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Centre Number

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Student Number

2020

Year 12

Mathematics Extension 1

Task 3
19/06/2020

Q	Marks
MC	/3
3	/11
4	/13
5	/12
Total	/39

**General
Instructions:**

- Reading time – 5 minutes
- Working time – 55 minutes
- Write using blue or black pen
- NESA approved calculators may be used
- Show relevant mathematical reasoning and/or calculations

Total Marks:
39

Section I – 3 marks

- Allow about 5 minutes for this section

Section II – 36 marks

- Allow about 50 minutes for this section

This question paper must not be removed from the examination room.

This assessment task constitutes 25% of the course.

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Section I

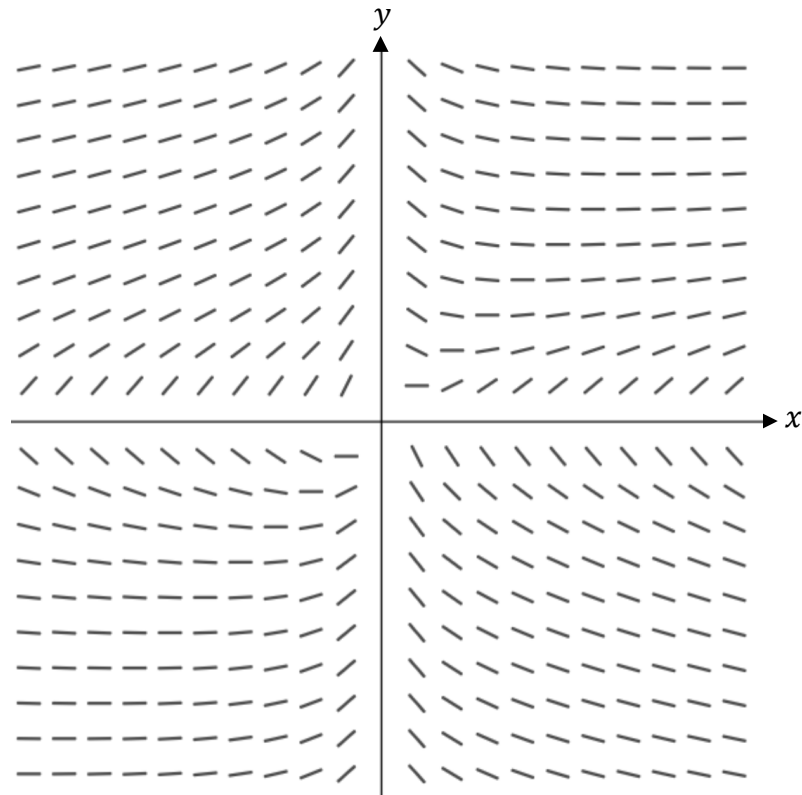
3 marks

Attempt Questions 1 – 3

Allow about 5 minutes for this section

Use the multiple-choice answer sheet for questions 1 – 3 (Detach from paper)

1



The differential equation that best represents the slope field above is:

(A) $\frac{dy}{dx} = \frac{x - y}{xy}$

(B) $\frac{dy}{dx} = \frac{y - x}{xy}$

(C) $\frac{dy}{dx} = \frac{x + y}{xy}$

(D) $\frac{dy}{dx} = \frac{x^2 - y^2}{xy}$

Section I continues on next page

2 $\int_{-\pi}^{-\frac{\pi}{2}} \cos^3(x) \sin(x) dx$ can be written as:

(A) $-\int_{-\pi}^{-\frac{\pi}{2}} u^3 du$

(B) $\int_{-1}^0 u^3 du$

(C) $\int_0^{-1} u^3 du$

(D) $\int_{-1}^0 u^4 du$

3 The derivative of $y = x^2 \sin^{-1}(2x)$ is:

(A) $2x \sin^{-1}(2x) + \frac{2x^2}{\sqrt{1-4x^2}}$

(B) $2x \sin^{-1}(2x) + \frac{x^2}{\sqrt{1-4x^2}}$

(C) $2x \sin^{-1}(2x) - \frac{4x^2}{\sqrt{1-4x^2}}$

(D) $2x \sin^{-1}(2x) + \frac{x^2}{\sqrt{1-2x^2}}$

End of Section I

Section II

36 marks

Attempt Questions 5 – 7

Allow about 50 minutes for this section

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

In Questions 5 – 7, your responses should include relevant mathematical reasoning and/or calculations.

