2021 **Year 12 Assessment Task 1**

Mathematics Extension 1

General Instructions	• Reading time – 10 minutes • Working time – 1 hour • Write using black pen • Calculators approved by NESA may be used • A reference sheet is provided at the back of this paper • In Questions 6 – 8, show relevant mathematical reasoning and/or calculations
Total marks: 32	**Section I – 5 marks** (pages 2 – 4) • Attempt Questions 1 – 5 • Allow about 10 minutes for this section **Section II – 30 marks** (pages 5 – 7) • Attempt Questions 6 – 8 • Allow about 50 minutes for this section

Section I

5 marks
Attempt Questions 1 – 10
Allow about 10 minutes for this section

Use the multiple-choice answer page in the writing booklet for Questions 1 – 10.

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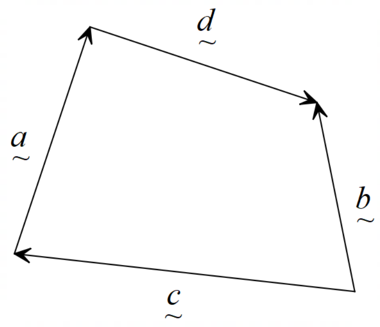
1 Consider the diagram below.



Which of the following is a correct statement?

A. $\underset{\sim}{a}+\underset{\sim}{b}+\underset{\sim}{c}=\underset{\sim}{d}$

B. $\underset{\sim}{b}+\underset{\sim}{c}+\underset{\sim}{d}=\underset{\sim}{a}$

C. $\underset{\sim}{c}+\underset{\sim}{d}+\underset{\sim}{a}=\underset{\sim}{b}$

D. $\underset{\sim}{d}+\underset{\sim}{a}+\underset{\sim}{b}=\underset{\sim}{c}$

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2 Which of the following vectors point in the same direction of $4\underset{\sim}{i}+2\underset{\sim}{j}$

A. $-4\underset{\sim}{i}-2\underset{\sim}{j}$

B. $\underset{\sim}{i}+2\underset{\sim}{j}$

C. $\underset{\sim}{i}+\frac{1}{2}\underset{\sim}{j}$

D. $-2\underset{\sim}{i}-4\underset{\sim}{j}$

3 Which of the following statements regarding non-zero vectors $\underset{\sim}{u}$ and $\underset{\sim}{v}$ is <u>**NOT ALWAYS TRUE**</u>?

A. $\left|\underset{\sim}{u}+\underset{\sim}{v}\right| \leq \left|\underset{\sim}{u}\right| + \left|\underset{\sim}{v}\right|$

B. $\underset{\sim}{u} \cdot \underset{\sim}{v} \leq \left|\underset{\sim}{u}\right| \times \left|\underset{\sim}{v}\right|$

C. $\left|\operatorname{proj}_{\underset{\sim}{v}} \underset{\sim}{u}\right| \leq \left|\underset{\sim}{u}\right|$

D. $\left|\hat{\underset{\sim}{u}}\right| \leq \left|\underset{\sim}{u}\right|$

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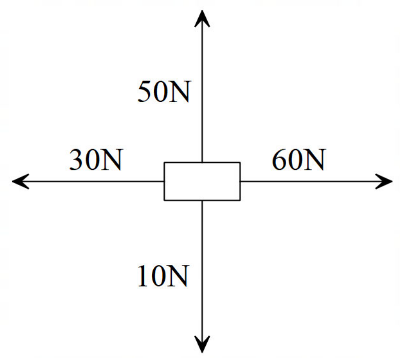
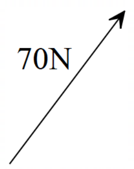
4 The following diagram shows the direction and magnitude of four forces acting on an object.



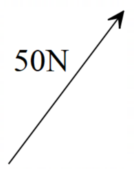
LENGTH OF ARROWS NOT DRAWN TO SCALE

What is the direction of magnitude of the net force on this object?

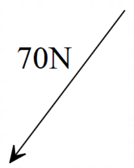
A.



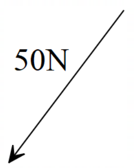
B.



C.



D.



5 Consider two non-zero vectors $\underset{\sim}{a}$ and $\underset{\sim}{b}$. It is given that $|\underset{\sim}{a}|=|\underset{\sim}{b}|$, which of the following statement is **<u>ALWAYS TRUE</u>**?

