}{2^3}$

$= 9$

$\therefore m_T = 9$

$\therefore$ equation of tangent: $y - 8 = 9(x-2)$

$y = 9x - 18 + 8$

$y = 9x - 10$ — ① correct equation of tangent (CFPE)

- Many students did not know how to differentiate $\frac{1}{x^2}$. Please revise!
- $1\frac{1}{2}$ marks awarded if only found the **gradient** of the tangent at (2, 8).

NO HALF MARKS FOR THIS QUESTION – ONE MARK EACH

MARKER'S COMMENTS

Q13)

x	0	2	3	5
$P(X = x)$	0.3	~~2k~~ 0.24	~~3k~~ 0.36	0.1

a) $0.3 + 2k + 3k + 0.1 = 1$

$5k = 0.6$

$k = 0.12$

b) $E(X) = 0 \times 0.3 + 2 \times 0.24 + 3 \times 0.36 + 5 \times 0.1$

$= 2.06$

c) $E(X^2) = 0^2 \times 0.3 + 2^2 \times 0.24 + 3^2 \times 0.36 + 5^2 \times 0.1$

$= 6.7$

$Var(X) = E(X^2) - [E(X)]^2$

$= 6.7 - 2.06^2$

$= 2.4564$

$\sigma = \sqrt{2.4564}$

$= 1.567290...$

This part was poorly done.

Many students did not know how to find the **standard deviation** or used the **incorrect formula** for variance. Please check the Reference Sheet.

(or use the STAT mode on your calculator!)

d) $P(X \text{ is odd}) = P(X = 3) + P(X = 5)$

$= 0.36 + 0.1$

$= 0.46$

0 is neither odd nor even!

MARKER'S COMMENTS

Q14)

$$\int_0^1 \frac{x}{\sqrt{x^2+2}}\,dx = \int_0^1 x(x^2+2)^{-\frac{1}{2}}\,dx$$

$$= \frac{1}{2}\int 2x(x^2+2)^{-\frac{1}{2}}\,dx$$

$$= \frac{1}{2}\left[\frac{(x^2+2)^{\frac{1}{2}}}{\frac{1}{2}}\right]_0^1$$

$$= \left[(x^2+2)^{\frac{1}{2}}\right]_0^1$$

$$= \sqrt{3} - \sqrt{2}$$

$$= 0.3178372\ldots$$

$$\approx 0.3178$$

$\frac{1}{2}$ mark	Multiplying x by 2 and multiplying the integral by $\frac{1}{2}$.
$1\frac{1}{2}$ marks	Correctly integrating using Reverse Chain Rule.
2 marks	Correct substitution and final answer.

- This question was very poorly attempted.
- Many students did not recognise this as a **reverse chain rule** question and incorrectly integrated as a $\ln x$ function. Please revise this concept and use the Reference Sheet to check your formula!

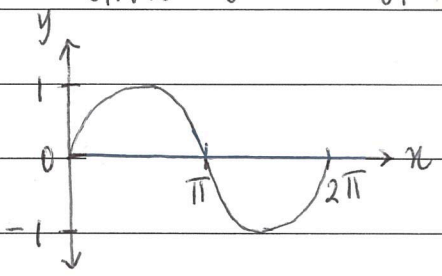
MARKER'S COMMENTS

Q15)

$\sin^2 x - \sqrt{3}\sin x \cos x = 0$ for $0 \leq x \leq 2\pi$

$\sin x(\sin x - \sqrt{3}\cos x) = 0$

$\sin x = 0$ or $\sin x - \sqrt{3}\cos x = 0$



$\sin x = \sqrt{3}\cos x$

$\tan x = \sqrt{3}$

✓ ✓

$\therefore x = 0, \pi, 2\pi$

$\therefore x = \frac{\pi}{3}, \frac{4\pi}{3}$

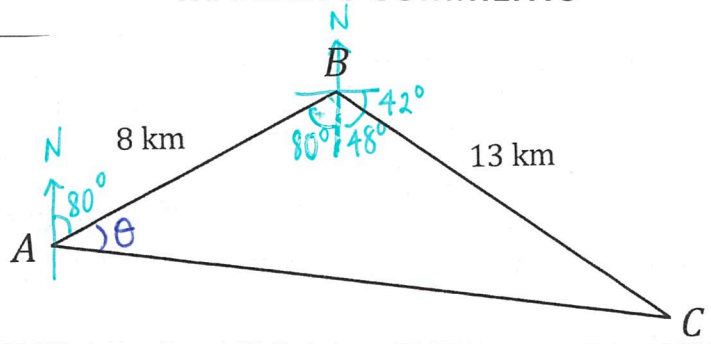
$\therefore x = 0, \frac{\pi}{3}, \pi, \frac{4\pi}{3}, 2\pi$

1 mark	Factorising $\sin x$ first.
2 marks	One set of solutions correct.
3 marks	Both sets of solutions correct.

- This question was very poorly attempted.
- Many students divided both sides of the equation by $\sin x$, and consequently, lost a set of solutions. Students who did this and solved $\tan x = \sqrt{3}$ correctly only received $1\frac{1}{2}$ marks.
- When solving $\sin x = 0$, please **sketch the graph**! Many students missed one or two solutions and just said $x = 0$.
- If too many solutions were given, $\frac{1}{2}$ mark was deducted.

Q16)

MARKER'S COMMENTS



a) $\angle ABC = 80 + 48$

$= 128°$

no half marks

b) $AC = \sqrt{8^2 + 13^2 - 2(8)(13)\cos 128°}$ — ① correct cosine rule to find AC. (CFPE from part a))

$= 19.00151538...$ km

Let $\angle BAC = \theta$.

$\frac{\sin\theta}{13} = \frac{\sin 128°}{AC}$

$\sin\theta = \frac{13\sin 128°}{AC}$

$\theta = \sin^{-1}\left(\frac{13\sin 128°}{AC}\right)$ — ① correct sine rule / cosine rule to find $\angle BAC$.

$= 32°37'26.06''$

$\approx 33°$

- Be careful when typing long expressions into your calculators! There were many calculator errors in this part of the question.
- Check that your calculator is on **degrees** mode for any bearings questions.
- Remember to check that your answers make sense in the context of the question! E.g. If $\angle ABC = 128°$, then $\angle BAC \neq 100°$, etc.
  No CFPE marks awarded if the angle didn't work in the question.

$\therefore$ Bearing $= 80 + 33$

$= 113°$ — ① correct bearing

Q17)

MARKER'S COMMENTS

$f'(x) = 3x^2 - 4x$

$f(x) = \frac{3x^3}{3} - \frac{4x^2}{2} + C$ — ① correct integration

$\therefore f(x) = x^3 - 2x^2 + C$

$f(2) = 5 = 2^3 - 2(2)^2 + C$

$\therefore C = 5$ — ① correct value of c

$\therefore f(x) = x^3 - 2x^2 + 5$

$f(3) = a = 3^3 - 2(3)^2 + 5$

$\therefore a = 14$ — ① correct value of a

- Mostly done well, although some students need to revisit this concept (given the derivative, find the original function).
- $\frac{1}{2}$ mark deducted if no constant of integration $(+C)$.
- $2\frac{1}{2}$ marks awarded if value of a is wrong but showed correct working/substitution.

MARKER'S COMMENTS

Q18.

