Sydney Girls High School
2026
TRIAL HIGHER SCHOOL CERTIFICATE

EXAMINATION

Mathematics Advanced

General Instructions

- Reading time – 10 minutes
- Working time – 3 hours
- Write using blue or black pen
- Calculators approved by NESA may be used
- A reference sheet is provided
- For questions in Section II, show relevant mathematical reasoning and/or calculations

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Total Marks:

100

Section I – 10 marks

- Attempt Questions 1–10
- Allow about 15 minutes for this section

Section II – 90 marks

- Attempt Questions 11–32
- Allow about 2 hours and 45 minutes for this section

THIS IS A TRIAL PAPER ONLY

It does not necessarily reflect the format or the content of the 2026 HSC Examination.

Questions	M.C.	Q11-16	Q17-20	Q21-27	Q28-29	Q30-32	Total
Total	/10	/14	/14	/30	/15	/17	/100

Section I

10 marks

Attempt Questions 1-10

Allow about 15 minutes for this section

Use the multiple-choice answer sheet for questions 1 – 10

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1 The domain of the function $y = \frac{\sqrt{x}}{x-1}$ is ?

A) $[0, \infty)$

B) $(1, \infty)$

C) $[0, 1) \cup (1, \infty)$

D) $(0,1) \cup (1, \infty)$

2 Consider the function $f(x) = |4 + 2x|$, where $-6 \leq x \leq 2$.
Which of the following is NOT true about $f(x)$?

A) The range of $f(x)$ is $0 \leq y \leq 8$

B) The line $x = -2$ is an axis of symmetry of $f(x)$.

C) $f(x)$ is a continuous function over the entire domain $-6 \leq x \leq 2$.

D) $f(x)$ is a differentiable function over the entire domain $-6 \leq x \leq 2$.

3 Simplify $\tan(\pi + x)\cos(-x) + \cos\left(\frac{\pi}{2} - x\right)$.

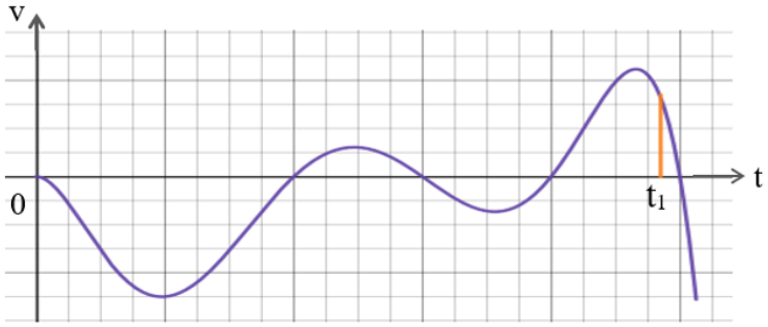
A) $-2\sin x$

B) $\sin x + \cos x$

C) 0

D) $2\sin x$

4 The graph shows the velocity of a particle that starts to move from the origin along the t axis.



Which statement describes the motion of the particle at the point where $t = t_1$?

A) The displacement is positive and the acceleration is negative.

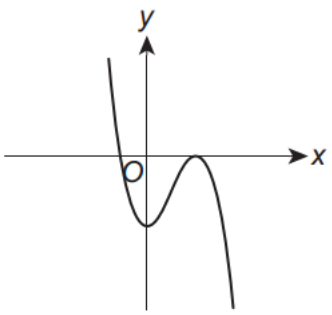
B) The displacement is negative and the acceleration is positive.

C) The displacement is positive and the acceleration is positive.

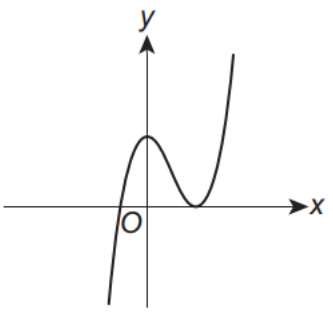
D) The displacement is negative and the acceleration is negative.

5 Which of the following graphs best represents the function $f(x) = b(x - b)(x + 2b)^2$, where $b < 0$?

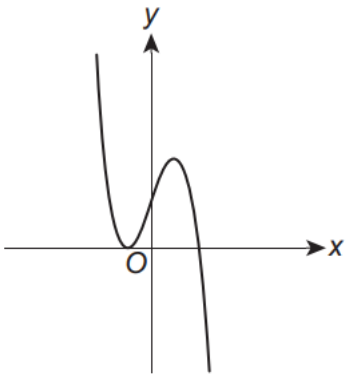
A.



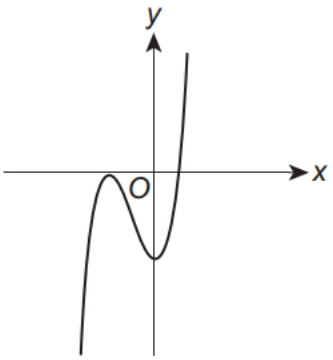
B.



C.



D.



6 A food critic scores restaurants each year, where a positive score means the restaurant exceeded expectations and a negative score means it fell below expectations. The scores follow a normal distribution where the mean score is equal to twice the standard deviation.

Scores were recorded for all restaurants reviewed over one year.

What percentage of the restaurants received a score below 0?

A) 50 %

B) 16 %

C) 2.5 %

D) 0.15 %

7 At a busy cafe, 70% of customers order coffee and 45% order food. A customer who orders food has a 60% chance of also ordering coffee. A customer is chosen at random. What is the probability that they order coffee but not food?

A) 0.27

B) 0.31

C) 0.43

D) 0.6

8 The functions $P(x), Q(x)$ and $R(x)$ are defined for all real x and the function $R(x) = P[Q(x)]$.
Given that $Q(2) = 5$ and $Q'(2) = 8$ and $P'(5) = -3$, what is the gradient of the normal to the curve $R(x)$ at $x = 2$?

A) 24

B) $\frac{1}{24}$

C) $-\frac{1}{24}$

D) -24

9 Find all real values of x satisfying:

$$\log_2(x) - 1 = \frac{3 - \log_2 x}{2} + \left(\frac{3 - \log_2 x}{2}\right)^2 + \left(\frac{3 - \log_2 x}{2}\right)^3 + \cdots$$

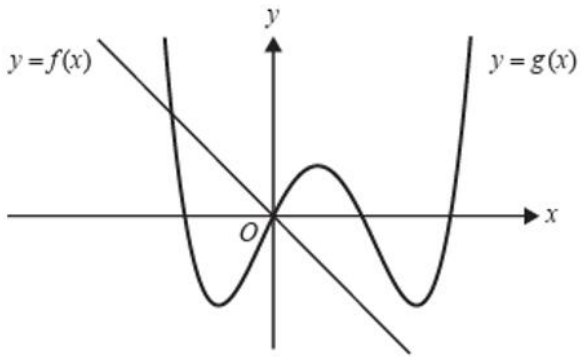
A) $x = 4$

B) $x = \frac{1}{2}$

C) $x = \frac{1}{2},\ x = 4$

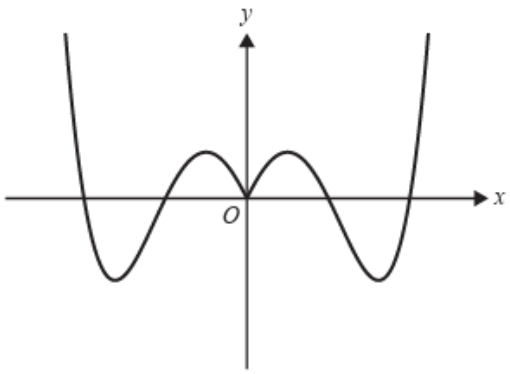
D) $x = 2,\ x = 4$

10 The graphs of $y = f(x)$ and $y = g(x)$ are sketched on the same set of axes below.

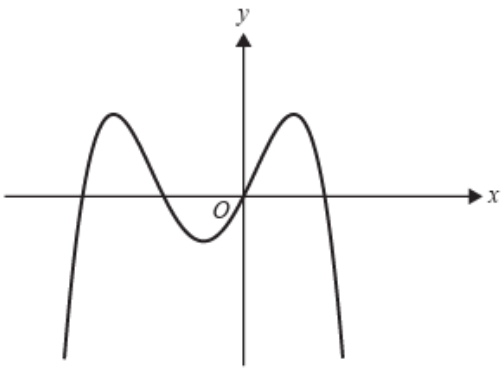


Which of the following could be the graph of $y = g\big(f(x)\big)$?

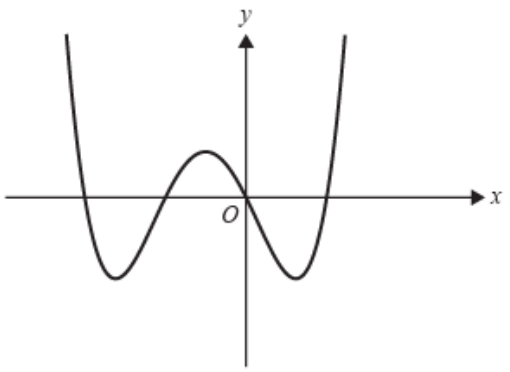
A.



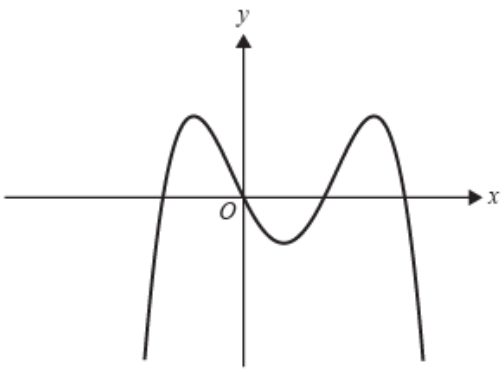
B.



C.



D.



Mathematics

Advanced Section II

90 marks

Attempt Questions 11 – 32

Allow about 2 hours and 45 minutes for this section

Instructions

- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
- Your responses should include relevant mathematical reasoning and/or calculations.
- Please clearly mark the questions written on the papers provided at the end of the paper.

Question 11 (2 marks)

The first term of an arithmetic series is -7 and the sum of the first 20 terms is 430. Find the common difference. **2**

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Question 12 (2 marks)

Find the coordinates of the centre and the radius of the circle with equation: **2**

$$x^2 + y^2 - 8x + 10y + 5 = 0$$

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Question 13 (2 marks)

Evaluate the definite integral below, leaving your answer in exact form. **2**

$$\int_0^{\frac{\pi}{4}} \sin(2x) \cdot e^{\cos 2x}\, dx$$

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Question 14 (4 marks)

Differentiate the following with respect to x :

(a) $y = \ln \sqrt{x}$ **2**

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(b) $y = x^2 e^{-x}$ **2**

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Question 15 (2 marks)

Prove that **2**

$$\frac{\sin\theta}{1-\cos\theta} - \frac{\sin\theta}{1+cos\theta} = 2\cot\theta$$

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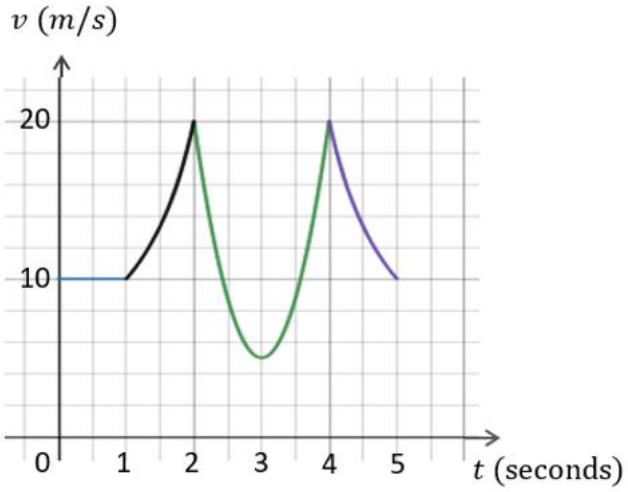
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Question 16 (2 marks)

The graph below shows the velocity (in metres per second) of a particle over a 5 second period. The particle reaches its lowest velocity of 5m/s after 3 seconds.



