Centre Number | 1 | 2 | 5 |

CRANBROOK
SCHOOL

Student Number | | | | | | | | |

2020

HSC Assessment Task

Assessment Task 4

Mathematics Extension 1

Reading time	5 minutes
Writing time	50 minutes
Total Marks	25
Task weighting	15%

General Instructions

- Write using blue or black pen
- A Board-approved calculator may be used
- The Reference sheet is at the back of this booklet
- Use the Multiple-Choice Answer Sheet provided
- Each student will need a minimum of 3 booklets to answer Questions 6-8. Please use a separate booklet for each question

Additional Materials Needed

- Multiple Choice Answer Sheet
- 3 writing booklets

Structure & Suggested Time Spent

Section I
Multiple Choice Questions

- Answer Q1 – 5 on the multiple choice answer sheet
- Allow 10 minutes for this section

Section II
Extended response Questions

- Attempt all questions in this section in separate booklets
- Allow about 40 minutes for this section

This paper must not be removed from the examination room

Disclaimer
The content and format of this paper does not necessarily reflect the content and format of the HSC examination paper.

Section I

5 Marks

Allow about 10 minutes for this section

Use the multiple choice answer sheet for Questions 1 – 5.

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1 $3^n + 7^{n+1}$ for $n \in \mathbb{Z}^+$ is divisible by:

(A) 3

(B) 4

(C) 5

(D) 10

2 If $\begin{bmatrix} 3 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} b \\ -1 \end{bmatrix}$ are perpendicular. The value of b is:

(A) -3

(B) 3

(C) $\frac{1}{3}$

(D) $-\frac{1}{3}$

3 The graph $y = f(x)$ is reflected about the x-axis and dilated horizontally with a scale factor of 3. The transformed graph has the equation:

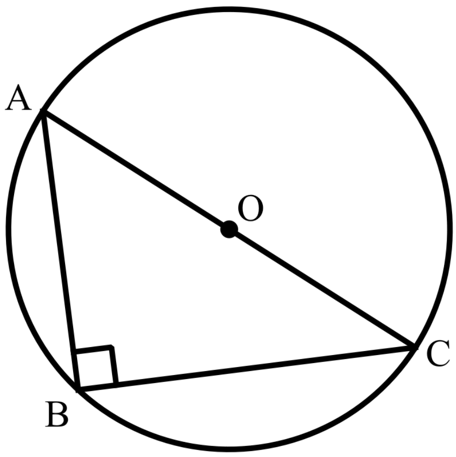
(A) $y = -f\left(\frac{x}{3}\right)$

(B) $y = f\left(\frac{-x}{3}\right)$

(C) $y = -f(x-3)$

(D) $y = -f(3x)$

4 Consider the circle with centre O and diameter $\overrightarrow{AC} = \underset{\sim}{a}$. Point B is on the circumference of the circle, where $\overrightarrow{BC} = \underset{\sim}{b}$ and $\angle ABC = 90^\circ$



Which of the following statements is true?

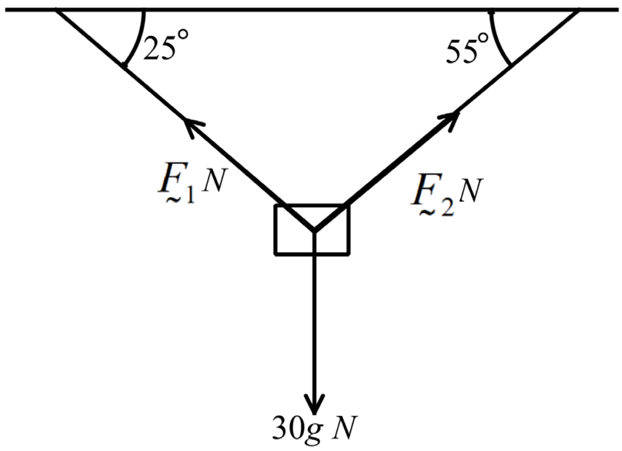
(A) $|\underset{\sim}{a}|^2 = 2|\underset{\sim}{b}|^2$

(B) $\underset{\sim}{a} \cdot \underset{\sim}{b} = 0$

(C) $|\underset{\sim}{a}|^2 - |\underset{\sim}{b}|^2 = \underset{\sim}{a} \cdot \underset{\sim}{b}$

(D) $|\underset{\sim}{b}|^2 = \underset{\sim}{a} \cdot \underset{\sim}{b}$

5 A particle of mass 30kg is suspended by two strings attached to two points in the same horizontal plane. The two strings make angles of 25° and 55° respectively to the horizontal. The magnitude of acceleration due to gravity is $10\,ms^{-2}$.



