Student Name: _____

Teacher: _____

Sydney Technical High School

Mathematics ADVANCED Trial HSC 2026

General Instructions

- Reading time – 10 minutes
- Working time – 3 hours
- Write using black pen
- Approved calculators may be used
- A NESA reference sheet is provided
- For questions in Section II, show relevant mathematical reasoning and/or calculations

Total marks : 100

Section I – 10 marks (pages 3 – 7)

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section
- **Answer on the Multiple-choice answer sheet on page 37**

Section II – 90 marks (pages 8 – 29)

Markers only:

Questions 1 – 10	Questions 11 - 17	Questions 18 - 22	Questions 23 – 27	Questions 28 – 31	Questions 32 – 34	TOTAL
/10	/18	/18	/18	/18	/18	/100

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Section I

10 marks
Attempt Questions 1–10
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Allow about 15 minutes for this section

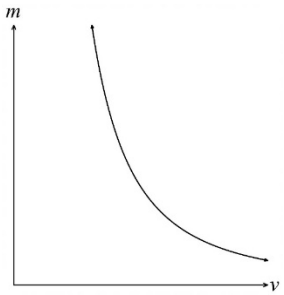
Use the multiple-choice answer sheet for Questions 1 – 10

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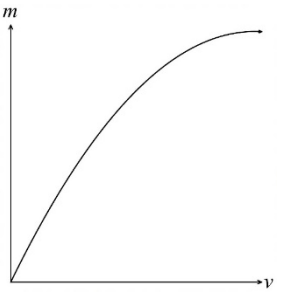
1. The mass of an object is graphed against the velocity of it's motion.

Which of the following shows $\frac{dm}{dv} < 0 \; and \; \frac{d^2m}{dv^2} = 0$?

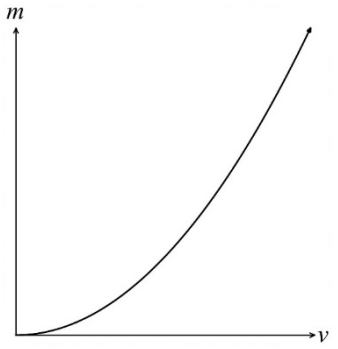
A.



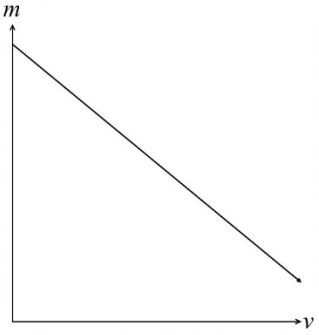
B.



C.



D.



2. Consider the graph of $y = \frac{1}{x-5} - 5$

What is the equation of the horizontal asymptote?

A. $y = -5$
B. $y = 5$
C. $x = 5$
D. $x = -5$

3. Consider the function $f(x) = \sqrt{5 - x} + \ln(2x + 1)$

The domain of the function is:

A. $\left[\frac{1}{2}, 5\right]$

B. $\left[-\frac{1}{2}, 5\right]$

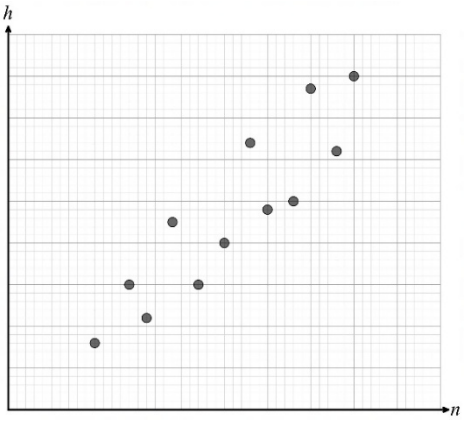
C. $\left(-\frac{1}{2}, 5\right]$

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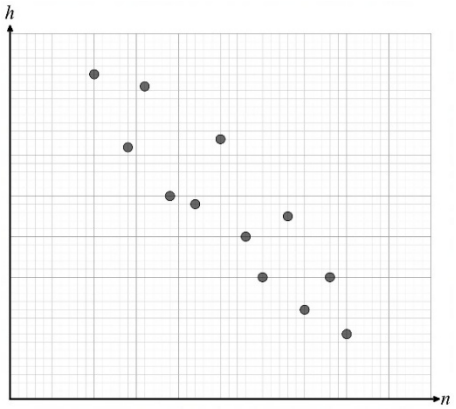
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Which scatterplot could represent this data?

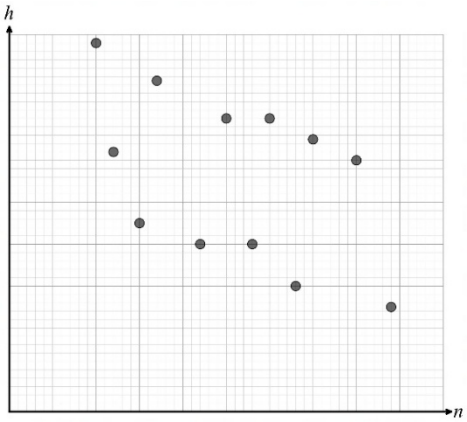
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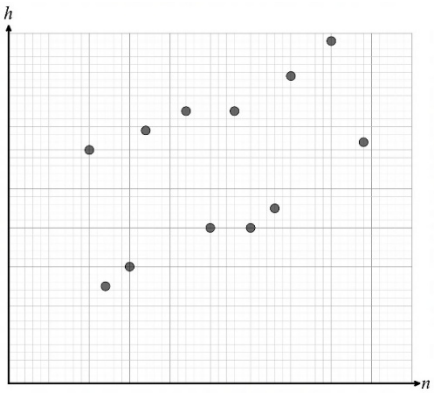
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5. The table shows the function values of $y = f(x)$ and $y = g(x)$

x	-1	0	1	2	3
$f(x)$	3	0	3	12	27
$g(x)$	4	3	2	1	0

What is the value A if $f(g(A)) = 0$

A. 0

B. 1

C. 2

D. 3

6. A box contains 5 red, 4 blue, and 3 green counters. Two counters are drawn without replacement. The value of P(red / not green) is;

A. $\frac{4}{9}$

B. $\frac{5}{12}$

C. $\frac{5}{9}$

D. $\frac{5}{7}$

7. Given $f(1) = 9,\ f'(1) = -8,\ g(1) = 1$ and $g'(1) = 6$

$h(x) = x\, f[g(x)]$ then $h'(1) =$

A. -48

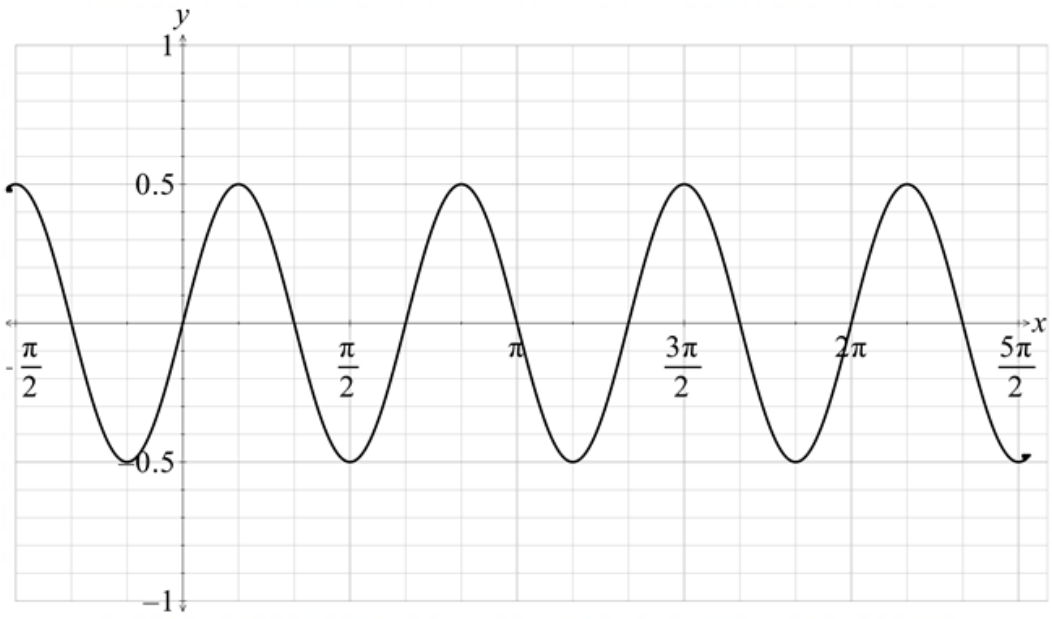
B. -39

C. 9

D. 54

8. The curve $y = f(x)$ is graphed below:

What is the equation of the curve $y = g(x)$, where $g(x) = f'(x)$



A. $g(x) = \frac{1}{2} sin3x$

B. $g(x) = \frac{1}{2} cos3x$

C. $g(x) = -\frac{3}{2} sin3x$

D. $g(x) = \frac{3}{2} cos3x$

9. Which of the following definite integrals has the largest value?

A. $\int_{-1}^{1} x^5 \, dx$

B. $\int_{-1}^{1} 2sinx \, dx$

C. $\int_{-1}^{1} x^4 \, dx$

D. $\int_{-1}^{1} e^x - e^{-x} \, dx$

10. For the angle θ, $tan\theta = \frac{9}{40}$ and $sin\theta < 0$

What is the value of $\cos(90° - \theta) + \cos(180° + \theta)$?