Use the trapezoidal rule with 5 applications to determine an estimate for the distance the particle has travelled over the 5 seconds. **2**

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Question 17 (2 marks)

Solve the following equation for x. **2**

$$2\log_2 x - \log_2(x+3) = 2$$

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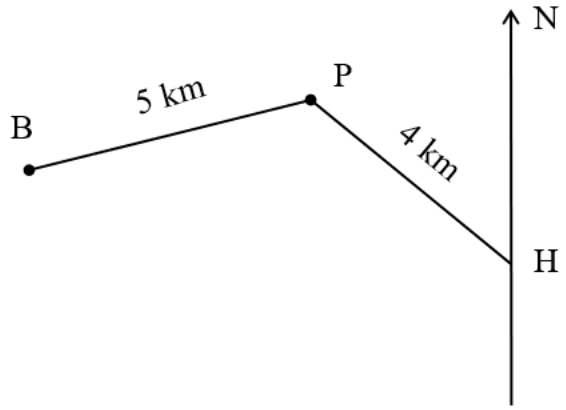
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Question 18 (5 marks)

Katie starts riding her bike from her house *H*.

She rides 4 km on a bearing of $320°$ to the park P then rides 5 km on a bearing of $250°$ to the beach *B*.



(a) Use the diagram to show that angle HPB is $110°$. **1**
No geometric reasoning is required.

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(b) Find the distance BH, correct to 3 decimal places. **2**

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(c) Hence, find the bearing of Kate's house *H* from the beach *B,* to the nearest minute. **2**

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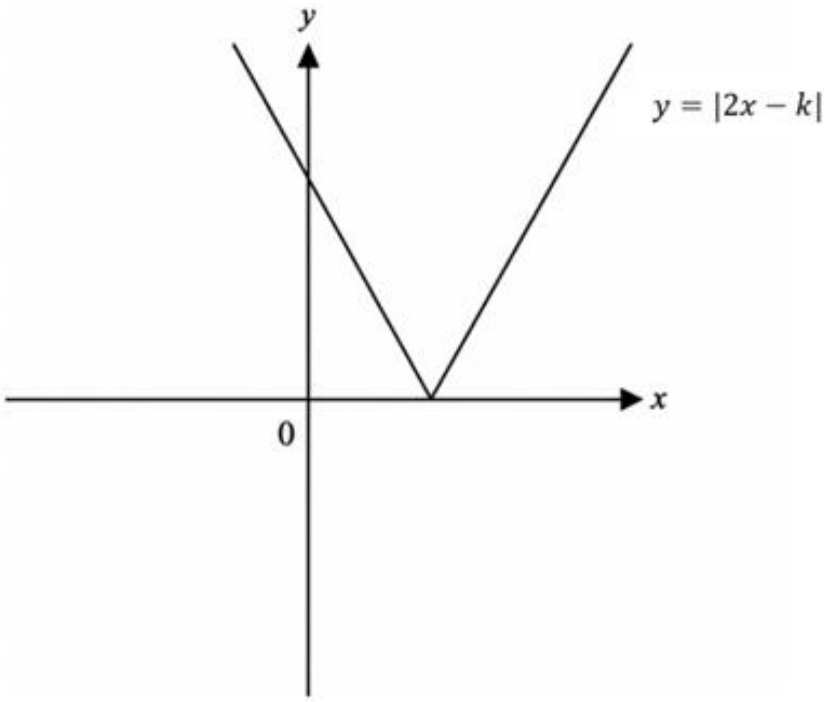
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Question 19 (4 marks)

The diagram below shows the graph of the function, $y = |2x - k|$ for some constant k.



(a) On the diagram, mark in terms of k, the intercepts on the axes. **2**

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(b) On the same diagram, sketch the function $y = k - |2x - k|$ showing in terms of k any intercepts on the axes and the coordinates of the vertex. **2**

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Question 20 (3 marks)

After a patient receives an injection, the concentration of the drug $C(t)$ in their bloodstream (in mg/L) is modelled by:

$$C(t) = Ae^{-0.14t}$$

where t is the time in hours after the injection. The drug becomes ineffective when the concentration falls below 2 mg/L. A doctor measures the concentration to be 12.5 mg/L exactly 4 hours after the injection.

(a) Find the initial concentration A at the time of injection, correct to one decimal place. **1**

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(b) Find the time t after the injection at which the drug becomes ineffective. Give your answer to the nearest minute. **2**

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Question 21 (3 marks)

A table of future values for an annuity of $1 is shown below.

Table of future value interest factors								
	Interest rate per period							
Period	0.50%	1.00%	1.50%	2.00%	3.00%	4.00%	5.00%	6.00%
2	2.0050	2.0100	2.0150	2.0200	2.0300	2.0400	2.0500	2.0600
3	3.0150	3.0301	3.0452	3.0604	3.0909	3.1216	3.1525	3.1836
4	4.0301	4.0604	4.0909	4.1216	4.1836	4.2465	4.3101	4.3746
5	5.0503	5.1010	5.1523	5.2040	5.3091	5.4163	5.5256	5.6371
6	6.0755	6.1520	6.2296	6.3081	6.4684	6.6330	6.8019	6.9753
7	7.1059	7.2135	7.3230	7.4343	7.6625	7.8983	8.1420	8.3938
8	8.1414	8.2857	8.4328	8.5830	8.8923	9.2142	9.5491	9.8975

(a) Sam contributes $3,000 at the end of each quarter into his superannuation account, which earns interest at 6% p.a compounded quarterly, for 2 years. What is the total amount of interest earned? **2**

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(b) Elena makes equal contributions of $\$M$ at the end of every six months into her investment account, which earns interest at 4% p.a compounded six-monthly. What is the value of M if the future value is $37, 848.60 after 3 years? **1**

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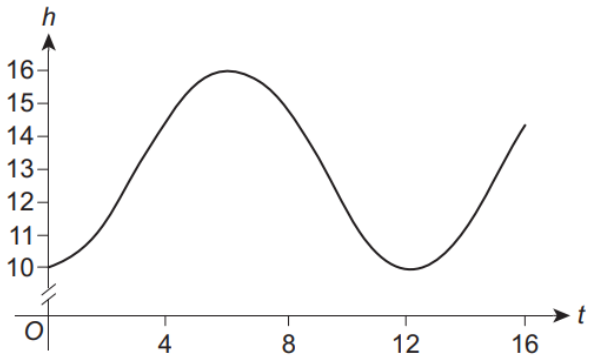
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Question 22 (4 marks)

The water level of a harbour rises and falls according to changes in the tide. Ships can enter the harbour when the water is at a suitable level.

The harbour's water level can be modelled by the function $h(t) = A\cos(Bt) + 13$, where h is the water level in metres above the seabed and t is the number of hours after 8 am on a Saturday.

Serena is planning to enter the harbour in a ship. She observes the water level from 8 am until midnight on Saturday. At 8am during a low tide, the water level is 10m. At 2 pm during a high tide, the water level is 16m. The graph shows the changes in water level that Serena observed during the 16- hour period.



(a) Find the values of A and B. **2**

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(b) Serena's ship requires the water level to be at least 11.5m to enter safely. At what times between 8am and midnight on Saturday could Serena safely enter the harbour? **2**

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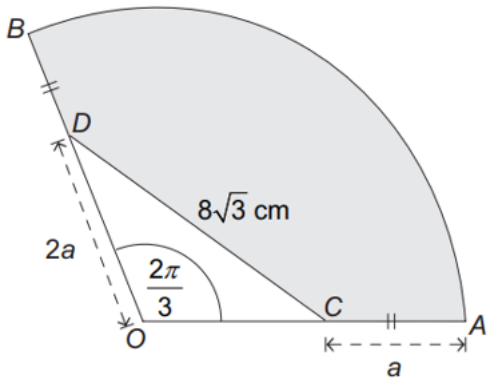
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Question 23 (3 marks)

The triangle COD is inside a sector that has a central angle of $\frac{2\pi}{3}$, as shown in the diagram.



The lengths of the sector and triangle are as follows.

$$DC = 8\sqrt{3}\ ,\ AC = BD\ ,\ OC = 2AC$$

Find the exact area of the shaded region. **3**

Question 24 (4 marks)

A statistician is analysing whether chess players perform better when they stay in a hotel with a higher rating. Hotels are rated by online reviews on a scale from 3 to 5, with 3 being the worst and 5 being the best rating that can be achieved. Chess players are rated by the Elo score ranging from 1000 to 2800. The higher the Elo score the better the performance by the chess player.

The table below shows the hotel rating and the Elo score for chess players.

Hotel rating (x)	5	4.3	3.9	3.5	4.2	4.8	3.1	4.5	4.7	3
Elo rating (y)	2650	2700	2100	2000	2500	2600	1800	2450	2750	1900

(a) Determine the equation of the least-squares regression line and correlation coefficient for this data. Give answers to 2 decimal places. **2**

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(b) Interpret the gradient of the least squares line in the context of the question. **1**

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(c) Describe the correlation coefficient in terms of its strength and direction. **1**

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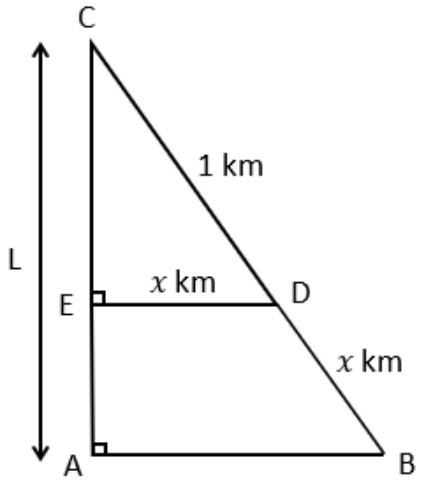
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Question 25 (4 marks)

ABC is a right-angled triangle at A.

The point D lies on the hypotenuse BC such that $BD = x$ and $DC = 1$ km.

The point E lies on AC such that DE is parallel to BA and $DE = x$ km as shown below.



(a) It is given that $\frac{CA}{CE} = \frac{CB}{CD}$.

