SYDNEY BOYS HIGH SCHOOL

Name:

NESA Number:

Tick next to your teacher's name

MZ	
JC	
RCW	
WB	

2026

YEAR 12
TASK 4
TRIAL HSC

Mathematics Advanced

General Instructions

- Reading time – 10 minutes
- Working time – 3 hours
- Write using black pen
- Only NESA approved calculators may be used
- A reference sheet is provided with this paper
- All answers, unless otherwise stated, should be left in simplified, exact form
- Marks may **NOT** be awarded for messy or badly arranged work
- For questions in Section II, show ALL relevant mathematical reasoning and/or calculations as answers that rely totally on calculator technology may not necessarily receive full marks.

Total Marks: 100

Section I – 10 marks (pages 2 – 8)

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section

Section II – 90 marks (pages 9 – 47)

- Attempt all Questions in Section II
- Allow about 2 hour and 45 minutes for this section

Examiner:
External Examiner

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Section I

10 marks
Attempt Questions 1 – 10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1 – 10.

Question 1

Evaluate $\sin \frac{\pi}{7}$ to 3 significant figures.

A. 0.00783

B. 0.008

C. 0.433

D. 0.434

Question 2

Which of the following is equal to $\frac{9}{2^4} + \frac{27}{2^6} + \frac{81}{2^8} + \frac{243}{2^{10}} + \ldots$?

A. 4

B. $\frac{9}{4}$

C. $\frac{9}{7}$

D. $\frac{16}{7}$

Question 3

The number of collisions on a highway is inversely proportional to the number of lanes. If 6 collisions occurred on a 4 lane road, how many collisions are expected on a 2 lane road?

A. 12

B. 6

C. 3

D. 2

Question 4

Which of the following is the domain of $y = \ln\left(\sqrt{x^2} - \sqrt{3}\right)$?

A. $\left(-\infty, -\sqrt{3}\,\right] \cup \left[\sqrt{3}, \infty\right)$

B. $\left(-\infty, -\sqrt{3}\,\right) \cup \left(\sqrt{3}, \infty\right)$

C. $\left(\sqrt{3}, \infty\right)$

D. $\left[\sqrt{3}, \infty\right)$

Question 5

What is the future value interest factor for \$1 invested at the end of each period, at 4% per period over 3 periods?

A. 1.124864

B. 2.04

C. 3.12

D. 3.1216

Question 6

Which of the following is equal to the derivative of $y = f(g(x))$?

A. $f'(g(x))$

B. $f'(g(x))g(x)$

C. $f'(g(x))g'(x)$

D. $f'(g'(x))$

Question 7

Two standard dice are rolled and the numbers at the top are multiplied together. What is the difference in the probability that at least one of the dice shows a 6, with and without the information that the product is larger than 20?

A. $\frac{19}{36}$

B. $\frac{23}{72}$

C. $\frac{11}{36}$

D. $\frac{1}{9}$

Question 8

Which of the following options is not equal to the others?

A. $\lim_{h\to 0}\left[\frac{f(x+h)-f(x-h)}{h}\right]$

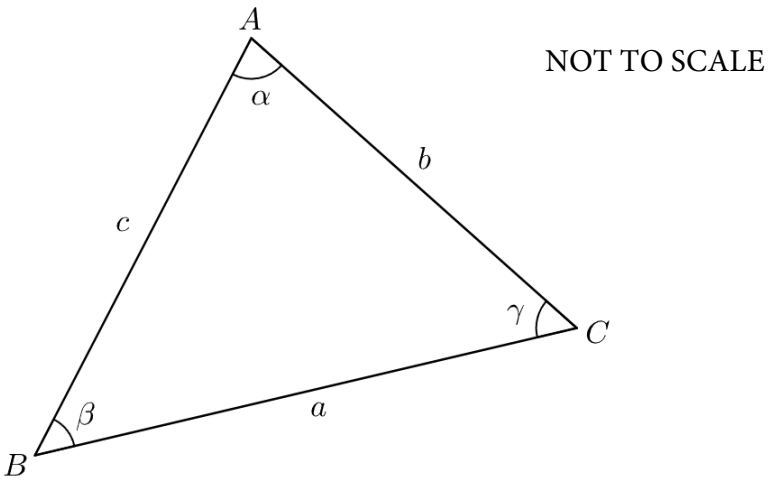
B. $\lim_{h\to 0}\left[\frac{f(x-h)-f(x-2h)}{h}\right]$

C. $\lim_{h\to 0}\left[\frac{f(x)-f(x-h)}{h}\right]$

D. $\lim_{h\to 0}\left[\frac{f\left(x+3h\right)-f\left(x+h\right)}{2h}\right]$

Question 9

In the triangle $\triangle ABC$, let $\angle ABC = \beta, \angle BAC = \alpha$ and $\angle ACB = \gamma$. Also, let the side lengths BC, AC, AB be a, b and c respectively.



NOT TO SCALE

Consider the following scenarios.

- Given side lengths a, b, c.
- Given side lengths a, b and angle γ
- Given side lengths a, b and angle α
- Given side length a and angles α, β

In how many of the above situations can the exact area of the triangle be calculated, without any additional information?

A. 1

B. 2

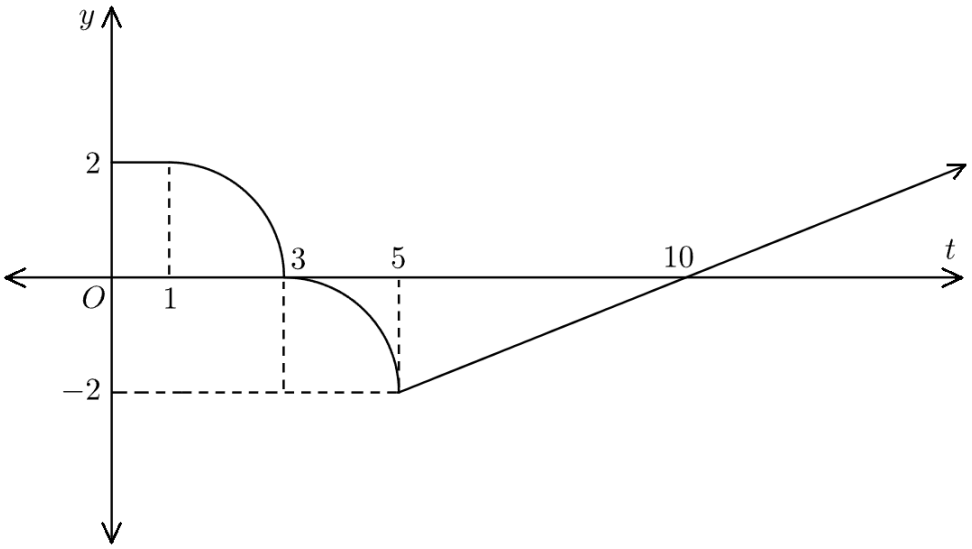
C. 3

D. 4

Question 10

The graph shows the function $y = f(t)$.

The curves between $x = 1$ and $x = 5$ are quadrants of circles.



How many x-intercepts does the function $A(x) = \int_0^x f(t)\ dt$ have for $0 \leq x \leq 10$?

A. 0

B. 1

C. 2

D. 3

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Y12 Mathematics Advanced

HSC Task 4 THSC

Part A

Section II

Part A 15 marks
Attempt Questions 11 – 16

Answer each question in the space provided.
A blank page is provided at the end of this question to allow rewriting of a part.
Your responses should include relevant mathematical reasoning and/or calculations.

Question 11

Find 345° in radians. **1**

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Question 12

(a) Prove that $y = r(x) = \dfrac{x}{1 + \cos x}$ is an odd function. **1**

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(b) Find its derivative. **2**

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Question 13

Find:

(a) $\int \left(e^x + \frac{1}{\sqrt{x}}\right)\, dx.$ **2**

(b) $\int_4^8 \frac{0.5 - x}{5 + x - x^2} dx.$ **2**

Question 14

Find the equation of the normal to $y = 3x - \ln x$ where the tangent is parallel to $y = 2x - 3$. **2**

Question 15

A cart containing n bags of artichokes is pushed around a market. Every customer that comes across it takes 10 more bags than the previous. If the first customer takes 20 bags, and the cart sells out after 9 more people, evaluate n. **2**

Question 16

Prove that $(\cot\phi + \operatorname{cosec}\phi)^2 = \dfrac{1+\cos\phi}{1-\cos\phi}$. **3**

Use this space to re-write any questions for Part A.

NESA Number										

Y12 Mathematics Advanced

HSC Task 4 THSC

Part B

Section II

Part B 15 marks
Attempt Questions 17 – 22

Answer each question in the space provided.
A blank page is provided at the end of this question to allow rewriting of a part.
Your responses should include relevant mathematical reasoning and/or calculations.

Question 17

Solve for x if $\ln(x^3 + 19) = 3\ln(x + 1)$. **2**

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Question 18

The graph of $y = f(x)$ has a y-intercept of 3. **2**

Find the corresponding point on the graph $y = -\frac{3}{2}f(2x+3)+4$.

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Question 19

The concentration of dirt after t minutes in a flowing river follows $\frac{dc}{dt} = -0.8c$, where c is the concentration in g/L. **2**

Initially, the concentration is 45 g/L. Let $c = Ae^{-kt}$, where k is a positive constant.

By finding the values of A and k, when does the concentration of dirt reach 25 mg/L, to the nearest minute?

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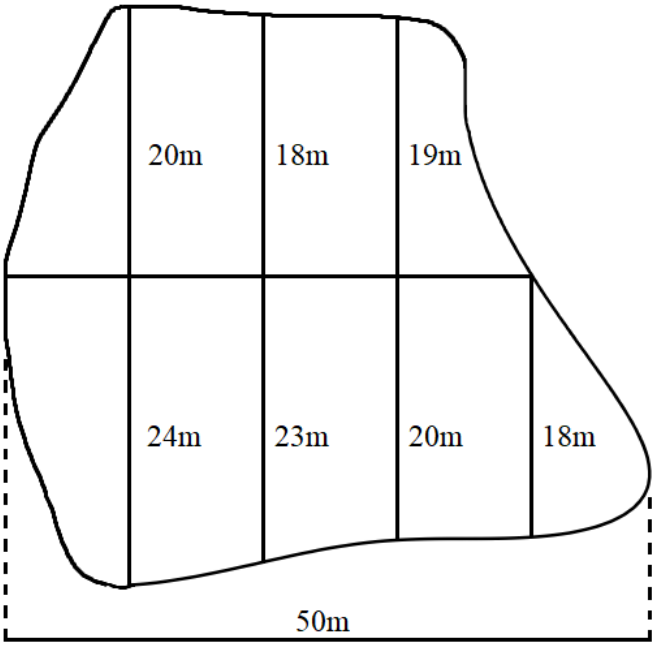
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Question 20

A river 50 metres wide is surveyed at 10 metre intervals for the width as shown. **2**



Using the trapezoidal rule, estimate the surface area of the river.

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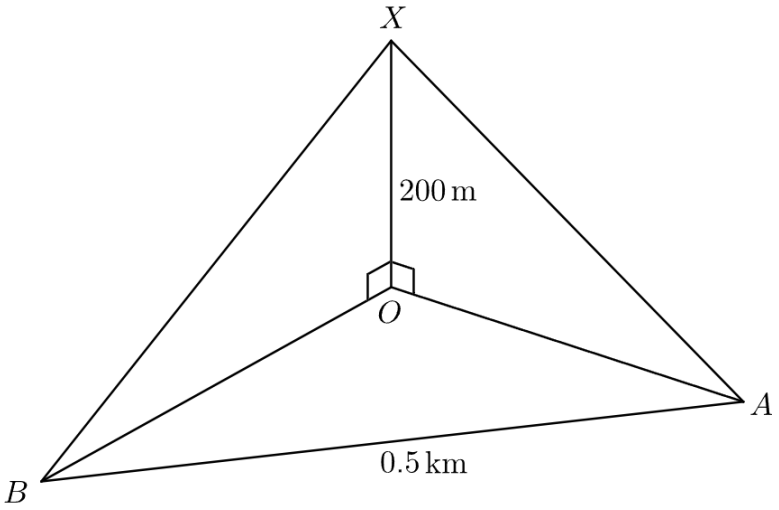
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Question 21

Find the equations of tangents to $y = x^2 - 4x + 5$ that have a y-intercept of 1. **3**

Question 22

An observatory on top of Mount Middleton sits 200 m above sea level. A ship is initially spotted at A, due North at an angle of depression of 30°. The ship then travels half a kilometre and is then spotted again at B on a bearing of S30°E from the observatory. **4**



Find the angle of elevation from which the ship sees the observatory from B.

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Use this space to re-write any questions for Part B.

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HSC Task 4 THSC

Part C

Section II

Part C 14 marks
Attempt Questions 23 – 24

Answer each question in the space provided.
A blank page is provided at the end of this question to allow rewriting of a part.
Your responses should include relevant mathematical reasoning and/or calculations.

Question 23

(a) Find $\frac{d}{dt}(te^t)$. **1**

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(b) A particle has acceleration $a = te^t$ m/s^2, where t is the time measured in seconds after it is released. Hence, or otherwise, if the particle is stationary 1 second after release, find the initial velocity of the particle. **2**

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Question 23 continues on page 23

(c) Find the total distance traveled by the particle in the first 3 seconds. **3**

End of Question 23

Question 24

Mira invests \$100 000 in a savings account that pays 12 percent interest per annum. The bank pays her monthly interest, and she will withdraw \$8000 every three months.

Let A_n be the amount in the bank after n months.