$LHS = \sec x + \tan x$

$= \frac{1}{\cos x} + \frac{\sin x}{\cos x}$ —— 1' $\sec x = \frac{1}{\cos x}$ $\frac{1}{2}$'

$= \frac{1+\sin x}{\cos x}$ $\tan x = \frac{\sin x}{\cos x}$ $\frac{1}{2}$'

$= \frac{(1+\sin x)\cos x}{\cos^2 x}$

$= \frac{(1+\sin x)\cos x}{1-\sin^2 x}$ —— 1' Use $\sin^2 x + \cos^2 x = 1$

$= \frac{(1+\sin x)\cos x}{(1+\sin x)(1-\sin x)}$ for a clear purpose or right direction

$= \frac{\cos x}{1-\sin x}$ —— 1' Factorise and cancelling

$= RHS$

Marker's comments

This question was not done well overall. Most students recognised the first step but then struggled to apply the Pythagorean identity to progress with the solution. More practice with trigonometric proofs is required.

Q19. (a) $r = \frac{1800}{2700} = \frac{2}{3}$ —— 1'

(b) $T_8 = 2700 \times \left(\frac{2}{3}\right)^7$

$= 158.024691$

$\doteqdot \$158$ (nearest dollar) —— 1'

(c) $S_{13} = \frac{2700 \times \left(1 - \left(\frac{2}{3}\right)^{13}\right)}{1 - \frac{2}{3}}$

$\doteqdot \$8058$ (can round to other decimal places as well) —— 1'

No half mark rewarded for part (a)–(c)

Marker's comments

Part (a)–(c) were done very well.

MARKER'S COMMENTS

(d) since $-1<r<1$, there is a limiting sum.

$$S_\infty = \frac{a}{1-r}$$

$$= \frac{2700}{1-\frac{2}{3}}$$

$= \$8100$ —— 1' (Show $S_\infty = \$8100$)

Since S_{13} is only \$42 less than the limiting sum, it makes sense to start a new fundraiser after week 13.

—— 1' (compare \$8000 with S_{13} from part (c) to justify that the sum is already close to \$8100)

(or find T_{14} and justify it's a small amount)

no marks rewarded if students only stated that there is a limiting sum or the drive gets smaller and smaller, they have to compare with week 13.

Marker's comments:

This question was not done particularly well. Most students correctly used the limiting sum formula to show that the maximum amount was \$8100. However, many did not use the result from part (c) to compare the current balance and justify that there was very little room left for further accumulation, meaning a new drive should be started. Instead, many simply stated that the savings would continue to increase and eventually reach the limit of \$8100, without providing the justification required by the question.

MARKER'S COMMENTS

Q20.

(a) $t = \frac{9}{12} = 0.75$ ($t = \frac{273}{365}$ was accepted)

$V = 1500 \times e^{-0.06 \times 0.75}$

$\doteqdot 1434$ (or 1433) people —— 1' (no half mark rewarded)

Marker's comments:

Overall, this question was done okay. However, some students did not determine the correct value of t, which led to an incorrect answer.

(b) $\frac{dV}{dt} = 1500 \times (-0.06 e^{-0.06t})$

$= -90 e^{-0.06t}$ —— 1' differentiation

When $t = 5$

$\frac{dV}{dt} = -90 e^{-0.06 \times 5}$

$= -66.673639...$

$\doteqdot -67$ people/year —— 1' If students used $t=6$ $\frac{1}{2}$' was deducted.

or decreasing at a rate of 67 people/year.

Marker's comments:

This question was done well.

Q21. Method 1: $\log_3 x + \log_3 y = 8$

$\log_3 xy = 8$

$xy = 3^8$ —— 1'

$\log_x y = 3$

$x^3 = y$ —— 1'

Sub $x^3 = y$ into $xy = 3^8$:

MARKER'S COMMENTS

$x \cdot x^3 = 3^8$

$x^4 = 3^8$

$\therefore x = \sqrt[4]{3^8}$

$= 3^2$

$\therefore x = 9$ — $\frac{1}{2}$' solve for x

Then $y = x^3 = 9^3$

$= 729$ — $\frac{1}{2}$' solve for y

Method 2: $\frac{\log_3 y}{\log_3 x} = 3$

$\log_3 y = 3\log_3 x$ — 1'

sub into $\log_3 x + \log_3 y = 8$:

$\log_3 x + 3\log_3 x = 8$

$4\log_3 x = 8$

$\log_3 x = 2$ — 1'

$\therefore x = 3^2 = 9$ — $\frac{1}{2}$'

$\therefore \log_3 y = 3\log_3 9 = 3 \times 2$

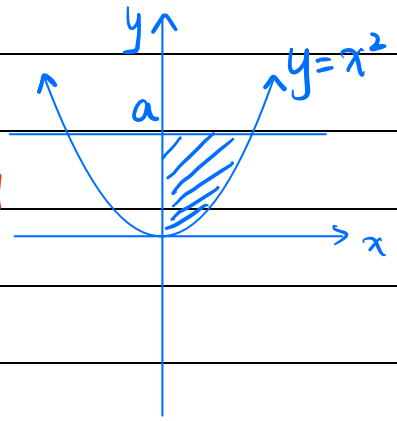
$\therefore y = 3^6 = 729$ — $\frac{1}{2}$'

Marker's comments:

This question was done well overall. However, a few students forgot to solve for y.

MARKER'S COMMENTS

Q22. Method 1: make x the subject: $x=\sqrt{y}$



$10=\int_0^a y^{\frac{1}{2}}\,dy$

$=\left[\frac{2}{3}y^{\frac{3}{2}}\right]_0^a$ —— 1' integrating correctly

$10=\frac{2}{3}a^{\frac{3}{2}}-0$

$15=a^{\frac{3}{2}}$ —— 1' substitution

$\sqrt{a^3}=15$

$a^3=225$

$\therefore a=\sqrt[3]{225}$

$\doteq 6.08$ (2 d.p.) —— 1' solve for a

No marks were rewarded if students used $\int_0^a x^2\,dy$ to find the area as all the equivalent steps become easier.

Method 2: When $y=a$

$x^2=a$

$\therefore x=\pm\sqrt{a}$

But $x>0$

$\therefore x=\sqrt{a}$

$\int_0^{\sqrt{a}}(a-x^2)\,dx=10$ —— 1' for the correct bounds

$\left[ax-\frac{1}{3}x^3\right]_0^{\sqrt{a}}=10$ —— 1' for integrating correctly

$a^{\frac{3}{2}}-\frac{1}{3}a^{\frac{3}{2}}=10$

$\frac{2}{3}a^{\frac{3}{2}}=10$ —— $\frac{1}{2}$' substitution

$a^{\frac{3}{2}}=15$

$\therefore a\doteq 6.08$ (2 d.p.) —— $\frac{1}{2}$' solve for a

MARKER'S COMMENTS

Marker's comment:

This question was completed reasonably well. Most students recognised the need to use the y-bounds or, when using the x-bounds, to find the area by subtracting the region from a rectangle. However, some students did not recognise these approches. When using the x-bounds, some students forgot to determine the corresponding x-bound values first. And some students solved the exponential equation at the end incorrectly.