The magnitude of the tension force $\underset{\sim}{F}_1$, correct to the nearest newton, N, is given by:

(A) 17 N

(B) 28 N

(C) 175 N

(D) 276 N

END OF SECTION I

Section II

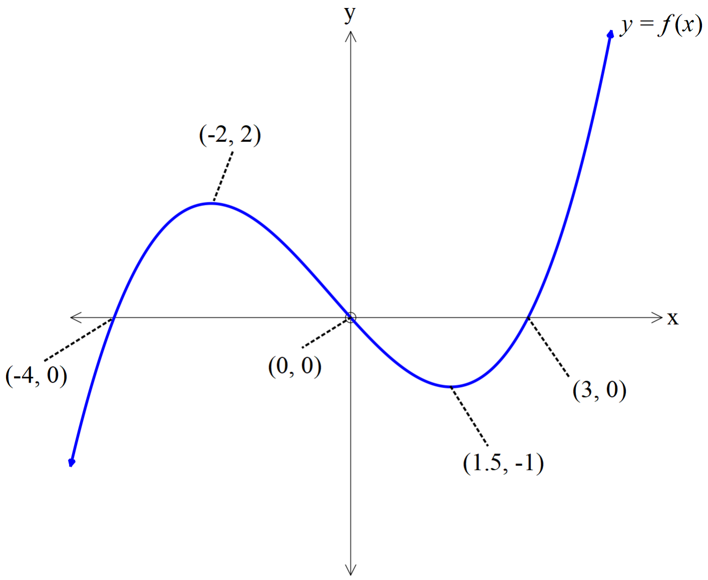
20 Marks

Allow about 40 minutes for this section.

Answer questions 6 – 8 in separate booklets.

Question 6 **START A NEW BOOKLET** **6 Marks**

(a) Consider the sketch of $y = f(x)$ below



Please answer the following questions using the attached answer sheets.

Sketch the following showing all key features:

(i) $y = f(3-x)+1$ **2**

(ii) $y = f\left(\left|\frac{x}{2}\right|\right)$ **2**

(iii) $y = \frac{1}{f(-x)}$ **2**

END OF QUESTION 6

Question 7 **START A NEW BOOKLET** **6 Marks**

(a) Prove that $2\times1! + 5\times2! + 10\times3! + \ldots + \left(n^2+1\right)n! = n\left(n+1\right)!$ for $n\in\mathbb{Z}^+$ by mathematical induction. **3**

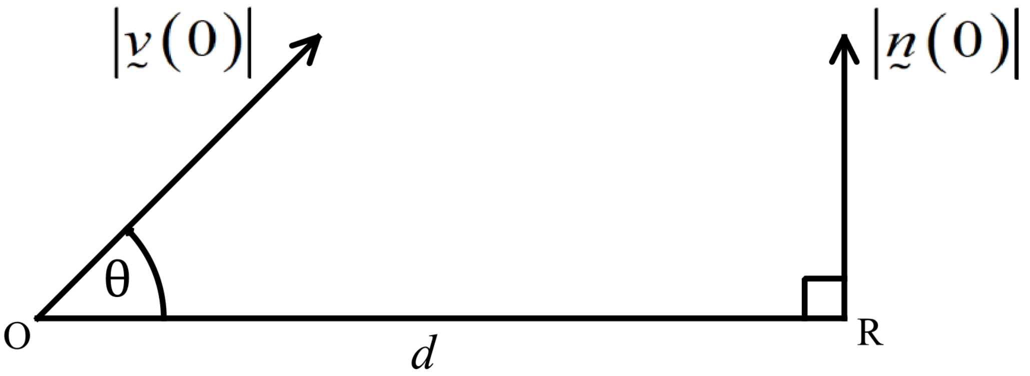
(b) Prove that $x^{2n}-y^{2n}$ is divisible by $\left(x+y\right)$ for $n\in\mathbb{Z}^+$ by mathematical induction where $x, y\in\mathbb{Z}$. **3**

END OF QUESTION 7

Question 8 **START A NEW BOOKLET** **8 Marks**

(a) O and R are two points d metres apart on horizontal ground. A rocket is projected from O with speed $\left|\underset{\sim}{v}(0)\right|$ m/s at an angle of θ above the horizontal, where $0 < \theta < \frac{\pi}{2}$.