(a) Write down expressions for the balance after the first, second and third month. **1**

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(b) Show that balance after $3n$ months is **2**

$$A_{3n} = 100\ 000 \times 1.01^{3n} - 8000 \times \left(1 + 1.01^3 + 1.01^6 + \cdots + 1.01^{3(n-1)}\right).$$

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Question 24 continues on page 25

(c) Hence, find the month in which she will withdraw her last sum of money to clear her account and reach a balance of zero. **3**

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(d) How much interest did the bank pay to Mira overall? **2**

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End of Question 24

Use this space to re-write any questions for Part C.

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HSC Task 4 THSC

Part D

Section II

Part D 16 marks
Attempt Questions 25 – 28

Answer each question in the space provided.
A blank page is provided at the end of this question to allow rewriting of a part.
Your responses should include relevant mathematical reasoning and/or calculations.

Question 25

Let $P(A) = 0.2$ and $P(B) = 0.4$.

(a) If $P(A \cup B) = 0.5$, are A and B independent events? **1**

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(b) Evaluate $P\left(\overline{A} \cup B\right)$. **2**

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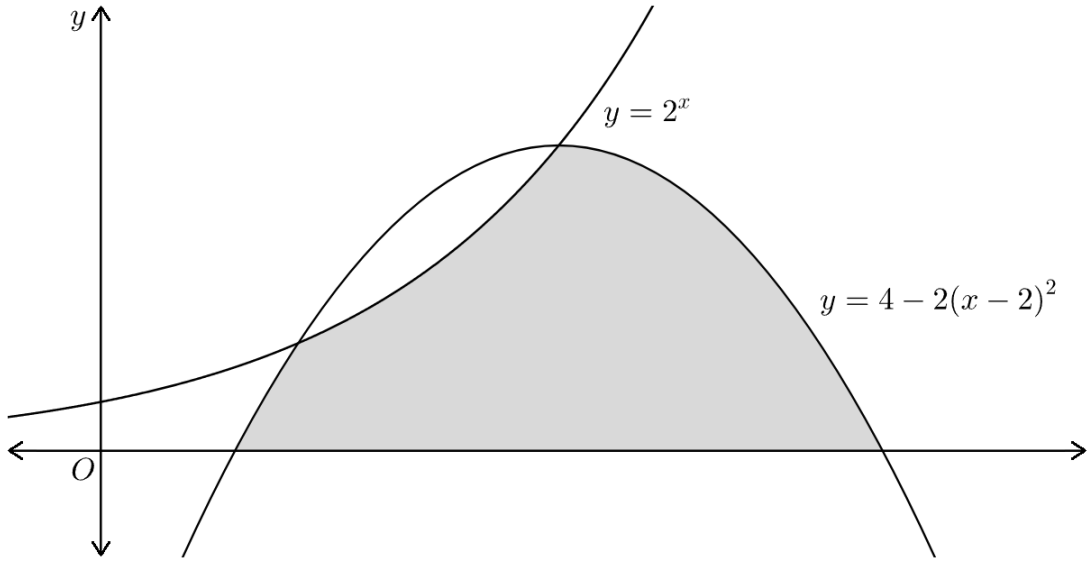
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Question 26

The graphs of $y = 2^x$ and $y = 4 - 2(x-2)^2$ are shown.



(a) Prove that the graphs intersect at $(1, 2)$ and $(2, 4)$. **1**

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Question 26 continues on page 31

(b) Calculate the area of the shaded region. **4**

End of Question 26

Question 27

Let $y = x^3 - 3x + 2$.

(a) Show that $y = (x-1)^2(x+2)$. **1**

(b) Find the coordinates of any stationary points, and determine their nature. **2**

Question 27 continues on page 33

(c) Find the coordinates of any inflexion points. **1**

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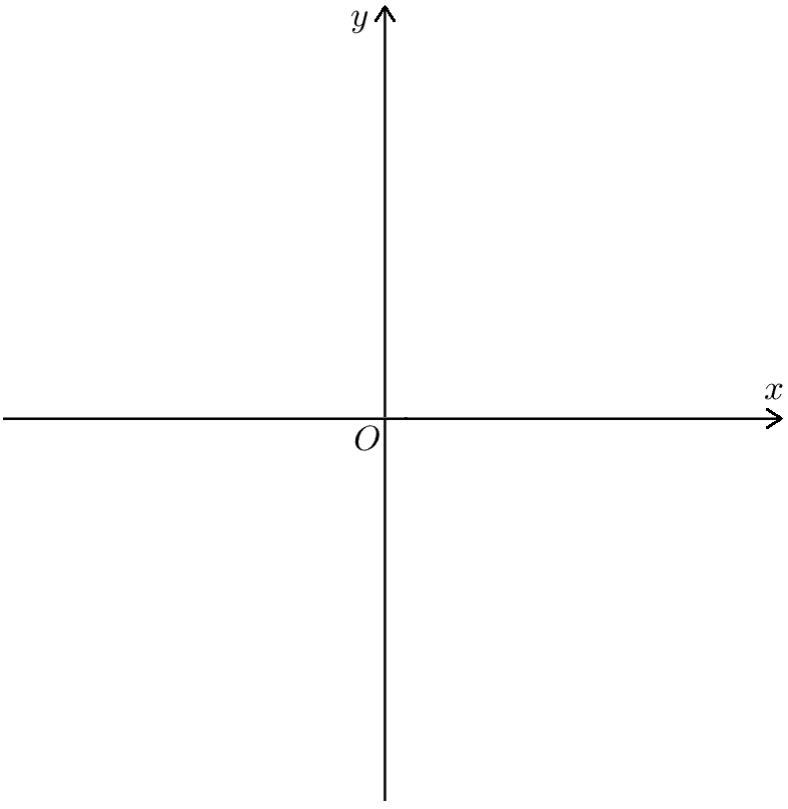
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(d) Hence, or otherwise, sketch the curve. **2**

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End of Question 27

Question 28

For some real number $p > 1$, it is given that $\log_p 2 = x$ and $\log_p 3 = y$. **2**

Find in terms of x and y, the value of $\ln\left(12^{\frac{1}{\ln p}}\right)$.

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Use this space to re-write any questions for Part D.

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HSC Task 4 THSC

Part E

Section II

Part E 16 marks
Attempt Questions 29 – 33

Answer each question in the space provided.
A blank page is provided at the end of this question to allow rewriting of a part.
Your responses should include relevant mathematical reasoning and/or calculations.

Question 29

An investigation into the relationship between the grams of steak s a person eats in a meal and their weight w in kilograms was conducted, and their results plotted in the table below.

s	310	180	420	270	240	390	250	360	290	320
w	85	55	90	75	50	80	65	95	60	70

(a) Which best describes the correlation between the quantities, **1**

Weak, Moderate or Strong?

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(b) Find the line of best fit for the data. **1**

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Question 30

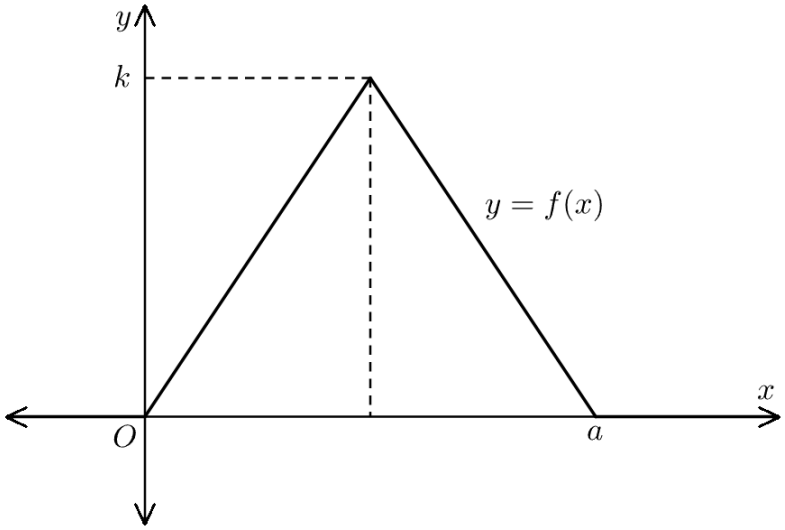
A discrete random variable X has the following probability distribution. **3**

x	1	3	4	5
$P(X = x)$	a	b	0.1	0.4

If $\text{Var}(X) = 1.94$ and $E(X^2) = 14.9$, calculate the values of a and b.

Question 31

A continuous random variable X with probability density function $f(x)$ follows the symmetrical triangle distribution between 0 and a, as shown in the diagram below.



(a) Find the value of k. **1**

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(b) Now, $f(x) = \dfrac{2k}{a}x$, for $0 \leq x < \dfrac{a}{2}$, where k is defined from (a). **1**

Write down the probability density function $f(x)$ for $0 \leq x \leq a$.

Your answer should be in terms of a and x only.

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Question 31 continues on page 40

(c) Hence, find the inter-quartile range for values found from this distribution. **3**

(d) Prove that all values that the random variable X takes are not outliers. **2**

End of Question 31

Question 32

The waiting time for a train can be modeled by a continuous random variable T, which has cumulative density function **2**

$$G(t) = \begin{cases} 0 & , t < 0, \\ 1 - e^{-t^2} & , t \geq 0. \end{cases}$$

Find the mode for the distribution of times.

Question 33

99.7% of the scores of a recent medical test are found to exist symmetrically between 60 and 96. Assuming the scores are normally distributed, what proportion of scores are going to exist between 66 and 84? **2**

Use this space to re-write any questions for Part E.

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Y12 Mathematics Advanced

HSC Task 4 THSC

Part F

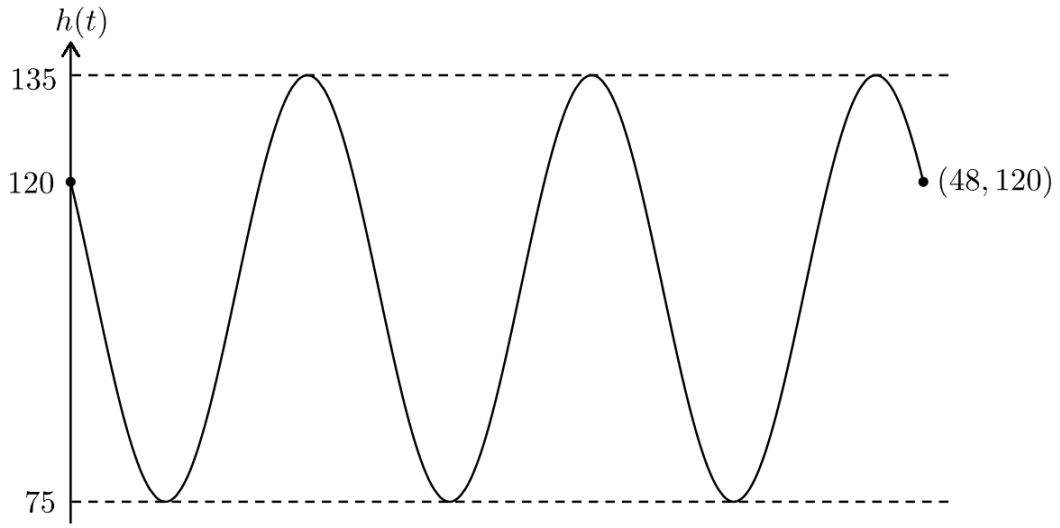
Section II

Part F 14 marks
Attempt Questions 34 – 36

Answer each question in the space provided.
A blank page is provided at the end of this question to allow rewriting of a part.
Your responses should include relevant mathematical reasoning and/or calculations.

Question 34

The rise and fall of the average heart rate $h(t)$ of an athlete through a race is modeled by a sinusoidal curve over the interval $0 \leq t \leq 48$, where t is the time in seconds from the start of the race.



The equation of the curve is $h(t) = a \sin\left[\frac{\pi}{8}(t-c)\right] + k$. The heart rate reaches a maximum of 135 beats per minute and a minimum of 75 beats per minute.

(a) Find the values of a and k, where a and k are positive constants. **2**

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Question 34 continues on page 46

(b) The athlete started with an initial heart rate of 120 beats per minute. **1**

What is the value of c?