Question 5 (11 marks) Use a SEPARATE writing booklet	Marks
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(a) Differentiate:	2
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$$\frac{\cos^{-1} x}{x}$$

(b) (i) Differentiate:	2
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$$\left(\tan^{-1}\left(\frac{x}{3}\right)\right)^2$$

(ii) Hence, find the exact value of	2
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$$\frac{1}{\pi} \int_0^{\sqrt{3}} \frac{\tan^{-1}\left(\frac{x}{3}\right)}{x^2 + 9} dx$$

(c) (i) Evaluate:	2
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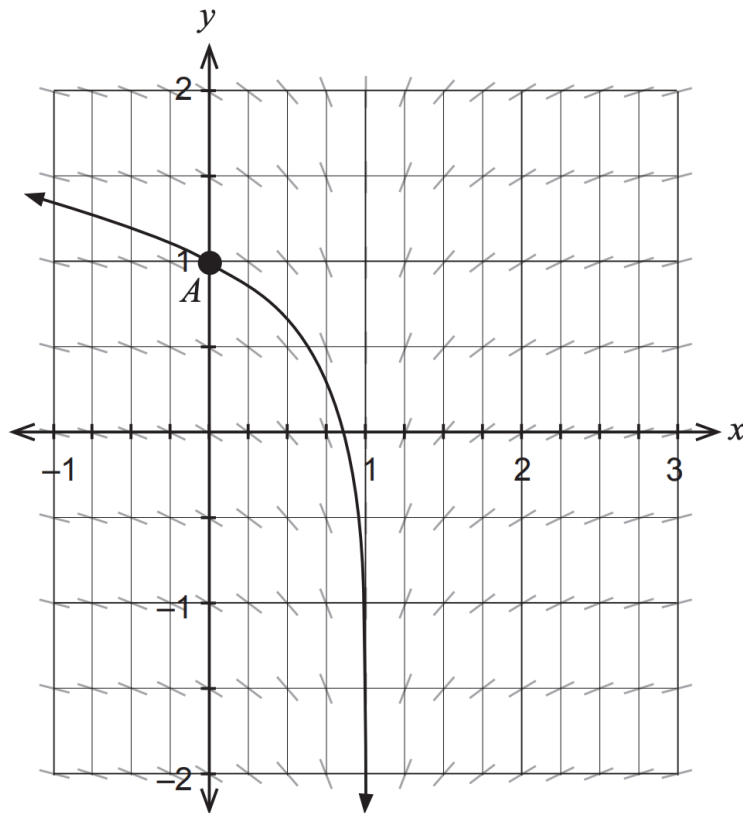
$$\int_0^{\frac{1}{2}} \frac{dx}{\sqrt{1-x^2}}$$

(ii) Use the substitution $u = 3x^2$ to find:	3
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$$\int \frac{x}{\sqrt{1-9x^4}} dx$$

End of Question 5

- (a) A series of magnets is placed under a glass pane and some iron fillings are sprinkled onto the glass. The orientation or slope of the iron fillings, as determined by the magnetic field, is shown below. One of the lines of magnetic force that passes through the point $A(0, 1)$ is also shown.



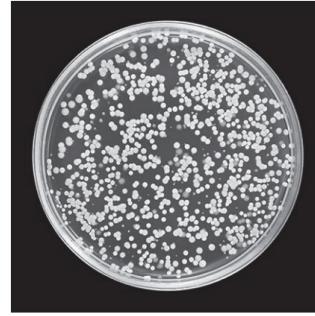
- | | | |
|-------|---|----------|
| (i) | The slope field is given by $\frac{dy}{dx} = \frac{1}{2x-2}$, $x \neq 1$.
Determine the value of the slope field at the point $(0, 1)$. | 2 |
| (ii) | Explain the orientation of the iron fillings at $x = 1$. | 1 |
| (iii) | Determine the equation for the line of force that passes through the point $A(0, 1)$. | 3 |

Question 6 continues on next page

- (b) In an experiment one type of bacterium called alpha was grown in a Petri dish. The rate of change of the area in the Petri dish that is covered by alpha bacteria can be modelled by the differential equation

$$\frac{dA}{dt} = \frac{1}{2}A \left(\frac{50 - A}{50} \right)$$

Where A is area in cm^2 and t is time in days.