At the same instant, another rocket is projected vertically from R with speed $\left|\underset{\sim}{n}(0)\right|$ m/s.



After time t seconds, the rocket from O has horizontal and vertical displacements from O, while the rocket from R has vertical displacement from R and a constant horizontal displacement from O.

The two rockets collide after T seconds.

The vector equation of the projectile fired from O is given by $\underset{\sim}{r}(t) = \left(\left|\underset{\sim}{v}(0)\right| t\cos\theta\right)\underset{\sim}{i} + \left(\left|\underset{\sim}{v}(0)\right| t\sin\theta - \frac{1}{2}gt^2\right)\underset{\sim}{j}$ where $g\ ms^{-2}$ is the acceleration due to gravity. *(You do not need to prove this).*

Question 8 continues on the next page

(i) Show that the vector equation for the displacement of the rocket fired from R can be given by $\underset{\sim}{s}(t) = d\underset{\sim}{i} + \left(|\underset{\sim}{n}(0)|t - \frac{1}{2}gt^2 \right)\underset{\sim}{j}$.

(<u>Hint</u>: let the vector equation for the acceleration of the rocket projected from R be $\underset{\sim}{a}(t) = 0\underset{\sim}{i} - g\underset{\sim}{j}$*)* **2**

(ii) Hence, show that $d = |\underset{\sim}{v}(0)|T\cos\theta$ and $|\underset{\sim}{n}(0)| = |\underset{\sim}{v}(0)|\sin\theta$ **2**

(iii) Explain why $|\underset{\sim}{v}(0)| > |\underset{\sim}{n}(0)|$ for the rockets to collide **1**

(iv) Show that $T = \dfrac{d}{\sqrt{|\underset{\sim}{v}(0)|^2 - |\underset{\sim}{n}(0)|^2}}$ **2**

(v) If the two rockets collide at the highest points of their respective flights, show that $d = \dfrac{|\underset{\sim}{n}(0)| \times \sqrt{|\underset{\sim}{v}(0)|^2 - |\underset{\sim}{n}(0)|^2}}{g}$ **1**

END OF QUESTION 8

END OF ASSESSMENT

NSW Education Standards Authority

2020 HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Advanced
Mathematics Extension 1
Mathematics Extension 2

REFERENCE SHEET

Measurement

Length

$$l = \frac{\theta}{360} \times 2\pi r$$

Area

$$A = \frac{\theta}{360} \times \pi r^2$$

$$A = \frac{h}{2}(a + b)$$

Surface area

$$A = 2\pi r^2 + 2\pi rh$$

$$A = 4\pi r^2$$

Volume

$$V = \frac{1}{3}Ah$$

$$V = \frac{4}{3}\pi r^3$$

Functions

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For $ax^3 + bx^2 + cx + d = 0$:

$$\alpha + \beta + \gamma = -\frac{b}{a}$$

$$\alpha\beta + \alpha\gamma + \beta\gamma = \frac{c}{a}$$

and $\alpha\beta\gamma = -\frac{d}{a}$

Relations

$$(x - h)^2 + (y - k)^2 = r^2$$

Financial Mathematics

$$A = P(1 + r)^n$$

Sequences and series

$$T_n = a + (n - 1)d$$

$$S_n = \frac{n}{2}[2a + (n - 1)d] = \frac{n}{2}(a + l)$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(1 - r^n)}{1 - r} = \frac{a(r^n - 1)}{r - 1}, \ r \neq 1$$

$$S = \frac{a}{1 - r}, \ |r| < 1$$

Logarithmic and Exponential Functions

$$\log_a a^x = x = a^{\log_a x}$$

$$\log_a x = \frac{\log_b x}{\log_b a}$$

$$a^x = e^{x \ln a}$$

Trigonometric Functions

$\sin A = \dfrac{\text{opp}}{\text{hyp}}, \quad \cos A = \dfrac{\text{adj}}{\text{hyp}}, \quad \tan A = \dfrac{\text{opp}}{\text{adj}}$