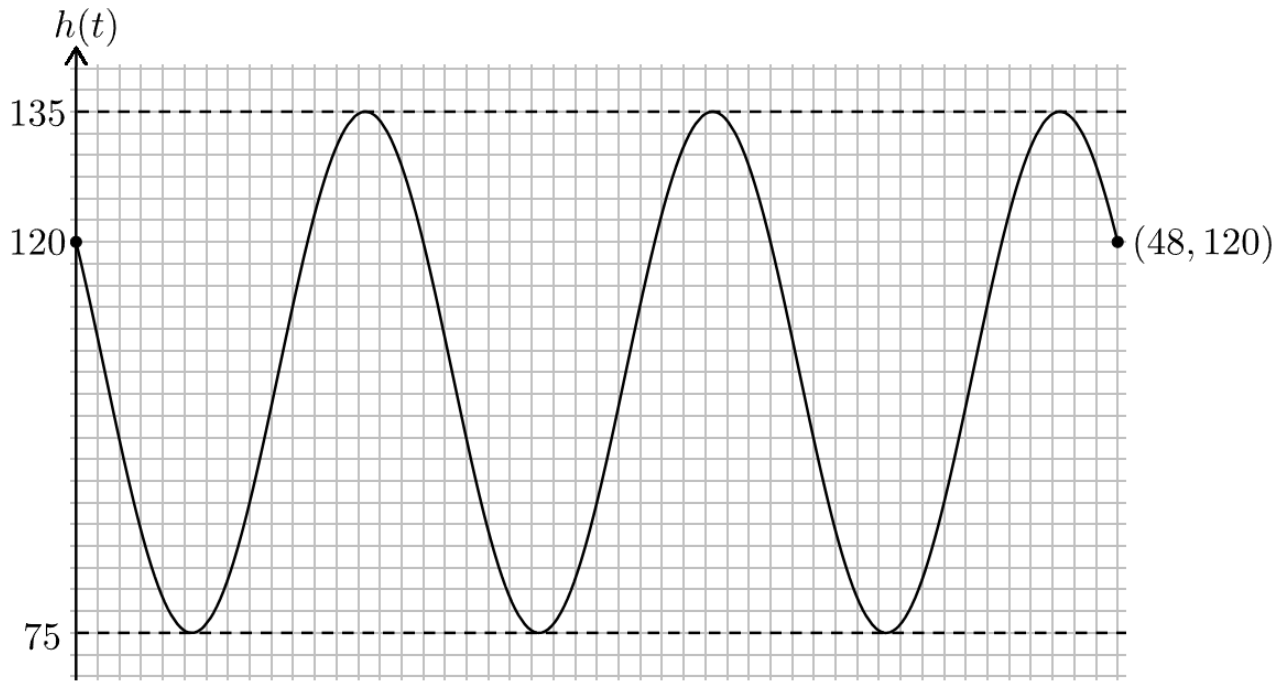
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(c) A better trained athlete has the same minimum heart rate, but a maximum heartbeat of 105 beats per minute. The better athlete also has an initial heart rate of 75 beats per minute, and his heart rate per minute will oscillate with a period 50% longer than the original. Sketch his heart rate on the diagram provided, and state the equation. **3**



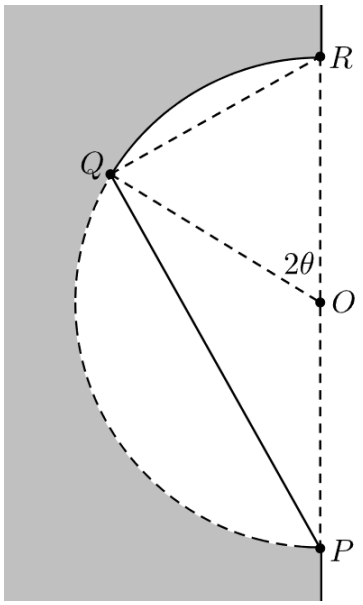
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End of Question 34

Question 35

A data center is stationed on the edge of a semi-circular bay of radius 4 km at P. A server is at R, positioned directly across the bay's diameter. An underwater connection could be laid from P to a relay server at Q, then wrapped along the curved semi-circular boundary above ground from Q to R.



The diameter of the bay is 8 km, centered at O.

$\angle PQR$ is a right angle (do NOT prove).

Laying a direct internet connection underwater costs 6 million dollars per kilometer, whereas a direct internet connection above ground costs a quarter as much.

Let $\angle QOR = 2\theta$.

(a) Prove the total cost in millions for laying the wires is $C = 12(4\cos\theta + \theta)$ dollars. **2**

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Question 35 continues on page 48

(b) Find the minimum cost to lay a connection from P to R. **4**

End of Question 35

Question 36

Let $y = f(x) = \begin{cases} 0 & , x < 0, \\ x & , x \geq 0. \end{cases}$ **2**

Express $y = f(x)$ as a single function without a piecemeal definition.

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End of Paper

Use this space to re-write any questions for Part F.

SYDNEY BOYS HIGH SCHOOL

2026

YEAR 12

HSC TASK 4

Mathematics Advanced

Sample Solutions

NOTE: This process of checking your mark is about reading the solutions and the comments. Before putting in an appeal re marking, first consider that the mark is not linked to the amount of writing you have done.

Just because you have shown some 'working' does not justify that your solution is worth any marks.

Students who used pencil, an erasable pen and/or whiteout, may NOT be able to appeal.

MC Answers

1	2	3	4	5	6	7	8	9	10
D	B	A	B	D	C	A	A	C	C

Question 11

Find 345° in radians. **1**

$$1° = \frac{\pi}{180}$$

$$\therefore 345° = \frac{345\pi}{180} = \frac{23\pi}{12}$$

Criteria	Marks
Provides correct solution	**1**

Marker's Comment: No half mark.

Accept $\frac{345\pi}{180}$ or 6.0212385919 (or equivalent).

Question 12

(a) Prove that $y = r(x) = \frac{x}{1+\cos x}$ is an odd function. **1**

$$r(-x) = \frac{-x}{1+\cos(-x)} = \frac{-x}{1+\cos x} \text{ since } \cos(-x) = \cos x$$

$$= -r(x)$$

Criteria	Marks
Provide correct solution	**1**
State $\cos(-x) = \cos x$	**0.5**

Marker's Comment:

Simply stating $r(-x) = -r(x)$ is not worth half marks.

(b) Find its derivative. **2**

$$\frac{dy}{dx} = \frac{(1+\cos x)(1) - x(-\sin x)}{(1+\cos x)^2}$$

$$= \frac{x\sin x + \cos x + 1}{(1+\cos x)^2}$$

Criteria	Marks
Provide correct solution	**2**
Attempt to use the quotient rule correctly	**1**

Marker's Comment: No half mark.

Students should not be using chain rule and product rule for this question.

Students need to look at the formula for the quotient rule from the reference sheet.

Question 13

Find:

(a) $\int \left(e^x + \frac{1}{\sqrt{x}}\right)\, dx.$ **2**

$$\int \left(e^x + \frac{1}{\sqrt{x}}\right) dx = \int e^x\, dx + \int x^{-\frac{1}{2}}\, dx$$

$$= e^x + \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + C$$

$$= e^x + 2\sqrt{x} + C$$

Criteria	Marks
Provide correct solution	**2**
Correct integration for e^x	**0.5**
$+ C$	**0.5**

Marker's Comment:

Generally ok, some students forgot the $+ C$ or could not integrate $x^{\frac{-1}{2}}$ correctly.

(b) $\int_4^8 \frac{0.5 - x}{5 + x - x^2} dx.$ **2**

$f(x) = 5 + x - x^2$ and $f'(x) = 1 - 2x = 2(0.5 - x)$

$$\therefore \int_4^8 \frac{0.5 - x}{5 + x - x^2}\, dx = \frac{1}{2}\int_4^8 \frac{2(0.5 - x)}{5 + x - x^2}\, dx$$

$$= \frac{1}{2}\left[\ln\left|5 + x - x^2\right|\right]_4^8$$

$$= \frac{1}{2}(\ln 51 - \ln 7)$$

$$= \ln\sqrt{\frac{51}{7}}$$

Criteria	Marks
Provide correct solution	**2**
Provide correct solution with decimal answer, leave answer in exact form as per instructions. Or incorrection substitutions with the limits.	**1.5**
Correct integration with ln with the absolute value sign only	**0.5**
Correct manipulation of the integral with a factor of $\frac{1}{2}$	**0.5**

Marker's Comment:

Generally ok, some students forgot the $+ C$ or could not integrate $x^{\frac{-1}{2}}$ correctly.

Accepted $\frac{1}{2}(\ln 51 - \ln 7)$ or $\frac{1}{2}\ln\frac{51}{7}$.

Question 14

Find the equation of the normal to $y = 3x - \ln x$ where the tangent is parallel to $y = 2x - 3$. **2**

$$y' = 3 - \frac{1}{x}$$

Tangent parallel to $y = 2x - 3 \quad \therefore m = 2$

$$3 - \frac{1}{x} = 2$$
$$\frac{1}{x} = 1$$
$$x = 1$$

$$\therefore y = 3(1) - \ln(1)$$
$$y = 3$$

$$m_{\text{normal}} = \frac{1}{m_{\text{tangent}}} = -\frac{1}{2}$$

$$y - 3 = -\frac{1}{2}(x - 1)$$
$$2y - 6 = -x + 1$$
$$\therefore \quad x + 2y - 7 = 0$$

or $$y = -\frac{1}{2}x + \frac{7}{2}$$

Criteria	Marks
Provide correct solution	**2**
Provide correct coordinates of the point of contact (no half mark for correct x or y coordinate)	**1**
Correct gradient of the normal	**0.5**

Marker's Comment:

Students had problems understanding the questions.

They need to read the questions carefully to get the correct gradient.

Question 15

A cart containing n bags of artichokes is pushed around a market. Every customer that comes across it takes 10 more bags than the previous. If the first customer takes 20 bags, and the cart sells out after 9 more people, evaluate n. **2**

20, 30, 40, …

AP : $a = 20$, $d = 10$, number of terms = 10

$$n = S_n = \frac{10}{2}\left(2(20) + 9 \times 10\right)$$

$$\therefore n = 650 \text{ bags}$$

Criteria	Marks
Provide correct solution	**2**
Provide correct expression for S_n	**1**
Any 2 correct variables	**0.5 each**

Marker's Comment:

Students had problems understanding the questions.

They need to read the questions carefully and make good use of the reference sheet provided.

Question 16

Prove that $(\cot\phi + \operatorname{cosec}\phi)^2 = \dfrac{1+\cos\phi}{1-\cos\phi}$. **3**

METHOD 1

$$\text{LHS} = \left(\frac{\cos\phi}{\sin\phi} + \frac{1}{\sin\phi}\right)^2 = \left(\frac{1+\cos\phi}{\sin\phi}\right)^2$$
$$= \frac{(1+\cos\phi)^2}{\sin^2\phi}$$
$$= \frac{(1+\cos\phi)^2}{1-\cos^2\phi}$$
$$= \frac{(1+\cos\phi)^2}{(1-\cos\phi)(1+\cos\phi)}$$
$$= \frac{(1+\cos\phi)}{(1-\cos\phi)} \quad = \quad \text{RHS}$$

METHOD 2

$$\text{LHS} = \cot^2\phi + 2\cot\phi\operatorname{cosec}\phi + \operatorname{cosec}^2\phi = \frac{\cos^2\phi}{\sin^2\phi} + 2\frac{\cos\phi}{\sin\phi}\frac{1}{\sin\phi} + \frac{1}{\sin^2\phi}$$
$$= \frac{\cos^2\phi + 2\cos\phi + 1}{\sin^2\phi}$$
$$= \frac{(1+\cos\phi)^2}{1-\cos^2\phi}$$
$$= \frac{(1+\cos\phi)^2}{(1-\cos\phi)(1+\cos\phi)}$$
$$= \frac{(1+\cos\phi)}{(1-\cos\phi)} \quad = \quad \text{RHS}$$

Criteria	Marks
Provide correct solution by correctly factorising $1-\cos^2\phi$	**3**
Show $\sin^2\phi = 1-\cos^2\phi$	**1**
Express LHS with the common denominator of $\sin\phi$	**1**

Marker's Comment:

Students need to look at the RHS to see where to start with this question.

They struggled to put the expression into a single fraction.

Students using method 2 did not know how to factorise $\cos^2\phi + 2\cos\phi + 1$.

They should aim to reduce the expression using cosine, not cotangent or tangent, and avoid expanding it further, which only complicates it.

End of Part A solutions

Question 17

Solve for x if $\ln(x^3 + 19) = 3\ln(x + 1)$. **2**

$$\ln\left(x^3+19\right)=3\ln\left(x+1\right)$$
$$\ln\left(x^3+19\right)=\ln\left(x+1\right)^3$$
$$\left(x^3+19\right)=\left(x+1\right)^3$$
$$x^3+19=x^3+3x^2+3x+1$$
$$3x^2+3x-18=0$$
$$x^2+x-6=0$$
$$\left(x-2\right)\left(x+3\right)=0$$
$$x=2 \qquad \text{since } x>-1$$

1 mark for applying relevant algebraic operations to obtain a quadratic equation
1 mark for solving for x correctly, noting the domain constraints

Done quite well.

The main errors were:

- Not recognising that $x=-3$ is an extraneous solution
- Incorrect application of log laws
- Incorrect expansion of $\left(x+1\right)^3$

Question 18

The graph of $y = f(x)$ has a y-intercept of 3. **2**

Find the corresponding point on the graph $y = -\frac{3}{2}f(2x+3)+4$.

The corresponding point will satisfy the substitution $f(2x+3)=f(0)=3$.

The x-coordinate will be $x=-\frac{3}{2}$.

The y-coordinate will be $y=-\frac{3}{2}(3)+4=-\frac{1}{2}$.

Thus, the corresponding point is $\left(-\frac{3}{2},-\frac{1}{2}\right)$.

1 mark for correctly determining the x-coordinate
1 mark for correctly determining the y-coordinate

Done poorly.

Most students were able to find either the x- or y-coordinate, but not both.
This was primarily due to an incorrect identification of the order in which the transformations were applied.

Very few students used the algebraic method shown in the model solution above- it is recommended that all students take some time to understand this method, as it is very efficient (and can be used to quickly plot points of a transformed graph for easy sketching).

Question 19

The concentration of dirt after t minutes in a flowing river follows $\frac{dc}{dt} = -0.8c$, where c is the concentration in g/L. **2**

Initially, the concentration is 45 g/L. Let $c = Ae^{-kt}$, where k is a positive constant.

By finding the values of A and k, when does the concentration of dirt reach 25 mg/L, to the nearest minute?

Since $\frac{dc}{dt} = (-k)Ae^{-kt} = (-k)c = -0.8c$, therefore $k = 0.8$.

At $t = 0$, $c = 45 = Ae^{-0.8\times 0} = A$, therefore $A = 45$.

To find the time when concentration reaches 25 mg/L, we substitute in $c = 0.025$ and solve for t:

$$\begin{aligned} 0.025 &= 45e^{-0.8t} \\ \frac{1}{1800} &= e^{-0.8t} \\ 1800 &= e^{0.8t} \\ \ln(1800) &= 0.8t \\ t &= \frac{\ln(1800)}{0.8} \\ &\approx 9.37 \end{aligned}$$

Since want the nearest minute for when the concentration **reaches** 25 mg/L, we round **up** to 10 mins.