A. $\underset{\sim}{a}=\underset{\sim}{b}$

B. $\underset{\sim}{a}-\underset{\sim}{b}=\underset{\sim}{b}-\underset{\sim}{a}$

C. $\text{proj}_{\underset{\sim}{a}}\underset{\sim}{b}=\text{proj}_{\underset{\sim}{b}}\underset{\sim}{a}$

D. $\left|\text{proj}_{\underset{\sim}{a}}\underset{\sim}{b}\right|=\left|\text{proj}_{\underset{\sim}{b}}\underset{\sim}{a}\right|$

End of Section I

Section II

30 marks
Attempt Questions 6 – 8
Allow about 50 minutes for this section

Answer each question in the appropriate section of the writing booklet. Extra writing paper is available.

In Questions 6 – 8, your responses should include relevant mathematical reasoning and/or calculations.

Question 6 (10 marks) Use the Question 6 section of the writing booklet.

(a) Given $\underset{\sim}{a} = 3\underset{\sim}{i} - 2\underset{\sim}{j}$ and $\underset{\sim}{b} = 4\underset{\sim}{i} + 2\underset{\sim}{j}$, find:

(i) $\underset{\sim}{a} + \underset{\sim}{b}$ **1**

(ii) $\underset{\sim}{b} - \underset{\sim}{a}$ **1**

(iii) $\left| \underset{\sim}{a} - \underset{\sim}{b} \right|$ **1**

(iv) $\underset{\sim}{a} \cdot \underset{\sim}{b}$ **1**

(b) A mass of 2 kg is hanging from the ceiling by a light inextensible string. Assuming $g = 9.8 \text{ m/s}^2$, calculate the tension in the string. **1**

(c) Consider the vectors $\underset{\sim}{p} = 3\underset{\sim}{i} + 4\underset{\sim}{j}$ and $\underset{\sim}{q} = -5\underset{\sim}{i} + 2\underset{\sim}{j}$.

(i) Find $\hat{\underset{\sim}{p}}$ in component form. **1**

(ii) Hence, or otherwise, find $\text{proj}_{\underset{\sim}{p}}\underset{\sim}{q}$. **2**

(iii) Show algebraically that $\text{proj}_{\underset{\sim}{p}}\left(k\underset{\sim}{q}\right) = k\left(\text{proj}_{\underset{\sim}{p}}\underset{\sim}{q}\right)$, where k is a real number, and hence find the value of k such that $\text{proj}_{\underset{\sim}{p}}\left(k\underset{\sim}{q}\right) = -\underset{\sim}{p}$. **2**

End of Question 6

Question 7 (10 marks) Use the Question 7 section of the writing booklet.

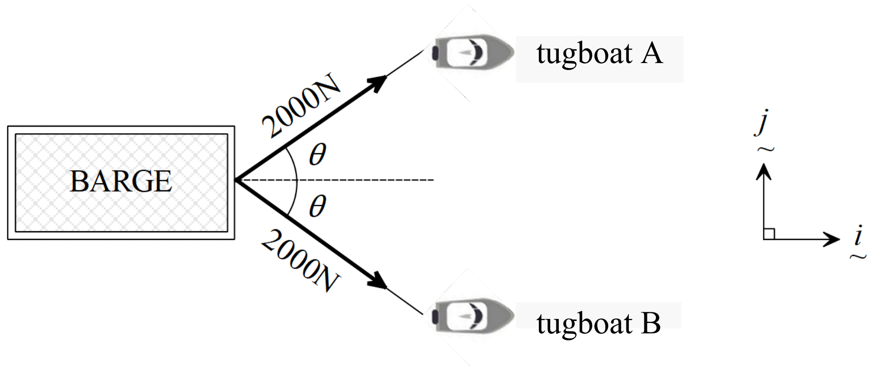
(a) Prove that $n^3 + 2n$ is divisible by 3 for all positive integers n. **3**

(b) Use the principle of mathematical induction to show that for all integers $n \geq 1$, **3**

$$1\times 3 + 2\times 3^2 + 3\times 3^3 + \cdots + n\times 3^n = \frac{(2n-1)3^{n+1}+3}{4}.$$

(c) A barge is being pulled by two tugboats. Each tugboat applies a force of 2000 newtons at an angle of θ to the horizontal, symmetrically as shown in the diagram.

Let $\underset{\sim}{i}$ and $\underset{\sim}{j}$ be the unit vectors in the directions as shown.