$A = \dfrac{1}{2}ab\sin C$

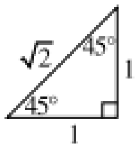
$\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

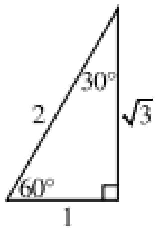
$c^2 = a^2 + b^2 - 2ab\cos C$

$\cos C = \dfrac{a^2 + b^2 - c^2}{2ab}$

$l = r\theta$

$A = \dfrac{1}{2}r^2\theta$





Trigonometric identities

$\sec A = \dfrac{1}{\cos A}, \ \cos A \neq 0$

$\operatorname{cosec} A = \dfrac{1}{\sin A}, \ \sin A \neq 0$

$\cot A = \dfrac{\cos A}{\sin A}, \ \sin A \neq 0$

$\cos^2 x + \sin^2 x = 1$

Compound angles

$\sin(A + B) = \sin A\cos B + \cos A\sin B$

$\cos(A + B) = \cos A\cos B - \sin A\sin B$

$\tan(A + B) = \dfrac{\tan A + \tan B}{1 - \tan A\tan B}$

If $t = \tan\dfrac{A}{2}$ then $\sin A = \dfrac{2t}{1 + t^2}$

$\cos A = \dfrac{1 - t^2}{1 + t^2}$

$\tan A = \dfrac{2t}{1 - t^2}$

$\cos A\cos B = \dfrac{1}{2}\left[\cos(A - B) + \cos(A + B)\right]$

$\sin A\sin B = \dfrac{1}{2}\left[\cos(A - B) - \cos(A + B)\right]$

$\sin A\cos B = \dfrac{1}{2}\left[\sin(A + B) + \sin(A - B)\right]$

$\cos A\sin B = \dfrac{1}{2}\left[\sin(A + B) - \sin(A - B)\right]$

$\sin^2 nx = \dfrac{1}{2}(1 - \cos 2nx)$

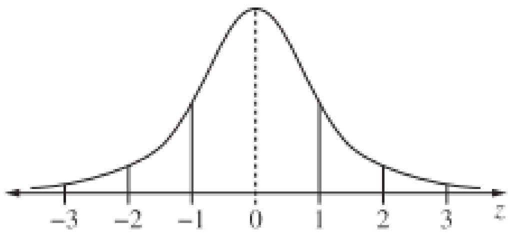
$\cos^2 nx = \dfrac{1}{2}(1 + \cos 2nx)$

Statistical Analysis

$z = \dfrac{x - \mu}{\sigma}$

An outlier is a score
less than $Q_1 - 1.5 \times IQR$
or
more than $Q_3 + 1.5 \times IQR$

Normal distribution



- approximately 68% of scores have z-scores between –1 and 1
- approximately 95% of scores have z-scores between –2 and 2
- approximately 99.7% of scores have z-scores between –3 and 3

$E(X) = \mu$

$\operatorname{Var}(X) = E\left[(X - \mu)^2\right] = E\left(X^2\right) - \mu^2$

Probability

$P(A \cap B) = P(A)P(B)$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A|B) = \dfrac{P(A \cap B)}{P(B)}, \ P(B) \neq 0$