1 mark for finding the correct values for A and k.
1 mark for determining when the concentration reaches 25 mg/L

Done ok.

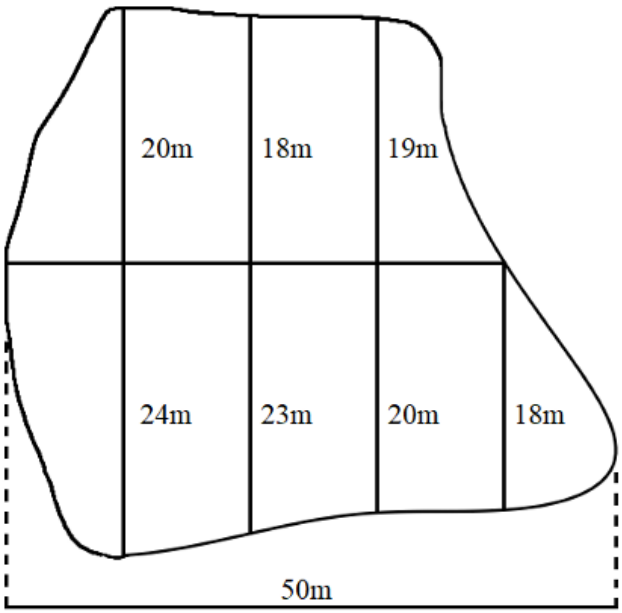
The main errors were:

- Substituting the initial conditions into the derivative $\frac{dc}{dt}$, rather than into c
- Not noticing that the target concentration was 25 **mg**/L (i.e. need to substitute $c = 0.025$)

Many students also had difficulty accessing the question because they didn't recognise that k can be found by differentiating the exponential decay function.

Question 20

A river 50 metres wide is surveyed at 10 metre intervals for the width as shown. **2**



Using the trapezoidal rule, estimate the surface area of the river.

$$A \approx \frac{50}{2(5)}\left(0+0+2\left[44+41+39+18\right]\right)$$
$$= 1420 \text{ m}^2$$

1 mark for attempting to use the trapezoidal rule
1 mark for the correct answer

Done ok.

It was almost impossible to get 0/2 marks for this question – please note that past HSC trapezoidal rule questions typically reward a mark for participation, so make sure you always attempt these problems.

The consistent error among responses was not recognising that the first and last function values for this approximation should be zero (since the ends of the river have zero width).

It was interesting that many students opted to split the area into a ‘top’ and ‘bottom’ to make the problem more familiar- this is a good strategy.

Question 21

Find the equations of tangents to $y = x^2 - 4x + 5$ that have a y-intercept of 1. **3**

A line with a y-intercept of 1 will have the equation form $y = mx + 1$.

To find the tangents, we need the values of m such that

(1) $y = mx + 1$

(2) $y = x^2 - 4x + 5$

have exactly one point of intersection (i.e. they have only one simultaneous solution).

$$x^2 - 4x + 5 = mx + 1$$
$$x^2 - (4 + m)x + 4 = 0$$

For exactly one intersection, we set the discriminant to be equal to zero and solve:

$$\begin{aligned}\Delta &= (4+m)^2 - 4(4)(1) \\ &= 16 + 8m + m^2 - 16 \\ &= m(8+m) \\ &= 0\end{aligned}$$

Thus, the gradients of the tangents with y-intercept of 1 are $m = 0, -8$.

The equations of the tangents are:

$$y = 1$$
$$y = -8x + 1$$

1 mark for constructing the correct simultaneous equation
1 mark for setting the discriminant equal to zero and solving for m
1 mark for the correct tangent equations

Done ok.

The most common error was trying to substitute the y-intercept into the general tangent equation. This lead absolutely nowhere.

Students that used the method shown above tended to do well.

However, many students opted to use the calculus method shown on the next page. (tsk tsk)

Question 21 (continued)

Alternate (much less efficient) method

Using the first derivative of the parabola, we know that the gradient of the tangent will be:

$$m = y' = 2x_0 - 4$$

where x_0 is the *x*-coordinate of a point of tangency.

This gives a general tangent equation of

$$y = (2x_0 - 4)x + 1.$$

Therefore, the point of tangency (x_0, y_0) is the simultaneous solution of the equations

$$(1) \qquad y_0 = (2x_0 - 4)x_0 + 1$$

$$(2) \qquad y_0 = x_0^2 - 4x_0 + 5.$$

Solving this gives

$$\begin{aligned}(2x_0 - 4)x_0 + 1 &= x_0^2 - 4x_0 + 5\\ 2x_0^2 - 4x_0 + 1 &= x_0^2 - 4x_0 + 5\\ x_0^2 &= 4\\ x_0 &= \pm 2.\end{aligned}$$

Therefore, the points of tangency are at $x_0 = \pm 2$, and thus the gradients of those tangents are

$$\begin{aligned}m &= 2(\pm 2) - 4\\ &= 0, -8.\end{aligned}$$

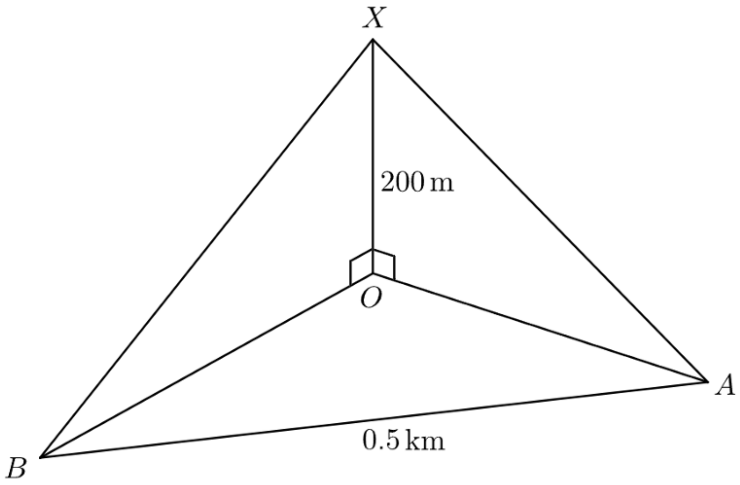
The equations of the tangents are:

$$y = 1$$
$$y = -8x + 1$$

Students that attempted this method often failed due to a lack of understanding of what the derivative represented.

Question 22

An observatory on top of Mount Middleton sits 200 m above sea level. A ship is initially spotted at A, due North at an angle of depression of 30°. The ship then travels half a kilometre and is then spotted again at B on a bearing of S30°E from the observatory. **4**



Find the angle of elevation from which the ship sees the observatory from B.

From the question, we know that $\angle OAX = 30°$ and $\angle AOB = 150°$.

We can also calculate $OA = \dfrac{200}{\tan 30°} = 200\sqrt{3}$.

Therefore, by cosine rule (letting $OB = x$), we have:

$$500^2 = \left(200\sqrt{3}\right)^2 + x^2 - 2\left(200\sqrt{3}\right)x\left(\cos 150°\right)$$

$$250000 = 120000 + x^2 + 600x$$

$$x^2 + 600x - 130000 = 0$$

$$x = \frac{-600 + \sqrt{600^2 + 4(130000)}}{2} \qquad \text{since } x > 0$$

$$= 169.04$$

Thus, $\angle OBX = \tan^{-1}\left(\dfrac{200}{169.04}\right) = 49.80°$.

The angle of elevation from B to the observatory is $49.80°$.

1 mark for determining OA
2 marks for determining OB
1 mark for finding the correct angle of elevation

[comments on the page B #8]

Question 22 (continued)

Done ok.

The main errors were:

- Putting the angle of depression as $\angle OXA = 30°$, when it should be $\angle OAX = 30°$ (as the alternate angle of inclination)

- Misinterpreting the bearing S30°E, setting $\angle AOB = 120°$ when instead it should be $150°$

- Not recognising that AB needs to be converted into metres

Interestingly, most students opted to do two applications of sine rule, rather than a single application of chain rule.
In this question $\angle AOB$ is obtuse so the ambiguous case is avoided, but it should be noted that this is not always the case.

End of Part B solutions

Question 23

(a) Find $\frac{d}{dt}(te^t)$. **1**

$$\frac{d}{dt}(te^t) = t \times \frac{d}{dt}(e^t) + \frac{d}{dt}(t) \times e^t$$

$$= te^t + e^t$$

Comments

Generally this was done very well by students who realised they needed the 'product rule'.

Question 23 (continued)

(b) A particle has acceleration $a = te^t$ m/s^2, where t is the time measured in seconds after it is released. Hence, or otherwise, if the particle is stationary 1 second after release, find the initial velocity of the particle. **2**

From (a): $te^t = \frac{d}{dt}(te^t) - e^t$

Method 1:

$$\frac{dv}{dt} = te^t \Rightarrow \int_1^v dv = \int_0^t te^t\, dt$$

$$\therefore v - 1 = \int_0^t \frac{d(te^t)}{dt} - e^t\, dt$$

$$\therefore v - 1 = \left[te^t - e^t\right]_0^t$$

$$\therefore v - 1 = \left(te^t - e^t\right) - \left(0 - e^0\right)$$

$$\therefore v = te^t - e^t$$

$$\therefore v(0) = 0 - e^0 = 1$$

Method 2:

$$v(t) = \int te^t\, dt$$

$$= \int \left[\frac{d}{dx}(te^t) - e^t\right] dt$$

$$= te^t - e^t + C$$

$$[v(1) = 0]$$

$$\therefore 0 = 1 \times e^1 - e^1 + C \Rightarrow C = 0$$

$$\therefore v(t) = te^t - e^t$$

$$\therefore v(0) = 0 \times e^0 - e^0 = -1$$

Comments

Generally this was done well by students who had practised examples of "differentiate, hence integrate'. To those who hadn't this was horribly done.

One student found the change in velocity in the first second i.e. $\int_0^1 te^t\, dt = 1$ and reasoned that the initial velocity was –1 m/s.

The major error was with students who didn't write down their substitutions, they didn't see that they were taking away a negative term.

Question 23 (continued)

(c) Find the total distance traveled by the particle in the first 3 seconds. **3**

NOTE: $t = 1$, $v = 0$, $a = 2e$

$\therefore$ the particle is initially moving to the left, it stops at $t = 1$.
As the acceleration is positive at $t = 1$, it now moves (backwards) in the positive direction.

$\therefore$ Distance travelled $= \left| \int_0^1 v\ dt \right| + \int_1^3 v\ dt$.

Using part (a) and part (b): $x = \int v\ dt = \int \left(te^t - e^t\right) dt = \int \left(\frac{d}{dt}\left(te^t\right) - 2\,e^t \right) dt$

Distance travelled $= \left| \left[te^t - 2e^t \right]_0^1 \right| + \left[te^t - 2e^t \right]_1^3$

$= \left| (e - 2e - (0 - 2)) \right| + \left(3e^3 - 2e^3\right) - (e - 2e)$

$= \left| 2 - e \right| + e^3 + e$

$= e - 2 + e^3 + e$

$= e^3 + 2e - 2$

Comments

Students who evaluated $\int_0^3 v\ dt$ could only get a maximum of 1 mark.
They showed much less understanding of the problem at hand and had much less work to do.

Students who opted to do the indefinite integrals and assumed $x(0) = 0$, could not get full marks.
They also opted for easier (and less) working as they didn't have to have the "+ C" in their expressions.

Similar errors to part (b) where students were not tracking that they had substitutions involving the taking away of negative terms.
Also some students either didn't fully simplify and/or had an approximation as their answer.

Too many students in parts (b) and (c) were opting to have multiple integrals in their working. This is a sure way to not be consistent and cause an error.

ECF was applied where possible, but in some cases it made the question quite easy/trivial.

Question 24

Mira invests \$100 000 in a savings account that pays 12 percent interest per annum. The bank pays her monthly interest, and she will withdraw \$8000 every three months.

Let A_n be the amount in the bank after n months.

(a) Write down expressions for the balance after the first, second and third month. **1**

12% p.a. = 1% per month.

Let $R = 1.01$

$A_1 = 100\,000R$

$A_2 = A_1R$

$= 100\,000R^2$

$A_3 = A_2R - 8000$

$= 100\,000R^3 - 8000$

Comments

Working not required, but recommended.

Some students opted to just write the final answer, this then was no help in showing development of the formula in part (b).
Others also wrote correct statements but would have to significantly increase the time in simplifying them for part (b).

Some students did not defined their pronumerals.