(i) If the resultant force is 3000 newtons purely in the $\underset{\sim}{i}$ direction, calculate the size of angle θ correct to the nearest degree. **2**

(ii) Suppose now that the angle at which the tugboats pull is kept symmetrically at 30° to the horizontal each, compared to the scenario in part (i), would the magnitude of the resultant force increase or decrease, and by how much? Express your answer correct to the nearest newton. **2**

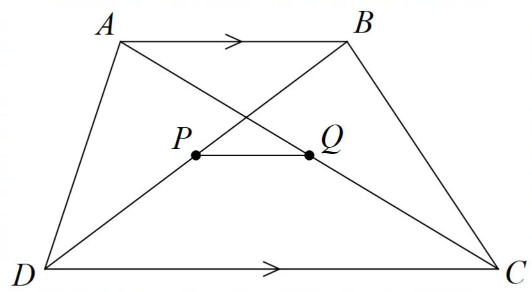
End of Question 7

Question 8 (10 marks) Use the Question 8 section of the writing booklet.

(a) The vector $\begin{pmatrix} -4h \\ 3h-2 \end{pmatrix}$ is perpendicular to $\begin{pmatrix} 2h \\ 4h+4 \end{pmatrix}$, where h is a constant, find all such pairs of vectors. **3**

(b) Prove using mathematical induction that $2^{n+2}+3^{2n+1}$ is divisible by 7 for all positive integers n. **3**

(c) The diagram below shows a trapezium $ABCD$, where $AB \parallel DC$ with $|AB|<|DC|$ as shown. The points P and Q are the midpoints of the diagonals BD and AC respectively.



Let $\underset{\sim}{a}$, $\underset{\sim}{b}$, $\underset{\sim}{c}$, $\underset{\sim}{d}$ be the position vectors of A, B, C and D respectively.

(i) Find the position vector of P in terms of the given position vectors. **1**

(ii) Use vector methods to show that PQ is parallel to the parallel sides of the trapezium and is equal to half of the difference in their lengths. **3**

End of Paper

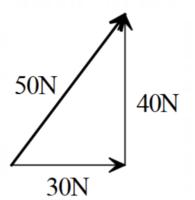
YEAR 12 ASSESSMENT TASK 1 2021
MATHEMATICS EXTENSION 1
MARKING GUIDELINES

Section I

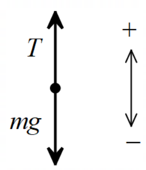
Multiple-choice Answer Key

Question	Answer
1	C
2	C
3	D
4	B
5	D

Questions 1 – 10

Sample solution	
1.	Vector addition
2.	$\underset{\sim}{i}+\frac{1}{2}\underset{\sim}{j}=\frac{1}{4}\left(4\underset{\sim}{i}+2\underset{\sim}{j}\right)$, therefore $\underset{\sim}{i}+\frac{1}{2}\underset{\sim}{j} \,//\, 4\underset{\sim}{i}+2\underset{\sim}{j}$ and since the $\frac{1}{4}$ means it's a quarter of the length, the direction remains unchanged.
3.	Statement A is always true by the triangle inequality. Statement B is always true because $\underset{\sim}{u}\cdot\underset{\sim}{v}=\lvert\underset{\sim}{u}\rvert\lvert\underset{\sim}{v}\rvert\cos\theta$ and $\cos\theta$ takes on a maximum value of 1. Statement C is always true by the geometry of vector projections, the projection of a vector can only be as long as the vector itself, never longer. Statement D is not always true, the inequality is false $\lvert\underset{\sim}{u}\rvert>1$.
4.	Horizontal and vertical forces sum as follows, the resultant force is 50 newtons in the direction indicated. 
5.	Statement D is always true as: $\lvert\text{proj}_{\underset{\sim}{a}}\underset{\sim}{b}\rvert=\lvert\underset{\sim}{b}\cdot\hat{\underset{\sim}{a}}\rvert$ $=\dfrac{\lvert\underset{\sim}{b}\cdot\underset{\sim}{a}\rvert}{\lvert\underset{\sim}{a}\rvert}$ $=\dfrac{\lvert\underset{\sim}{a}\cdot\underset{\sim}{b}\rvert}{\lvert\underset{\sim}{b}\rvert}$ $\left(\text{since } \underset{\sim}{a}\cdot\underset{\sim}{b}=\underset{\sim}{b}\cdot\underset{\sim}{a} \text{ and given } \lvert\underset{\sim}{a}\rvert=\lvert\underset{\sim}{b}\rvert\right)$ $=\lvert\underset{\sim}{a}\cdot\hat{\underset{\sim}{b}}\rvert$ $=\lvert\text{proj}_{\underset{\sim}{b}}\underset{\sim}{a}\rvert$