Continuous random variables

$P(X \le x) = \displaystyle\int_a^x f(x)\,dx$

$P(a < X < b) = \displaystyle\int_a^b f(x)\,dx$

Binomial distribution

$P(X = r) = {}^nC_r p^r(1 - p)^{n - r}$

$X \sim \operatorname{Bin}(n, p)$

$\Rightarrow \ P(X = x)$

$= \dbinom{n}{x}p^x(1 - p)^{n - x}, \ x = 0, 1, \ldots, n$

$E(X) = np$

$\operatorname{Var}(X) = np(1 - p)$

Differential Calculus

Function	Derivative
$y = f(x)^n$	$\frac{dy}{dx} = nf'(x)[f(x)]^{n-1}$
$y = uv$	$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$
$y = g(u)$ where $u = f(x)$	$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$
$y = \frac{u}{v}$	$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$
$y = \sin f(x)$	$\frac{dy}{dx} = f'(x)\cos f(x)$
$y = \cos f(x)$	$\frac{dy}{dx} = -f'(x)\sin f(x)$
$y = \tan f(x)$	$\frac{dy}{dx} = f'(x)\sec^2 f(x)$
$y = e^{f(x)}$	$\frac{dy}{dx} = f'(x)e^{f(x)}$
$y = \ln f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{f(x)}$
$y = a^{f(x)}$	$\frac{dy}{dx} = (\ln a)f'(x)a^{f(x)}$
$y = \log_a f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{(\ln a)f(x)}$
$y = \sin^{-1} f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{\sqrt{1-[f(x)]^2}}$
$y = \cos^{-1} f(x)$	$\frac{dy}{dx} = -\frac{f'(x)}{\sqrt{1-[f(x)]^2}}$
$y = \tan^{-1} f(x)$	$\frac{dy}{dx} = \frac{f'(x)}{1+[f(x)]^2}$

Integral Calculus

$$\int f'(x)[f(x)]^n dx = \frac{1}{n+1}[f(x)]^{n+1} + c$$

where $n \neq -1$

$$\int f'(x)\sin f(x)dx = -\cos f(x) + c$$

$$\int f'(x)\cos f(x)dx = \sin f(x) + c$$

$$\int f'(x)\sec^2 f(x)dx = \tan f(x) + c$$

$$\int f'(x)e^{f(x)}dx = e^{f(x)} + c$$

$$\int \frac{f'(x)}{f(x)}dx = \ln|f(x)| + c$$

$$\int f'(x)a^{f(x)}dx = \frac{a^{f(x)}}{\ln a} + c$$

$$\int \frac{f'(x)}{\sqrt{a^2-[f(x)]^2}}dx = \sin^{-1}\frac{f(x)}{a} + c$$

$$\int \frac{f'(x)}{a^2+[f(x)]^2}dx = \frac{1}{a}\tan^{-1}\frac{f(x)}{a} + c$$

$$\int u\frac{dv}{dx}dx = uv - \int v\frac{du}{dx}dx$$

$$\int_a^b f(x)dx$$

$$\approx \frac{b-a}{2n}\left\{f(a) + f(b) + 2\left[f(x_1) + \cdots + f(x_{n-1})\right]\right\}$$

where $a = x_0$ and $b = x_n$

Combinatorics

$${}^{n}P_{r} = \frac{n!}{(n-r)!}$$

$$\binom{n}{r} = {}^{n}C_{r} = \frac{n!}{r!\,(n-r)!}$$

$$(x+a)^{n} = x^{n} + \binom{n}{1}x^{n-1}a + \cdots + \binom{n}{r}x^{n-r}a^{r} + \cdots + a^{n}$$

Vectors

$$\left|\underset{\sim}{u}\right| = \left|x\underset{\sim}{i} + y\underset{\sim}{j}\right| = \sqrt{x^{2}+y^{2}}$$

$$\underset{\sim}{u}\cdot\underset{\sim}{v} = \left|\underset{\sim}{u}\right|\left|\underset{\sim}{v}\right|\cos\theta = x_{1}x_{2} + y_{1}y_{2},$$

where $\underset{\sim}{u} = x_{1}\underset{\sim}{i} + y_{1}\underset{\sim}{j}$

and $\underset{\sim}{v} = x_{2}\underset{\sim}{i} + y_{2}\underset{\sim}{j}$

$$\underset{\sim}{r} = \underset{\sim}{a} + \lambda\underset{\sim}{b}$$

Complex Numbers

$$\begin{aligned} z = a + ib &= r(\cos\theta + i\sin\theta) \\ &= re^{i\theta} \end{aligned}$$

$$\begin{aligned} \left[r(\cos\theta + i\sin\theta)\right]^{n} &= r^{n}(\cos n\theta + i\sin n\theta) \\ &= r^{n}e^{in\theta} \end{aligned}$$