A. $-\frac{40}{41}$

B. $\frac{31}{41}$

C. $\frac{40}{41}$

D. $-\frac{49}{41}$

End of section 1

Section II - 90 marks

Attempt Questions 11 – 34 (pages 8 – 29)

Allow about 2 hours and 45 minutes for this section

Instructions

- Answer the question in the spaces provided. These spaces provide guidance for the expected length of response.
- Your responses should include relevant mathematical reasoning and or calculations.
- Extra writing space is provided on pages 30 to 34. If you use this space clearly indicate which question you are answering. Indicate which page the marker needs to refer to.

Question 11 (2 marks)

Solve the equation $7 - \frac{2-3x}{2} = 4$

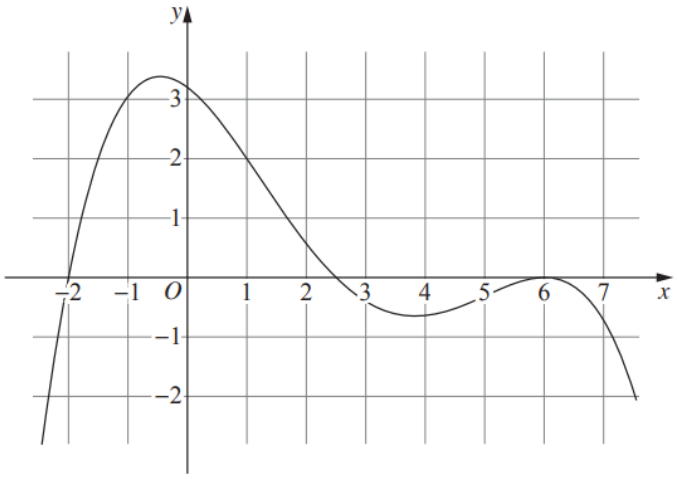
Question 12 (2 marks)

Find the first term of the series $-12 - 8 - 4 + \cdots$ that is greater than 150.

Question 13 (2 marks)

The diagram shows the graph of $y = f'(x)$

Describe the feature of the curve $y = f(x)$ at $x = 6$. Justify your feature mathematically.



Question 14 (3 marks)

Find the equation of the tangent to the curve $y = \sqrt{2x + 1}$ at the point where $x = 4$

Question 15 (2 marks)

Solve the equation $|4 - 3x| = 13$

Question 16 (4 marks)

Find,

i. $\frac{d}{dx}\left(\frac{x^2}{sinx}\right)$ **2**

ii. $\frac{d}{dx}(\,2^x log_2 x)$ **2**

Question 17 (3 marks)

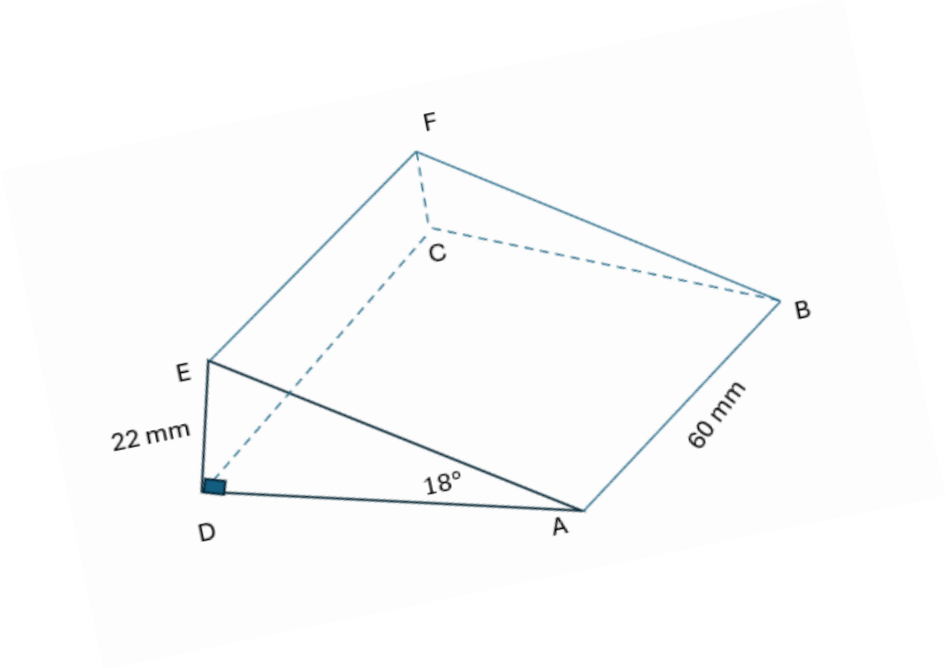
A particle moves along the x-axis with its displacement, in metres from O, at time t seconds, given by the equation $x = 2t^2 - \ln t$.

Where and when does the particle come to rest?

Question 18 (3 marks)

A triangular prism *ABCDEF* is made as a door wedge.

It has a height of 22 mm and w width of 60 mm. The angle of the wedge is $18°$



Find the angle, correct to the nearest degree, between the diagonal *BE* and the diagonal of the base *ABCD*.

Question 19 (2 marks)

Solve the equation $2log_7 5 = 2log_7 x - log_7 3$

Question 20 (6 marks)

Find:

i. $\int (\ln 3)x^2 \ dx$ **2**

ii. $\int \frac{3x}{3-x^2} \ dx$ **2**

iii. $\int_{-1}^{2} \frac{dx}{(4-x)^2}$ **2**

Question 21 (4 marks)

i. $\frac{d}{dx}(x^2 - 6)^8$ **2**

ii. Hence, evaluate $\int_0^2 x(x^2 - 6)^7 \, dx$ **2**

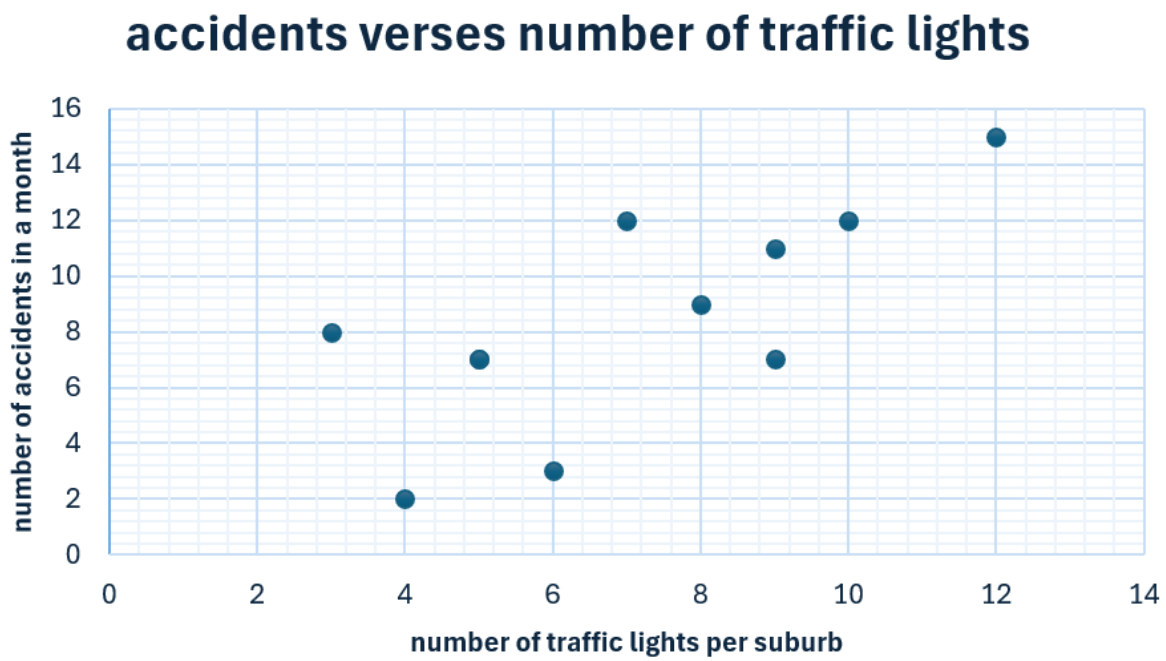
Question 22 (3 marks)