MARKER'S COMMENTS

23) In $\triangle ODC$, $(8\sqrt{3})^2 = (2a)^2 + (2a)^2 - 2(2a)(2a)\cos\left(\frac{2\pi}{3}\right)$

$192 = 4a^2 + 4a^2 - 8a^2 \times -\frac{1}{2}$

$= 8a^2 + 4a^2$

$= 12a^2$

$a^2 = 16$

$\therefore a = 4$ since $a > 0$

Shaded region = A of sector - A of triangle

$= \frac{1}{2} r^2\theta - \frac{1}{2} ab \sin C$

$= \frac{1}{2}(3a)^2\left(\frac{2\pi}{3}\right) - \frac{1}{2}(2a)(2a)\sin\left(\frac{2\pi}{3}\right)$

$= 3\pi a^2 - 2a^2 \times \frac{\sqrt{3}}{2}$

$= 3\pi a^2 - \sqrt{3}a^2$

as $a = 4$ $= 3 \times 4^2\pi - \sqrt{3} \times 4^2$

$= 48\pi - 16\sqrt{3}$ cm^2

1 mark for correctly applying the sine/cosine rule to find a or correctly applying the area of a sector or correctly applying the area of a triangle using the sine area formula.

1 mark for finding the correct value of a

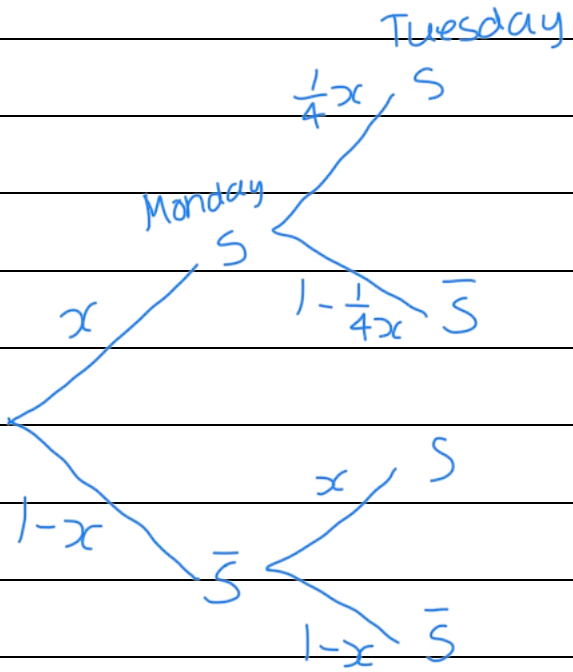
1 mark for finding the exact value of the shaded area.

MARKER'S COMMENTS

* Many students wrote the area of a sector as $\pi r^2\theta$. Students needed to use either $\frac{1}{2} \times r^2 \times \theta$ or $\frac{\theta}{360} \times r^2 \times \pi$.

* Many students did not find the value of a.

24)a)



$1\frac{1}{2}$ marks if there was one error, or a pair was incorrect but they added to 1.

1 mark for more than 1 error.

* Many students wrote $\frac{3}{4}x$ instead of $1-\frac{1}{4}x$.

MARKER'S COMMENTS

24) b) P(Snow on Tuesday) = P(S,S) + P($\bar{S}$,S) (1) (1)

$= x \times \frac{1}{4}x + (1-x)x$

$= \frac{1}{4}x^2 + x - x^2$

$= x - \frac{3}{4}x^2$

1 mark for finding P(S,S) within the expression.

1 mark for finding P($\bar{S}$,S) within the expression.

No marks were deducted for errors in simplifying.

24) c) $P = x - \frac{3}{4}x^2$ concave down parabola

$\therefore x = -\frac{b}{2a}$

$= -\frac{1}{2\times(-3/4)} = \frac{2}{3}$ (1)

$P = \frac{2}{3} - \frac{3}{4} \times \left(\frac{2}{3}\right)^2 = \frac{1}{3}$ (1)

MARKER'S COMMENTS

1 mark for finding the value of x when the probability is maximised using a valid method such as finding the vertex, differentiation, graphing or finding the x-intercepts and the midpoint.

1 mark by substituting $x = \frac{2}{3}$ into the probability expression found in part b.

* Some students found the x-intercepts and substituted this into the expression.

25) d = centre = 6 $(\frac{1}{2})$

period = 12

$\therefore \frac{2\pi}{b} = 12$ $\frac{1}{2}$

$b = \frac{2\pi}{12} = \frac{\pi}{6}$

a = amplitude = 2 $\frac{1}{2}$

To find c

$y = 2\sin[\frac{\pi}{6}(t+c)]+6$

sub $(0, 7.59)$

$7.59 = 2\sin[\frac{\pi}{6}(0+c)]+6$

$0.759 = \sin(\frac{\pi}{6}c)$

$\frac{\pi}{6}c = \sin^{-1}(0.795)$

(c=2 was also accepted) $\therefore c = \frac{\sin^{-1}(0.795)}{\pi/6}$

$c \approx 1.76$ $\frac{1}{2}$

MARKER'S COMMENTS
$\frac{1}{2}$ for finding a
$\frac{1}{2}$ for finding b
$\frac{1}{2}$ for finding d
$\frac{1}{2}$ for finding c

Question 26 **MARKER'S COMMENTS**

a)

Since $g(x)$ has the same shape as $f(x)$, and the vertex is at the same x-value, the transformations required are a reflection and a vertical translation. These 2 transformation can be done in either order, but the order does affect the direction of the vertical translation. The y-intercept of $g(x)$ needs to be at $+5$, as per the diagram.

Method 1 : Expand $f(x)$

$f(x) = (1+x)(5-x) = 5 - x + 5x - x^2$

$= -x^2 + 4x + 5$

Option A:

1. Reflect in x-axis: $-f(x) = -(-x^2+4x+5)$

   $= x^2 - 4x - 5$

   then

2. Vertical translation up 10: $-f(x)+10 = x^2 - 4x - 5 + 10$

   $= x^2 - 4x + 5$

Option B:

1. Vertical translation down 10: $f(x) - 10 = -x^2 + 4x + 5 - 10$

   $= -x^2 + 4x - 5$

   then

2. Reflect in x-axis $-[f(x)-10] = -(-x^2+4x-5)$

   $= x^2 - 4x + 5$

$\therefore g(x) = x^2 - 4x + 5$

Made with Goodnotes

Question 26 **MARKER'S COMMENTS**

a)

Method 2: Leave $f(x)$ as is

$f(x) = (1+x)(5-x)$

Option A:

1. Reflect in x-axis: $-f(x) = -(1+x)(5-x)$
   then
2. Vertical translation up 10: $-f(x)+10 = -(1+x)(5-x)+10$

Option B:

1. Vertical translation down 10: $f(x)-10 = (1+x)(5-x)-10$
   then
2. Reflect in x-axis $-[f(x)-10] = -[(1+x)(5-x)-10]$
   $= -(1+x)(5-x)+10$

$\therefore g(x) = -(1+x)(5-x)+10$

Made with Goodnotes

MARKER'S COMMENTS

Method 3: Complete the square

$$f(x) = (1+x)(5-x)$$
$$= 5 - x + 5x - x^2$$
$$= -x^2 + 4x + 5$$
$$= -(x^2 - 4x - 5)$$
$$= -(x^2 - 4x + 4 - 4 - 5)$$
$$= -[(x-2)^2 - 9]$$
$$= -(x-2)^2 + 9$$

Option A:

1. Reflect in x-axis: $-f(x) = -[-(x-2)^2 + 9]$
$= (x-2)^2 - 9$

2. Vertical translation up 10: $-f(x) + 10 = (x-2)^2 - 9 + 10$
$= (x-2)^2 + 1$

Option B:

1. Vertical translation down 10: $f(x) - 10 = -(x-2)^2 + 9 - 10$
$= -(x-2)^2 - 1$

2. Reflect in x-axis $-[f(x) - 10] = -[-(x-2)^2 - 1]$
$= (x-2)^2 + 1$

$\therefore g(x) = (x-2)^2 + 1$

Made with Goodnotes

MARKER'S COMMENTS

1 mark – for recognising that a reflection over the x-axis and a vertical translation need to occur.