Section II
Question 6

Sample solution			Suggested marking criteria
(a)	(i)	$\underset{\sim}{a}+\underset{\sim}{b}=\left(3\underset{\sim}{i}-2\underset{\sim}{j}\right)+\left(4\underset{\sim}{i}+2\underset{\sim}{j}\right)$ $=7\underset{\sim}{i}$	• 1 – correct answer
	(ii)	$\underset{\sim}{b}-\underset{\sim}{a}=\left(4\underset{\sim}{i}+2\underset{\sim}{j}\right)-\left(3\underset{\sim}{i}-2\underset{\sim}{j}\right)$ $=\underset{\sim}{i}+4\underset{\sim}{j}$	• 1 – correct answer
	(iii)	$\lvert\underset{\sim}{a}-\underset{\sim}{b}\rvert=\lvert\underset{\sim}{b}-\underset{\sim}{a}\rvert$ $=\sqrt{1^2+4^2}$ $=\sqrt{17}$	• 1 – correct answer
	(iv)	$\underset{\sim}{a}\cdot\underset{\sim}{b}=3\times4+(-2)\times2$ $=8$	• 1 – correct answer
(b)		 $T-mg=0$ $T=mg$ $=2\times9.8$ $=19.6\text{ N}$	• 1 – correct answer
(c)	(i)	$\underset{\sim}{\hat{p}}=\dfrac{\underset{\sim}{p}}{\lvert\underset{\sim}{p}\rvert}$ $=\dfrac{3\underset{\sim}{i}+4\underset{\sim}{j}}{\sqrt{3^2+4^2}}$ $=\dfrac{3}{5}\underset{\sim}{i}+\dfrac{4}{5}\underset{\sim}{j}$	• 1 – correct answer
	(ii)	$\text{proj}_{\underset{\sim}{p}}\underset{\sim}{q}=\left(\underset{\sim}{q}\cdot\underset{\sim}{\hat{p}}\right)\underset{\sim}{\hat{p}}$ $=\left[\left(-5\underset{\sim}{i}+2\underset{\sim}{j}\right)\cdot\left(\dfrac{3}{5}\underset{\sim}{i}+\dfrac{4}{5}\underset{\sim}{j}\right)\right]\left(\dfrac{3}{5}\underset{\sim}{i}+\dfrac{4}{5}\underset{\sim}{j}\right)$ $=\left(-5\times\dfrac{3}{5}+2\times\dfrac{4}{5}\right)\left(\dfrac{3}{5}\underset{\sim}{i}+\dfrac{4}{5}\underset{\sim}{j}\right)$ $=-\dfrac{7}{5}\left(\dfrac{3}{5}\underset{\sim}{i}+\dfrac{4}{5}\underset{\sim}{j}\right)$ $=-\dfrac{21}{25}\underset{\sim}{i}-\dfrac{28}{25}\underset{\sim}{j}$ OR $\text{proj}_{\underset{\sim}{p}}\underset{\sim}{q}=\left(\dfrac{\underset{\sim}{q}\cdot\underset{\sim}{p}}{\underset{\sim}{p}\cdot\underset{\sim}{p}}\right)\underset{\sim}{p}$ $=\left[\dfrac{\left(-5\underset{\sim}{i}+2\underset{\sim}{j}\right)\cdot\left(3\underset{\sim}{i}+4\underset{\sim}{j}\right)}{\left(3\underset{\sim}{i}+4\underset{\sim}{j}\right)\cdot\left(3\underset{\sim}{i}+4\underset{\sim}{j}\right)}\right]\left(3\underset{\sim}{i}+4\underset{\sim}{j}\right)$ $=\dfrac{-5\times3+2\times4}{3\times3+4\times4}\left(3\underset{\sim}{i}+4\underset{\sim}{j}\right)$ $=-\dfrac{7}{25}\left(3\underset{\sim}{i}+4\underset{\sim}{j}\right)$ $=-\dfrac{21}{25}\underset{\sim}{i}-\dfrac{28}{25}\underset{\sim}{j}$	• 2 – correct solutions • 1 – finds $\underset{\sim}{q}\cdot\underset{\sim}{\hat{p}}$ or $\dfrac{\underset{\sim}{q}\cdot\underset{\sim}{p}}{\underset{\sim}{p}\cdot\underset{\sim}{p}}$