Prove that

$$\frac{\sin\theta}{1 + cos\theta} + \frac{1 + cos\theta}{sin\theta} = 2\, cosec\theta$$

Question 23 (3 marks)

The number of traffic lights in 10 suburbs and the number of accidents in a month are shown in the scatter plot below



Find the equation of the least squares regression line and comment on its use to predict the number of traffic lights in a suburb that had no accidents in the given month.

Question 24 (3 marks)

The random variable X has the following probability distribution

x	2	3	5	7	9
$P(X) = x$	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{4}$	p	q

i. Find the value of p, q given $P(X = \{prime \cap odd\}) = \frac{31}{48}$ **2**

ii. Find $E(X)$ **1**

Question 25 (3 marks)

The graph of $y = \sqrt{x}$ has been dilated both horizontally and vertically by a factor of 2 and then translated to the left 3 units before being reflected in the x – axis, to produce the equation $y = g(x)$

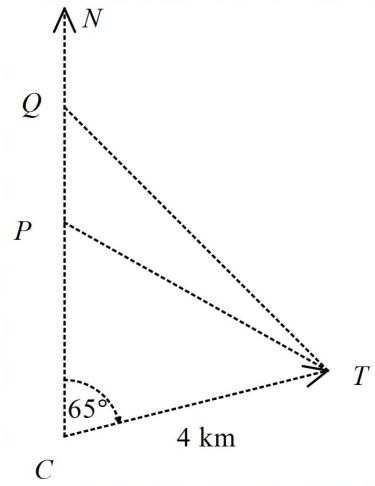
Write down the equation of $y = g(x)$

Question 26 (5 marks)

Two mountains, *P* and *Q* lie directly north of a campground *C*.

Rehan walks from *C* on a bearing of $065°$, to a point T, a distance of 4 km.

The diagram below shows the relative positions, when viewed from above.



NOT TO SCALE

i. From point T, the base of the mountain P is on a bearing of 285° whilst the base of mountain Q is on a bearing of 315°. **1**

Explain why angle QTC = 70°

ii. Find the distance CQ, correct to 1 decimal place **2**

iii. Hence or otherwise find the distance PQ. **2**

Question 27 (4 marks)

Thomas makes up a card game where he gets points for the type of card chosen randomly from a well shuffled deck of 52 playing cards.

He replaces the card chosen after each draw.

The game is scored as follows;

- He scores 10 points if he draws an Ace, King, Queen or Jack
- He scores 3 points if he draws a card less than 10 &
- He scores -10 points if the card drawn is a 10 of any suit.

i. What is the probability that Thomas will score zero in his first 2 draws. **2**

ii. After playing his game a few times, Thomas decides to change the rules. His new rules are:

- He wins if he draws an Ace, King, Queen or Jack and the game stops
- He plays again if he draws a card less than 10 &
- He loses and the game stops if he draws a 10 of any suit.

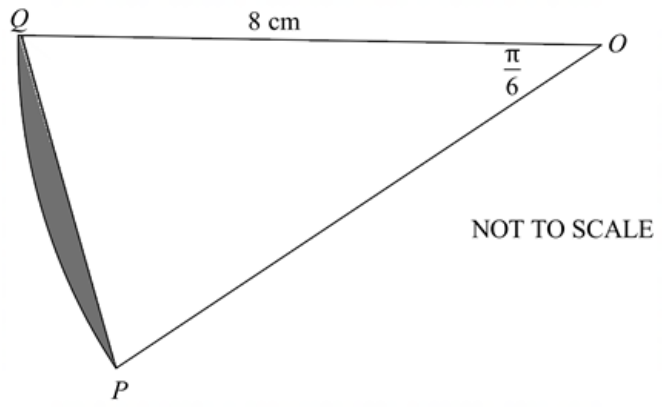
What is the probability that he wins the game? **2**

Question 28 (3 marks)

A sector OPQ of a circle, is shown in the diagram.

The circle has its centre at *O* and a radius of 8 cm. **3**

$\angle QOP = \frac{\pi}{6}$.



Find the perimeter of the shaded segment, leave in exact form.

Question 29 (3 marks)

Mathsville's population at the start of 2000 is unknown. However, by the beginning of 2020 it had grown to 2.5 million.

Assuming that the population of Mathsville can be modelled exponentially, using the equation, $P = Ae^{0.0144t}$,where A is a constant, and t is measured in years from the beginning of 2000.

i. Find the value of A **1**

ii. A newspaper article, published in 1980, forecast that the population of Mathsville would reach 5 million by the year 2050.

Using calculations to justify your response, state whether you agree or disagree with this article. **2**

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Question 30 (9 marks)

For the curve $f(x) = x^4 - 8x^2$

i. Show that $y = f(x)$ represents an even function.

1

ii. Find the coordinates of all the stationary points and determine their nature.

3

iii. Find any points of inflexion. **2**

iv. Sketch the curve in the domain, showing all stationary points as well as any inflexions and intercepts with the coordinate axes.

3

Question 31 (3 marks)

Evaluate $\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{\cos^2\left(\frac{\pi}{2}-x\right)}{\cos^2 x}\, dx$

Question 32 (6 marks)

Jerome starts working on January 1, 2027. The advertised wage is $56 000 p.a. After the first year his wage will increase to $62 000 then increase by the same amount each year for the first 10 years of his employment, after that it will increase by the CPI of 3% p.a.

i. Show that Jerome earns $74000 at the start of 2030. **1**

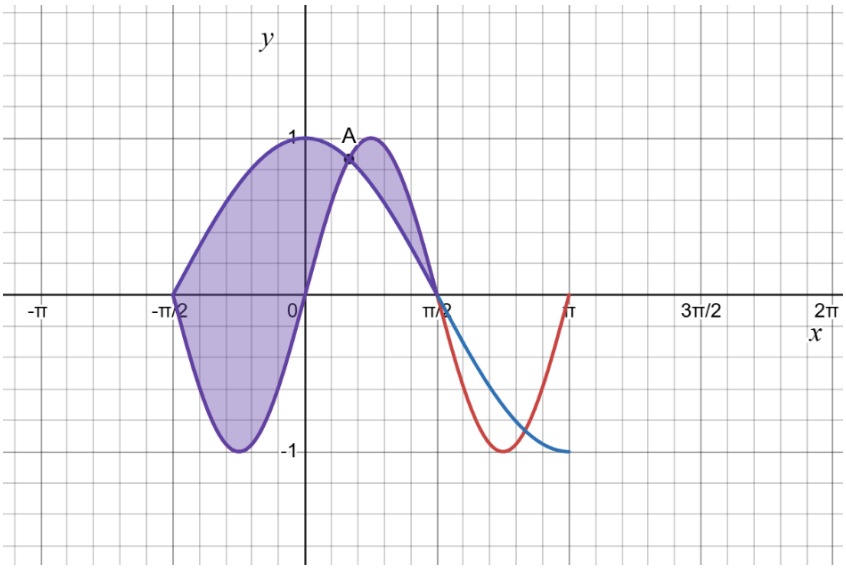
ii. How much will Jerome earn for the first 10 years. **2**

iii. How many years will Jerome need to work before the total he has earned is over 2 million dollars.