1 mark – for listing the sequence of transformations fully together and in correct order (lose mark if the transformations are worked out but not written together clearly and fully - IN A LIST!

1 mark – for correct equation for $g(x)$ (not awarded if transformation above was less complex)

* Many students found this difficult. They knew that it was a reflection and a translation, but they struggled to visualise that a translation of 2 x the y-intercept was required.
* Many students failed to list the sequence of transformations fully and clearly together.

Made with Goodnotes

MARKER'S COMMENTS

26b) Find points of intersection of $f(x)$ and $g(x)$.

$$f(x) = g(x)$$

$$(1+x)(5-x) = -(1+x)(5-x)+10$$

$$5 - x + 5x - x^2 = -(5 - x + 5x - x^2) + 10$$

$$5 + 4x - x^2 = -5 - 4x + x^2 + 10$$

$$-x^2 + 4x + 5 = x^2 - 4x + 5$$

$$-2x^2 + 8x = 0$$

$$-2x(x - 4) = 0$$

$$\therefore x = 0, +4 \quad \text{—— 1}$$

$$A = \int_0^4 f(x) - g(x)\, dx$$

$$= \int_0^4 (-x^2+4x+5) - (x^2-4x+5)\, dx \quad \text{— 1}$$

CFPE allowed

$$= \int_0^4 -2x^2 + 8x \, dx$$

$$= \left[-\frac{2x^3}{3} + 4x^2 \right]_0^4$$

$$= \left[\frac{-2\times4^3}{3} + 4\times4^2 \right] - \left[\frac{-2\times0^3}{3} + 4\times0^2 \right]$$

$$= \frac{64}{3} \text{ units}^2 = 21\tfrac{1}{3} \text{ units}^2 \quad \text{— 1}$$

CFPE allowed.

Made with Goodnotes

MARKER'S COMMENTS

* Some students took the integral of $g(x) - f(x)$ instead of $f(x) - g(x)$.
* Some students made algebra errors and should take greater care in future with negatives.

Made with Goodnotes

Advanced 2026 Trial Examination

MARKER'S COMMENTS Q27 – Q29

27a)

a) $\frac{d}{dx}(2x\cos x)$

let $u = 2x$ $\quad v = \cos x$

$\frac{du}{dx} = 2$ $\quad \frac{dv}{dx} = -\sin x$

$\therefore \frac{d}{dx}(2x\cos x) = u\frac{dv}{dx} + v\frac{du}{dx}$

$= 2x \times -\sin x + \cos x \times 2$

$= -2x\sin x + 2\cos x$ or

$= 2\cos x - 2x\sin x$ or

1mk

Any subsequent error was ignored.

This question was generally done well. A few students need to re-visit the product rule formula.

MARKER'S COMMENTS

27 b) Hence, find $\int x \sin x \, dx$.

Method 1

From part (a) $\frac{d}{dx}(2x\cos x) = 2\cos x - 2x\sin x$

Now, integrate all the expressions:

$\int \frac{d}{dx} 2x\cos x \, dx = \int 2\cos x \, dx - \int 2x\sin x \, dx$

Since integration and differentiation are inverse operations of each other then we get:

$2x\cos x + C = 2\int \cos x \, dx - 2\int x\sin x \, dx$

Now, we need to find $\int x\sin x \, dx$

∴ Now, divide the expressions on the LHS and the RHS by 2 which is a common factor.

$x\cos x + C = \int \cos x \, dx - \int x\sin x \, dx$

By rearranging the expressions, we obtain:

$\int x\sin x \, dx = \int \cos x \, dx - x\cos x + C$

$\therefore \int x\sin x \, dx = \sin x - x\cos x + C$

Note that C is just a constant so we don't have to worry about it too much in terms of algebraic manipulations. You can simply add it at the end once all the working has been organised.

MARKER'S COMMENTS

Method 2

$\frac{d}{dx} 2x\cos x = 2\cos x - 2x\sin x$

Rearranging the expressions we obtain:

$2x\sin x = 2\cos x - \frac{d}{dx}(2x\cos x)$

Dividing all the expressions by 2 we obtain:

$x\sin x = \cos x - \frac{d}{dx}(x\cos x)$

Now, integrating both sides gives us:

$\int x\sin x\,dx = \int \cos x\,dx - \int \frac{d}{dx} x\cos x\,dx$

$\therefore \int x\sin x\,dx = \sin x - x\cos x + C$

Method 3

$\int x\sin x$

$= \int (x\sin x + \cos x - \cos x)\,dx$ ← adding and subtracting by the same term

equivalent to each other. (Multiplying by 1)

$= -\frac{1}{2}\int [-2x\sin x - 2\cos x + 2\cos x]\,dx$

$= -\frac{1}{2}\int [-2x\sin x + 2\cos x - 2\cos x]\,dx$

$= \frac{d}{dx}(2x\cos x) = 2\cos x - 2x\sin x$ (from part (a))

$= -\frac{1}{2}\int \frac{d}{dx}(2x\cos x)\,dx + \int \cos x\,dx$

$= -x\cos x + \sin x + C$

MARKER'S COMMENTS

1mk → for obtaining the following:
$\int 2\cos x - 2x\sin x\,dx = 2x\cos x$ (with or without c)
½ mk – for appropriate algebraic manipulation, particularly the ± signs, dividing by 2 etc.
½ mk → correct integration of cos x and placing the constant of integration in an appropriate context.
Poorly attempted by students. Students need to re-visit this concept. Some students forgot to place the constant of integration.

Question 28

a) $h(t) = 12 + \frac{5}{2}\cos\left[\frac{\pi}{183}(t-22)\right]$ for

$1 \leq t \leq 366$

For maximum number of hours:

$$\cos\left[\frac{\pi}{183}(t-22)\right] = 1$$

$$\therefore h(t) = 12 + \frac{5}{2} \times 1$$

$$= 14\tfrac{1}{2} \text{ hours.} - 1\text{mk (no } \tfrac{1}{2} \text{ marks were awarded)}$$

This question was not attempted as expected. You don't have to always use Calculus methods to find maximum or minimum.

MARKER'S COMMENTS

b) maximum daylight occurs when:

$\cos\left[\frac{\pi}{183}(t-22)\right] = 1$ (from part c)

$\therefore \frac{\pi}{183}(t-22) = \cos^{-1}(1) = 0$

$\frac{\pi}{183}(t-22) = 0$

$t - 22 = 0$

$\therefore t = 22$ — ½ mk

$\therefore$ 22nd January. — ½ mk.

Generally attempted well.

c) $h(t) = 10.75$

Need to solve $h(t) \geq 10.75$

$12 + \frac{5}{2}\cos\left[\frac{\pi}{183}(t-22)\right] = 10.75$ — 1 mk

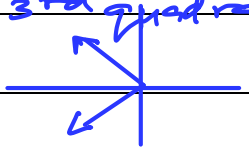
$\cos\left[\frac{\pi}{183}(t-22)\right] = -\frac{1}{2}$

Ignoring the negative sign, we can obtain the related angle: $\frac{\pi}{183}(t-22) = \cos^{-1}\left(\frac{1}{2}\right)$

$\therefore$ related angle $= \pi/3$

But $\cos\alpha$ is negative in the 2nd and 3rd quadrants.