Question 24 (continued)

(b) Show that balance after $3n$ months is **2**

$$A_{3n} = 100\ 000 \times 1.01^{3n} - 8000 \times \left(1 + 1.01^3 + 1.01^6 + \cdots + 1.01^{3(n-1)}\right).$$

Method 1: Recognising the pattern:

$A_3 = 100\ 000R^3 - 8000$ [From part (a)]

$$\begin{aligned} A_6 &= A_3R^3 - 8000 \\ &= \left(100\ 000R^3 - 8000\right)R^3 - 8000 \\ &= 100\ 000R^6 - 8000\left(1 + R^3\right) \end{aligned}$$

$$\begin{aligned} A_9 &= A_6R^3 - 8000 \\ &= \left(100\ 000R^6 - 8000\left(1 + R^3\right)\right)R^3 - 8000 \\ &= 100\ 000R^9 - 8000\left(1 + R^3 + R^6\right) \end{aligned}$$

So generalising: $A_{3n} = 100\ 000R^{3n} - 8000\left(1 + R^3 + R^6 + \ldots + R^{3(n-1)}\right)$

Method 2: Not quite recognising the pattern:

$$\begin{aligned} A_4 &= A_3R \\ &= 100\ 000\ R^4 - 8000\ R \end{aligned} \qquad \begin{aligned} A_7 &= A_6R \\ &= 100\ 000\ R^7 - 8000\ R\left(1 + R^3\right) \end{aligned}$$

$$\begin{aligned} A_5 &= A_4R \\ &= 100\ 000\ R^5 - 8000\ R^2 \end{aligned} \qquad \begin{aligned} A_8 &= A_7R \\ &= 100\ 000\ R^7 - 8000\ R^2\left(1 + R^3\right) \end{aligned}$$

$$\begin{aligned} A_6 &= A_5R - 8000 \\ &= 100\ 000\ R^6 - 8000\ R^3 - 8000 \\ &= 100\ 000\ R^6 - 8000\left(1 + R^3\right) \end{aligned} \qquad \begin{aligned} A_9 &= A_8R - 8000 \\ &= 100\ 000\ R^9 - 8000\ R^3\left(1 + R^3\right) - 8000 \\ &= 100\ 000\ R^9 - 8000\left(1 + R^3 + R^6\right) \end{aligned}$$

So generalising: $A_{3n} = 100\ 000R^{3n} - 8000\left(1 + R^3 + R^6 + \ldots + R^{3(n-1)}\right)$

Comments: This was done well by the students who knew they had to show three terms and those terms were A_3, A_6, and A_9.

This is a "Show that" question in that students have to show development of the formula, not "show" how they can just substitute into a given formula.

Students who effectively just wrote down the following could only score ½ marks.

$$A_3 = 100\,000\,R^3 - 8000\,(1)$$

$$A_6 = 100\,000\,R^6 - 8000\,(1 + R^3)$$

$$A_9 = 100\,000\,R^9 - 8000\,(1 + R^3 + R^6)$$

$$\vdots$$

$$A_{3n} = 100\,000\,R^{3n} - 8000\,(1 + R^3 + R^6 + \ldots + R^{3(n-1)})$$

In some past questions, students just had to show A_1, A_2, and A_3, but the formula they were trying to "Show" was A_n.

Some students did show to A_6 properly, but as a result their maximum mark was 1.

Some students rewrote their part (a) in part (b). This is time inefficient and totally not necessary.

Question 24 (continued)

(c) Hence, find the month in which she will withdraw her last sum of money to clear her account and reach a balance of zero. **3**

From (b):

$$A_{3n} = 100\,000\,R^{3n} - 8000\left(\underbrace{1 + R^3 + R^6 + \ldots + R^{3(n-1)}}_{\text{GP: } a = 1,\ r = R^3,\ n \text{ terms}}\right)$$

$$A_{3n} = 100\,000\,R^{3n} - 8000 \times \frac{(R^3)^n - 1}{R^3 - 1}$$

$$= 100\,000\,R^{3n} - 8000 \times \frac{R^{3n} - 1}{R^3 - 1}$$

$$= 0$$

$$\therefore R^{3n}\left(100\,000 - \frac{8000}{R^3 - 1}\right) = \frac{-8000}{R^3 - 1}$$

$$\therefore R^{3n} = \frac{-8000}{R^3 - 1} \times \frac{R^3 - 1}{100\,000\left(R^3 - 1\right) - 8000}$$

$$= \frac{-8000}{100\,000\left(R^3 - 1\right) - 8000}$$

$$\therefore R^{3n} = 1.60969\ldots$$

$$\therefore 3n \approx 47.8418\ldots$$

$$= 48 \text{ (nearest month)}$$

Comments: If students got line 2 (above) correct, they generally didn't make any mistakes until they got to the number of months in the second last line.

Common errors: The denominator was written as (1.01 – 1).
These students could only get a maximum of 1½ marks.

The number of terms varied from 'n', so the formula in line 2 above had a few variations.

Students didn't realise that $3n$ is the correct answer and needs to be rounded up. Some students found n, rounded and then multiplied by 3 and got the wrong number of months.

Rounding.

Question 24 (continued)

(d) How much interest did the bank pay to Mira overall? **2**

Interest = $Amount of Mira's payments – $100 000

How much was in Mira's account before the last payment of $8000 in the 48th month?

Payments are only made every three years.

Method 1:
$A_{45} = 7361$
Start of 48th month= $A_{45} \times R^3 = 7584.09$

Method 2:
$A_{48} = -415.91$
Start of 48th month = $A_{48} + 8000 = 7584.09$

As this is less than $8000 then the last full $8000 payment was for A_{45}
i.e. 15 full payments of $8000

Total paid = 15 × $8000 + $7584.09 = $127 584.09

∴ Interest = $127 584.09 – $1000 000 = $27 584.09

So the interest paid is $27 584.09

Comments: This was not done well.

Common errors: There has to be a whole number of repayments of $8000.
Students who chose not to do this could not get many marks.

Rounding.

End of Part C solutions

Question 25

Let $P(A) = 0.2$ and $P(B) = 0.4$.

(a) If $P(A \cup B) = 0.5$, are A and B independent events? **1**

Method 1:

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)} = \frac{0.2 + 0.4 - 0.5}{0.4} = 0.25 \neq P(A)$$

$\therefore$ A and B are not independent events

Method 2:

$$P(A) \times P(B) = 0.2 \times 0.4$$
$$= 0.08$$
$$P(A \cap B) = P(A) + P(B) - P(A \cup B)$$
$$= 0.2 + 0.4 - 0.5$$
$$= 0.1$$

$\therefore$ A and B are not independent events as $P(A) \times P(B) \neq P(A \cap B)$

Method 3:

If *A and B are independent events,*

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$= 0.2 + 0.4 - 0.2 \times 0.4$$
$$= 0.52$$
$$\neq 0.5$$

$\therefore$ A and B are not independent events

Generally well done.

The meaning of independent events is that the occurrence of one event does not affect the probability of the other.

Independence does not mean $P(A) \times P(B) = 0$ *or* $P(A \cap B) = 0$

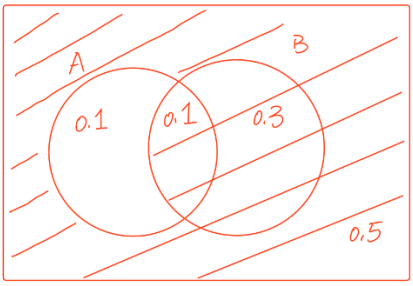
Only when A and B are mutually exclusive events, we can have $P(A \cap B) = 0$ *because the two events cannot occur together. Therefore,* $P(A \cup B) = P(A) + P(B)$ *cannot be used to show that A and B are independent events, it only shows that they are mutually exclusive.*

*Only stating "*A and B are not independent events*" was not sufficient to receive any marks.*
A valid justification was required.

Question 25 (continued)

(b) Evaluate $P\left(\overline{A} \cup B\right)$. **2**

Method 1:



$P(\overline{A} \cup B) = 1 - 0.1 = 0.9$

Method 2:

$$P(\overline{A}) = 1 - P(A) = 0.8$$
$$P(\overline{A} \cap B) = P(B) - P(\overline{A} \cap B)$$
$$= 0.4 - 0.1$$
$$= 0.3$$
$$P(\overline{A} \cup B) = P(\overline{A}) + P(B) - P(\overline{A} \cap B)$$
$$= 0.8 + 0.4 - 0.3$$
$$= 0.9$$

Method 3:

$$P(\overline{A} \cup B) = P(\overline{A}) + P(A \cap B)$$
$$= 0.8 + 0.1$$
$$= 0.9$$

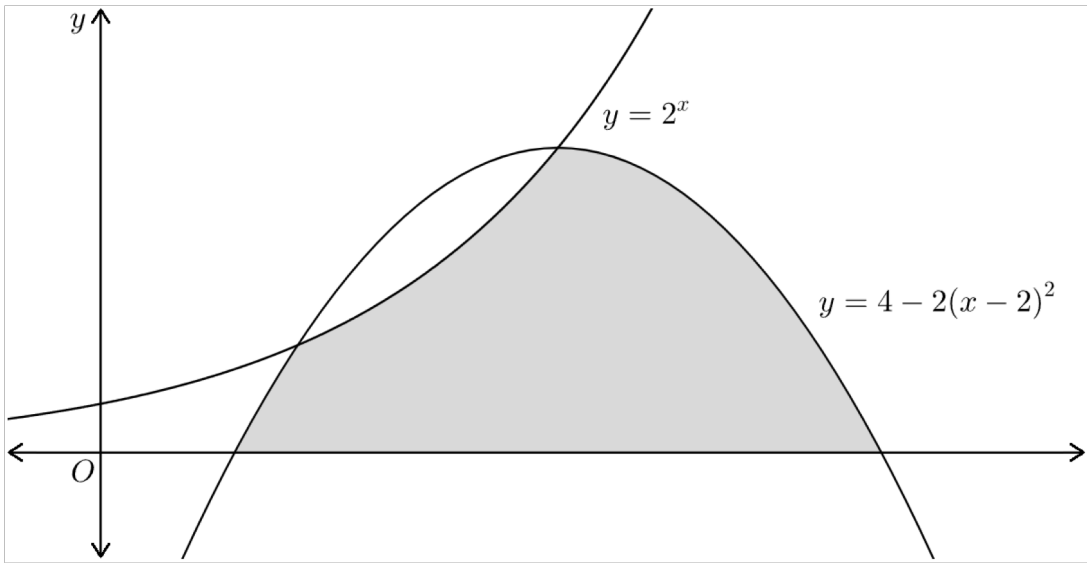
Poorly done.

The Venn Diagram was the most successful approach and was the recommended method.

The most common incorrect answer was 0.88, which resulted from an error in calculating $P(\overline{A} \cap B) = 0.8 \times 0.4 = 0.32$ *using the second method.*

Question 26

The graphs of $y = 2^x$ and $y = 4 - 2(x - 2)^2$ are shown.



(a) Prove that the graphs intersect at $(1, 2)$ and $(2, 4)$. **1**

Let $f(x)=2^x, g(x)=4-2(x-2)^2$

$f(1)=2,\ g(1)=4-2(1-2)^2=2$

$f(2)=4,\ g(2)=4-2(2-2)^2=0$

Therefore, the graphs intersect at $(1,2)$ and $(2,4)$

Well done.

No half marks

This question was expected to be a high-success question; however, many students found it more challenging than anticipated.

A common issue was overcomplicating the problem and attempting to solve for x in $2^x=4-2(x-2)^2$.

(b) Calculate the area of the shaded region. **4**

Find the x-intercepts of $y=4-2(x-2)^2$

$4-2(x-2)^2=0$

$(x-2)^2=2$

$\therefore x=2\pm\sqrt{2}$

$$A=2\int_2^{2+\sqrt{2}}\left[4-2(x-2)^2\right]dx-\int_1^2\left[4-2(x-2)^2-2^x\right]dx$$

$$=2\left[4x-\frac{2}{3}(x-2)^3\right]_2^{2+\sqrt{2}}-\left[4x-\frac{2}{3}(x-2)^3-\frac{2^x}{\ln 2}\right]_1^2$$

$$=2\left[4\times(2+\sqrt{2})-\frac{2}{3}(2+\sqrt{2}-2)^3-\left(4(2)-\frac{2}{3}(0)^3\right)\right]-\left[8-\frac{2}{3}(0)^3-\frac{2^2}{\ln 2}-\left(4-\frac{2}{3}(-1)^3-\frac{2^1}{\ln 2}\right)\right]$$

$$=2\left(8+4\sqrt{2}-\frac{4\sqrt{2}}{3}-8\right)-\left(8-\frac{4}{\ln 2}-4-\frac{2}{3}+\frac{2}{\ln 2}\right)$$

$$=\frac{16\sqrt{2}}{3}-\frac{10}{3}+\frac{2}{\ln 2}$$

Alternatively:

$$A=\int_{2-\sqrt{2}}^1\left[4-2(x-2)^2\right]dx+\int_1^2 2^x\,dx+\int_2^{2+\sqrt{2}}\left[4-2(x-2)^2\right]dx$$

$$=\left[4x-\frac{2}{3}(x-2)^3\right]_{2-\sqrt{2}}^1+\left[\frac{2^x}{\ln 2}\right]_1^2+\left[4x-\frac{2}{3}(x-2)^3\right]_2^{2+\sqrt{2}}$$

$$=\left[4-\frac{2}{3}(1-2)^3-\left(4(2-\sqrt{2})-\frac{2}{3}(2-\sqrt{2}-2)^3\right)\right]$$
$$+\left(\frac{2^2}{\ln 2}-\frac{2^1}{\ln 2}\right)+\left[4\times(2+\sqrt{2})-\frac{2}{3}(2+\sqrt{2}-2)^3-\left(4\times 2-\frac{2}{3}(2-2)^3\right)\right]$$

$$=\frac{8\sqrt{2}-10}{3}+\frac{2}{\ln 2}+\frac{8\sqrt{2}}{3}$$

$$=\frac{16\sqrt{2}}{3}-\frac{10}{3}+\frac{2}{\ln 2}$$

Question 26 (b) (continued)

Generally well done.

To receive full marks, students were required to

- correctly find the x-intercepts of $y = 4 - 2(x-2)^2$
- correctly establish the integrand required to find the area of the shaded region
- correctly finds the antiderivatives of $4 - 2(x-2)^2$ and 2^x
- obtain the correct final answer

Most students were able to address the first three points above. However, there were numerous calculation errors, which led to the incorrect final answer.

Instead of expanding $4 - 2(x-2)^2$, using the reverse chain rule to find the antiderivative is recommended, as this reduces the likelihood of making careless algebraic mistakes.

Some students were unable to find the antiderivative of 2^x. However, the relevant formula was provided on the reference sheet, so students should make effective use of the formulas available to them.