Mechanics

$$\frac{d^{2}x}{dt^{2}} = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^{2}\right)$$

$$x = a\cos(nt + \alpha) + c$$

$$x = a\sin(nt + \alpha) + c$$

$$\ddot{x} = -n^{2}(x - c)$$

CRANBROOK
SCHOOL

ESSE QUAM VIDERI

Student Name/Number: ______________________

Teacher: ______________________

2020

HSC Examination
Assessment Task 4

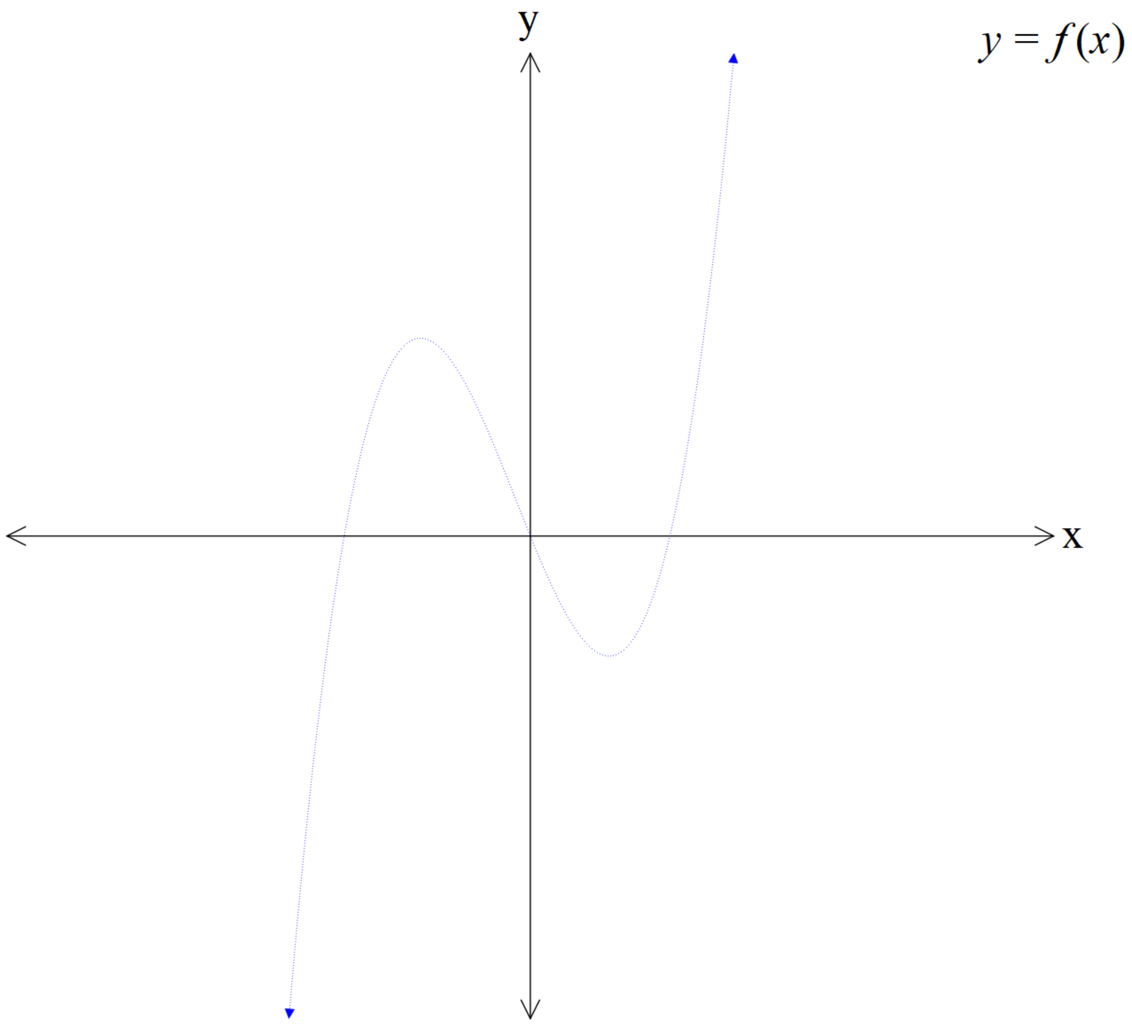
Section I – Multiple Choice

Mathematics Extension 1

Question	**1**	A ○	B ○	C ○	D ○
	2	A ○	B ○	C ○	D ○
	3	A ○	B ○	C ○	D ○
	4	A ○	B ○	C ○	D ○
	5	A ○	B ○	C ○	D ○