3

Question 33 (5 marks)

The graphs of $y = sin2x$ and $y = cosx$ in the domain $-\frac{\pi}{2} \leq x \leq \pi$ are shown below



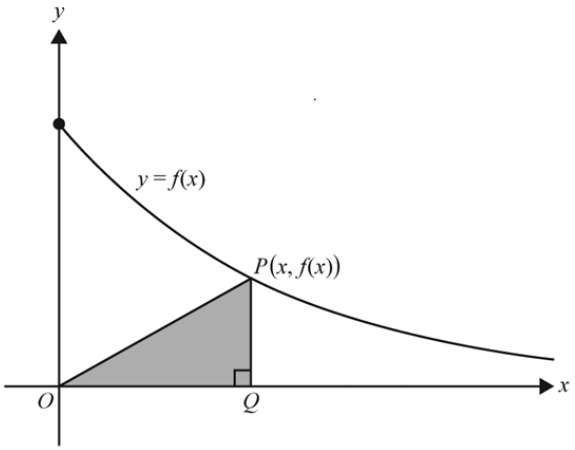
i. Show that the coordinates of the point A, the point of intersection of the two curves in the first quadrant is $(\frac{\pi}{6}, \frac{\sqrt{3}}{2})$ **2**

ii. Find the shaded area. **3**

Question 34 (7 marks)

A point P$(x, f(x))$ lies on the curve $f(x) = \frac{2}{\sqrt[5]{e^x}}$

The perpendicular from P to the x axis, at Q, is shown and triangle OPQ is shaded.

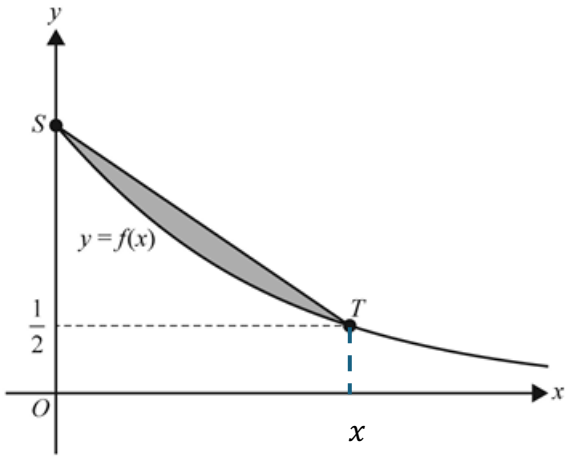


i. Find the area of triangle OPQ in terms of x. **2**

ii. Find the x –coordinate of P, for which the triangle OPQ has a maximum area. **3**

Question 35 continues the next page ……

iii. Let S be the point where the curve $y = f(x)$ crosses the y – axis and let T be the point on the curve where $y = \frac{1}{2}$



α. Find the exact value of x at the point T. **2**

Question 35 continues the next page ……

β. Find the area of the shaded region, held between the line ST and the curve $y = f(x)$. **3**

END OF ASSESSMENT TASK

EXTRA WRITING SPACE

Indicate the question and page that any working refers to

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Suggested Solutions.

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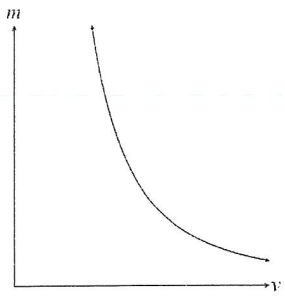
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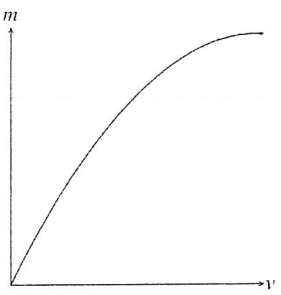
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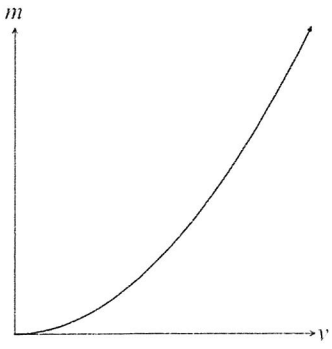
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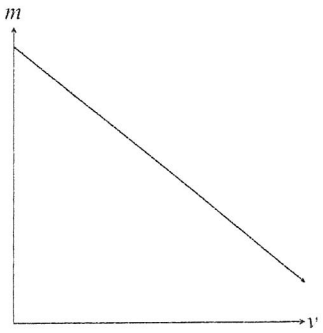
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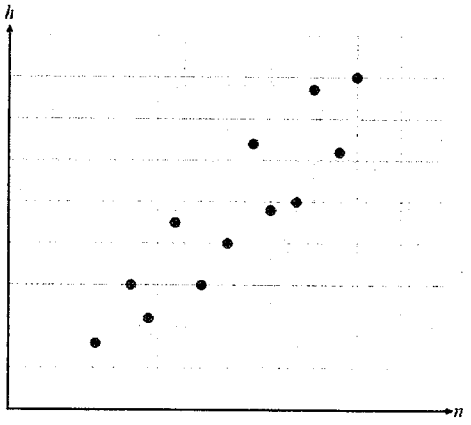
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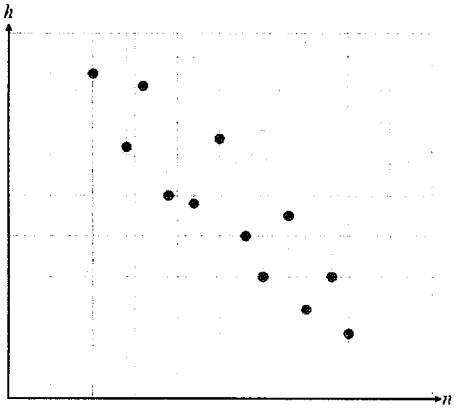
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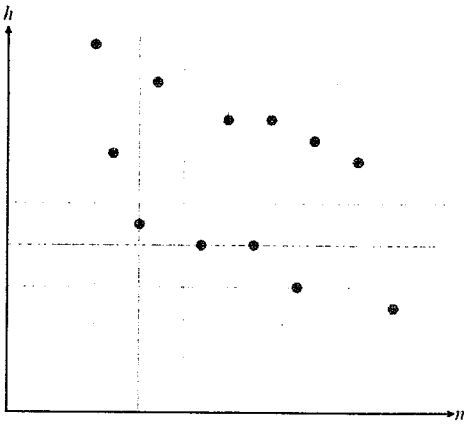
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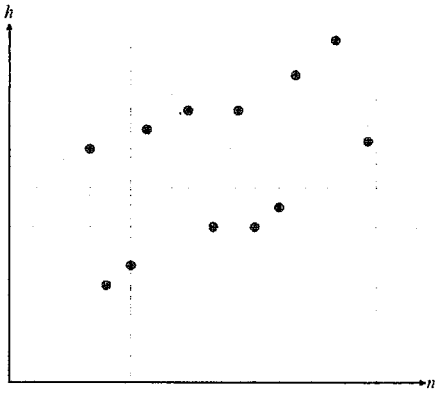
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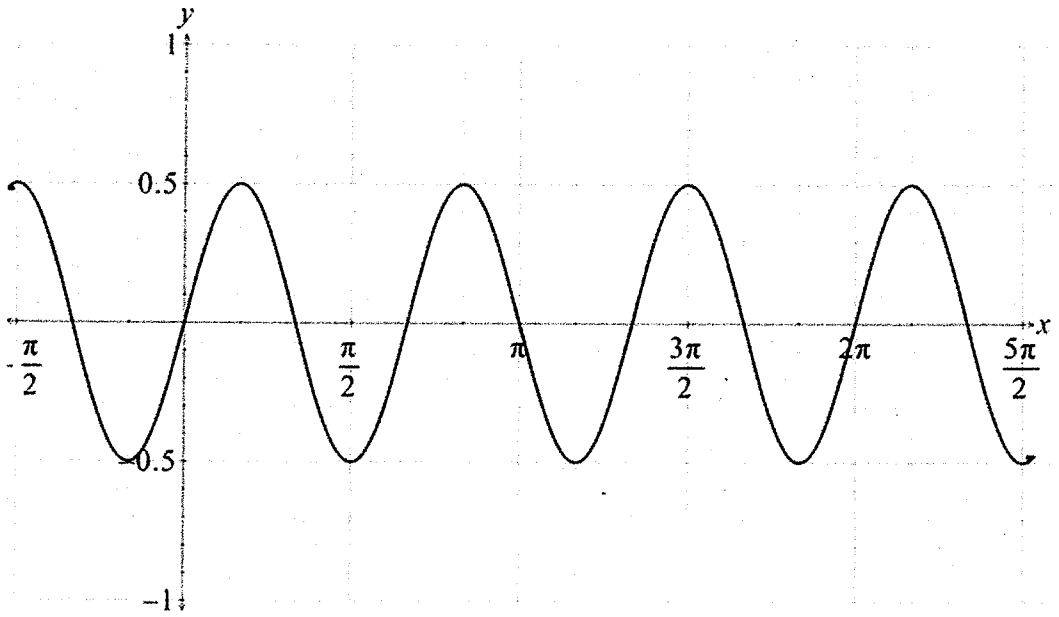
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What is the value of $\cos(90° - \theta) + \cos(180° + \theta)$?