A common error was incorrectly calculating the area of the unshaded region as $\int_1^2 \left[2^x - \left(4 - 2(x-2)^2 \right) \right] dx$.

Question 27

Let $y = x^3 - 3x + 2$.

(a) Show that $y = (x-1)^2(x+2)$. **1**

Method 1:

$$\begin{aligned}(x-1)^2(x+2) &= (x^2-2x+1)(x+2)\\ &= x^3+2x^2-2x^2-4x+x+2\\ &= x^3-3x+2\end{aligned}$$

Method 2:

$$\begin{aligned}x^3-3x+2 &= x^3-2x^2+x+2x^2-4x+2\\ &= x(x^2-2x+1)+2(x^2-2x+1)\\ &= (x+2)(x-1)^2\end{aligned}$$

Well done.

No half marks

Sufficient working was required to receive the mark, as this is a "show that" question.

(b) Find the coordinates of any stationary points, and determine their nature. **2**

$y' = 3x^2 - 3$

$y'' = 6x$

Let $y' = 0$

$x = \pm 1$

When $x = 1$:

$y'' = 6 > 0$

$y = 0$

$\therefore (1,0)$ is the minimum turning point

When $x = -1$:

$y'' = -6 < 0$

$y = 4$

$\therefore (-1,4)$ is the maximum turning point

Well done.

Common mistakes:

- *A few students were unable to state the nature of the stationary points.*
- *A few students only found the x-coordinates of the stationary points, rather than giving the corresponding coordinates of the points.*

Question 27 (continued)

(c) Find the coordinates of any inflexion points. **1**

$y'' = 6x$
Let $y'' = 0$
$x = 0$
$\therefore y = 2$

x	-1	0	1
y''	-6	0	6
Concavity	$\cap$	$\bullet$	$\cup$

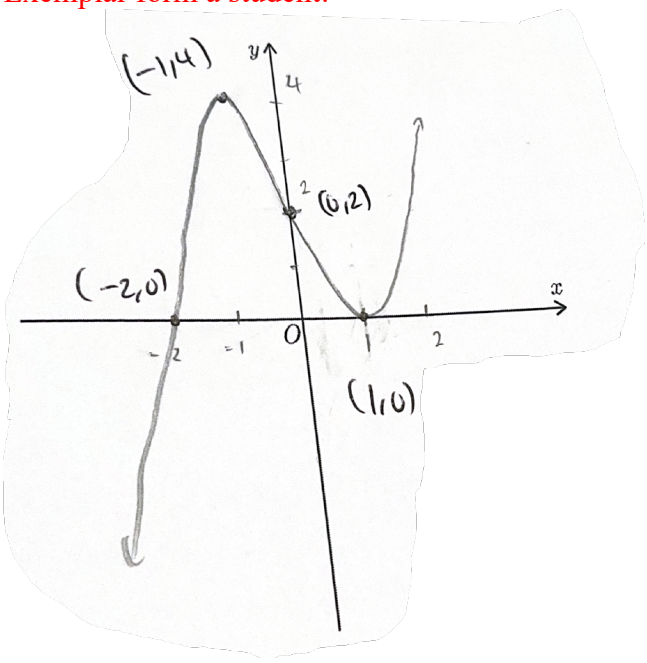
$\therefore$ The inflection point is $(0,2)$

Well done.

Students needed to check the concavity to demonstrate the existence of a point of inflexion.
A few students only found the x-coordinate of the point rather than giving the coordinates of the points.

(d) Hence, or otherwise, sketch the curve. **2**

Exemplar form a student:



Well done.

The concavity changes at $(0,2)$ from concave down to concave up. Some students sketched the graph in the opposite orientation.

All key points should be clearly labelled on the graph.

Question 28

For some real number $p > 1$, it is given that $\log_p 2 = x$ and $\log_p 3 = y$. **2**

Find in terms of x and y, the value of $\ln\left(12^{\frac{1}{\ln p}}\right)$.

$$\ln\left(12^{\frac{1}{\ln p}}\right) = \frac{1}{\ln p}\ln 12$$

$$= \frac{\ln 12}{\ln p}$$

$$= \log_p 12$$

$$= \log_p\left(2^2 \times 3\right)$$

$$= 2\log_p 2 + \log_p 3$$

$$= 2x + y$$

Poorly done.

A few students were able to show $\log_p 12 = 2x + y$, but they did not receive full marks because they did not demonstrate $\ln\left(12^{\frac{1}{\ln p}}\right) = \frac{1}{\ln p}\ln 12$.

End of Part D solutions

Question 29

An investigation into the relationship between the grams of steak s a person eats in a meal and their weight w in kilograms was conducted, and their results plotted in the table below.

s	310	180	420	270	240	390	250	360	290	320
w	85	55	90	75	50	80	65	95	60	70

(a) Which best describes the correlation between the quantities, Weak, Moderate or Strong? **1**

Calculator

Pearson's coefficient, $r = 0.8114835107$

Strong

Comments:
Well done.
Common mistake:
Some students didn't justify. They just stated 'Strong'. No half mark for this question.

(b) Find the line of best fit for the data. **1**

$y = A + Bx$

Calculator

$A = 21.53717975$
$B = 0.1681941262$

Comments: Well done.
Common mistakes:

Some students wrote the values of A and B in reverse.

Question 30

A discrete random variable X has the following probability distribution. **3**

x	1	3	4	5
$P(X = x)$	a	b	0.1	0.4

If $\text{Var}(X) = 1.94$ and $E(X^2) = 14.9$, calculate the values of a and b.

Method 1:

$a + b + 0.5 = 1$ (sum of probabilities)

$\therefore a + b = 0.5$

$$\begin{aligned} \text{E}(X) &= 1\times a + 3\times b + 4\times 0.1 + 5\times 0.4 \\ &= a + 3b + 2.4 \\ &= 0.5 - b + 3b + 2.4 \\ &= 2b + 2.9 \end{aligned}$$

$$\text{Var}(X) = \text{E}(X^2) - [\text{E}(X)]^2$$

$$\therefore 1.94 = 14.9 - (2b + 2.9)^2$$

$$\therefore (2b + 2.9)^2 = 12.96$$

$$\therefore 2b + 2.9 = 3.6 \qquad [0 \le b \le 1]$$

$$\therefore b = 0.35 \Rightarrow a = 0.15$$

Method 2:

$a + b + 0.5 = 1$

$\therefore a + b = 0.5$

$\text{Var}(x) = E(x^2) - [E(x)]^2$

$1.94 = 14.9 - [E(x)]^2$

$E(x) = 3.6$

$a + 3b + 4\times 0.1 + 5\times 0.4 = 3.6$

$\therefore a + 3b = 1.2$

$$\begin{cases} a + b = 0.5 & ① \\ a + 3b = 1.2 & ② \end{cases}$$

② − ①: $2b = 0.7$

$b = 0.35$

So $a = 0.5 - 0.35$

$= 0.15$

Method 3:

For those who substituted the other way:

$$\begin{aligned} \text{E}(X) &= a + 3b + 2.4 \\ &= a + 3(0.5 - a) + 2.4 \\ &= 3.9 - 2a \end{aligned}$$

$$\text{Var}(X) = \text{E}(X^2) - [\text{E}(X)]^2$$

$$\therefore 1.94 = 14.9 - (3.9 - 2a)^2$$

$$\therefore (3.9 - 2a)^2 = 12.96$$

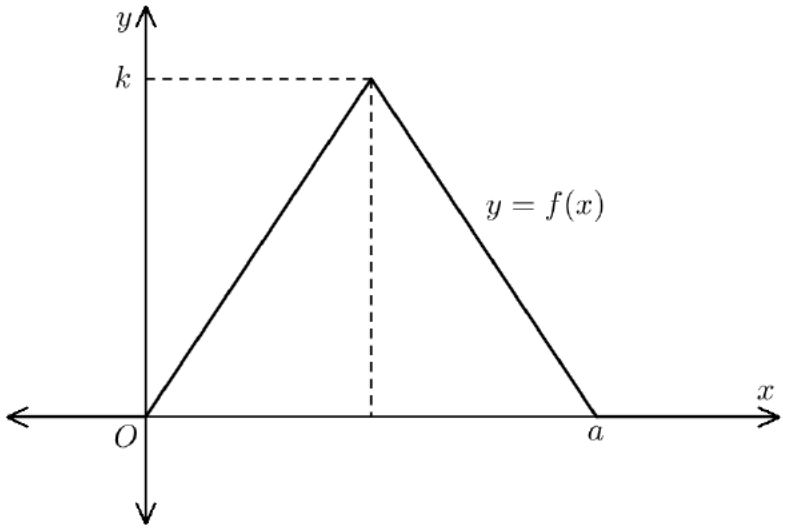
$$\therefore 3.9 - 2a = 3.6 \qquad [0 \le a \le 1]$$

$$\therefore a = 0.15 \Rightarrow b = 0.35$$

Comments:
Students did okay on this one.
Common mistakes:
Some students are very poor at algebraic techniques, such as substitution and solving simultaneous equations.

Question 31

A continuous random variable X with probability density function $f(x)$ follows the symmetrical triangle distribution between 0 and a, as shown in the diagram below.



(a) Find the value of k. **1**

The area of the triangle must equal 1.

$$\frac{1}{2} \times a \times k = 1$$

$$\therefore k = \frac{2}{a}$$

Comments:
Well done.
Common mistakes:
$k = 0.5$; $k = \frac{a}{2}$.

Question 31 (continued)

(b) Now, $f(x) = \frac{2k}{a}x$, for $0 \leq x < \frac{a}{2}$, where k is defined from (a). **1**

Write down the probability density function $f(x)$ for $0 \leq x \leq a$.

Your answer should be in terms of a and x only.

Due to the symmetry the gradient of the other side of the triangle is $-\frac{2k}{a} = -\frac{4}{a^2}$

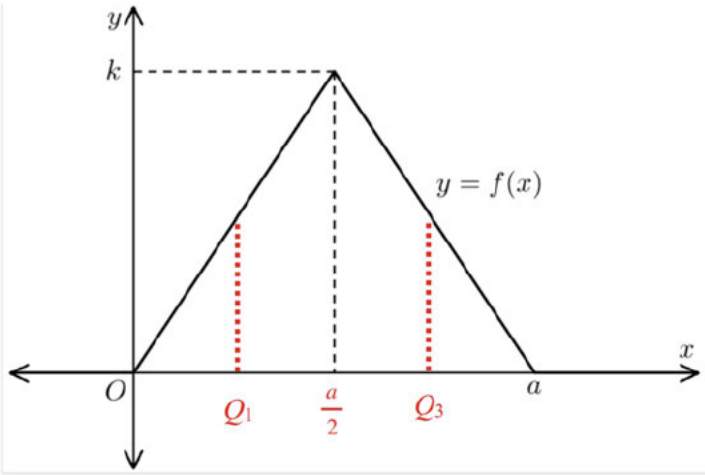
So the line has equation $y - 0 = -\frac{4}{a^2}(x - a) \Rightarrow y = -\frac{4}{a^2}(x - a)$

$$\therefore f(x) = \begin{cases} \frac{4}{a^2}x, & 0 \leq x < \frac{a}{2} \\ -\frac{4}{a^2}(x - a), & \frac{a}{2} \leq x \leq a \end{cases}$$

Comments:
Poorly done.
Common mistakes:
1. Some students left *k* in their solution.
2. Most students could get the equation of the first segment right, but many either only wrote the first one or couldn't get the second one correct.
3. Some students didn't write it as a piecewise function.

(c) Hence, find the inter-quartile range for values found from this distribution. **3**

Interquartile range is the middle 50% of scores.



Q_1 is where the area under the graph is a ¼ (of the area = 1)

i.e. $\int_0^{Q_1} f(x)\,dx = \frac{1}{4}$

$\therefore \frac{1}{2} \times Q_1 \times f(Q_1) = \frac{1}{4}$ (Area of triangle)

$\frac{1}{2} \times Q_1 \times \frac{4}{a^2}Q_1 = \frac{1}{4}$ (Using (b))

$Q_1^2 = \frac{a^2}{8}$

$\therefore Q_1 = \frac{a}{2\sqrt{2}} > 0$ (1)

Note that by symmetry, $a - Q_3 = Q_1 - 0$

$\therefore Q_3 = a - Q_1 = a - \frac{a}{2\sqrt{2}}$ (2)

$\therefore \text{IQR} = Q_3 - Q_1$

$= a - \frac{a}{2\sqrt{2}} - \frac{a}{2\sqrt{2}}$ (From (1) and (2))

$= a - \frac{a}{\sqrt{2}}$

$= \frac{\sqrt{2}-1}{\sqrt{2}}a$

Comments: Very poorly done. Common mistakes:

1. Many students had no clue on how to do this one.
2. Some students got stuck after getting Q1, they didn't notice and use the symmetry to get Q3 and left a mess in their calculation.