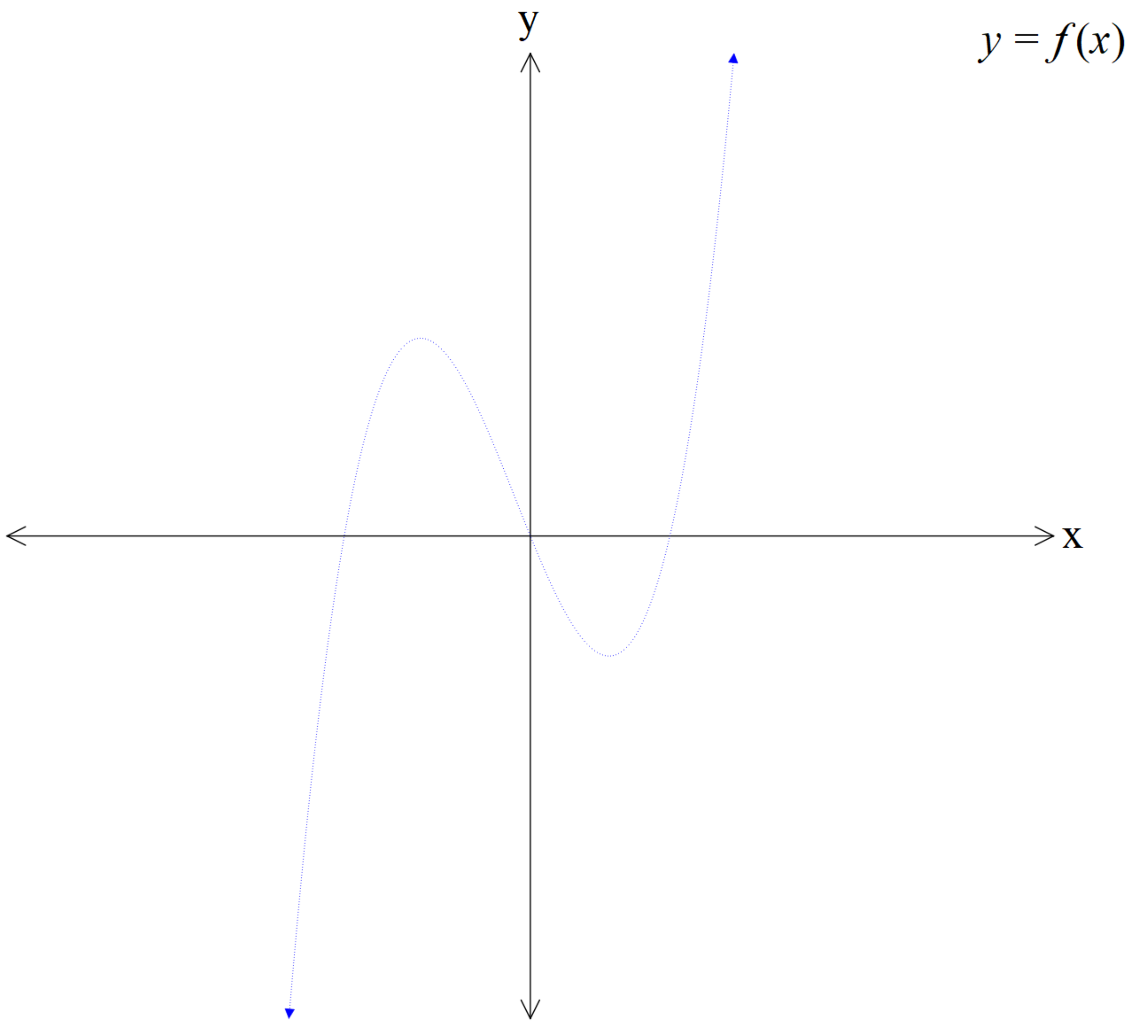
Student Name/Number: ____________________ Teacher: ________________

Question: 6 (a) (i) Answer



Student Name/Number: ____________________ Teacher: _______________

Question: 6 (a) (ii) Answer



Student Name/Number: ____________________ Teacher: ________________

Question: 6 (a) (iii) Answer