Show that the length of side AC is $L = (1 + x)\sqrt{1 - x^2}$ **1**

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Question 25 (continued)

(b) Find the exact maximum length L of side AC. Use mathematical reasoning to justify that this is the maximum length of L. **3**

Question 26 (8 marks)

Consider the function $f(x) = xe^{-\frac{x}{2}}$

(a) Find all stationary points and determine their nature. **3**

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(b) Find any points of inflection. **2**

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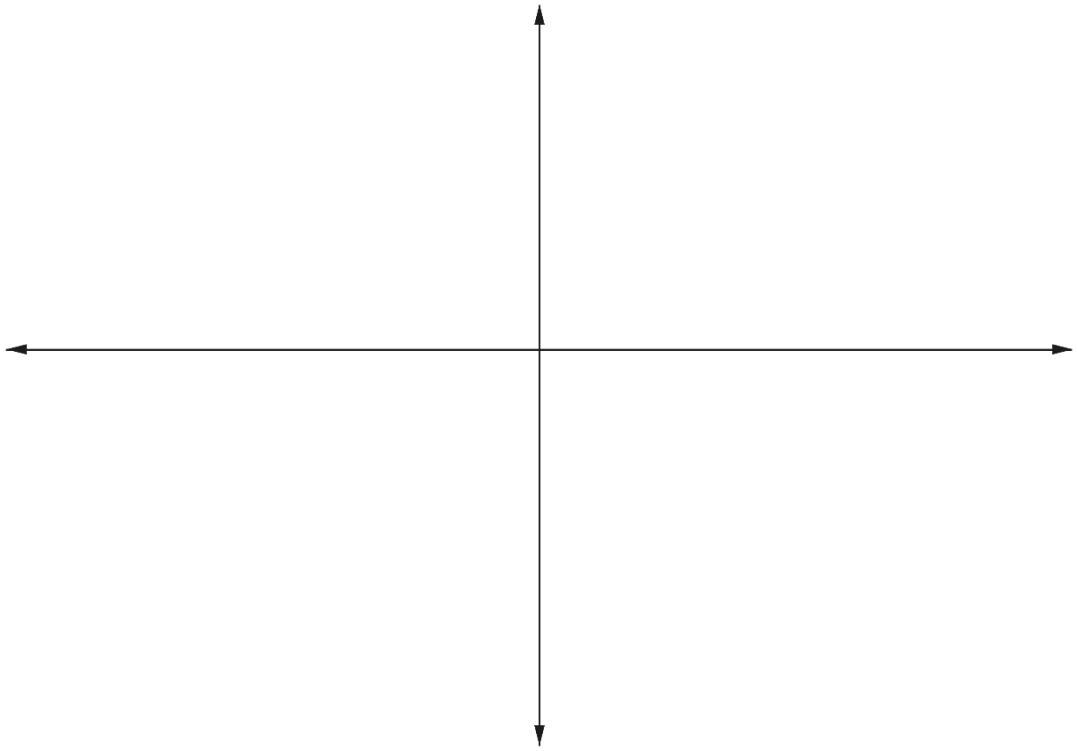
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Question 26 (continued)

(c) Sketch $y = f(x)$ labelling all key features. **3**



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Question 27 (4 marks)

The table below shows the probability that a random variable z, is less than Z.

z	−2	−1.5	−1	−0.5	0	0.5	1	1.5	2
$P(z < Z)$	0.0228	0.0668	0.1587	0.3085	0.5	0.6915	0.8413	0.9332	0.9772

For a particular airline, the number of minutes that the arrival time of a flight varies from its scheduled arrival time is normally distributed. A negative x score indicates that the flight arrived early, and a positive x score indicates that the flight arrived late.

(a) For this airline, 30.85% of flights arrive more than 6 minutes early, while 2.28% of flights arrive more than 14 minutes late. Find the mean and standard deviation of this distribution. **2**

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(b) Effie is on a flight that has a scheduled arrival time of 16:14. Using the table above determine the percentage chance that her flight will arrive before 16:00? Give your answer correct to two decimal places. **2**

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Question 28 (7 marks)

A particle P starts from a point O and moves along a horizontal straight line. Its velocity v after t seconds is given by:

$$v(t) = \begin{cases} 4t - 1, & \text{for } 0 \leq t \leq 1 \\ 3\sqrt{t}, & \text{for } 1 < t \leq 9 \end{cases}$$

(a) Does the particle ever change direction? Justify your answer fully. **2**

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(b) Find the total distance travelled by the particle in the first 9 seconds. **3**

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Question 28 (continued)

(b)

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(c) At $t = 0$ a second particle Q starts from the same point O and moves along the same line with velocity $w(t) = kt$, where k is a positive constant. Given that P and Q have the same displacement from O at $t = 9$, find the value of k. **2**

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Question 29 (8 marks)

(a) Find the x coordinates of the points of intersection of $y = -3\cos x$ and $y = \tan x$ for $-\pi \leq x \leq \pi$. Give answers to 2 decimal places. **3**

Question 29 (continued)

(b) For $-\pi \leq x \leq \pi$, sketch the graph of the equations $y = -3\cos x$ and $y = \tan x$ on the same set of axes. **2**

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(c) Hence, find the area enclosed between the two curves and the y −axis. **3**

Give your answer to 2 decimal places

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Question 30 (7 marks)

The duration of telemarketing calls to mobile phone users is a continuous random variable t hours, with probability density function:

$$f(t) = \begin{cases} at \sin\left(\frac{\pi}{2}t\right), & \text{for } 0 \leq t \leq 1 \\ 0, & \text{otherwise} \end{cases}$$

(a) Show $\frac{d}{dt}\left(\frac{4}{\pi^2}\sin\frac{\pi}{2}t - \frac{2}{\pi}t\cos\frac{\pi}{2}t\right) = t\sin\frac{\pi}{2}t$ **2**

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(b) Hence, show that $a = \frac{\pi^2}{4}$. **1**

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Question 30 (continued)

(c) Show that the probability that a call lasts for more than 30 minutes is: **2**

$$1 - \frac{1}{\sqrt{2}} + \frac{\pi}{4\sqrt{2}}$$

Question 30 (continued)

(d) The telemarketing company also surveys customers on how the call made them feel, rating their experience from 1 (very negative) to 5 (very positive).

The survey results were normally distributed with a mean of 3 and a standard deviation of 0.5.

The probability that a call received a rating of 2 or lower given that the call lasted more than 30 minutes is 1.5%. Use the empirical rule when required to answer the following.

Find the probability that a call lasted more than 30 minutes, given that the customer gave a rating of 2 or lower. **2**

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Question 31 (6 marks)

Elsha is planning to save \$110 000 over 5 years by depositing her monthly salary of $\$M$ at the beginning of each month into a savings account. The account pays interest at 6% p.a compounded monthly. She will need to withdraw \$500 at the end of each month to cover her living expenses.

Let A_n represent the balance in the account at the end of month n.

(a) Show that at the end of the second month, immediately after making her \$500 withdrawal, the balance is given by: **1**

$$A_2 = M(1.005^2 + 1.005) - 500(1.005 + 1)$$

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(b) Show that $A_n = 201M(1.005^n - 1) - 100\,000(1.005^n - 1)$. **2**

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Question 31 (continued)

(c) Hence, calculate Elsha's monthly salary, $\$M$. Give your answer to the nearest cent. **1**

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(d) After the 12^{th} withdrawal Elsha deposits \$6000 she received from her grandparents. She decides she can now afford to withdraw a total of \$600 at the end of each month for her living costs. **2**

How many more complete months will it take Elsha to reach her \$110,000 goal?

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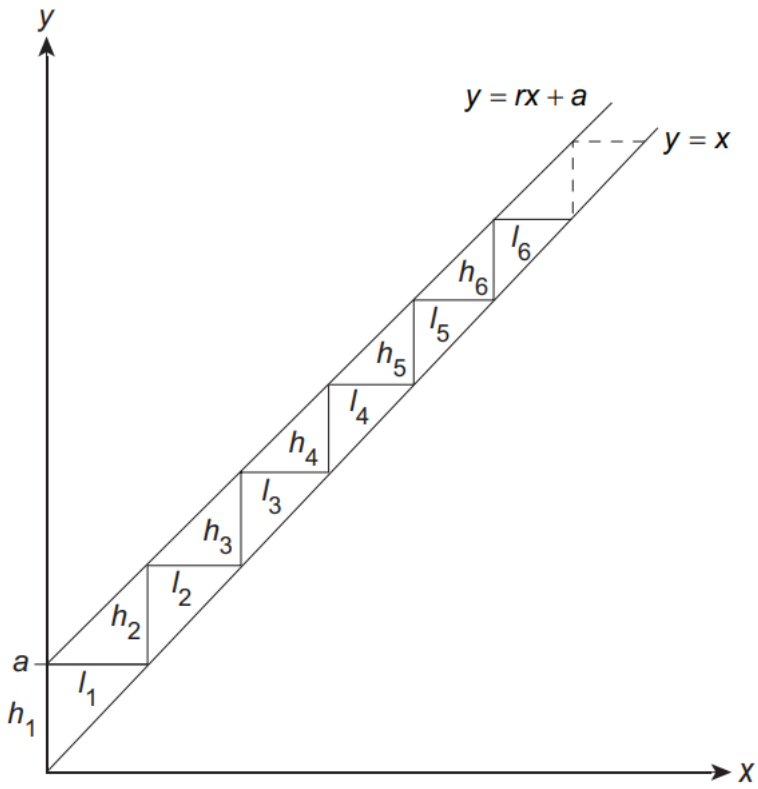
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Question 32 (4 marks)

Consider the lines $y = x$ and $y = rx + a$, where $r > 0$ and $a > 0$.

The two lines intersect at an unknown point in the first quadrant. A sequence of right-angled triangles is constructed between the two lines. The heights (h)and lengths (l) of the triangles are labelled as $h_1, l_1, h_2, l_2, h_3, l_3$.....

The graph shows these two lines and the triangles between them.



(a) State the range of the values of r for which the two lines intersect in the first quadrant. **1**

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(b) Find the x- coordinate of the intersection point in terms of r and a. **1**

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Question 32 (continued)

(c) Using infinite series only, find the total area of the triangles between the lines. **2**

Give your answers in terms of a and r.

End of paper

Section II extra writing space

If you use this space, clearly indicate which question you are answering.

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Section I

10 marks

Attempt Questions 1-10

Allow about 15 minutes for this section

Use the multiple-choice answer sheet for questions 1 – 10

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1 The domain of the function $y = \frac{\sqrt{x}}{x-1}$ is ?

$x \neq 1 \quad x \geq 0$

A) $[0, \infty)$

B) $(1, \infty)$

C) $[0, 1) \cup (1, \infty)$

D) $(0,1) \cup (1, \infty)$

2 Consider the function $f(x) = |4 + 2x|$, where $-6 \leq x \leq 2$.
Which of the following is NOT true about $f(x)$?

A) The range of $f(x)$ is $0 \leq y \leq 8$

B) The line $x = -2$ is an axis of symmetry of $f(x)$.

C) $f(x)$ is a continuous function over the entire domain $-6 \leq x \leq 2$.

D) $f(x)$ is a differentiable function over the entire domain $-6 \leq x \leq 2$.

3 Simplify $\tan(\pi + x)\cos(-x) + \cos\left(\frac{\pi}{2} - x\right)$.

A) $-2\sin x$

B) $\sin x + \cos x$

C) 0

D) $2\sin x$

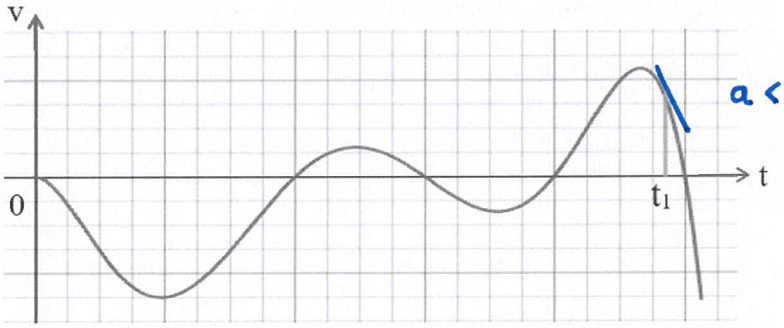
even f(n) ↙ comp. ↘ LS.

$= \tan x \times \cos x + \sin x$

$= \frac{\sin x}{\cos x} \times \cos x + \sin x$

$= 2\sin x$

4 The graph shows the velocity of a particle that starts to move from the origin along the t axis.



$a < 0$

area under graph)

area above

$\therefore$ disp < 0

Which statement describes the motion of the particle at the point where $t = t_1$?

A) The displacement is positive and the acceleration is negative.

B) The displacement is negative and the acceleration is positive.

C) The displacement is positive and the acceleration is positive.