A. $-\frac{40}{41}$

B. $\frac{31}{41}$

C. $\frac{40}{41}$

D. $-\frac{49}{41}$

End of section 1

Section II - 90 marks

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Allow about 2 hours and 45 minutes for this section

Instructions

- Answer the question in the spaces provided. These spaces provide guidance for the expected length of response.
- Your responses should include relevant mathematical reasoning and or calculations.
- Extra writing space is provided on pages 30 to 34. If you use this space clearly indicate which question you are answering. Indicate which page the marker needs to refer to.

Question 11 (2 marks)

Solve the equation $7 - \frac{2-3x}{2} = 4$

$14 - (2-3x) = 8$

$12 + 3x = 8$

$3x = -4$

$x = -4/3$

Question 12 (2 marks)

Find the first term of the series $-12 - 8 - 4 + \cdots$ that is greater than 150.

AP $a = -12$ $d = 4$ $T_n = a + (n-1)d$

$-12 + 4(n-1) > 150$

$4n - 4 > 162$

$4n > 166$

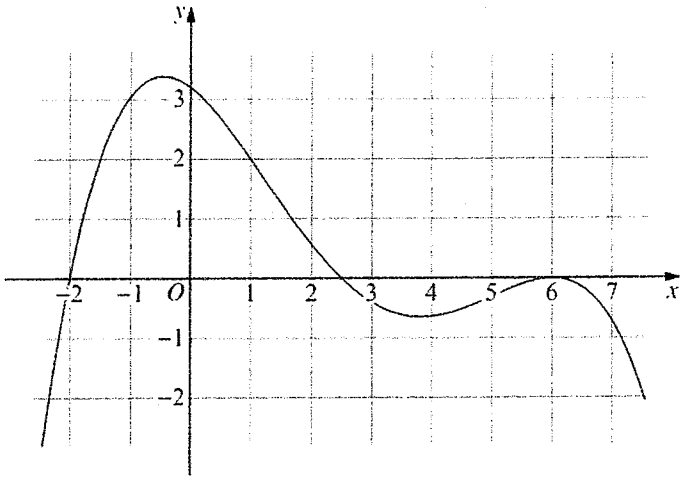
$n > 41.5$

$\therefore T_{42} = 152$ which is > 150

Question 13 (2 marks)

The diagram shows the graph of $y = f'(x)$

Describe the feature of the curve $y = f(x)$ at $x = 6$. Justify your feature mathematically.



Horizontal pt of inflection as
$x < 6$ $f'(x) < 0$ at $x = 6$ $f'(x) = 0$
and $x >$ $f'(x) < 0$

Question 14 (3 marks)

Find the equation of the tangent to the curve $y = \sqrt{2x + 1}$ at the point where $x = 4$

$y = 3$

$dy/dx = \frac{1}{2}(2x+1)^{-1/2} \times 2$

$= \frac{1}{\sqrt{2x+1}}$ at $x = 4$

$M_T = 1/\sqrt{9} = 1/3$

eqn $y - y_1 = m(x - x_1)$

$y - 3 = 1/3(x - 4)$

$3y - 9 = x - 4$

$x - 3y + 5 = 0$ either

OR $y = 1/3x + 5/3$

Question 15 (2 marks)

Solve the equation $|4 - 3x| = 13$

$-(4-3x)=13$ or $4-3x=13$

$-4+3x=13$ $\quad -3x=9$

$3x=17$ $\quad x=-3$

$x=17/3$

$\therefore x=-3$ or $x=17/3$

Question 16 (4 marks)

Find,

i. $\frac{d}{dx}\left(\frac{x^2}{sinx}\right)$ **2**

$$\frac{d}{dx}\left(\frac{x^2}{\sin x}\right) = \frac{(\sin x)(2x) - x^2(\cos x)}{(\sin x)^2}$$

$$= \frac{2x\sin x - x^2\cos x}{\sin^2 x}$$

ii. $\frac{d}{dx}(2^x log_2 x)$ $\quad vu' + uv'$ **2**

$$= 2^x \times \left(\frac{1}{x\ln 2}\right) + (2^x \times \ln 2) \times \log_2 x$$

$$= \frac{2^x}{x\ln 2} + 2^x \ln 2 \log_2 x$$

Question 17 (3 marks)

A particle moves along the x-axis with its displacement, in metres from O, at time t seconds, given by the equation $x = 2t^2 - \ln t$.

Where and when does the particle come to rest?

$x = 2t^2 - \ln t$

$v = 4t - 1/t$

$v = 0$

$4t^2 - 1 = 0$

$(2t-1)(2t+1) = 0$

$t = 1/2$ or $t = -1/2$

as $t > 0$

$\therefore t = 1/2$

$x = 2(1/2)^2 - \ln(1/2)$

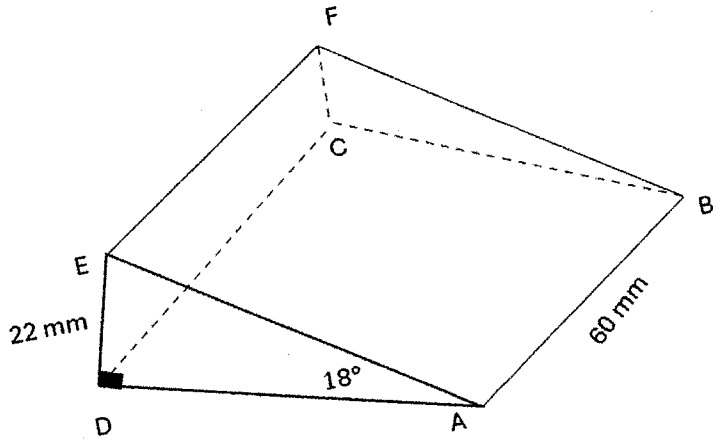
$= 2/4 + \ln 2$

$= 1/2 + \ln 2$ or $1/2 - \ln 1/2$

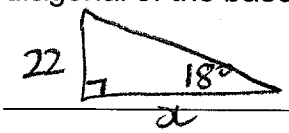
Question 18 (3 marks)

A triangular prism *ABCDEF* is made as a door wedge.

It has a height of 22 mm and w width of 60 mm. The angle of the wedge is 18°



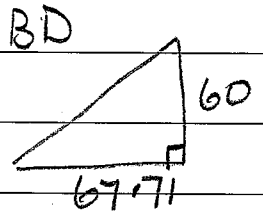
Find the angle, correct to the nearest degree, between the diagonal *BE* and the diagonal of the base *ABCD*.



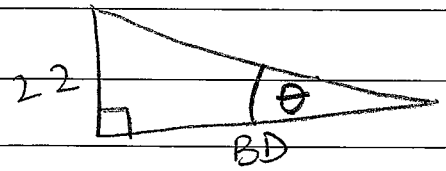
$\tan 18^\circ = 22/x$

$x = 22/\tan 18^\circ$

$= 67.71$ mm.