$\therefore \frac{\pi}{183}(t-22) = \frac{2\pi}{3}, \frac{4\pi}{3}$

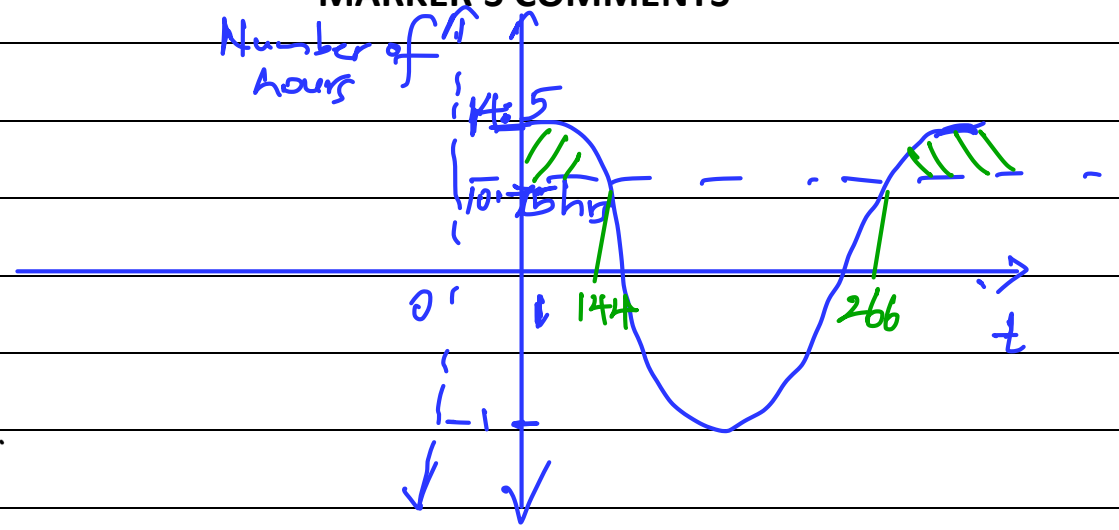


$\therefore t = 144, 266$

Any one of the solutions with appropriate working — 1 mk

The second solution was awarded a ½ mk.

MARKER'S COMMENTS



$\therefore t \in [1, 144] \cup [266, 366]$

$144 - 1 + 1 = 144$

$366 - 266 + 1 = 101$

$\therefore$ 144 days + 101 days = 245 days.

$\frac{1}{2}$ mk

Poorly attempted. Students should have realised that the number of days between 266 – 366 is 101 days and not 100 days.

MARKER'S COMMENTS

Question 29

a)

x	0	1	2	3
$f(x)$	3	4	3	12

} 1 mk

From HSC Reference Sheet:

$$A \simeq \frac{b-a}{2n}\left\{ f(a) + f(b) + 2\left[f(x_1) + \ldots + f(x_{n-1}) \right] \right\}$$

$$\simeq \frac{3-0}{2(3)}\left\{ 3 + 12 + 2(4+3) \right\}$$ 1 mk

$$= \frac{3}{6}\left\{ 3 + 12 + 2(4+3) \right\}$$

$$= \frac{1}{2}\left\{ 15 + 14 \right\}$$

$$= \frac{29}{2} \text{ units}^2 \text{ or } 14\tfrac{1}{2} \text{ units}^2$$

2 marks - fully correct working out.
Minus 1 mark for any error, for example forgetting to include 3 in the 3 application of Trapezoidal rule.

Generally, well attempted. Students are encouraged to write units^2 for area related Trapezoidal Rule questions. Trapezoidal Rule is an approximate method for calculating the area hence the symbol $\simeq$ is used. A few students did not understand what 3 applications of Trapezoidal rule meant. (did Trapezoidal rule 3 times).

MARKER'S COMMENTS

b) $\int_{-1}^{2}(x^4 - 2x^2 + 4)\,dx \rightarrow \int_{0}^{3}[(x-1)^4 - 2(x-1)^2 + 4]\,dx$

↑ Horizontal translation by 1 unit to the right

$\int_{0}^{3}[(x-1)^4 - 2(x-1)^2 + 4]\,dx \rightarrow \int_{1}^{2}(x^4 - 2x^2 + 4)\,dx$

↑ Horizontal translation by 1 unit to the left.

→ ½ mk

This represents a horizontal shift to the left by 1 unit so the bounds are also shifted. —½ mk

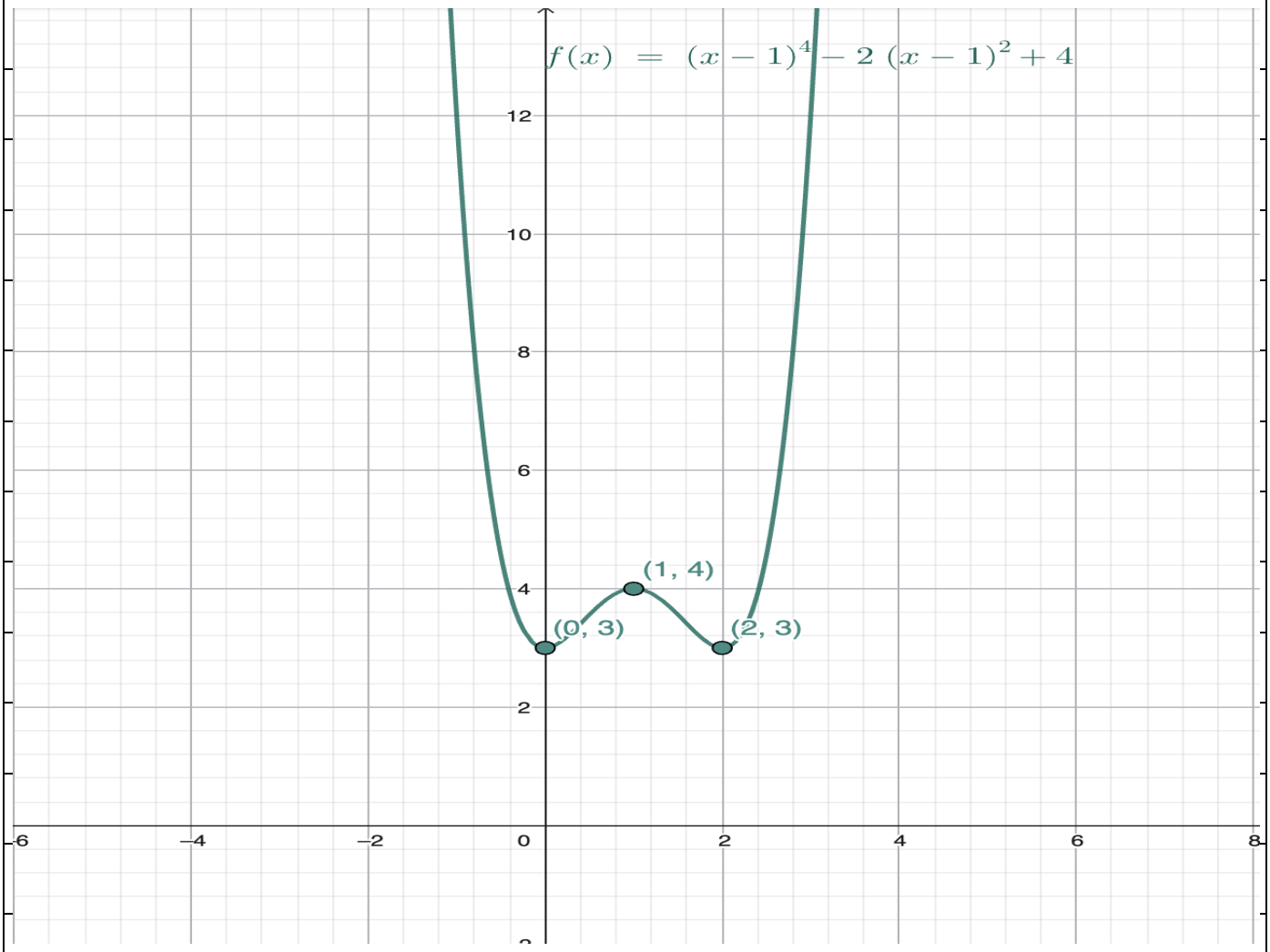
This implies that the area indicated by the integral remains unchanged.