- (i) At $t = 0$, the area in the Petri dish that is covered by alpha bacteria is 1 cm^2 . Draw the solution curve on the **provided slope field on page 11 (on the back of the Multiple Choice answer sheet)**. **2**

- (ii) Show that **1**

$$\frac{50}{A(50 - A)} = \frac{1}{A} + \frac{1}{50 - A}$$

- (iii) Hence, solve the differential equation **3**

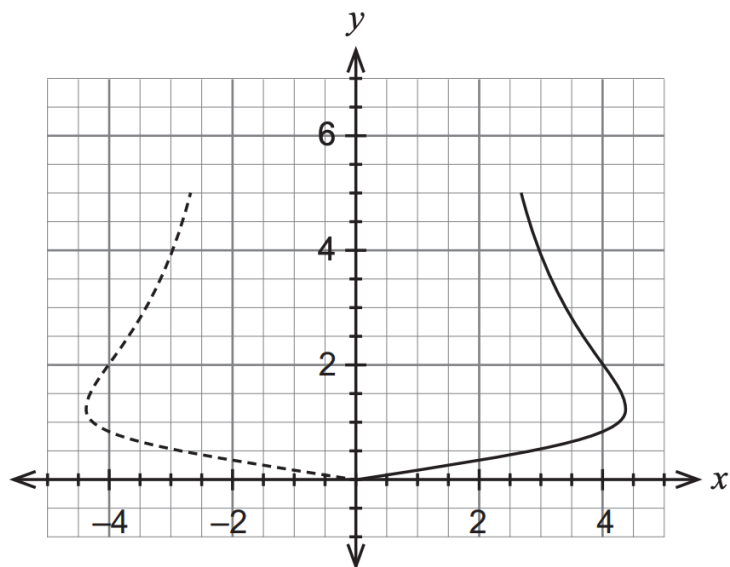
$$\frac{dA}{dt} = \frac{1}{2}A \left(\frac{50 - A}{50} \right)$$

With the initial condition $A(0) = 1$.

- (iv) State the maximum area that is available for bacterial growth. **1**

End of Question 6

- (a) The top part of a wine glass is modelled by rotating the graph of $x^2 = y^2(36 - x^2y)$ from $y = 0$ to $y = 5$ about the y -axis as shown below. Dimensions are measured in centimetres.

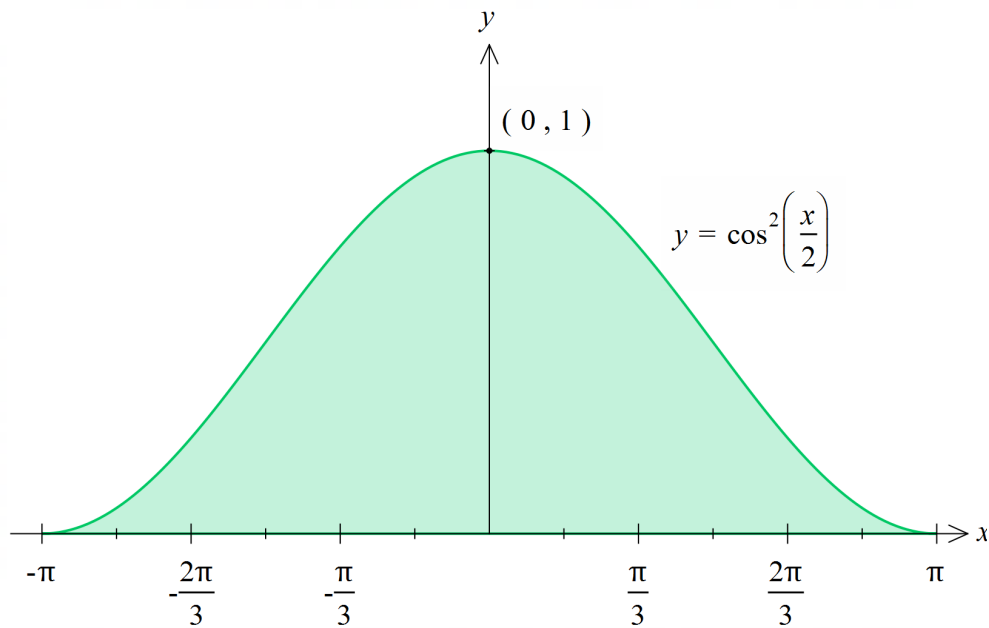


Determine the exact volume of the solid of revolution $V \text{ cm}^3$.