$BD = \sqrt{60^2 + 67.71^2} = 90.47$



$\tan\theta = 22/BD$

$\theta \approx 14^\circ$

Question 19 (2 marks)

Solve the equation $2log_7 5 = 2log_7 x - log_7 3$

$\log_7 5^2 = \log_7 \left(\frac{x^2}{3}\right)$ equating

$25 = x^2/3$

$x^2 = 75$ Now $x > 0$

$\therefore x = \sqrt{75}$ $(= 5\sqrt{3})$

Question 20 (6 marks)

Find:

i. $\int (\ln 3)x^2 \, dx$ **2**

$\frac{\ln 3 \cdot x^3}{3} + C$ (penalise +C here)

ii. $\int \frac{3x}{3-x^2} \, dx$ **2**

$-3/2 \ln|3 - x^2| + C$ (Needs Abs value sign)

iii. $\int_{-1}^{2} \frac{dx}{(4-x)^2}$ **2**

$= \int_{-1}^{2} (4-x)^{-2} \, dx$

$= \frac{1}{4-x} \Big]_{-1}^{2}$

$= 1/2 - (1/5)$

$= 3/10$

Question 21 (4 marks)

i. $\frac{d}{dx}(x^2-6)^8$ **2**

$= 8(x^2-6)^7 \cdot 2x$

$= 16x(x^2-6)^7$

ii. Hence, evaluate $\int_0^2 x(x^2-6)^7\,dx$ **2**

$= 1/16 \int_0^2 16x(x^2-6)^7\,dx$

$= \frac{1}{16}(x^2-6)^8 \Big]_0^2$

$= \frac{1}{16}\left[(4-6)^8 - (-6)^8\right]$

$= 104992$

Question 22 (3 marks)

Prove that

$$\frac{\sin\theta}{1+\cos\theta} + \frac{1+\cos\theta}{\sin\theta} = 2\,\mathrm{cosec}\theta$$

$LHS = \frac{\sin\theta}{1+\cos\theta} + \frac{1+\cos\theta}{\sin\theta}$

$= \frac{\sin^2\theta + (1+\cos\theta)^2}{(1+\cos\theta)\sin\theta}$ [common den.]

$= \frac{\sin^2\theta + 1 + 2\cos\theta + \cos^2\theta}{(1+\cos\theta)\sin\theta}$

$[\sin^2\theta + \cos^2\theta = 1]$

$= \frac{1 + 1 + 2\cos\theta}{(1+\cos\theta)\sin\theta}$

$= \frac{2(1+\cos\theta)}{(1+\cos\theta)\sin\theta}$

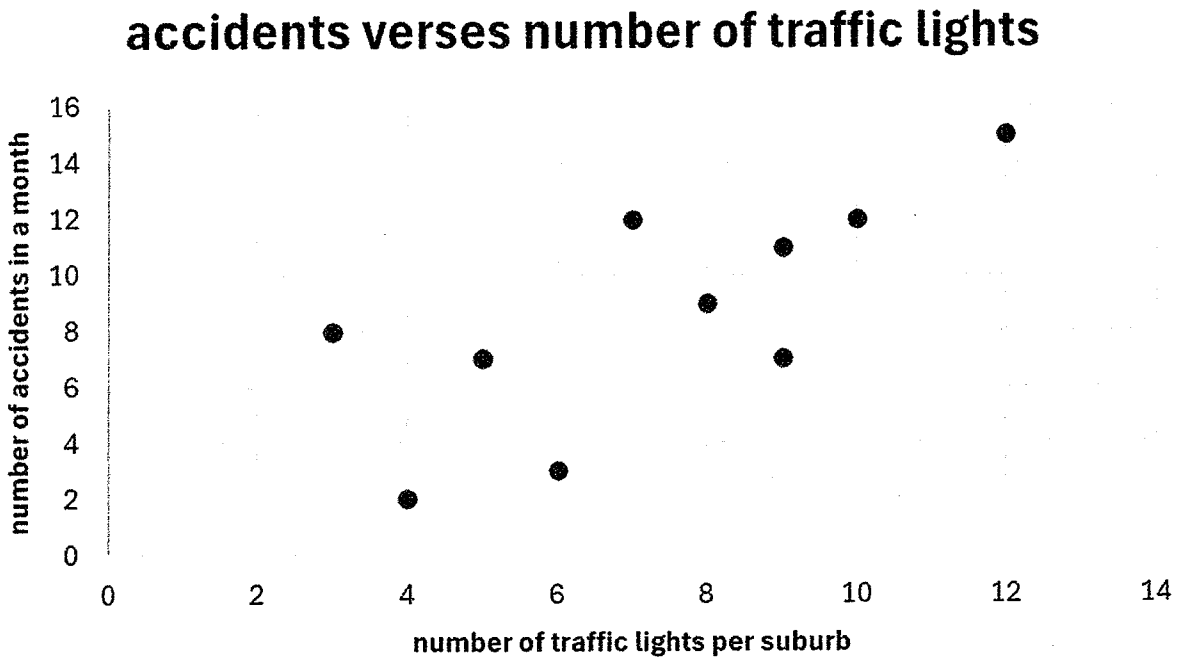
$= \frac{2}{\sin\theta}$ $\left[\frac{1}{\sin\theta} = \mathrm{cosec}\theta\right]$

$= 2\mathrm{cosec}\theta$

$= RHS$

Question 23 (3 marks)

The number of traffic lights in 10 suburbs and the number of accidents in a month are shown in the scatter plot below



Find the equation of the least squares regression line and comment on its use to predict the number of traffic lights in a suburb that had no accidents in the given month.

enter on calculator $y = A + Bx$

$A = 0.99$ $B = 1.04$

$\therefore$ $y = 1.04x + 0.99$

If $y = 0$ then x would be negative so it is not suitable to predict in this scenario.

Question 24 (3 marks)

The random variable X has the following probability distribution

x	2	3	5	7	9
$P(X) = x$	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{4}$	p	q

i. Find the value of p, q given $P(X = \{prime \cap odd\}) = \frac{31}{48}$ **2**

$1/6 + 1/3 + 1/4 + p + q = 1$ (Total prob)

also $1/3 + 1/4 + p = 31/48$

$\therefore$ solving $p = 1/16$ $q = 3/16$

ii. Find $E(X)$ Calculator or $\sum x p(x)$ **1**

$E(X) = 2/6 + 3/3 + 5/4 + 7/16 + 27/16$

$= 113/24$ or $4\frac{17}{24}$

Question 25 (3 marks)

The graph of $y = \sqrt{x}$ has been dilated both horizontally and vertically by a factor of 2 and then translated to the left 3 units before being reflected in the x – axis, to produce the equation $y = g(x)$

Write down the equation of $y = g(x)$

$x \to x/2$ $\quad y \to y/2$ $\quad y/2 = \sqrt{x/2}$

then $\frac{x}{2} \to \frac{(x+3)}{2}$ $\quad \frac{y}{2} = \sqrt{\frac{x+3}{2}}$

then reflect

$y = -2\sqrt{\frac{x+3}{2}}$ ← (1)

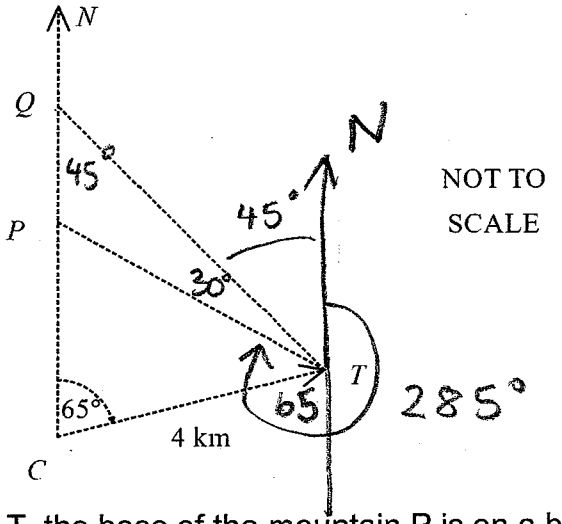
(1) ↑ (1)

Question 26 (5 marks)

Two mountains, *P* and *Q* lie directly north of a campground *C*.

Rehan walks from *C* on a bearing of 065°, to a point *T*, a distance of 4 km.

The diagram below shows the relative positions, when viewed from above.