Question 6 (continued)

Sample solution			Suggested marking criteria
(b)	(iii)	$\text{proj}_{\underset{\sim}{p}}\left(k\underset{\sim}{q}\right)=\left(k\underset{\sim}{q}\cdot\hat{\underset{\sim}{p}}\right)\hat{\underset{\sim}{p}}$ $=k\left(\underset{\sim}{q}\cdot\hat{\underset{\sim}{p}}\right)\hat{\underset{\sim}{p}}$ $=k\left(\text{proj}_{\underset{\sim}{p}}\underset{\sim}{q}\right)$ OR $\text{proj}_{\underset{\sim}{p}}\left(k\underset{\sim}{q}\right)=\left(\dfrac{k\underset{\sim}{q}\cdot\underset{\sim}{p}}{\underset{\sim}{p}\cdot\underset{\sim}{p}}\right)\underset{\sim}{p}$ $=k\left(\dfrac{\underset{\sim}{q}\cdot\underset{\sim}{p}}{\underset{\sim}{p}\cdot\underset{\sim}{p}}\right)\underset{\sim}{p}$ $=k\left(\text{proj}_{\underset{\sim}{p}}\underset{\sim}{q}\right)$ $\text{proj}_{\underset{\sim}{p}}\left(k\underset{\sim}{q}\right)=-\underset{\sim}{p}$ $k\left(\text{proj}_{\underset{\sim}{p}}\underset{\sim}{q}\right)=-\underset{\sim}{p}$ $k\left(-\dfrac{21}{25}\underset{\sim}{i}-\dfrac{28}{25}\underset{\sim}{j}\right)=-\left(3\underset{\sim}{i}+4\underset{\sim}{j}\right)$ $-\dfrac{7k}{25}\left(3\underset{\sim}{i}+4\underset{\sim}{j}\right)=-\left(3\underset{\sim}{i}+4\underset{\sim}{j}\right)$ $-\dfrac{7k}{25}=-1$ $k=\dfrac{25}{7}$	• 2 – correct solution • 1 – correctly shows the required identity

Question 7

<table>
<tr><th colspan="2">Sample solution</th><th>Suggested marking criteria</th></tr>
<tr><td>(a)</td><td>Let $S(n)$ be the statement that n^3+2n is divisibly by 3.

<u>Show $S(1)$ is true:</u>
$1^3+2\times1=3,$ clearly divisible by 3.
$\therefore S(1)$ is true.

<u>Assume $S(k)$ is true, i.e.:</u>
$k^3+2k=3M$ for some integer M.

<u>Show $S(k+1)$ is true, i.e.:</u>
$(k+1)^3+2(k+1)=3N$ for some integer N.

$\begin{aligned}\text{LHS} &= (k+1)^3+2(k+1)\\ &= k^3+3k^2+3k+1+2k+2\\ &= k^3+2k+3k^2+3k+3\\ &= 3M+3(k^2+k+1)\\ &= 3N, \text{ where } N=M+k^2+k+1 \text{ is an integer.}\\ &= \text{RHS}\end{aligned}$
$\therefore S(k+1)$ is true if $S(k)$ is assumed true.

Since $S(1)$ is shown true, by the principle of mathematical induction, $S(n)$ is true for all positive integers n.</td><td>• 3 – correct induction
• 2 – attempts to use the assumption to induce the required result
• 1 – correctly shows base case</td></tr>
</table>

Question 7 (continued)

<table>
<tr><th colspan="2">Sample solution</th><th>Suggested marking criteria</th></tr>
<tr><td>(b)</td><td>

Let $S(n)$ be the statement that $1\times 3+2\times 3^2+3\times 3^3+\cdots+n\times 3^n=\dfrac{(2n-1)3^{n+1}+3}{4}$.

<u>Show $S(1)$ is true:</u>

$$\text{LHS}=1\times 3^1 \qquad \text{RHS}=\frac{(2\times 1-1)\times 3^{1+1}+3}{4}$$
$$=3 \qquad =\frac{1\times 9+3}{4}$$
$$=\frac{12}{4}$$
$$=3$$

Since LHS = RHS, $\therefore S(1)$ is true.