D) The displacement is negative and the acceleration is negative.

neg ∴ A or C

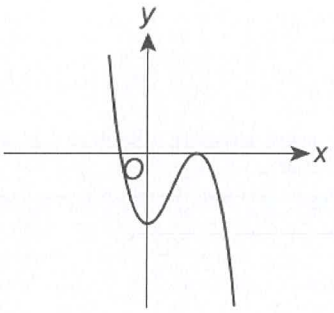
5 Which of the following graphs best represents the function $f(x) = b(x - b)(x + 2b)^2$, where $b < 0$?

Let $b = -3$

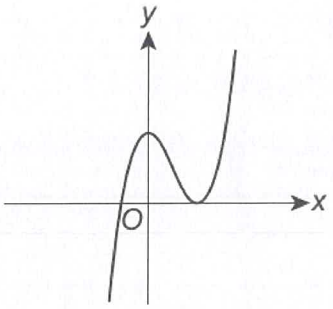
$f(x) = -3(x+3)(x-6)^2$

∴ A

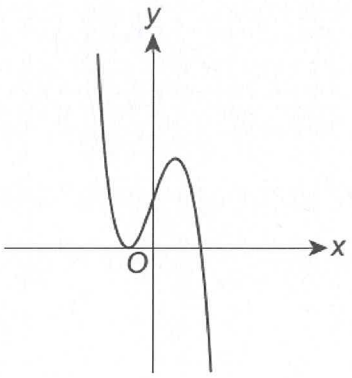
A.



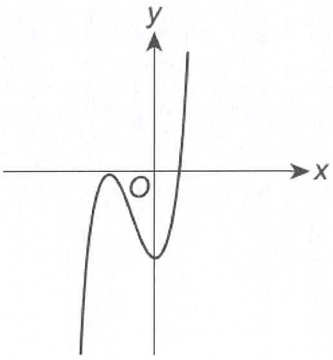
B.



C.



D.



6 A food critic scores restaurants each year, where a positive score means the restaurant exceeded expectations and a negative score means it fell below expectations. The scores follow a normal distribution where the mean score is equal to twice the standard deviation.

Scores were recorded for all restaurants reviewed over one year.

What percentage of the restaurants received a score below 0?

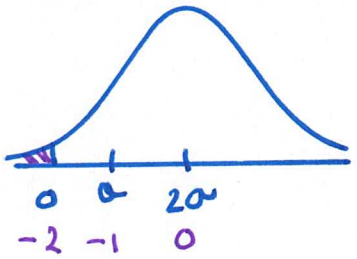
this is an x-score not a z-score

A) 50 %

B) 16 %

C) 2.5 %

D) 0.15 %



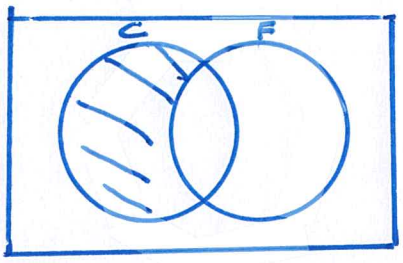
7 At a busy cafe, 70% of customers order coffee and 45% order food. A customer who orders food has a 60% chance of also ordering coffee. A customer is chosen at random. What is the probability that they order coffee but not food?

A) 0.27

B) 0.31

C) 0.43

D) 0.6



$P(C) = 0.7$

$P(F) = 0.45$

$P(C|F) = 0.6$

want $P(C) - P(C \cap F)$

$= 0.7 - 0.27$

$= 0.43$

$P(C|F) = \frac{P(C \cap F)}{P(F)}$

$P(C \cap F) = 0.45 \times 0.6$

$= 0.27$

8 The functions $P(x), Q(x)$ and $R(x)$ are defined for all real x and the function $R(x) = P[Q(x)]$.

Given that $Q(2) = 5$ and $Q'(2) = 8$ and $P'(5) = -3$, what is the gradient of the normal to the curve $R(x)$ at $x = 2$?

A) 24

B) $\frac{1}{24}$

C) $-\frac{1}{24}$

D) -24

$R'(x) = P'(Q(x)) \times Q'(x)$

$R'(2) = P'(Q(2)) \times Q'(2)$

$= P'(5) \times 8$

$= -3 \times 8$

$= -24$

$\therefore m_N = \frac{1}{24}$

9 Find all real values of x satisfying:

$$\log_2(x) - 1 = \frac{3 - \log_2 x}{2} + \left(\frac{3 - \log_2 x}{2}\right)^2 + \left(\frac{3 - \log_2 x}{2}\right)^3 + \cdots$$

A) $x = 4$

B) $x = \frac{1}{2}$

C) $x = \frac{1}{2},\ x = 4$

D) $x = 2,\ x = 4$

Let $t = \log_2 x$ $\therefore a = \frac{3-t}{2}$ $r = \frac{3-t}{2}$

$S_\infty = \frac{a}{1-r}$

$t - 1 = \frac{3-t}{2} \div \left(1 - \frac{3-t}{2}\right)$

$t - 1 = \frac{3-t}{2} \div \frac{2-3+t}{2}$

$t - 1 = \frac{3-t}{2} \times \frac{2}{t-1}$

$3 - t = t^2 - 2t + 1$

$t^2 - t - 2 = 0$

$(t-2)(t+1) = 0$

$\therefore t = 2$ or $t = -1$ ← but for L.S

$-1 < r < 1$

$\therefore -1 < \frac{3-t}{2} < 1$

$-2 < 3 - t < 2$

$-5 < -t < -1$

$1 < t < 5$

$\therefore$ reject Soln.

Sub in $t = \log_2 x$

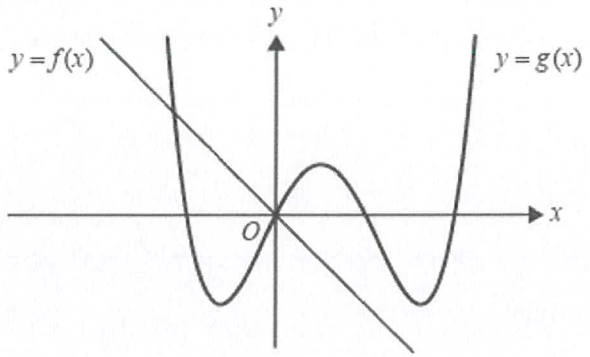
$2 = \log_2 x$

$x = 2^2$

$x = 4$

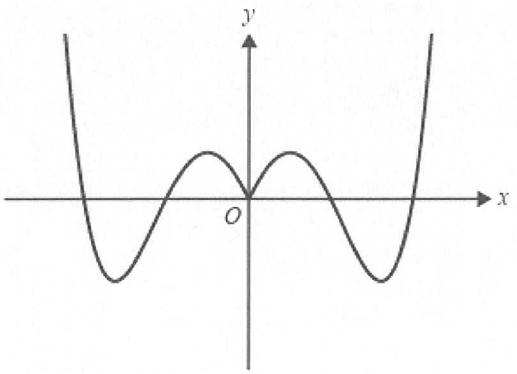
$\therefore$ A

10 The graphs of $y = f(x)$ and $y = g(x)$ are sketched on the same set of axes below.

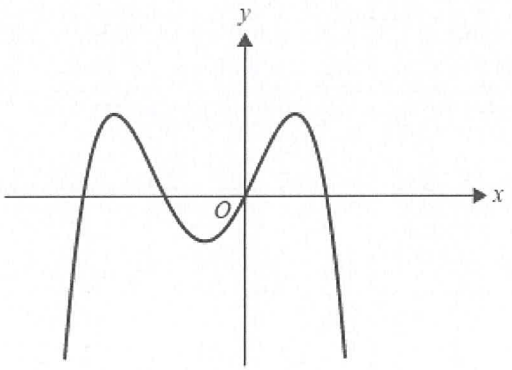


Which of the following could be the graph of $y = g(f(x))$?

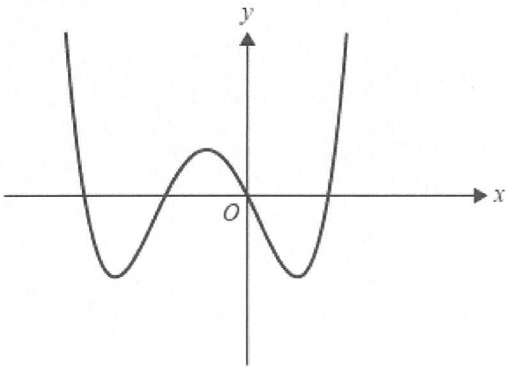
A.



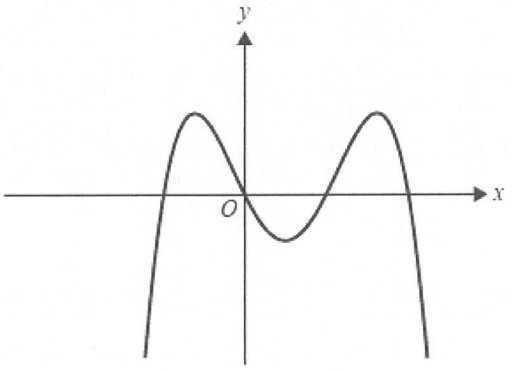
B.



C.



D.



$y = f(x) \div -x$

$\therefore g(f(x)) \rightarrow g(-x)$

which is a reflection over y-axis $\therefore$ C

Question 11 (2 marks)

The first term of an arithmetic series is -7 and the sum of the first 20 terms is 430. Find the common difference. **2**

$S_n = \frac{n}{2}[2a+(n-1)d]$

$430 = \frac{20}{2}[-14+19d]$ ✓ correctly applied formula

$43 = -14+19d$

$19d = 57$

$d = 3$ ✓ correct answer

Some students used the nth term instead of the sum formula.

Question 12 (2 marks)

Find the coordinates of the centre and the radius of the circle with equation: **2**

$$x^2 + y^2 - 8x + 10y + 5 = 0$$

$x^2 - 8x + 4^2 + y^2 + 10y + 5^2 = -5 + 5^2 + 4^2$

$(x-4)^2 + (y+5)^2 = 36$ ✓ correctly completed the squares.

centre $= (4, -5)$ radius $= 6$ ✓ correct answer

Question 13 (2 marks)

Evaluate the definite integral below, leaving your answer in exact form. **2**

$$\int_0^{\frac{\pi}{4}} \sin(2x) \cdot e^{\cos 2x}\, dx$$

$= -\frac{1}{2}\left[e^{\cos 2x}\right]_0^{\pi/4}$ ✓ correct integration

$= -\frac{1}{2}\left[e^{\cos\frac{\pi}{2}} - e^{\cos 0}\right]$

$= -\frac{1}{2}(e^0 - e^1)$

$= \frac{1}{2}(e-1)$ ✓ correct answer

Question 14 (4 marks)

Differentiate the following with respect to x :

(a) $y = \ln \sqrt{x}$ 2

$y = \frac{1}{2} \ln x$ ✓

$\therefore y' = \frac{1}{2x}$ ✓

Some students overcomplicated the question

(b) $y = x^2 e^{-x}$ 2

$u = x^2 \quad v = e^{-x}$
$u' = 2x \quad v' = -e^{-x}$

$y' = uv' + vu'$

$= x^2 \times -e^{-x} + e^{-x} \times 2x$ ✓✓ correctly applied product rule.

$= x e^{-x}(2 - x)$

Question 15 (2 marks)

Prove that 2

$$\frac{\sin\theta}{1-\cos\theta} - \frac{\sin\theta}{1+\cos\theta} = 2\cot\theta$$

$L.H.S = \dfrac{\sin\theta(1+\cos\theta) - \sin\theta(1-\cos\theta)}{1-\cos^2\theta}$

$= \dfrac{\sin\theta + \sin\theta\cdot\cos\theta - \sin\theta + \sin\theta\cdot\cos\theta}{\sin^2\theta}$ ✓

$= \dfrac{2\sin\theta\cdot\cos\theta}{\sin^2\theta}$

Some students overcomplicate the question.