315 − 285 = 30

i. From point T, the base of the mountain P is on a bearing of 285° whilst the base of mountain Q is on a bearing of 315°. **1**

Explain why angle QTC = 70° (answers vary)

mark diagram

$\angle CTP = 360° - 65° - 180° - 75° = 40°$

$\therefore \angle QTC = \angle PTQ + \angle CTP = 30° + 40° = 70°$

ii. Find the distance CQ, correct to 1 decimal place **2**

$$\frac{CQ}{\sin 70°} = \frac{4}{\sin 45°}$$

$CQ = 4\sin 70° \div \sin 45°$

$= 5.3$ km (1dp)

iii. Hence or otherwise find the distance PQ. **2**

$PC/\sin 40° = 4/\sin 75°$ in ΔPCT

$PC = 2.7$ (1dp)

$\therefore PQ = QC - CP \div 2.6$ (1dp) km

Question 27 (4 marks)

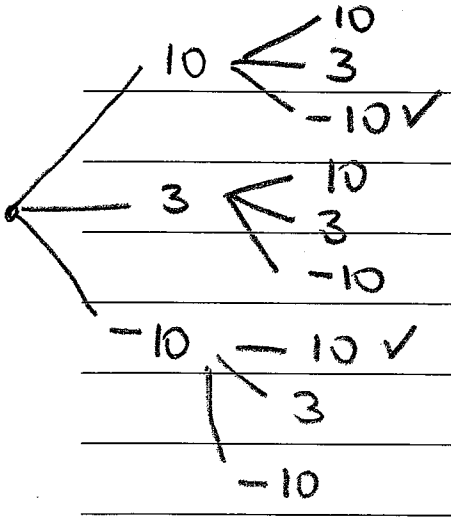
Thomas makes up a card game where he gets points for the type of card chosen randomly from a well shuffled deck of 52 playing cards.

He replaces the card chosen after each draw.

The game is scored as follows;

- He scores 10 points if he draws an Ace, King, Queen or Jack 16
- He scores 3 points if he draws a card less than 10 & 4
- He scores -10 points if the card drawn is a 10 of any suit.

i. What is the probability that Thomas will score zero in his first 2 draws. **2**



$$P(\text{zero}) = \frac{16}{52} \times \frac{4}{52} + \frac{4}{52} \times \frac{16}{52}$$

$$= \frac{128}{2704}$$

$$= \frac{8}{169}$$

ii. After playing his game a few times, Thomas decides to change the rules. His new rules are:

- He wins if he draws an Ace, King, Queen or Jack and the game stops
- He plays again if he draws a card less than 10 &
- He loses and the game stops if he draws a 10 of any suit.

W or PW or PPW or PPPW ----

What is the probability that he wins the game? **2**

$$P(W) = \frac{16}{52} + \frac{32}{52} \times \frac{16}{52} + \left(\frac{32}{52}\right)^2 \times \frac{16}{52} + \cdots$$

$$= 16/52 \left(1 + 32/52 + 32/52^2 + \cdots\right)$$

$$= 16/52 \times a/1-r \qquad a = 1 \quad r = 32/52$$

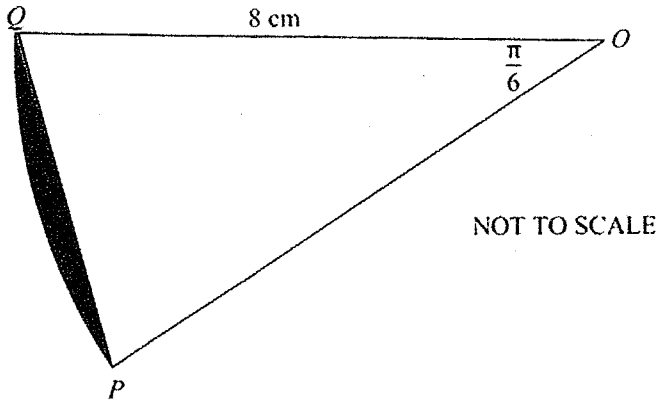
$$= \frac{4}{5}$$

Question 28 (3 marks)

A sector OPQ of a circle, is shown in the diagram.

The circle has its centre at O and a radius of 8 cm. **3**

$\angle QOP = \frac{\pi}{6}$.



Find the perimeter of the shaded segment, leave in exact form.

length side PQ — cosine rule

$PQ^2 = 8^2 + 8^2 - 2 \times 8 \times 8 \times \cos \pi/6$

$PQ^2 = 128 - 64\sqrt{3}$

Arc PQ $\Rightarrow$ $l = r\theta = 8 \times \pi/6$

$\therefore P = ARC + PQ$

$= \frac{4\pi}{3} + \sqrt{128 - 64\sqrt{3}}$ cm

Question 29 (3 marks)

Mathsville's population at the start of 2000 is unknown. However, by the beginning of 2020 it had grown to 2.5 million.

Assuming that the population of Mathsville can be modelled exponentially, using the equation, $P = Ae^{0.0144t}$,where A is a constant, and t is measured in years from the beginning of 2000.

i. Find the value of A **1**

$t = 10 \quad P = 2.5 \times 10^6$

$2.5 \times 10^6 = A e^{0.144}$

$A = 2164719.37$

$= 2164719$

ii. A newspaper article, published in 1980, forecast that the population of Mathsville would reach 5 million by the year 2050.

Using calculations to justify your response, state whether you agree or disagree with this article. **2**

$= 2164719 \times e^{0.0144 \times 50}$

$= 4447271$

∴ this is under the predicted 5 million

∴ the newspaper article was incorrect

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Question 30 (9 marks)

For the curve $f(x) = x^4 - 8x^2$

i. Show that $y = f(x)$ represents an even function. 1

$f(-x) = (-x)^4 - 8(-x)^2$

$= x^4 - 8x^2$

$= f(x)$ ∴ even fnc.

ii. Find the coordinates of all the stationary points and determine their nature. 3

$f'(x) = 4x^3 - 16x = 0$ for stat pts.

$4x(x^2 - 4) = 0$

$4x(x+2)(x-2) = 0$

$x=0$	$x=-2$	$x=2$
$y=0$	$y=-16$	$y=-16$

nature (either test ok - must have correct notation please)