<u>Assume $S(k)$ is true, i.e.:</u>

$$1\times 3+2\times 3^2+3\times 3^3+\cdots+k\times 3^k=\frac{(2k-1)3^{k+1}+3}{4}$$

<u>Show $S(k+1)$ is true, i.e.:</u>

$$1\times 3+2\times 3^2+3\times 3^3+\cdots+k\times k^n+(k+1)\times 3^{k+1}=\frac{[2(k+1)-1]3^{k+1+1}+3}{4}$$
$$=\frac{(2k+1)3^{k+1}+3}{4}$$

$$\begin{aligned}\text{LHS}&=1\times 3+2\times 3^2+3\times 3^3+\cdots+k\times 3^k+(k+1)\times 3^{k+1}\\&=\frac{(2k-1)3^{k+1}+3}{4}+(k+1)\times 3^{k+1}\\&=\frac{(2k-1)3^{k+1}+3+4(k+1)\times 3^{k+1}}{4}\\&=\frac{[(2k-1)+4(k+1)]3^{k+1}+3}{4}\\&=\frac{(6k+3)3^{k+1}+3}{4}\\&=\frac{(2k+1)3^{k+2}+3}{4}\\&=\text{RHS}\end{aligned}$$

$\therefore S(k+1)$ is true if $S(k)$ is assumed true.

Since $S(1)$ is shown true, by the principle of mathematical induction, $S(n)$ is true for all positive integers *n*.

</td><td>

- 3 – correct induction
- 2 – attempts to use the assumption to induce the required result
- 1 – correctly shows base case

</td></tr>
</table>

Question 12 (continued)

Sample solution			Suggested marking criteria
(d)	(i)	Summing the two forces applied by the tugboats: $2 \times (2000\cos\theta)\underset{\sim}{i} = 3000\underset{\sim}{i}$ Equating horizontal and vertical components: $4000\cos\theta = 3000$ $\cos\theta = \frac{3}{4}$ $\theta = 41^\circ$ (nearest degree)	• 2 – correct answer • 1 – correct horizontal component of the tugboat force
	(ii)	$2 \times (2000\cos 30^\circ) = 2000\sqrt{3}$ N This results in an increase of: $2000\sqrt{3} - 3000 = 464$ N (nearest newton)	• 2 – correct answer • 1 – correct magnitude of the resultant force

Question 8

<table>
<tr><th colspan="2">Sample solution</th><th>Suggested marking criteria</th></tr>
<tr><td>(a)</td><td>

$$\begin{pmatrix} -4h \\ 3h-2 \end{pmatrix} \cdot \begin{pmatrix} 2h \\ 4h+4 \end{pmatrix} = 0$$

$$-4h \times 2h + (3h-2)(4h+4) = 0$$

$$-8h^2 + 12h^2 + 4h - 8 = 0$$

$$4h^2 + 4h - 8 = 0$$

$$h^2 + h - 2 = 0$$

$$(h+2)(h-1) = 0$$

$$\therefore h = -2, h = 1$$

When $h = -2$, the vectors are

$$\begin{pmatrix} -4h \\ 3h-2 \end{pmatrix} = \begin{pmatrix} -4\times(-2) \\ 3\times(-2)-2 \end{pmatrix} = \begin{pmatrix} 8 \\ -8 \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} 2h \\ 4h+4 \end{pmatrix} = \begin{pmatrix} 2\times(-2) \\ 4\times(-2)+4 \end{pmatrix} = \begin{pmatrix} -4 \\ -4 \end{pmatrix}$$

When $h = 1$, the vectors are

$$\begin{pmatrix} -4h \\ 3h-2 \end{pmatrix} = \begin{pmatrix} -4\times 1 \\ 3\times 1-2 \end{pmatrix} = \begin{pmatrix} -4 \\ 1 \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} 2h \\ 4h+4 \end{pmatrix} = \begin{pmatrix} 2\times 1 \\ 4\times 1+4 \end{pmatrix} = \begin{pmatrix} 2 \\ 8 \end{pmatrix}$$

</td><td>

- 3 – correct solution
- 2 – one correct pair of vectors
- 1 – establishes a correct quadratic using the dot product

</td></tr>
</table>

Question 8 (continued)