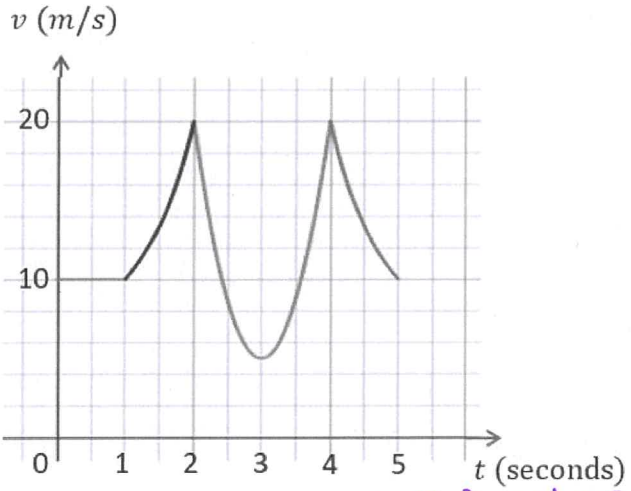
$= 2\cot\theta$ ✓

$= RHS.$

It is important to show your steps, including cancelling.

Question 16 (2 marks)

The graph below shows the velocity (in metres per second) of a particle over a 5 second period. The particle reaches its lowest velocity of 5m/s after 3 seconds.



6 f(n) values.

Use the trapezoidal rule with 5 applications to determine an estimate for the distance the particle has travelled over the 5 seconds. **2**

$D = \frac{h}{2}[f(1st) + f(last) + 2f(middle)]$

$= \frac{1}{2}[10 + 10 + 2(10 + 20 + 5 + 20)]$ ✓

$= \frac{1}{2}(130)$

$= 65$ m ✓

Question 17 (2 marks)

Solve the following equation for x. **2**

$$2\log_2 x - \log_2(x+3) = 2$$

Feedback for Q17-20 Year 12 Adv Trial 2026

Question 17

- This question in general was answered well.
- Note: that there is only one solution $x=6$ as $x>0$, therefore the solution $x=-2$ needs to be rejected. Several students did not do this, 1 mark deducted.

Question 18

- Mostly answered correctly.
- Most of the errors occurred in part (iii).
- For part (iii) several students found the incorrect angle ($\angle PHB$) for the bearing. The angle needed to find the bearing of H ***from*** B is $\angle PBH$. If the error was carried on correctly, then 1 mark still awarded.

Question 19

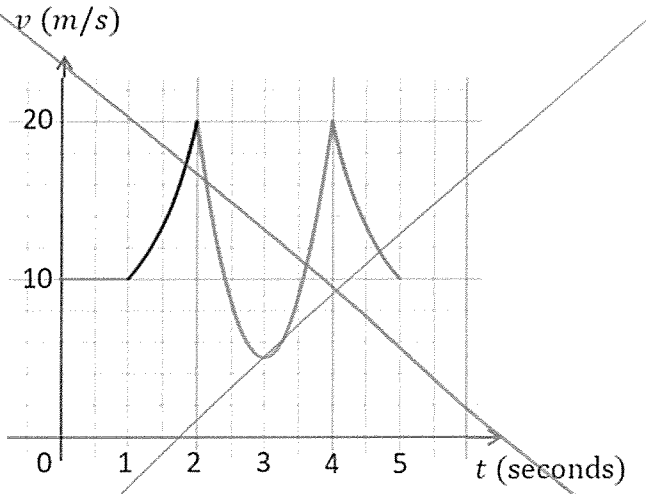
- Mostly answered correctly.
- Note the graphs intercepts need to be clearly shown on the diagram.
- The sketch for part (ii) needs to show the graph in the correct position, with graph lines that extend below the x-axis, as well as the intercepts and vertex.

Question 20

- Mostly answered correctly.
- Most of the errors occurred in part (ii).
- For part (ii) note that the drug was still effective up until 17 hours and 5 minutes and 24 seconds, therefore the drug becomes ineffective at 17 hours and 6 minutes (or 1026 minutes) after the injection (to the nearest minute).
- Note: 1 mark deducted if the final answer was 17 hours and 5 minutes.
- Note: 1 mark deducted if the final answer did not explicitly state the hours and minutes (or total minutes) but left as the calculator display.

Question 16 (? marks)

The graph below shows the velocity (in metres per second) of a particle over a 5 second period. The particle reaches it's lowest velocity of 5m/s after 3 seconds.



Use the trapezoidal rule with 5 applications to determine an estimate for the distance the particle has travelled over the 5 seconds. ?

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Question 17 (3 marks)

Solve the following equation for x. 2

$$2\log_2 x - \log_2(x+3) = 2$$

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$\log_2\left(\frac{x^2}{x+3}\right) = 2$

.....

.....

$\frac{x^2}{x+3} = 2^2$

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$x^2 = 4x + 12$

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$x^2 - 4x - 12 = 0$

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$(x-6)(x+2) = 0$

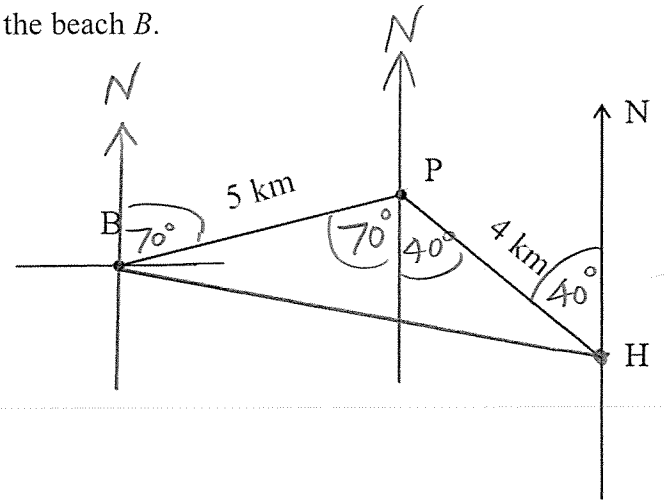
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$x = 6 \quad (x>0, x\neq -2)$

Question 18 (5 marks)

Katie starts riding her bike from her house *H*.

She rides 4 km on a bearing of 320° to the park P then rides 5 km on a bearing of 250° to the beach *B*.



(i) Use the diagram to show that angle *HPB* is 110°. **1**
No geometric reasoning is required.

$\angle HPB = 70 + 40 = 110°$

(ii) Find the distance *BH*, correct to 3 decimal places. **2**

$c^2 = a^2 + b^2 - 2ab\cos C$

$BH^2 = 5^2 + 4^2 - 2 \times 5 \times 4 \times \cos 110°$

$BH = \sqrt{41 - 40 \times \cos 110°}$

$\doteq 7.395$ km

(iii) Hence, find the bearing of Kate's house *H* from the beach *B*, to the nearest minute. **2**

$\cos C = \dfrac{a^2 + b^2 - c^2}{2ab}$

$= \dfrac{BH^2 + 5^2 - 4^2}{2 \times BH \times 5}$

$= 0.86117\ldots$

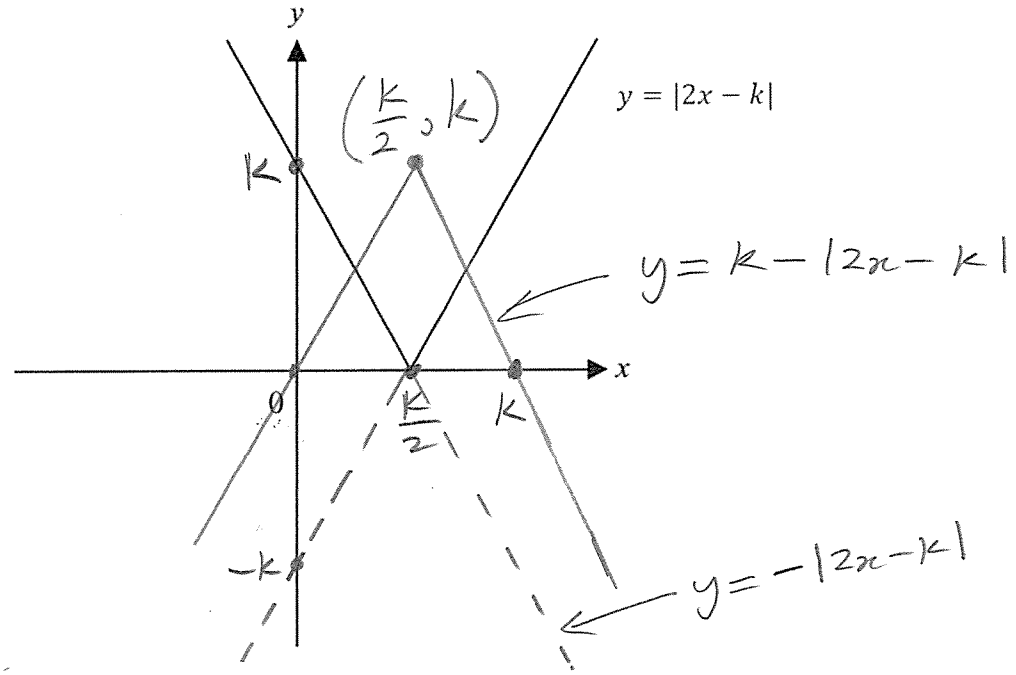
$C = \cos^{-1}(0.86117\ldots)$

$\therefore \angle PBH \doteq 30°33'$

$\therefore$ Bearing of H from B $= 70° + 30°33'$
$= 100°33'$ T

Question 19 (4 marks)

The diagram below shows the graph of the function, $y = |2x - k|$ for some constant k.



(i) On the diagram, mark in terms of k, the intercepts on the axes. **2**

$y = |2x - k|$

x-int. $y = 0$: $2x - k = 0$

$2x = k$

$x = \frac{k}{2}$

y-int. $x = 0$:

$y = |-k|$

$y = k$

(ii) On the same diagram, sketch the function $y = k - |2x - k|$ showing in terms of k any intercepts on the axes and the coordinates of the vertex. **2**

Question 20 (3 marks)

After a patient receives an injection, the concentration of the drug $C(t)$ in their bloodstream (in mg/L) is modelled by:

$$C(t) = Ae^{-0.14t}$$

where t is the time in hours after the injection. The drug becomes ineffective when the concentration falls below 2 mg/L. A doctor measures the concentration to be 12.5 mg/L exactly 4 hours after the injection.

(i) Find the initial concentration A at the time of injection, correct to one decimal place. **1**

$12.5 = Ae^{-0.14\times 4}$

$A = \dfrac{12.5}{e^{-0.56}}$

$\doteq 21.9 \text{ mg/L}$

(ii) Find the time t after the injection at which the drug becomes ineffective. Give your answer to the nearest minute. **2**

Ineffective when $C(t) < 2$

$Ae^{-0.14t} < 2$

$e^{-0.14t} < \dfrac{2}{A}$

$\dfrac{2}{A} > e^{-0.14t}$

$\log_e\left(\dfrac{2}{A}\right) > -0.14t$

$-0.14t < \log_e\left(\dfrac{2}{A}\right)$

$t > \dfrac{\log_e\left(\frac{2}{A}\right)}{-0.14}$

$t > 17^{\circ}\,5'\,24''$

$\therefore$ Time after injection $= 17$ hrs 6 mins

Question 21 (3 marks)

A table of future values for an annuity of $1 is shown below.