(d) Prove that all values that the random variable X takes are not outliers. **2**

$$\begin{aligned} Q_1 - 1.5 \times \text{IQR} &= \frac{a}{2\sqrt{2}} - 1.5 \times \left(\frac{\sqrt{2}-1}{\sqrt{2}} a \right) \qquad \text{(From (c))} \\ &= \frac{a}{2\sqrt{2}} - \frac{3}{2} \times \frac{\sqrt{2}-1}{\sqrt{2}} a \\ &= \frac{a}{2\sqrt{2}} \left[1 - 3(\sqrt{2}-1) \right] \\ &= \frac{a}{2\sqrt{2}} (4 - 3\sqrt{2}) \end{aligned}$$

$$\begin{aligned} Q_3 + 1.5 \times \text{IQR} &= a - \frac{a}{2\sqrt{2}} + 1.5 \times \left(\frac{\sqrt{2}-1}{\sqrt{2}} a \right) \qquad \text{(From (c))} \\ &= \frac{2\sqrt{2}-1}{2\sqrt{2}} a + \frac{3}{2} \times \frac{\sqrt{2}-1}{\sqrt{2}} a \\ &= \frac{a}{2\sqrt{2}} \left[(2\sqrt{2}-1) + 3(\sqrt{2}-1) \right] \\ &= \frac{a}{2\sqrt{2}} (5\sqrt{2} - 4) \end{aligned}$$

$\therefore$ For x to be an outlier, either

$x < Q_1 - 1.5 \times \text{IQR} = \dfrac{a}{2\sqrt{2}}(4 - 3\sqrt{2}) < 0$

or $x > Q_3 + 1.5 \times \text{IQR} = \dfrac{a}{2\sqrt{2}}(5\sqrt{2} - 4) > a$

However, $0 \leq x \leq a$, so no value of X is an outlier

Comments:
Poorly done, although ECF is allowed from (c).
Common mistakes:
This question asks students to prove, but many students:
1. didn't actually calculate the lower and upper boundaries of an outlier. They just stated the formula.
2. only calculated the lower boundary, but they didn't calculate the upper boundary instead they just used a 'similarly' argument.
3. didn't show $Q_1 - 1.5 \times IQR < 0$ and $Q_3 + 1.5 \times IQR > a$.

Question 32

The waiting time for a train can be modeled by a continuous random variable T, which has cumulative density function **2**

$$G(t) = \begin{cases} 0 & ,t < 0, \\ 1 - e^{-t^2} & ,t \geq 0. \end{cases}$$

Find the mode for the distribution of times.

Let $g(t)$ be the probability density function of T.

$$\begin{aligned} g(t) &= G'(t) \\ &= -(-2t)e^{-t^2} \\ \therefore g'(t) &= \frac{d}{dt}\left(2te^{-t^2}\right) \\ &= 2 \times e^{-t^2} + 2t \times (-2t)e^{-t^2} \\ &= 2e^{-t^2} - 4t^2e^{-t^2} \\ &= 2(1 - 2t^2)e^{-t^2} \end{aligned}$$

For the mode, set $g'(t) = 0$, so $2t^2 - 1 = 0 \Rightarrow t = \frac{1}{\sqrt{2}} > 0$

t	0.5	$\frac{1}{\sqrt{2}}$	1
$g'(t)$	0.78	0	−0.74

$\therefore$ Maximum turning point at $t = \frac{1}{\sqrt{2}}$ on $g(t)$

$\therefore$ Mode is $t = \frac{1}{\sqrt{2}}$ units

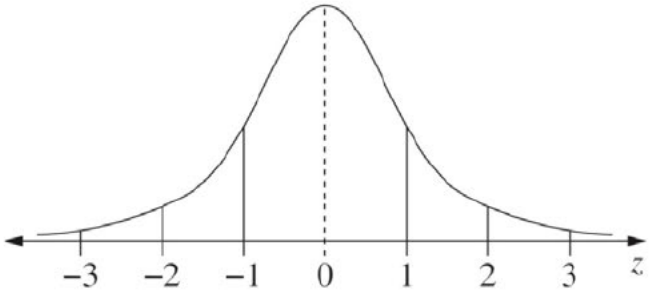
Comments:
Poorly done.
Common mistakes:

1. Many students didn't differentiate to get g'(t). They only got to $g(t) = -(-2t) \cdot e^{-t^2}$ and then solved g(t) = 0 to find the answer, which 0 mark is awarded.
2. Many students forgot to justify their answer using table or graph.
3. Some students made mistakes in differentiating.

Question 33

99.7% of the scores of a recent medical test are found to exist symmetrically between 60 and 96. Assuming the scores are normally distributed, what proportion of scores are going to exist between 66 and 84? **2**

From the Reference sheet:



- approximately 68% of scores have z-scores between −1 and 1
- approximately 95% of scores have z-scores between −2 and 2
- approximately 99.7% of scores have z-scores between −3 and 3

As the scores are normally distributed about the mean, then the mean μ is the mean of 60 and 96 = 78.
The score 96 represents $\mu + 3\sigma$, and so $\sigma = 6$

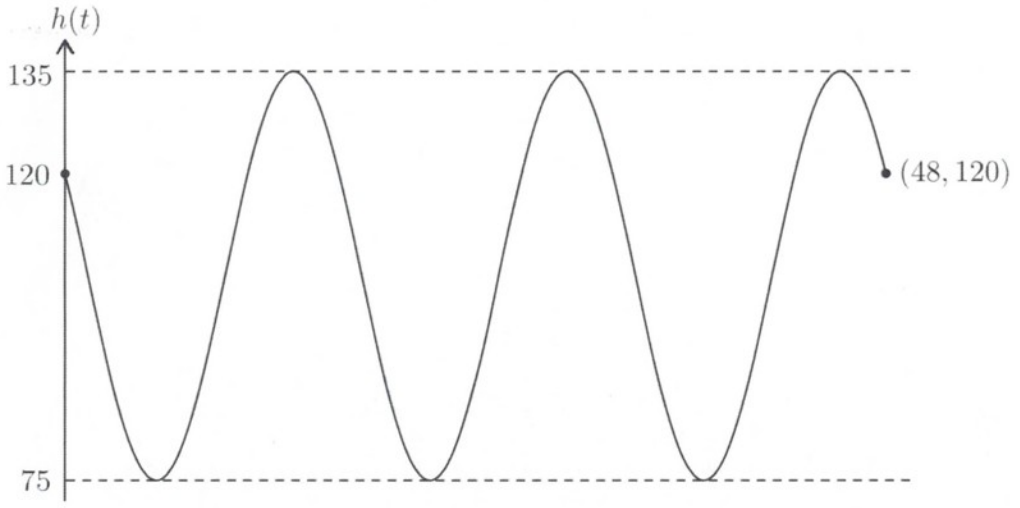
$\therefore 66 = \mu - 2\sigma$ and $84 = \mu + \sigma$

So the proportion of scores is $\frac{95\%}{2} + \frac{68\%}{2} = 81.5\%$

Comments:
Excellently done. Most students got full marks in this question.

Question 34

The rise and fall of the average heart rate $h(t)$ of an athlete through a race is modeled by a sinusoidal curve over the interval $0 \leq t \leq 48$, where t is the time in seconds from the start of the race.



The equation of the curve is $h(t) = a\sin\left[\frac{\pi}{8}(t-c)\right] + k$. The heart rate reaches a maximum of 135 beats per minute and a minimum of 75 beats per minute.

(a) Find the values of a and k, where a and k are positive constants. **2**

$$\text{Centre} = \frac{135+75}{2}$$

$$k = 105$$

$$a = 135-105 = 105-75$$

$$= 30$$

Comments: well done
k is the equilibrium point,
a is the amplitude

Question 34 (continued)

(b) The athlete started with an initial heart rate of 120 beats per minute. 1

What is the value of c?

$120 = 30\sin\left(\frac{\pi}{8}(0-c)\right)+105$

- from graph, $t=0$ is the second $\sin\theta = \frac{1}{2}$

$15 = 30\sin\left(\frac{\pi}{8}x - c\right)$

$\frac{15}{30} = \frac{1}{2} = \sin\left(\frac{\pi}{8}x - c\right)$



$-\frac{\pi}{8}c = \sin^{-1}\left(\frac{1}{2}\right)$

$= \frac{5\pi}{6}$ or $\frac{-7\pi}{6}$

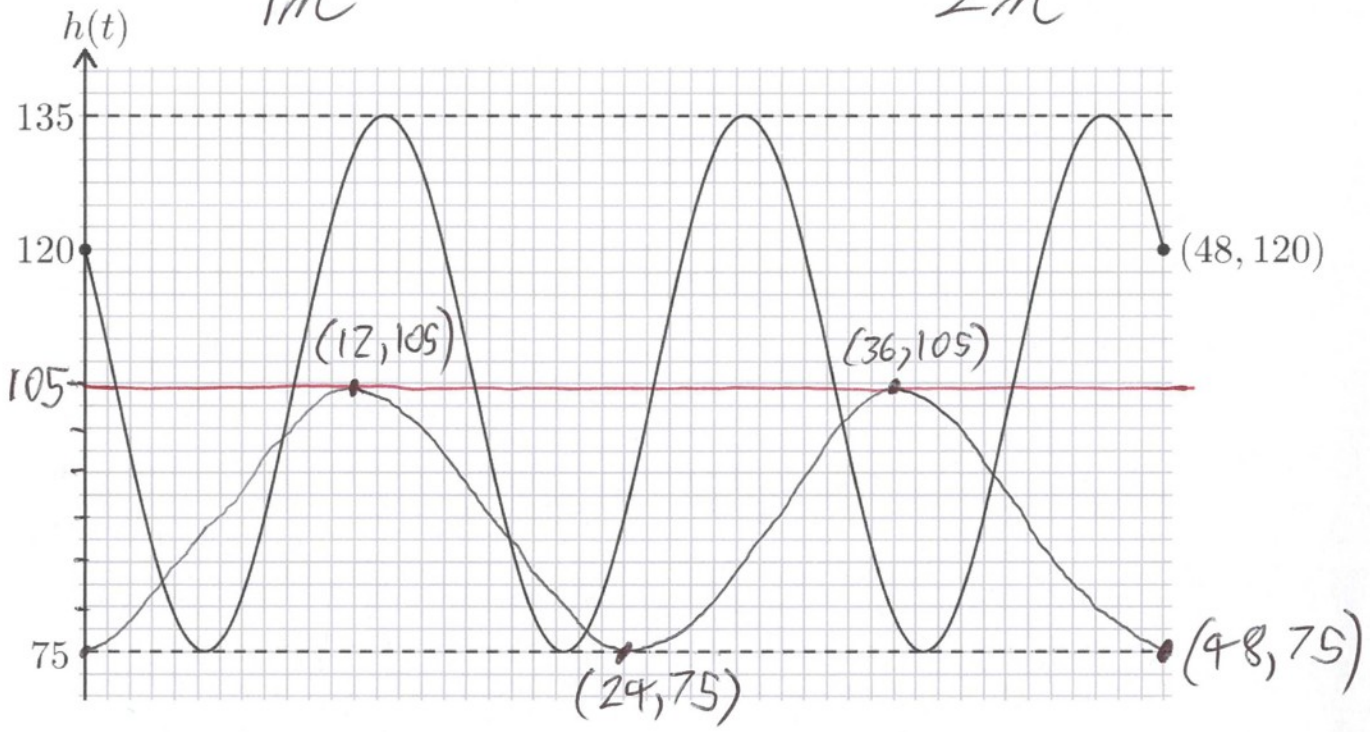
$c = -\frac{20}{3}$ or $\frac{28}{3}$

Comments: using desmos, these are the only 2 values of c that match the given curve. This is because the heart rate is decreasing and in front of the min. T.P.

- the most common mistake was using Q.1 $\sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$ ~~left~~ which gives $c = -\frac{4}{3}$

(c) A better trained athlete has the same minimum heart rate, but a maximum heartbeat of 105 beats per minute. The better athlete also has an initial heart rate of 75 beats per minute, and his heart rate per minute will oscillate with a period 50% longer than the original. Sketch his heart rate on the diagram provided, and state the equation. **3**

1m 2m



$T_{A1} = \frac{2\pi}{n} = \frac{2\pi}{\frac{\pi}{6}} = 16\,\text{sec}$

$T_{BTA} = 16 \times 1.5 = 24\,\text{s} \rightarrow \frac{2\pi}{n}$, $n = \frac{2\pi}{24} = \frac{\pi}{12}$

$h_{BTA}(t) = -15\cos\left(\frac{\pi}{12}t\right) + 90$

OR $+15\sin\left(\frac{\pi}{12}(t-6)\right) + 90$

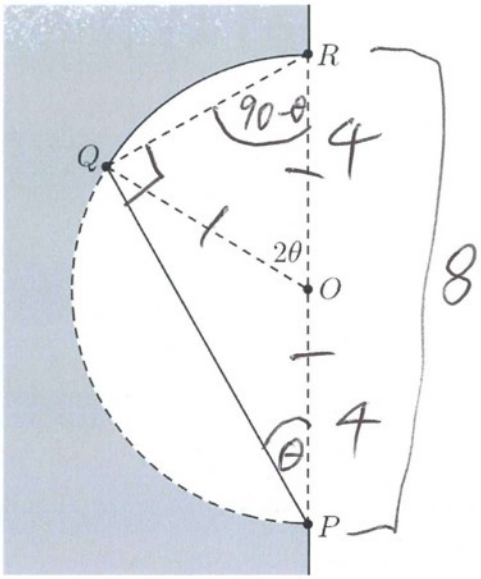
OR $+15\sin\left(\frac{\pi}{12}(t+18)\right) + 90$

because the sine curve is $\frac{1}{4}T$ behind cosine.

Comments: horribly drawn, & horribly answered.
- biggest issues: not starting & stopping at 75, not having 2 periods, not topping out at 105 (105 wrong place)

Question 35

A data center is stationed on the edge of a semi-circular bay of radius 4 km at P. A server is at R, positioned directly across the bay's diameter. An underwater connection could be laid from P to a relay server at Q, then wrapped along the curved semi-circular boundary above ground from Q to R.



The diameter of the bay is 8 km, centered at O.

$\angle PQR$ is a right angle (do NOT prove).

Laying a direct internet connection underwater costs 6 million dollars per kilometer, whereas a direct internet connection above ground costs a quarter as much.

Let $\angle QOR = 2\theta$.