4

Question 7 continues on next page

- (b) An area is bounded by $y = \cos^2\left(\frac{x}{2}\right)$ and the x -axis for $-\pi \leq x \leq \pi$. Find the exact volume of the solid formed when this area is rotated about the x -axis. 4



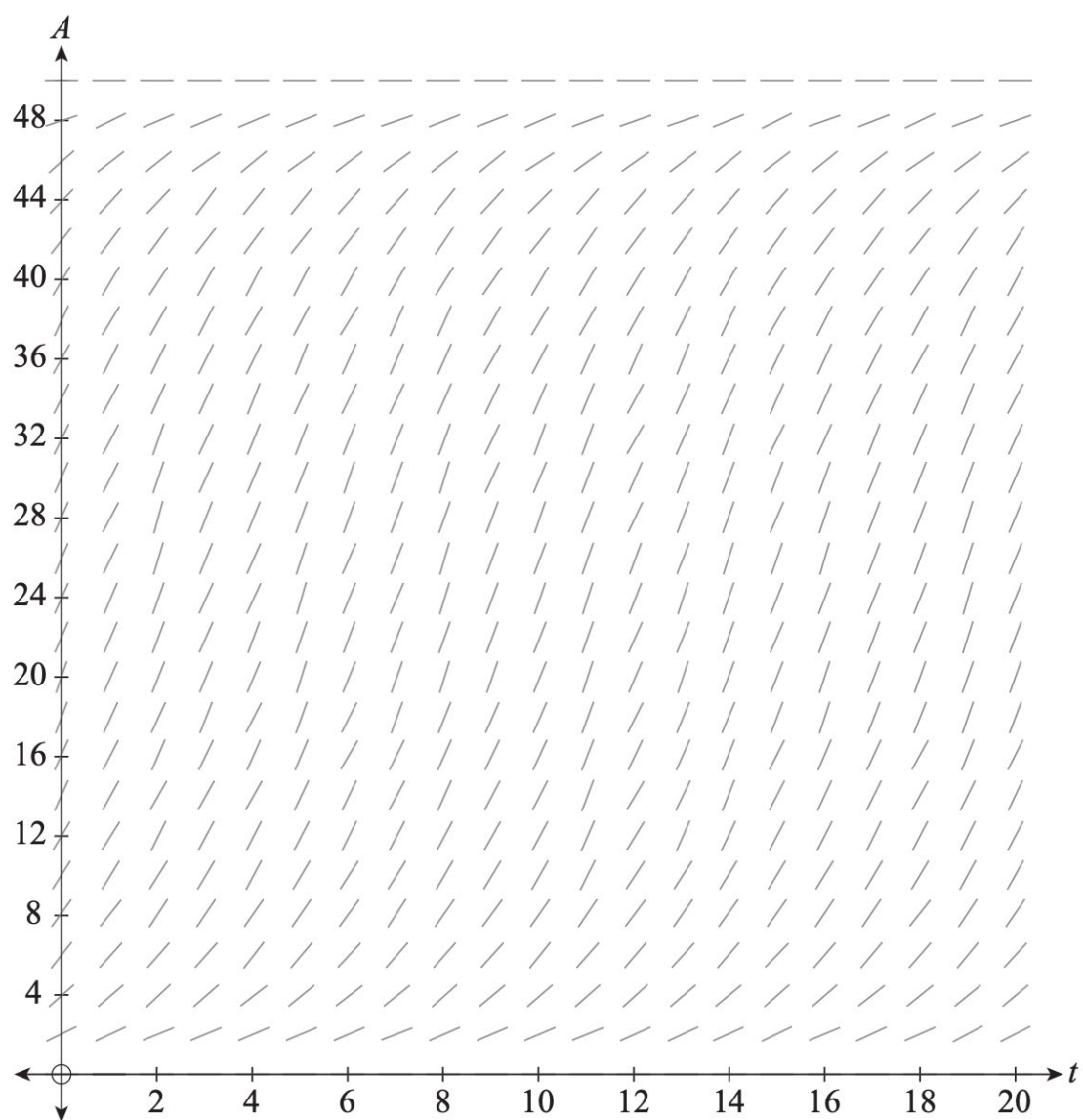
- (c) By using the substitution $u = \sin^2 x$, prove that 4

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sin 2x}{1 + \sin^4 x} dx = \tan^{-1}\left(\frac{8}{19}\right)$$

End of Section II and end of examination

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Slope Field for Question 6b part (i)



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Centre Number

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Student Number

Mathematics Extension 1

Section I – Multiple Choice Answer Sheet

Use this multiple-choice answer sheet for questions 1 – 3. **Detach this sheet.**

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample: $2 + 4 =$ (A) 2 (B) 6 (C) 8 (D) 9
A ☐ B ☒ C ☐ D ☐

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ☒ B ☒ C ☐ D ☐

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word *correct* and drawing an arrow as follows.

A ☒ B ☒ C ☐ D ☐
correct

**Start
Here**

→ 1. A ☐ B ☐ C ☐ D ☐

2. A ☐ B ☐ C ☐ D ☐

3. A ☐ B ☐ C ☐ D ☐