x	-3	-2	-1
$f'(x)$	-12×5	0	-4×3
	\	—	/

x	-1	0	1
$f'(x)$	/	0	4×-3
		—	\

x	1	2	3
$f'(x)$	\	0	12×5
		—	/

∴ $(-2,-16)$ and $(2,-16)$ are relative min TP's

$(0,0)$ is max T.P

iii. Find any points of inflexion. 2

$f''(x) = 12x^2 - 16 = 0$ for inflex

$x^2 = 16/12$

$x = \pm 4/2\sqrt{3}$ $y = -80/9$

check concavity changes

$f''(-3) = 12\times9 - 16 > 0$

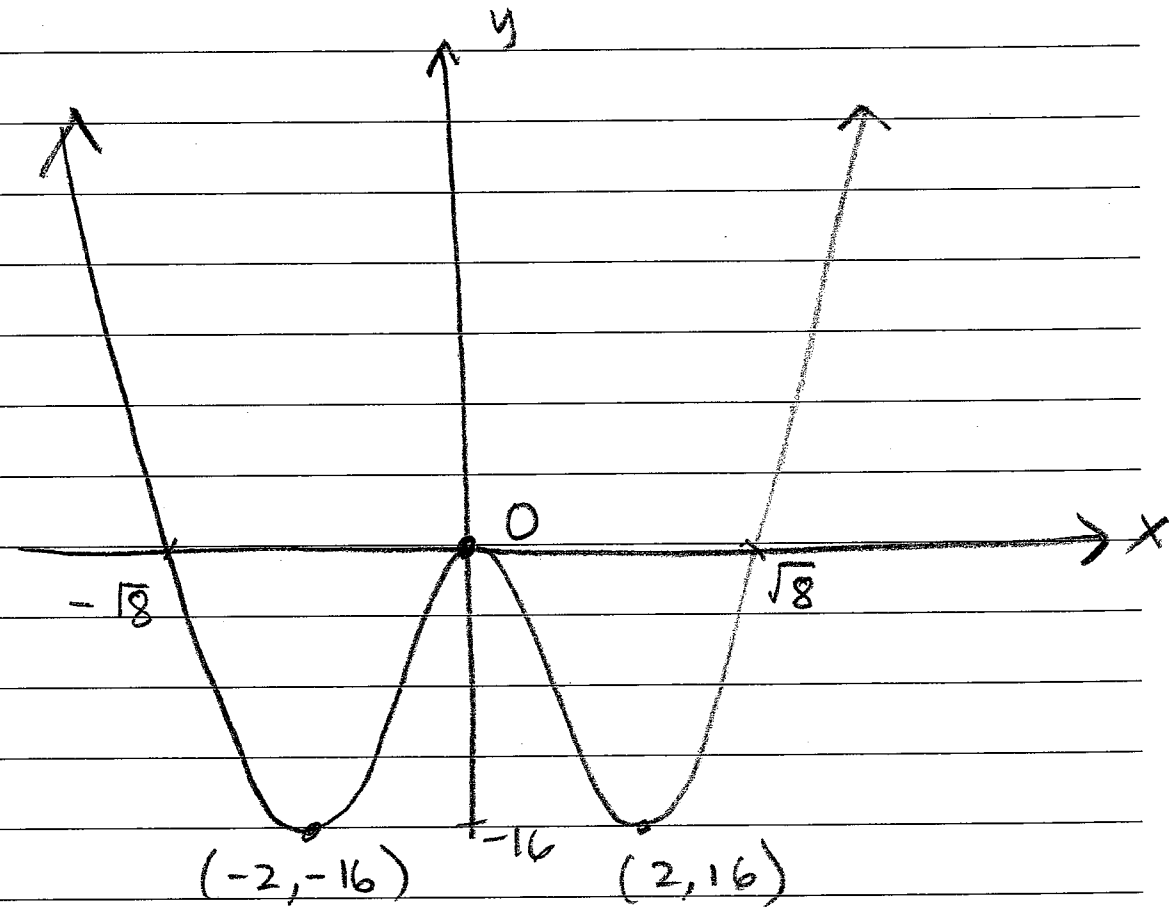
$f''(-1) = 12 - 16 < 0$

$f''(3) = 12\times9 - 16 > 0$

∴ inflections as concavity changes

iv. Sketch the curve in the domain, showing all stationary points as well as any inflexions and intercepts with the coordinate axes.

3



graph must be even ⇒ 1 mark

all pts correct ⇒ 1

intercepts on x-axis ⇒ 1 mark.

Question 31 (3 marks)

Evaluate $\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{\cos^2\left(\frac{\pi}{2}-x\right)}{\cos^2 x}\,dx$

$$= \int_{\pi/4}^{\pi/3} \frac{\sin^2 x}{\cos^2 x}\,dx$$

$$= \int \tan^2 x\,dx \qquad \frac{3\pi}{4} \quad \frac{4\pi}{3}$$

$$= \int \sec^2 x - 1\,dx$$

$$= \left[\tan x - x\right]_{\pi/4}^{\pi/3} = \tan\frac{\pi}{3} - \frac{\pi}{3} - \tan\frac{\pi}{4} + \frac{\pi}{4}$$

$$= \sqrt{3} - 1 - \pi/12$$

Question 32 (6 marks)

Jerome starts working on January 1, 2027. The advertised wage is \$56 000 p.a. After the first year his wage will increase to \$62 000 then increase by the same amount each year for the first 10 years of his employment, after that it will increase by the CPI of 3% p.a.

i. Show that Jerome earns \$74000 at the start of 2030. **1**

Jan 2027 56 000 → +6000 → Jan 2028 62 000 → +6000 → Jan 2029 68 000 → +6000 → Jan 2030 74 000

OR $T_n = a + (n-1)d \qquad a = 56000 \quad d = 6000$

$= 56000 + (4-1) \times 6000 \qquad n = 4$

$= \$74\,000$

ii. How much will Jerome earn for the first 10 years. **2**

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$= \frac{10}{2}\left[2 \times 56000 + 9 \times 6000\right]$$

$$= \$830\,000$$

iii. How many years will Jerome need to work before the total he has earned is over 2 million dollars. **3**

→ GP

$830000 + T_{11} + T_{12} + \dots$

$T_{11} = T_{10} \times 1.03$

$= [56000 + 9 \times 6000] \times 1.03$

$= 113300$

Now GP has $a = 113300$ $r = 1.03$ $n = ?$

$830000 + \frac{a(r^n - 1)}{r - 1} > 2000000$

-830000 -830000

$\frac{113300(1.03^n - 1)}{0.03} > 1170000$

$1.03^n - 1 > 0.30979\dots$

$1.03^n > 1.3097\dots$

$n \log 1.03 > \log 1.3097$

$n > 9.13\dots$

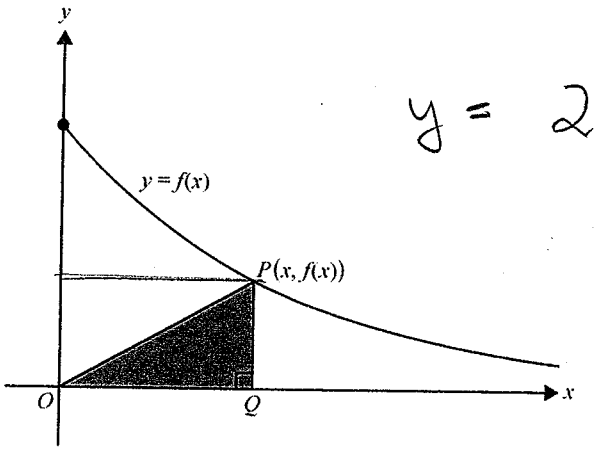
$\therefore n = 10$

$\therefore$ must work 20 yrs.

Question 34 (7 marks)

A point P$(x, f(x))$ lies on the curve $f(x) = \frac{2}{\sqrt[5]{e^x}}$

The perpendicular from P to the x axis, at Q, is shown and triangle OPQ is shaded.



$y = 2e^{-x/5}$

i. Find the area of triangle OPQ in terms of x. **2**

$A = \frac{1}{2} \times OQ \times PQ$

$= \frac{1}{2} \times x \times 2e^{-x/5}$

$A = xe^{-x/5}$ or equivalent

ii. Find the x –coordinate of P, for which the triangle OPQ has a maximum area. **3**

Max area $dA/dx = 0$ $dA/dx = x\left[-\frac{1}{5}e^{-x/5}\right] + 1\left[e^{-x/5}\right]$

$\frac{dA}{dx} = e^{-x/5}\left[-\frac{x}{5} + 1\right] = 0$

$\therefore x = 5$ as $e^{-x/5} \neq 0$

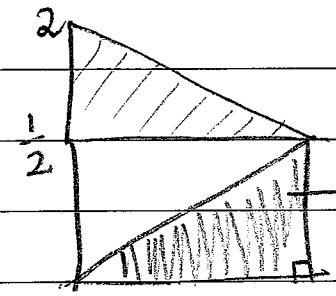
test nature (Sign determined by $-x/5 + 1$ as $e^{-x/5} > 0$ for all x)

x	4	5	6
dA/dx	$-4/5+1$	0	$-\frac{6}{5}+1$

$\therefore$ max Area when $x = 5$

Question 35 continues the next page .....

β. Find the area of the shaded region, held between the line ST and the curve $y = f(x)$. **3**



Δ OQP

$\rightarrow A = x e^{-x/5}$ when $x = 5\ln 4$

$$\therefore \text{rectangle} = 2 \times 5\ln 4 \, e^{-\frac{5\ln 4}{5}}$$

$$= 2 \times 5\ln 4 \times \frac{1}{4}$$

$$= \frac{5}{2} \ln 4 \qquad (4 = 2^2)$$

$$= 5 \ln 2$$

$$\Delta = \frac{1}{2} \times 3/2 \times 5\ln 4$$

$$= 15/4 \ln 4$$

$$= 15/2 \ln 2$$

$$\therefore \text{Shaded area} = \frac{15}{2}\ln 2 + 5\ln 2 - \int_0^{5\ln 4} 2e^{-x/5}\,dx$$

$$= \frac{25}{2}\ln 2 - \left[-10e^{-x/5}\right]_0^{5\ln 4}$$

$$= \frac{25}{2}\ln 2 - \left[-10 \times \frac{1}{4} - (-10e^0)\right]$$

$$= \frac{25}{2}\ln 2 + \frac{10}{4} - 10$$

$$= \frac{1}{2}(25\ln 2 - 15)\ u^2$$

END OF ASSESSMENT TASK