Poorly attempted with the majority of students discussing about even nature of the two curves.

This question does not involve the properties of even functions as $y = (x-1)^4 - 2(x-1)^2 + 4$ does not represent an even function.

27b continued

MARKER'S COMMENTS



Even functions can be reflected about the y-axis.

$f(x) = (x-1)^4 - 2(x-1)^2 + 4$

$f(-x) = (-x-1)^4 - 2(-x-1)^2 + 4$

$= [-(x+1)]^4 - 2[-(x+1)]^2 + 4$

$= (x+1)^4 - 2(x+1)^2 + 4$

$\neq f(x)$

Since $f(-x) \neq f(x)$ then $f(x)$ is NOT an even function.

MARKER'S COMMENTS

Method 1

29c) $A = \int_{-1}^{2} (x^4 - 2x^2 + 4)\, dx$

$= \left[\frac{x^5}{5} - \frac{2x^3}{3} + 4x\right]_{-1}^{2}$ } 1mk

$= \left[\frac{2^5}{5} - \frac{2(2)^3}{3} + 4(2)\right] - \left[\frac{(-1)^5}{5} - \frac{2(-1)^3}{3} + 4(-1)\right]$

$= \frac{63}{5}$ units^2 or 12.6 units^2 — 1mk

Method 2

or $\int_{0}^{3} [(x-1)^4 - 2(x-1)^2 + 4]\, dx$

$= \left[\frac{(x-1)^5}{5} - \frac{2(x-1)^3}{3} + 4x\right]_{0}^{3}$ } 1mk

$= \left[\frac{(3-1)^5}{5} - \frac{2(3-1)^3}{3} + 4(3)\right]$

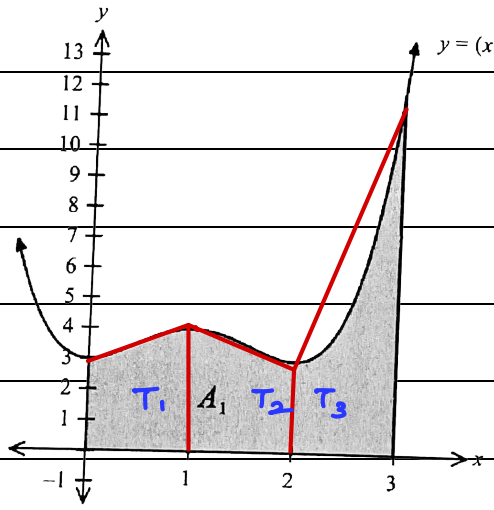
$- \left[\frac{(0-1)^5}{5} - \frac{2(0-1)^3}{3} + 4(0)\right]$

$= \frac{63}{5}$ or 12.6 units^2 — 1mk

Generally, well done. Students should have chosen Method 1 rather than 2.

MARKER'S COMMENTS

d)



T_1 and T_2 are very close approximations in the interval 0 – 2. – ½ mk

T_3 has an area much greater than the area under the curve obtained through integration as the curve is concave up. Thus, Trapezoidal rule gives an approximate area that is greater than the exact area as illustrated in the diagram above. – ½ mk

It is an 'explain' question so students should have explained what happens to the area in the region in the interval 0 to 2. Also, a number of students stated that the graph is concave up but it is not concave up throughout the entire domain.

MARKER'S COMMENTS

Question 30 (7 marks)

a) for stationary points, $\frac{dy}{dx} = 0$

$\therefore x^2e^x(x+3) = 0$

$x = 0$ or $x = -3$

when $x = -3$, $\frac{d^2y}{dx^2} = -3e^{-3}\left((-3)^2 + 6(-3) + 6\right)$

$= 9e^{-3} > 0$ ∪ (1)

$\therefore (-3, -27e^{-3})$ is a minimum turning point.

when $x = 0$ $\frac{d^2y}{dx^2} = 0$, $\therefore$ possible horizontal point of inflexion

x	-1	0	1
$\frac{d^2y}{dx^2}$	$-\frac{1}{e}$	0	e
	∩		∪

since change of concavity $(0,0)$ is a horizontal point of inflexion (1)

Accepted x values only for 1 mark.

If test for change of concavity missing deduct ½ mark.

* many student lost ½ mark for concavity check.

b) As $x \to \infty$ $x^3 \to \infty$ and $e^x \to \infty$ $\therefore y \to \infty$ (1)

As $x \to -\infty$ $x^3 \to -\infty$ and $e^x \to 0$ $\therefore y \to 0^-$ (1)

$\therefore y = 0$ is a horizontal asymptote for $x \to -\infty$.