(a) Prove the total cost in millions for laying the wires is $C = 12(4\cos\theta + \theta)$ dollars. **2**

$TC = 6 \times PQ + 6 \div 4 \times \text{arc } QR$

From external angle of iso. $\triangle OQP$,

$\angle OPQ + \angle OQP = 2\theta = 2\angle OPQ$

$\therefore \angle OPQ = \theta$ (marked on diagram)

in $\triangle PQR$, $\cos\theta = \frac{PQ}{8}$

$\therefore PQ = 8\cos\theta$

Question 35 (a) continues on page F #5

PTO

$$\text{arc } QR = l = r\theta$$
$$l = 4 \times 2\theta$$
$$= 8\theta$$

$$TC = 6 \times 8\cos\theta + \frac{3}{2} \times 8\theta$$
$$= 48\cos\theta + 12\theta$$
$$C = 12(4\cos\theta + \theta) \text{ as req.}$$

Comments: there is a much harder way of doing this, it req. $\cos 2\theta = 2\cos^2\theta - 1$
many students needed this formula but didn't use it, or they did, & met their doom on the rocky rocks of algebra

(b) Find the minimum cost to lay a connection from P to R. 4

$C = 48\cos\theta + 12\theta \quad , \quad 0 \leq \theta \leq \frac{\pi}{2}$

$C' = -48\sin\theta + 12 = 0 \quad (\text{or } 0 \leq 2\theta \leq \pi)$

$\sin\theta = \frac{1}{4}$

$\theta = \sin^{-1}(\frac{1}{4})$ in Q.1 & 2

$\theta \sim 0.253, \pi - 0.253\ldots \sim 2.8889$

x	0	0.253	1	2.889	π
$\frac{dC}{d\theta}$	+	0	−	0	+
slope	/	—	\	—	/

MAX MIN

- So $\theta = \sin^{-1}(\frac{1}{4})$ is a rad. T.P. & there is no need to calc. C because we want min. C.
- we don't want $\pi - \sin^{-1}(\frac{1}{4})$ either because this is in the 2nd Quad. & ∴ outside the valid Domain
- So we test the endpoints to find a local min.

$\theta = 0 \longrightarrow 12(4\cos 0 + 0) = 48 \times 1$

$= \$48M$

$\theta = \frac{\pi}{2} \longrightarrow 12(4\cos\frac{\pi}{2} + \frac{\pi}{2}) = 12 \times \frac{\pi}{2}$

$= 6\pi$

$= \$18.85M$

P.TO.

Question 35 (b) continues on page F #7

∴ the min. cost is $6π or $18.85M

Comments: horrible, horrible, horrible.

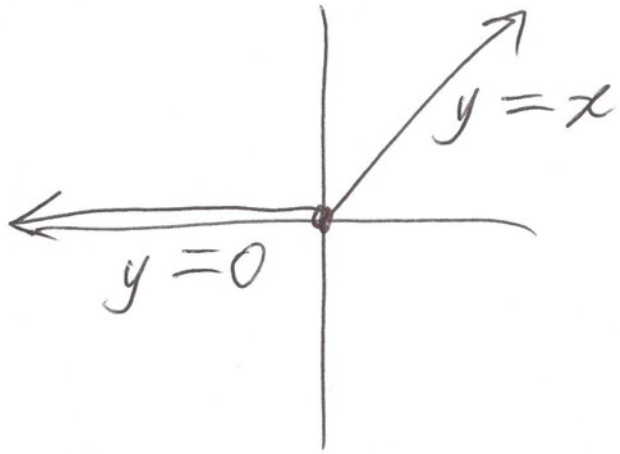
If I told you that going underwater was 6 times more expensive that going overground, then you would go overground, right? Well, this maths just proves it.

– there were mistakes in the first line of work
– there were mistakes in checking concavity, and saying the mad. T.P. was a min. T.P.
– there was a lack of insight in the domain ~~it~~ restriction in this context, & $\pi - \sin^{-1}(\frac{1}{4})$ was used as a valid min. T.P. BUT the (–) answer was ignored as (–), ie it was treated as (+) even though it was outside the valid domain.

Question 36

Let $y = f(x) = \begin{cases} 0 & , x < 0, \\ x & , x \geq 0. \end{cases}$ **2**

Express $y = f(x)$ as a single function without a piecemeal definition.



Q: what fn. when added to itself when $x<0$ cancels itself, & also is 'normal' when $x>0$

A: |absolute value|!

- so $x+|x|$ will cancel itself when $x \leq 0$
- but this $= 2x$ when $x>0$
- so we'll halve it: $f(x) = \dfrac{x+|x|}{2}$

OR $\dfrac{x}{2} + \dfrac{|x|}{2}$

Comments: anything with | | will be considered favourably, & $(\sqrt{x})^2$ is treated as | | equivalent

Section I Multiple Choice Solutions

1 Evaluate $\sin\frac{\pi}{7}$ to 3 significant figures.

A. 0.00783

B. 0.008

C. 0.433

D. 0.434

A	5
B	3
C	1
D	76

Need to be in RADIANS mode on calculator (and able to round)

2 Which of the following is equal to $\frac{9}{2^4}+\frac{27}{2^6}+\frac{81}{2^8}+\frac{243}{2^{10}}+\ldots$?

A. 4

B. $\frac{9}{4}$

C. $\frac{9}{7}$

D. $\frac{16}{7}$

A	1
B	77
C	6
D	1

$a=\frac{9}{2^4}$ and $r=\frac{3}{2^2}$

$S_\infty=\frac{a}{1-r}$

$=\frac{\frac{9}{16}}{1-\frac{3}{4}}$

$=\frac{9}{4}$

3 The number of collisions on a highway is inversely proportional to the number of lanes. If 6 collisions occurred on a 4 lane road, how many collisions are expected on a 2 lane road?

A. 12

B. 6

C. 3

D. 2

A	79
B	1
C	4
D	1

Method 1: Being inversely proportional, option A is the only possible answer.

Method 2: $c \propto \frac{1}{l} \Rightarrow cl = k$

$\therefore k = 6 \times 4 = 24$

$\therefore c = \frac{24}{2} = 12$

4 Which of the following is the domain of $y = \ln\left(\sqrt{x^2} - \sqrt{3}\right)$?

A. $(-\infty, -\sqrt{3}\,] \cup [\sqrt{3}, \infty)$

B. $(-\infty, -\sqrt{3}\,) \cup (\sqrt{3}, \infty)$

C. $(\sqrt{3}, \infty)$

D. $[\sqrt{3}, \infty)$

A	8
B	59
C	13
D	5

Options A and D are excluded due to $\pm\sqrt{3}$ [ln 0 is undefined]

Negative values of x can be included as y is an even function.

5 What is the future value interest factor for \$1 invested at the end of each period, at 4% per period over 3 periods?

A. 1.124864

B. 2.04

C. 3.12

D. 3.1216

A	43
B	1
C	3
D	38

What is \$1 worth after 3 periods?
Let A_n be the value after n periods.

$A_1 = 1$ [No interest gained]
$A_2 = A_1 \times 1.04 = 1.04$
$A_3 = A_2 \times 1.04 = 1.042 = 1.0816$

Future value = 1 + 1.04 + 1.0816 = 3.1216

6 Which of the following is equal to the derivative of $y = f(g(x))$?

A. $f'(g(x))$

B. $f'(g(x))g(x)$

C. $f'(g(x))g'(x)$

D. $f'(g'(x))$

A	8
B	5
C	71
D	1

For those who haven't memorised this:

let $u = g(x) \Rightarrow y = f(u)$

$$\frac{du}{dx} = g'(x); \frac{dy}{du} = f'(u)$$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$ [NESA Reference Sheet]

$$= f'(u) \times g'(x)$$

$$= f'(g(x)) \times g'(x)$$

7 Two standard dice are rolled and the numbers at the top are multiplied together. What is the difference in the probability that at least one of the dice shows a 6, with and without the information that the product is larger than 20?

A. $\frac{19}{36}$

B. $\frac{23}{72}$

C. $\frac{11}{36}$

D. $\frac{1}{9}$

A	46
B	13
C	17
D	9

×	1	2	3	4	5	6
1						
2						
3						
4					20	**24**
5				20	**25**	**30**
6				**24**	**30**	**36**

$P(\text{At least one 6}) = \frac{11}{36}$ (Yellow coloured cells)

$P(\text{At least one 6} \mid \text{Product} > 20) = \frac{5}{6}$ (Sample space = Red numbers)

$\text{Difference} = \frac{5}{6} - \frac{11}{36} = \frac{19}{36}$

8 Which of the following options is not equal to the others?

A. $\lim_{h\to 0}\left[\dfrac{f(x+h)-f(x-h)}{h}\right]$

B. $\lim_{h\to 0}\left[\dfrac{f(x-h)-f(x-2h)}{h}\right]$

C. $\lim_{h\to 0}\left[\dfrac{f(x)-f(x-h)}{h}\right]$

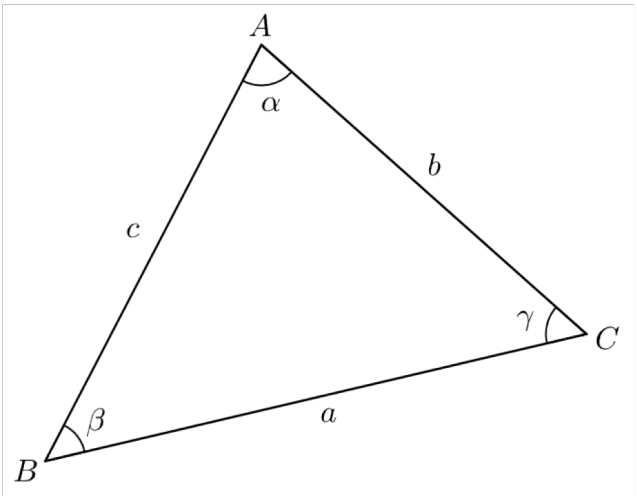
D. $\lim_{h\to 0}\left[\dfrac{f(x+3h)-f(x+h)}{2h}\right]$

A	34
B	14
C	17
D	20

Option A should have a $2h$. It is equal to $2f'(x)$.

The rest are all equal to $f'(x)$ by differentiation by first principles.

9 In the triangle $\triangle ABC$, let $\angle ABC = \beta, \angle BAC = \alpha$ and $\angle ACB = \gamma$. Also, let the side lengths BC, AC, AB be a, b and c respectively.



Consider the following scenarios.

- Given side lengths a, b, c.
  - ✓ Cosine rule to find an angle ⇒ Area of a triangle formula
    There is no 'ambiguous case' with the Cosine rule.

- Given side lengths a, b and angle γ
  - ✓ Area of a triangle formula

- Given side lengths a, b and angle α
  - ✗ Sine rule to find sin β, but then you have to determine γ.
    Need more information as you have to worry about the ambiguous case for β.

- Given side length a and angles α, β
  - ✓ Sine rule to get b ⇒ Area of a triangle formula

Basically all but the third dot point allowed you to determine congruence.

In how many of the above situations can the exact area of the triangle be calculated, without any additional information?

A. 1

B. 2

C. 3

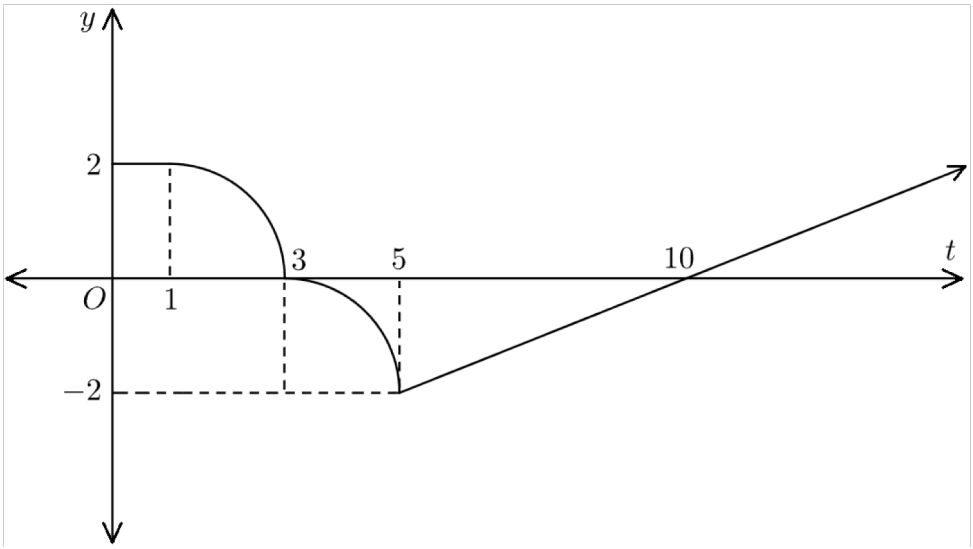
D. 4

A	5
B	6
C	32
D	42

MC #6

10 The graph shows the function $y = f(t)$.

The curves between $x = 1$ and $x = 5$ are quadrants of circles.



How many x-intercepts does the function $A(x) = \int_0^x f(t)\,dt$ have for $0 \leq x \leq 10$?

A. 0

B. 1

C. 2

D. 3

A	4
B	31
C	41
D	9

$A(0) = 0$

$A(1) = 2$

$A(3) = 2 + \frac{1}{4} \times \pi(2)^2 = 2 + \pi$

$A(5) = 2 + \pi + (\pi - 4) = 2\pi - 2$

$A(10) = 2\pi - 2 - \frac{1}{2} \times 2 \times 5 = 2\pi - 7 < 0$

So there must be an x-intercept for $5 < x < 10$.
For large values of x i.e. $x > 10$, $A(x) > 0$.
So there would be 3 in the domain of $A(x)$.

But for this question there are only 2.