Table of future value interest factors								
	Interest rate per period							
Period	0.50%	1.00%	1.50%	2.00%	3.00%	4.00%	5.00%	6.00%
2	2.0050	2.0100	2.0150	2.0200	2.0300	2.0400	2.0500	2.0600
3	3.0150	3.0301	3.0452	3.0604	3.0909	3.1216	3.1525	3.1836
4	4.0301	4.0604	4.0909	4.1216	4.1836	4.2465	4.3101	4.3746
5	5.0503	5.1010	5.1523	5.2040	5.3091	5.4163	5.5256	5.6371
6	6.0755	6.1520	6.2296	6.3081	6.4684	6.6330	6.8019	6.9753
7	7.1059	7.2135	7.3230	7.4343	7.6625	7.8983	8.1420	8.3938
8	8.1414	8.2857	8.4328	8.5830	8.8923	9.2142	9.5491	9.8975

(a) Sam contributes $3,000 at the end of each quarter into his superannuation account, which earns interest at 6% p.a compounded quarterly, for 2 years. What is the total amount of interest earned? **2**

$\frac{0.06}{4} = 0.015$ ie 1.5%.

for 8 periods

$\cdot 3000 \times 8.4328 = 25\,298.40$ — (1)

amount contributed $= 3000 \times 8$

$= 24\,000$

$\therefore$ Interest $= 25\,298.40 - 24000 = \$1298.40$ — (1)

(b) Elena makes equal contributions of $\$M$ at the end of every six months into her investment account, which earns interest at 4% p.a compounded six-monthly. What is the value of M if the future value is $37, 848.60 after 3 years? **1**

$\frac{0.04}{2} = 0.02$ ie 2%.

for 6 periods

$\therefore M \times 6.3081 = 37\,848.60$

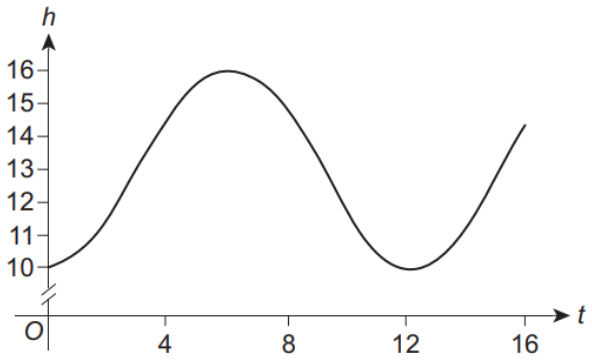
$\therefore M = \$6000$ — (1)

Question 22 (4 marks)

The water level of a harbour rises and falls according to changes in the tide. Ships can enter the harbour when the water is at a suitable level.

The harbour's water level can be modelled by the function $h(t) = A\cos(Bt) + 13$, where h is the water level in metres above the seabed and t is the number of hours after 8 am on a Saturday.

Serena is planning to enter the harbour in a ship. She observes the water level from 8 am until midnight on Saturday. At 8am during a low tide, the water level is 10m. At 2 pm during a high tide, the water level is 16m. The graph shows the changes in water level that Serena observed during the 16- hour period.



(a) Find the values of A and B. **2**

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(b) Serena's ship requires the water level to be at least 11.5m to enter safely. At what times between 8am and midnight on Saturday could Serena safely enter the harbour? **2**

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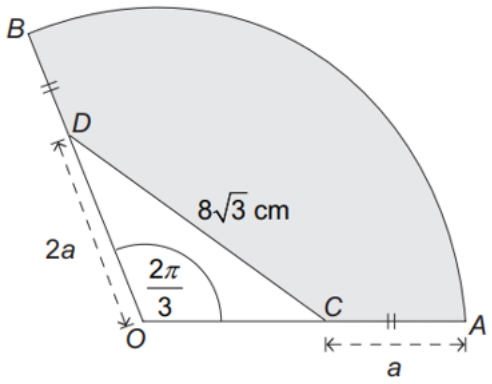
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Question 23 (3 marks)

The triangle COD is inside a sector that has a central angle of $\frac{2\pi}{3}$, as shown in the diagram.



The lengths of the sector and triangle are as follows.

$$DC = 8\sqrt{3}\ ,\ AC = BD\ ,\ OC = 2AC$$

Find the exact area of the shaded region. **3**

Question 24 (4 marks)

A statistician is analysing whether chess players perform better when they stay in a hotel with a higher rating. Hotels are rated by online reviews on a scale from 3 to 5, with 3 being the worst and 5 being the best rating that can be achieved. Chess players are rated by the Elo score ranging from 1000 to 2800. The higher the Elo score the better the performance by the chess player.

The table below shows the hotel rating and the Elo score for chess players.

Hotel rating (x)	5	4.3	3.9	3.5	4.2	4.8	3.1	4.5	4.7	3
Elo rating (y)	2650	2700	2100	2000	2500	2600	1800	2450	2750	1900

(a) Determine the equation of the least-squares regression line and correlation coefficient for this data. Give answers to 2 decimal places. **2**

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(b) Interpret the gradient of the least squares line in the context of the question. **1**

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(c) Describe the correlation coefficient in terms of its strength and direction. **1**

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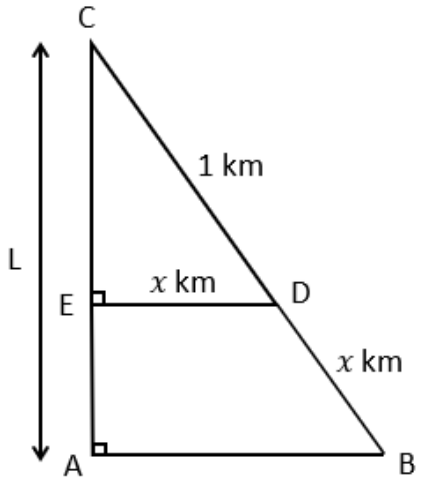
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Question 25 (4 marks)

ABC is a right-angled triangle at A.

The point D lies on the hypotenuse BC such that $BD = x$ and $DC = 1$ km.

The point E lies on AC such that DE is parallel to BA and $DE = x$ km as shown below.



(a) It is given that $\frac{CA}{CE} = \frac{CB}{CD}$.

Show that the length of side AC is $L = (1 + x)\sqrt{1 - x^2}$ **1**

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Question 25 (continued)

(b) Find the exact maximum length L of side AC. Use mathematical reasoning to justify that this is the maximum length of L. **3**

Question 26 (8 marks)

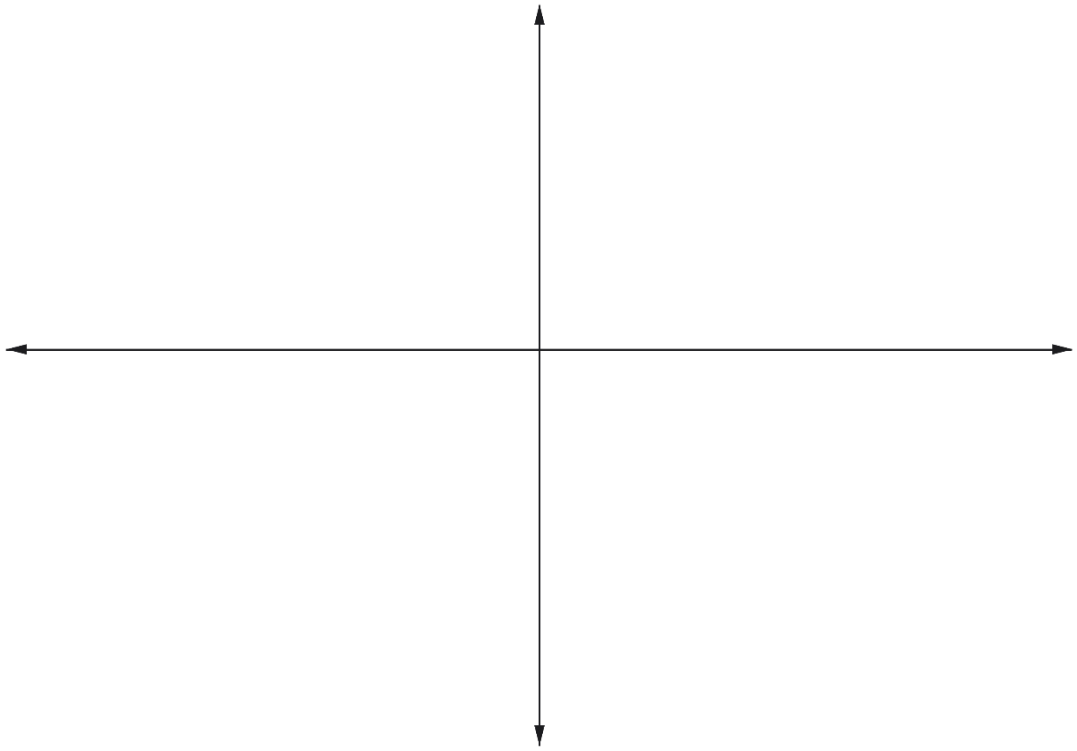
Consider the function $f(x) = xe^{-\frac{x}{2}}$

(a) Find all stationary points and determine their nature. **3**

(b) Find any points of inflection. **2**

Question 26 (continued)

(c) Sketch $y = f(x)$ labelling all key features. **3**



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Question 27 (4 marks)

The table below shows the probability that a random variable z, is less than Z.

z	−2	−1.5	−1	−0.5	0	0.5	1	1.5	2
$P(z < Z)$	0.0228	0.0668	0.1587	0.3085	0.5	0.6915	0.8413	0.9332	0.9772

For a particular airline, the number of minutes that the arrival time of a flight varies from its scheduled arrival time is normally distributed. A negative x score indicates that the flight arrived early, and a positive x score indicates that the flight arrived late.

(a) For this airline, 30.85% of flights arrive more than 6 minutes early, while 2.28% of flights arrive more than 14 minutes late. Find the mean and standard deviation of this distribution. **2**

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(b) Effie is on a flight that has a scheduled arrival time of 16:14. Using the table above determine the percentage chance that her flight will arrive before 16:00? Give your answer correct to two decimal places. **2**

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Question 28 (7 marks)

A particle P starts from a point O and moves along a horizontal straight line. Its velocity v after t seconds is given by:

$$v(t) = \begin{cases} 4t - 1, & \text{for } 0 \leq t \leq 1 \\ 3\sqrt{t}, & \text{for } 1 < t \leq 9 \end{cases}$$

(a) Does the particle ever change direction? Justify your answer fully. **2**

For $0 \leq t \leq 1$; $v(t) = 0$ at $t = \frac{1}{4}$

For $0 \leq t < \frac{1}{4}$, $v(t) < 0$

for $\frac{1}{4} < t \leq 1$, $v(t) > 0$

Also For $1 < t \leq 9$, $v(t) = 3\sqrt{t} > 0$

Hence velocity changes only once at $t = \frac{1}{4}$ s. from negative to positive.