Accepted $y \to 0^-$ and $y \to 0$

Many students struggled to separate $\pm\infty$

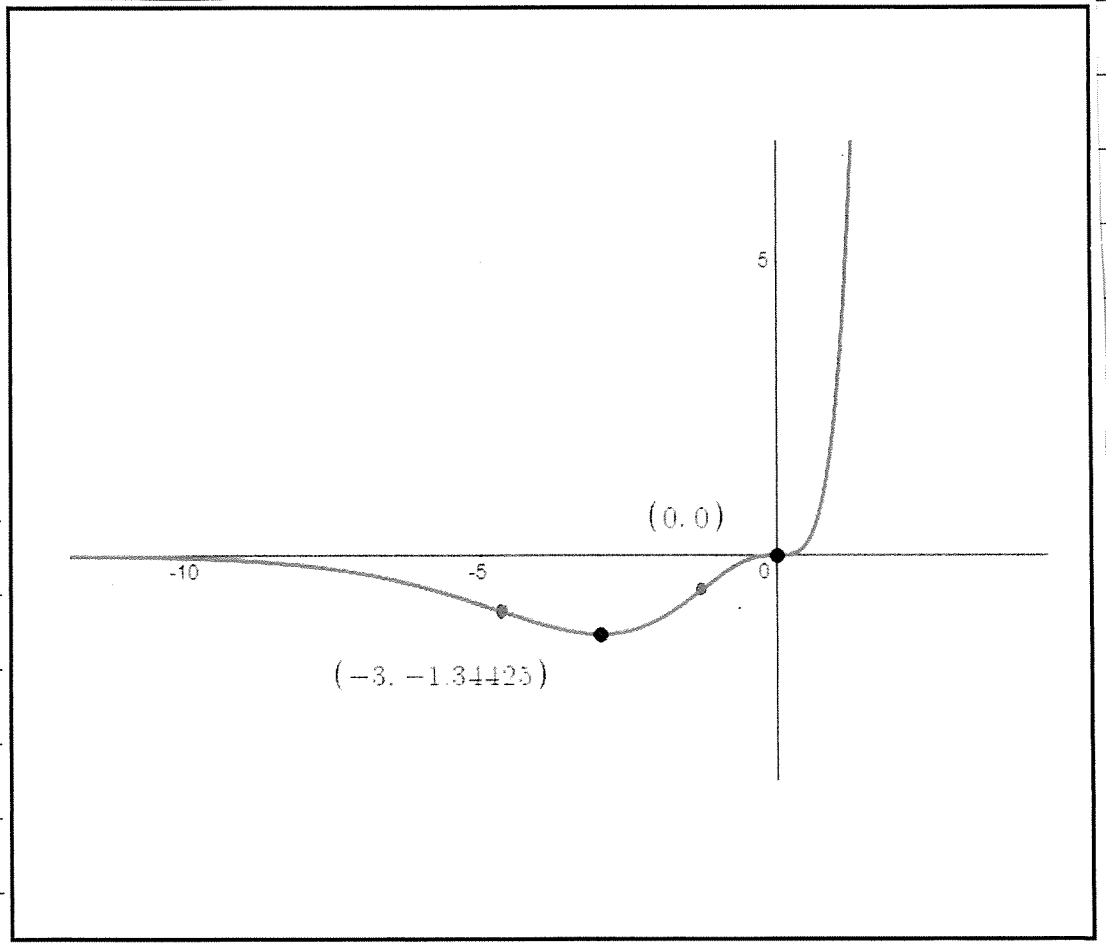
MARKER'S COMMENTS

30 (c) $-27e^{-3}$ or $\frac{-27}{e^3}$ or -1.34 (1)

$(-3, -27e^{-3})$ deduct ½ mark

Accept: Global minimum is when $x=-3$, $y=\frac{-27}{e^3}$

(d)



- ½ mark deducted for each incorrect point/direction
- if only $x=-3$ as stationary point in (a) maximum 1 mark unless (0, 0) plotted
- ½ mark given if something other than given P.O.I
- Students found the question challenging
  Difficulty plotting 2 given P.O.I

MARKER'S COMMENTS

Question 31 (5 marks)

a) $v = 8 + \dfrac{6}{2t+1}$

$= 8 + 6(2t+1)^{-1}$

$a = \dfrac{dv}{dx} = -6(2t+1)^{-2} \times 2$

$= -12$

$(2t+1)^2$ (1)

when $t = 2$, acceleration $= \dfrac{-12}{(2\times 2+1)^2}$

$= -\dfrac{12}{25}\ m/s^2$ (1)

wrong acceleration and correct substitution 1 mark.

substitution error eg. $-\dfrac{12}{5}\ m/s^2$ deduct ½ mark.

b) when $v = 0$ particle at rest.

$8 + \dfrac{6}{2t+1} = 0$

$\dfrac{6}{2t+1} = -8$

$6 = -16t - 8$

$-16t = 14$

$t = -7/8$

not possible at $t \geq 0$

∴ velocity is never at rest (1)

Deduct ½ mark if velocity is always ≥ but not mention of $t \geq 0$

MARKER'S COMMENTS

Q31 c) Distance $= \int_0^4 8 + \frac{6}{(2t+1)}\, dt$

$= \left[8t + 3\ln|(2t+1)|\right]_0^4$ (1)

$= 8\times4 + 3\ln(2\times4+1) - 8\times0 + 3\ln 1$

$= 32 + 3\ln 9 - 3\ln 1$

$= 32 + 3\ln 9$

or $= 32 + 3\ln 3^2$ (1)

or $= 32 + 6\ln 3$ m

If not a logarithmic function but evidence of 32 → ½ mark awarded.

Not exact deduct ½ mark.

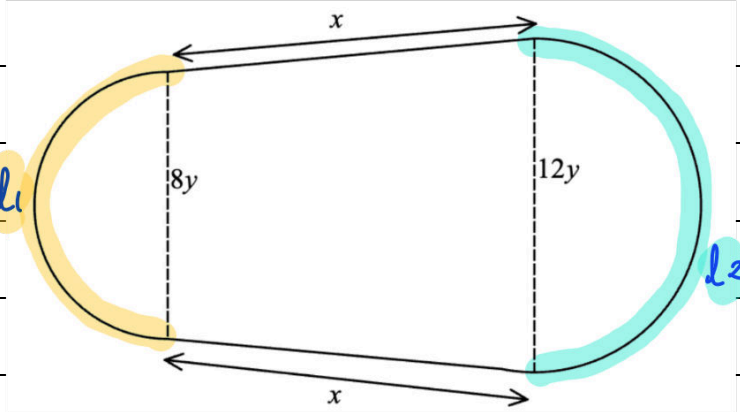
Minor numerical error: deduct ½ mark eg $3\ln 1 = 3$

Integration error (minor) - 1 mark eg $\frac{1}{3}\ln(2t+1)$

Question 31 was well answered by majority of students.

MARKER'S COMMENTS

32 a)



$c = 2\pi r$ or πd

$P = l_1 + x + l_2 + x$

$= \frac{\pi \times 8y}{2} + 2x + \frac{\pi \times 12y}{2}$

$= 4\pi y + 2x + 6\pi y$

$\therefore P = 10\pi y + 2x$

① working out that clearly leads to the desired result –

done well. some students were still risking the loss of marks by skipping steps in their working out. remember, the answer is provided so the skill being assessed is the ability to clearly communicate how the result was obtained.

MARKER'S COMMENTS

b) $6 = 10\pi y + 2x$

$10\pi y = 6 - 2x$

$y = \frac{6-2x}{10\pi}$

$= \frac{2(3-x)}{5 \cdot 10\pi}$

$\therefore y = \frac{3-x}{5\pi}$ (1)

(1) working out that clearly lead to the desired result –

Done well.

c) $S = 10\pi y(x^2+6x+5)$ (2)

sub (1) into (2)

$S = \frac{10\pi}{5\pi}(3-x)(x^2+6x+5)$ – (1) correctly substituting y from part (b)

(*) $= (6-2x)(x^2+6x+5)$

$= 6x^2 + 36x + 30 - 2x^3 - 12x^2 - 10x$

$= -2x^3 - 6x^2 + 26x + 30$

$\frac{dS}{dx} = -6x^2 - 12x + 26$ – (1) correct derivative

(*) Also possible to use product rule to find $\frac{dS}{dx}$ but expanding may be easier.