Sample solution		**Suggested marking criteria**
(b)	Let $S(n)$ be the statement that $2^{n+2}+3^{2n+1}$ is divisible by 7. <u>Show $S(1)$ is true:</u> $2^{1+2}+3^{2\times1+1} = 2^3+3^3$ $= 8+27$ $= 35$ $= 7\times5$, clearly divisible by 7. $\therefore S(1)$ is true. <u>Assume $S(k)$ is true, i.e.:</u> $2^{k+2}+3^{2k+1}=7M$, for some integer M. <u>Show $S(k+1)$ is true, i.e.:</u> $2^{k+1+2}+3^{2(k+1)+1}=7N$, for some integer N. $2^{k+3}+3^{2k+3}=7N$ $\text{LHS} = 2^{k+3}+3^{2k+3}$ $= 2\times2^{k+2}+3^2\times3^{2k+1}$ $= 2\times\left(7M-3^{2k+1}\right)+9\times3^{2k+1}$ $= 7\times2M-2\times3^{2k+1}+9\times3^{2k+1}$ $= 7\times2M+7\times3^{2k+1}$ $= 7\left(2M+3^{2k+1}\right)$ $= 7N$, where $N=2M+3^{2k+1}$ is an integer $= \text{RHS}$ $\therefore S(k+1)$ is true if $S(k)$ is assumed true. Since $S(1)$ is shown true, by the principle of mathematical induction, $S(n)$ is true for all positive integers *n*.	• 3 – correct induction • 2 – attempts to use the assumption to induce the required result • 1 – correctly shows base case

Question 8 (continued)

Sample solution			Suggested marking criteria
(c)	(i)	$\underset{\sim}{p} = \overrightarrow{OP}$ $= \overrightarrow{OD} + \overrightarrow{DP}$ $= \overrightarrow{OD} + \frac{1}{2}\overrightarrow{DB}$ $= \overrightarrow{OD} + \frac{1}{2}\left(\overrightarrow{OB} - \overrightarrow{OD}\right)$ $= \underset{\sim}{d} + \frac{1}{2}\left(\underset{\sim}{b} - \underset{\sim}{d}\right)$ $= \frac{1}{2}\left(\underset{\sim}{b} + \underset{\sim}{d}\right)$	• 1 – correct answer
	(ii)	Similarly, $\underset{\sim}{q} = \frac{1}{2}\left(\underset{\sim}{a} + \underset{\sim}{c}\right)$ $\overrightarrow{PQ} = \underset{\sim}{q} - \underset{\sim}{p}$ $= \frac{1}{2}\left(\underset{\sim}{a} + \underset{\sim}{c} - \underset{\sim}{b} - \underset{\sim}{d}\right)$ $= \frac{1}{2}\left[\left(\underset{\sim}{c} - \underset{\sim}{d}\right) - \left(\underset{\sim}{b} - \underset{\sim}{a}\right)\right]$ $= \frac{1}{2}\left(\overrightarrow{DC} - \overrightarrow{AB}\right)$ Noting $\overrightarrow{DC} = k\overrightarrow{AB}$, where $k > 1$ and therefore $\left\lvert\overrightarrow{DC}\right\rvert = \left\lvert k\overrightarrow{AB}\right\rvert = k\left\lvert\overrightarrow{AB}\right\rvert$ $\overrightarrow{PQ} = \frac{1}{2}\left(k\overrightarrow{AB} - \overrightarrow{AB}\right)$ $= \frac{k-1}{2}\overrightarrow{AB}$ $\therefore PQ \parallel AB \parallel DC$ $\left\lvert\overrightarrow{PQ}\right\rvert = \left\lvert\frac{k-1}{2}\overrightarrow{AB}\right\rvert$ $= \frac{k-1}{2}\left\lvert\overrightarrow{AB}\right\rvert$ $= \frac{1}{2}\left(k\left\lvert\overrightarrow{AB}\right\rvert - \left\lvert\overrightarrow{AB}\right\rvert\right)$ $= \frac{1}{2}\left(\left\lvert\overrightarrow{DC}\right\rvert - \left\lvert\overrightarrow{AB}\right\rvert\right)$ i.e. the length of PQ is half the difference in length of the two parallel sides of the trapezium.	• 3 – correct solution • 2 – shows PQ is parallel to the two parallel sides of the trapezium • 1 – expresses $\overrightarrow{PQ}$ in terms of the parallel sides of the trapezium