(b) Find the total distance travelled by the particle in the first 9 seconds. **3**

Total distance:

$$\left| \int_0^{1/4} (4t-1)\,dt \right| + \int_{1/4}^{1} (4t-1)\,dt + \int_1^9 3\sqrt{t}\,dt.$$

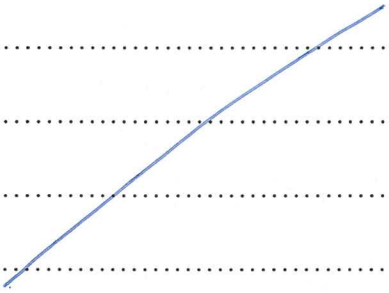
$$= \left[(t - 2t^2) \right]_0^{1/4} + \left[(2t^2 - t) \right]_{1/4}^{1} + 3 \left[\frac{2}{3} t^{3/2} \right]_1^9$$

$$= \left(\frac{1}{4} - \frac{1}{8} \right) + \left(1 + \frac{1}{8} \right) + 2(27 - 1)$$

$$= \frac{10}{8} + 52 = \frac{213}{4} \text{ units or } 53.25 \text{ m}$$

Question 28 (continued)

(b)



(c) At $t = 0$ a second particle Q starts from the same point O and moves along the same line with velocity $w(t) = kt$, where k is a positive constant. Given that P and Q have the same displacement from O at $t = 9$, find the value of k. **2**

$w(t) = kt$

$\int_0^9 kt\,dt = k\left[\frac{t^2}{2}\right]_0^9$

$= \left(\frac{81}{2}\right)k$

$\left(\frac{81}{2}\right)k = 53$

$\therefore k = \frac{106}{81}$

P's displacement:

$\int_0^1 (4t-1)\,dt + \int_1^9 3\sqrt{t}\,dt$

$= \left[2t^2 - t\right]_0^1 + 2\left[t^{3/2}\right]_1^9$

$= \left((2-1) - 0\right) + 2\left(9^{3/2} - 1\right)$

$= 53$

Question 29 (8 marks)

(a) Find the x coordinates of the points of intersection of $y = -3\cos x$ and $y = \tan x$ for $-\pi \leq x \leq \pi$. Give answers to 2 decimal places. **3**

points of intersection when both the curves intersect.

$-3\cos x = \tan x$

$-3\cos x = \frac{\sin x}{\cos x}$

$-3\cos^2 x = \sin x$

$-3(1-\sin^2 x) = \sin x$

$3\sin^2 x - \sin x - 3 = 0$

$\therefore \sin x = \frac{1 \pm \sqrt{37}}{6}$

$\sin x \neq \frac{1+\sqrt{37}}{6}$ (not a valid solution in the domain $[-\pi, \pi]$)

$\therefore \sin x = \frac{1-\sqrt{37}}{6}$

≈ -0.85

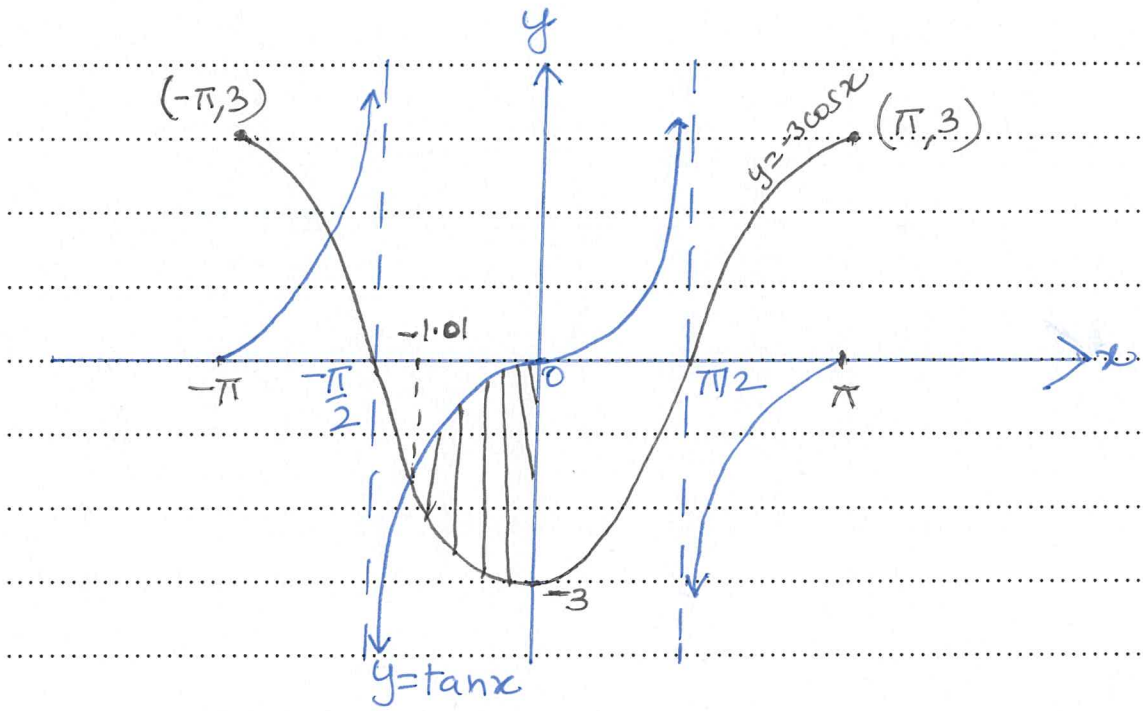
$\therefore x \approx -1.01$ and

$-\pi - (-1.01)$

$\doteq -2.13$

Question 29 (continued)

(b) For $-\pi \leq x \leq \pi$, sketch the graph of the equations $y = -3\cos x$ and $y = \tan x$ on the same set of axes. **2**



(c) Hence, find the area enclosed between the two curves and the y –axis. **3**

Give your answer to 2 decimal places

$$A = \left| \int_{-1.01}^{0} (\tan x + 3\cos x)\,dx \right|$$

$$= \left| \left[-\ln|\cos x| + 3\sin x \right]_{-1.01}^{0} \right|$$

$$= \left| 0 - \left(-\ln(0.532) - 2.541 \right) \right|$$

$$\approx 1.91$$

Marker's feedback for Q 28 and 29

Q28

a) This question was poorly performed by the cohort. Common error included not justifying mathematically that there was change in the direction of the velocity from negative to positive and for the first branch and positive for the second branch of the piecewise function. Approaches exhibited in ideal responses included substituting values around t=1/4 to justify a change in the direction or a correct diagram to support the reasoning. 1 mark was awarded to justify that the velocity changes from negative to positive for the first branch and 1 mark for showing that the velocity remains positive for the second branch. Few students used acceleration of the object to state that there is no change in the velocity which is incorrect. The object may be accelerating and still change direction, so this is not the correct approach. As this question had asked for full justification, stating that velocity changes at t=1/4 is not sufficient reasoning.

b) Mixed performance for this question. Common mistake included, not including the change in velocity around t=1/4 which means that the distance covered by the object must be calculated for three parts of the journey instead of directly calculating from t=0 to t=1 and then from t=1 to t=9. Other mistakes included calculation errors and incorrect methods such as substituting values in the velocity piecewise function. Few students calculated the areas of the triangles formed for the journey between t=0 and t=1 and received the correct answer for the distance travelled. 1 mark was awarded for setting up the correct expression for the distance travelled and 2 marks for correct working.

c) This question was again poorly performed. Common misconception included considering distance travelled same as the displacement. 1 mark was awarded for correct calculation of displacement and 1 mark for equating the displacements of both P and Q hence determining the value of k. The response must show the calculation of displacement and not just writing values without relevant working. There was some ambiguity around interpreting the question

where few students misinterpreted that the displacements are equal only at t=9, which was awarded appropriately provided the response has shown relevant working.

Q 29

a) This question was generally well answered by the cohort. Common mistakes included calculation mistakes in using the quadratic formula, not rejecting one solution for calculating the values of x, leaving the answer in degrees not in radians and not considering two possible solutions within the restricted domain $[-\pi,\pi]$. 1 mark was awarded for writing the equation in a quadratic form and applying the quadratic formula correctly, 1 mark was awarded for rejecting one solution with correct reasoning and accepting the other solution, 1 mark for finding two possible values of x within the restricted domain.

b) Mixed performance for this question too. Common mistakes included incorrect sketch for $y = \tan x$ which showed asymptotes at $x = \pi$, both the curves intersecting at wrong points and incorrect shapes of the curves. Part a) should have guided in sketching both the curves and keeping the points of intersection in mind. These sketches are fundamental curves in Trigonometry and must be practised as a core skill on this topic. Additionally, the diagram must be drawn in the restricted domain mentioned in the question. 1 mark was awarded for correct sketch of each curve. If you are drawing multiple diagrams, make sure you use extra working space at the end to sketch a neat diagram instead of drawing it in the margin space.

c) This question was not performed well by the cohort. Common mistakes included misinterpretation of the area included between the two curves and the y axis, incorrect expression for $\int \tan x \ dx$ and calculation errors. 1 mark was awarded for writing correct expression for the area, 1 mark for determining $\int \tan x \ dx$ and 1 mark for calculating the area. Appropriate carry forward marks were awarded

for legible working. Few students tried using inverse trigonometry formulae in this context however could not proceed further due to complicated working.

Question 30 (7 marks)

The duration of telemarketing calls to mobile phone users is a continuous random variable t hours, with probability density function:

$$f(t) = \begin{cases} at \sin\left(\frac{\pi}{2}t\right), & \text{for } 0 \leq t \leq 1 \\ 0, & \text{otherwise} \end{cases}$$

(a) Show $\frac{d}{dt}\left(\frac{4}{\pi^2}\sin\frac{\pi}{2}t - \frac{2}{\pi}t\cos\frac{\pi}{2}t\right) = t\sin\frac{\pi}{2}t$ **2**

(easier to keep this out)

L.H.S $= \frac{4}{\pi^2} \times \frac{\pi}{2}\cos\frac{\pi}{2}t - \frac{2}{\pi}\left[t \times -\frac{\pi}{2}\sin\frac{\pi}{2}t + \cos\frac{\pi}{2}t \times 1\right]$ ✓

$= \frac{2}{\pi}\cos\frac{\pi}{2}t - \frac{2}{\pi}\left(-\frac{\pi}{2}t\sin\frac{\pi}{2}t + \cos\frac{\pi}{2}t\right)$

$= \frac{2}{\pi}\cos\frac{\pi}{2}t + t\sin\frac{\pi}{2}t - \frac{2}{\pi}\cos\frac{\pi}{2}t$ ✓

$= t\sin\frac{\pi}{2}t$

all steps needed.

= RHS, as required.

(b) Hence, show that $a = \frac{\pi^2}{4}$. **1**

$\int_0^1 at\sin\frac{\pi}{2}t = 1$, by definition

$a\left[\frac{4}{\pi^2}\sin\frac{\pi}{2}t - \frac{2}{\pi}t\cos\frac{\pi}{2}t\right]_0^1 = 1$ from (a)

$a\left(\frac{4}{\pi^2}\sin\frac{\pi}{2} - \frac{2}{\pi}\cos\frac{\pi}{2} - \left(\frac{4}{\pi^2}\sin 0 - 0\right)\right) = 1$

$a\left(\frac{4}{\pi^2} - 0 - (0-0)\right) = 1$

substitution required to receive the 1 mk.

$\therefore a\left(\frac{4}{\pi^2}\right) = 1$ ✓

$\therefore a = \frac{\pi^2}{4}$, as required

Question 30 (continued)

(c) Show that the probability that a call lasts for more than 30 minutes is: **2**

$$1-\frac{1}{\sqrt{2}}+\frac{\pi}{4\sqrt{2}}$$

$P(t>30) = \int_{\frac{1}{2}}^{1} \frac{\pi^2}{4} t \sin\left(\frac{\pi}{2}t\right) dt$ 1

$= \frac{\pi^2}{4}\left[\frac{4}{\pi^2}\sin\frac{\pi}{2}t - \frac{2}{\pi}t\cos\frac{\pi}{2}t\right]_{\frac{1}{2}}^{1}$ ✓ ½

$= \left[\sin\frac{\pi}{2}t - \frac{\pi}{2}t\cos\frac{\pi}{2}t\right]_{\frac{1}{2}}^{1}$ 1

$= \left(\sin\frac{\pi}{2} - \frac{\pi}{2}\cos\frac{\pi}{2}\right) - \left(\sin\frac{\pi}{4} - \frac{\pi}{4}\cos\frac{\pi}{4}\right)$

$= 1 - 0 - \frac{1}{\sqrt{2}} + \frac{\pi}{4\sqrt{2}}$

$= 1 - \frac{1}{\sqrt{2}} + \frac{\pi}{4\sqrt{2}}$ ✓

alternatively $P(t>30) = 1 - P(t<30)$

again substitution needed to be shown.