MARKER'S COMMENTS

stationary pt when $\frac{dI}{dx} = 0$

i.e. $-6x^2 - 12x + 26 = 0$

$6x^2 + 12x - 26 = 0$

$3x^2 + 6x - 13 = 0$

$$x = \frac{-6 \pm \sqrt{6^2 - 4(3)(-13)}}{2(3)}$$

$$= \frac{-6 \pm \sqrt{192}}{6}$$

$$= \frac{-6 \pm 8\sqrt{3}}{6}$$

$$= \frac{-3 \pm 4\sqrt{3}}{3}$$

$\simeq 1.309\ldots$ or $-3.309\ldots$

But $x > 0$

$\therefore x = \frac{-3+4\sqrt{3}}{3}$ (1)

≈ 1.31 km (2dp)

$\frac{d^2I}{dx^2} = -12x - 12$

At $x = \frac{-3+4\sqrt{3}}{3}$, $\frac{d^2I}{dx^2} = -12\left(\frac{-3+4\sqrt{3}}{3}\right) - 12$

$= -27.71\ldots$ (< 0) ∩

$\therefore$ speed maximum when $x = \frac{-3+4\sqrt{3}}{3}$

MARKER'S COMMENTS

$-\frac{1}{2}$ mark for not eliminating $x = \frac{-3-4\sqrt{3}}{3}$ (or equivalent)

$-\frac{1}{2}$ mark for very minor arithmetic error that does not make question easier.

- this question was reasonably well-attempted
- common error included:
  - careless arithmetic error (very common) — please take care when simplifying complicated expressions with many terms / negative signs. Include an additional line of work, if needed, to avoid mistakes.
  - Forgetting to eliminate the negative solution. It is important to check the suitability of your solution(s) in the context of the problem (and justify your choice, eg. $x > 0$)
  - Attempting to differentiate S when the expression still in terms of x and y. The word 'Hence...' in the question gives you a clue that you will need to use an earlier result from part (a) and/or (b)
- It is good practice to check that your stationary point is indeed a maximum (using $\frac{d^2S}{dx^2}$ or a table of signs) but students were not penalised if they did not do that here —

MARKER'S COMMENTS

33a) $p_n = b \times 10^{n-1}$, $b > 0$

$B_n = \log_{10}(p_n)$

Method 1

$B_1 = \log_{10}(p_1)$

$= \log_{10}(b \times 10^0)$

$= \log_{10} b$ —— (1) Identify B_1 (or another term)

$B_2 = \log_{10}(p_2)$ ((1/2) only for: $B_n = \log_{10}(b \times 10^{n-1})$ or $p_1 = b \times 10^{1-1} = b$ (or another term))

$= \log_{10}(b \times 10^1)$

$= \log_{10} b + \log_{10} 10$

$= \log_{10} b + 1$

$B_3 = \log_{10}(p_3)$

$= \log_{10}(b \times 10^2)$

$= \log_{10} b + \log_{10} 10^2$

$= \log_{10} b + 2\log_{10} 10$

$= \log_{10} b + 2$

. . .

AP: $a = \log_{10} b$

$d = 1$

} (1) some work to indicate recognition of AP.

MARKER'S COMMENTS

⊛ $S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{10} = \frac{10}{2}[2(\log_{10} b + 9(1)]$

$= 5(2\log_{10} b + 9)$

But $S_{10} = 25$

$\therefore 5(2\log_{10} b + 9) = 25$

$2\log_{10} b + 9 = 5$

$2\log_{10} b = -4$

$\log_{10} b = -2$

$b = 10^{-2}$

$= 0.01$ — (1) working out that clearly leads to desired result.

⊛ can also find B_{10} and use $S_n = \frac{n}{2}(a+l)$.

Method 2 + nice method!

$B_n = \log_{10}(b \times 10^{n-1})$

$= \log_{10} b + \log_{10} 10^{n-1}$

$= \underbrace{\log_{10} b}_{a} + (n-1)(1)$ ← d

AP: $a = \log_{10} b$, $d = 1$ — (1) some work to indicate recognition of AP.

MARKER'S COMMENTS

$B_1 = \log_{10} b$ — (1)

$S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{10} = \frac{10}{2}[2(\log_{10} b + 9(1)]$

$= 5(2\log_{10} b + 9)$

... (remainder as with Method 1)

Method 3 very nice method!

$B_1 + B_2 + B_3 + \ldots + B_{10} = 25$

$\log_{10}(b \times 10^0) + \log_{10}(b \times 10^1) + \log_{10}(b \times 10^2) + \ldots \log_{10}(b \times 10^9) = 25$

$\log_{10} b + \log_{10} 10b + \log_{10} 10^2 b + \ldots + \log_{10} 10^9 b = 25$

(1)

$\log_{10}(b \times 10b \times 10^2 b \times \ldots \times 10^9 b) = 25$

$\log_{10}(b^{10} \times 10^{45}) = 25$

$10^{45} b^{10} = 10^{25}$

($\frac{1}{2}$ each)

$b^{10} = 10^{-20}$

$b = \sqrt[10]{10^{-20}} \quad (b > 0)$

$= 0.01$ (1)

MARKER'S COMMENTS

- this was a challenging question and many students had difficulty. students with the strongest responses were able to work confidently and flexibly with logarithms.
- common issues:
  - Not recognising that p_n needed to be substituted into B_n
  - Assuming that $B_1 + B_2 + \dots + B_n$ is an AP/GP without showing that this is the case (1 mark allocated to recognition of AP)
  - Incorrect application of log laws and indices — please review!
    - $\log_{10}(b \times 10^2) = 2\log_{10} 10b$
    - $\frac{\log_{10} 10b}{\log_{10} b} = 10$ or $9b$ (if treating as GP)

b) $S_n = \frac{n}{2}[2a + (n-1)d]$

$= \frac{n}{2}[2(\log_{10} b + (n-1)(1)]$

$= \frac{n}{2}(2(\log_{10} 0.01 + n - 1)$ — (1) correct substitution of $b = 0.01$ into relevant formula

$= \frac{n}{2}(-4 + n - 1)$

$= \frac{n}{2}(n-5)$

($\frac{1}{2}$) if simply sub. $B_n = \log_{10}(0.01 \times 10^{n-1})$

(0) if only $p_n = 0.01 \times 10^{n-1}$

↳ expression in terms of B_n)

For $S_n > 50$

$\frac{n}{2}(n-5) > 50$

$n(n-5) > 100$

$n^2 - 5n - 100 > 0$ — (1)

MARKER'S COMMENTS

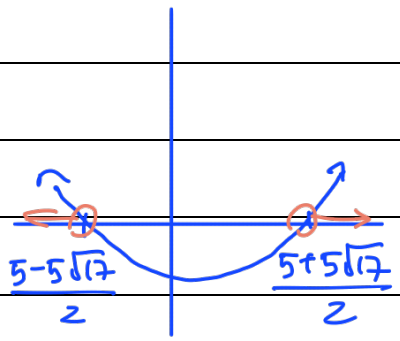
For $n^2 - 5n - 100 = 0$

$$n = \frac{5 \pm \sqrt{(-5)^2 - 4(1)(-100)}}{2(1)}$$

$$= \frac{5 \pm \sqrt{425}}{2}$$

$$= \frac{5 \pm 5\sqrt{17}}{2}$$

$$\approx 12.807\ldots, -7.807\ldots$$



$\therefore n < \frac{5-5\sqrt{17}}{2}$ or $n > \frac{5+5\sqrt{17}}{2}$

Actually not necessary to solve inequality in this problem since we can eliminate the negative solution—

BUT $n > 0$, n is an integer

$\therefore n = 13$ —— (1)

$-\frac{1}{2}$ marks for not eliminating $n = \frac{5-5\sqrt{17}}{2}$ or leaving n as an exact value/decimal.

CFPE from part (a) if treating as GP:

- (1) for correctly sub. $b = 0.01$ (i.e. $a = B_1 = \log_{10} 0.01 = -2$)
- (1) for correct calculations to find 'r' AND substit. into formula for S_n AND simplifying
- Final mark for solving quadratic is not available here—