Question 30 (continued)

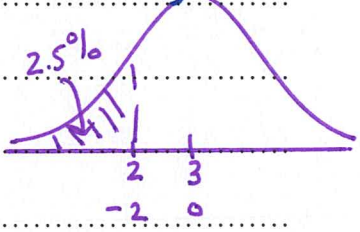
(d) The telemarketing company also surveys customers on how the call made them feel, rating their experience from 1 (very negative) to 5 (very positive).

The survey results were normally distributed with a mean of 3 and a standard deviation of 0.5.

The probability that a call received a rating of 2 or lower given that the call lasted more than 30 minutes is 1.5%. Use the empirical rule when required to answer the following.

Find the probability that a call lasted more than 30 minutes, given that the customer gave a rating of 2 or lower. **2**

We are given $P(\text{rating} \leq 2 \mid \text{call} > 30\text{mins}) = 0.015$



$P(\text{rating} \leq 2) = 2.5\% = 0.025$ (A)

$P(A \mid B) = \dfrac{P(A \cap B)}{P(B)}$

$0.015 = \dfrac{P(\text{call} > 30\text{ mins} \cap \text{rating} \leq 2)}{1 - \frac{1}{\sqrt{2}} + \frac{\pi}{4\sqrt{2}}}$

$P(\text{call} > 30\text{mins} \cap \text{rating} \leq 2) = 0.0127$ (B) ✓ 1 mark for (A) and (B)

$P(\text{call} > 30\text{mins} \mid \text{rating} \leq 2) = \dfrac{P(\text{call} > 30\text{mins} \cap \text{rating} \leq 2)}{P(\text{rating} \leq 2)}$

$= \dfrac{0.0127}{0.025}$

$= 0.509 \doteq 50.9\%$ ✓

Question 31 (6 marks)

Elsha is planning to save \$110 000 over 5 years by depositing her monthly salary of $\$M$ at the beginning of each month into a savings account. The account pays interest at 6% p.a compounded monthly. She will need to withdraw \$500 at the end of each month to cover her living expenses.

$R = \frac{6}{12}\% = 0.5\% = 0.005$

Let A_n represent the balance in the account at the end of month n.

(a) Show that at the end of the second month, immediately after making her \$500 withdrawal, the balance is given by: **1**

$$A_2 = M(1.005^2 + 1.005) - 500(1.005 + 1)$$

$A_1 = M(1+0.005)^1 - 500$

$= M(1.005) - 500$

$A_2 = (A_1 + M)(1.005) - 500$ ← crucial step required for the 1 mark.

$= (M(1.005) - 500 + M)(1.005) - 500$

$= M(1.005)^2 - 500(1.005) + M(1.005) - 500$

$= M(1.005^2 + 1.005) - 500(1.005 + 1)$ ✓

$\therefore A_3 = M(1.005^3 + 1.005^2 + 1.005) - 500(1.005^2 + 1.005 + 1)$

1 mark lost if students went straight from A_3 to G.P sum formula for n.

(b) Show that $A_n = 201M(1.005^n - 1) - 100\,000(1.005^n - 1)$. **2**

$\therefore A_n = M(1.005^n + 1.005^{n-1} + \ldots 1.005) - 500(1.005^{n-1} + \ldots 1.005 + 1)$

G.P $a = 1.005$ $r = 1.005$ $n = n$ ✓ G.P $a = 1$ $r = 1.005$ $n = n$

$A_n = M \times \frac{1.005(1.005^n - 1)}{1.005 - 1} - 500\left(\frac{1.005^n - 1}{1.005 - 1}\right)$

$= 201M(1.005^n - 1) - 100\,000(1.005^n - 1)$

This is a show question, You needed to apply the G.P sum formulae to A_3 before generalising it for A_n or establish the A_n pattern and then apply the G.P sum formulae.

Question 31 (continued)

(c) Hence, calculate Elsha's monthly salary, $\$M$. Give your answer to the nearest cent. **1**

$A_n = 201M(1.005^n - 1) - 100\,000(1.005^n - 1)$ from (b)

Let $A_n = 110{,}000$ $n = 5 \times 12 = 60$

$110{,}000 = 201M(1.005^{60} - 1) - 100000(1.005^{60} - 1)$

$M = \dfrac{110{,}000 + 100000(1.005^{60} - 1)}{201(1.005^{60} - 1)} = \$2066.28$ ✓

this is the only acceptable answer.

(d) After the 12th withdrawal Elsha deposits \$6000 she received from her grandparents. She decides she can now afford to withdraw a total of \$600 at the end of each month for her living costs. **2**

How many more complete months will it take Elsha to reach her \$110,000 goal?

$A_{12} = 201 \times 2066.28(1.005^{12} - 1) - 100000(1.005^{12} - 1) + 6000$

$= 19448.39 + 6000$

$= 25448.39$ (A) this doesn't earn a mark but it needs to be calculated.

Let B_k = amount in account after k more withdrawals.

$B_1 = (A + m)(1.005) - 600$

$= A(1.005) + m(1.005) - 600$

$B_2 = (B_1 + m)(1.005) - 600$

$= [A(1.005) + m(1.005) - 600 + m]1.005 - 600$

$= A(1.005)^2 + m(1.005^2 + 1.005) - 600(1 + 1.005)$

$B_k = A(1.005)^k + m(1.005^k + \cdots 1.005) - 600(1 + 1.005 \cdots 1.005^{k-1})$

↘ G.P $a = 1.005$, $r = 1.005$, $n = k$ ↘ G.P $a = 1$, $r = 1.005$, $k = n$.

Let $B_k = 110{,}000$. Find k

$\therefore 110\,000 = 25448.39(1.005)^k + 2066.28\left(\dfrac{1.005(1.005^k - 1)}{.005}\right) - 600\left(\dfrac{1.005^k - }{0.005}\right.$ ✓

1 mark for establishing correct pattern.

$$110\,000 = 25448.39(1.005)^k + 415322.28(1.005)^k - 415322.28 - 120000(1.005)^k + 120000$$

$$405322.28 = 320770.67(1.005)^k$$

$$1.005^k = 1.2635\ldots$$

$$k = \frac{\ln(1.2635\ldots)}{\ln 1.005}$$

$$\doteq 46.9$$

$\therefore$ 47 complete months ✓ 2nd mark.

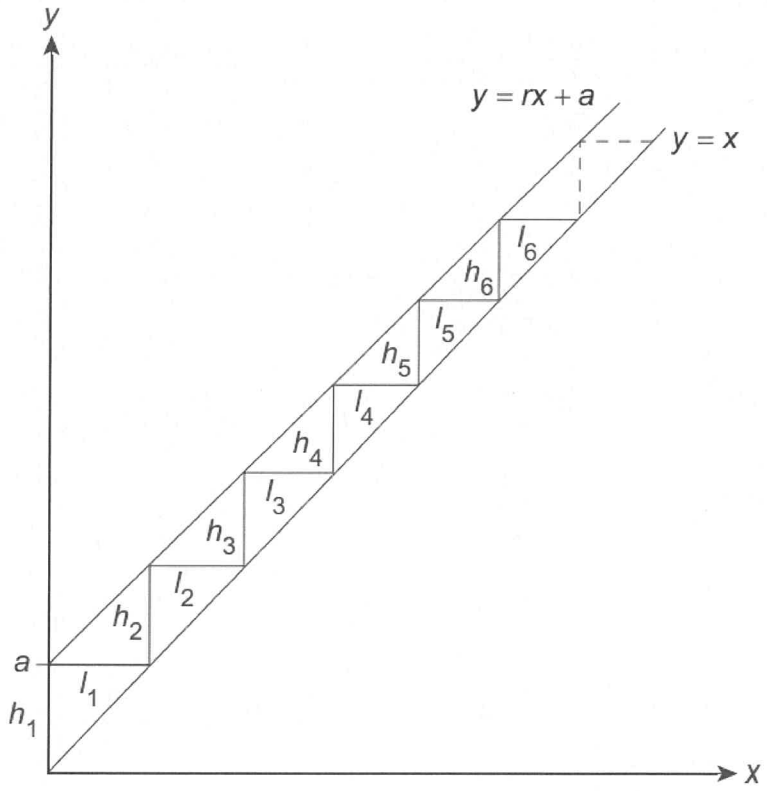
Some students disregarded the money already in the account which means they lost interest.

Question 32 (4 marks)

Consider the lines $y = x$ and $y = rx + a$, where $r > 0$ and $a > 0$.

The two lines intersect at an unknown point in the first quadrant. A sequence of right-angled triangles is constructed between the two lines. The heights (h)and lengths (l) of the triangles are labelled as $h_1, l_1, h_2, l_2, h_3, l_3$.....

The graph shows these two lines and the triangles between them.



(a) State the range of the values of r for which the two lines intersect in the first quadrant. **1**

$m_1 = r \quad (r > 0)$

$m_2 = 1$

$r < 1$ or lines would not intersect in Q_1

$\therefore \ 0 < r < 1$ ✓

(b) Find the x- coordinate of the intersection point in terms of r and a. **1**

$y = x \ (1)$ $\qquad x = rx + a$

$y = rx + a \ (2)$ $\qquad x - rx = a$

$x(1 - r) = a$

$x = \frac{a}{1-r}$ or $\frac{-a}{r-1}$

Question 32 (continued)

(c) Using infinite series only, find the total area of the triangles between the lines. **2**

Give your answers in terms of a and r.

The line $y = x$ makes an angle of 45° with x and y axis. $\therefore h_1 = l_1 \quad h_2 = l_2 \quad h_3 = l_3, \; h_1 = a$

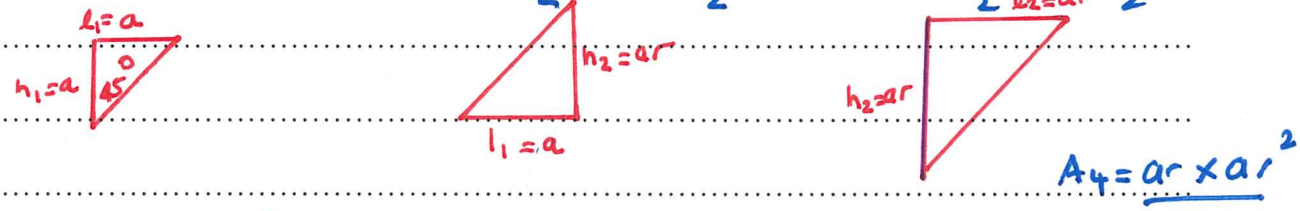
The triangles with hypontenuses on the line $y = rx + a$ have tangents equivalent to $m = r$

$$r = \frac{h_2}{l_1}$$

$\therefore h_2 = r l_1 \;, h_3 = r l_2 \;, h_4 = r l_3 \;\ldots\; l_1 = a$

1 mark

$$\therefore A_1 = \frac{a^2}{2} \qquad A_2 = \frac{a \times ar}{2} = \frac{a^2 r}{2} \qquad A_3 = \frac{ar \times ar}{2} = \frac{a^2 r^2}{2}$$



$$A_4 = \frac{ar \times ar^2}{2} = \frac{a^2 r^3}{2}$$

G.P $a = \frac{a^2}{2} \qquad r = r$

Adding the areas creates an infinite series since $0 < r < 1$

Hence total area $= \frac{a}{1-r}$

$$= \frac{\frac{a^2}{2}}{1-r} = \frac{a^2}{2(1-r)}$$

End of paper

A maximum of one mark awarded for correct answer acquired without infinite series.