



CRANBROOK  
SCHOOL

Centre Number

1 2 5

Student Number

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**2021**

**HSC Assessment Task**

**Assessment Task 2**

# Mathematics Extension 1

|                |            |
|----------------|------------|
| Reading time   | 5 minutes  |
| Writing time   | 90 minutes |
| Total Marks    | 48         |
| Task weighting | 25%        |

## General Instructions

- Write using blue or black pen
- Diagrams drawn using pencil
- A Board-approved calculator may be used
- The Reference sheet is on the last page of this booklet
- Use the Multiple-Choice Answer Sheet provided
- All relevant working should be shown for each question

## Additional Materials Needed

- Multiple Choice Answer Sheet
- 3 writing booklets

## Structure & Suggested Time Spent

### Section I

#### Multiple Choice Questions

- Answer Q1 – 5 on the multiple choice answer sheet
- Allow 9 minutes for this section

### Section II

#### Extended response Questions

- Attempt all questions in this section in a separate writing booklet
- Allow about 81 minutes for this section

This paper must not be removed from the examination room

### Disclaimer

*The content and format of this paper does not necessarily reflect the content and format of the HSC examination paper.*

# Section I

5 Marks

Allow about 9 minutes for this section

Use the multiple choice answer sheet for Questions 1 – 5.

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1 Evaluate  $\cos\left(\sin^{-1}\frac{\sqrt{3}}{2}\right)$ .

(A)  $\frac{\sqrt{3}}{2}$

(B)  $-\frac{\sqrt{3}}{2}$

(C)  $\frac{1}{2}$

(D)  $-\frac{1}{2}$

2 Evaluate  $\int \frac{2x^3 + 4x - 3}{x^3} dx$ .

(A)  $2 + \frac{4}{x^2} + \frac{3}{x^3} + C$

(B)  $2x - \frac{4}{x} + \frac{3}{2x^2} + C$

(C)  $x^2 + 4\ln x + \frac{1}{x^3} + C$

(D)  $\frac{x^3}{3} - \frac{2}{x^2} + \frac{3}{4x^3} + C$

3 Evaluate  $\int_{-1}^3 \left(\frac{x}{2} - 1\right)^3 dx$ .

(A)  $-\frac{5}{2}$

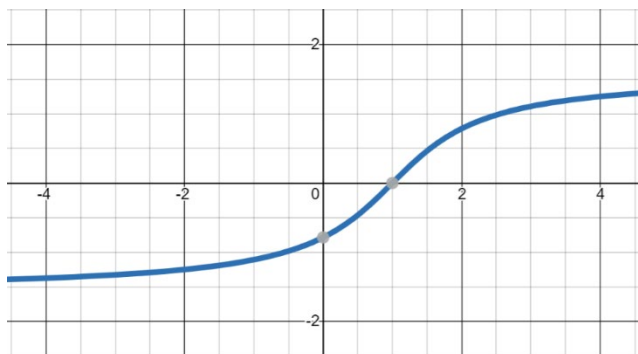
(B)  $\frac{5}{2}$

(C)  $-\frac{41}{16}$

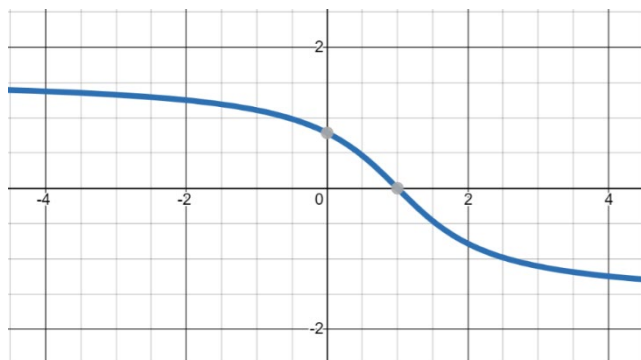
(D)  $\frac{41}{16}$

4 Which of the following is the curve  $y = -\frac{1}{2} \tan^{-1}(1+x)$ ?

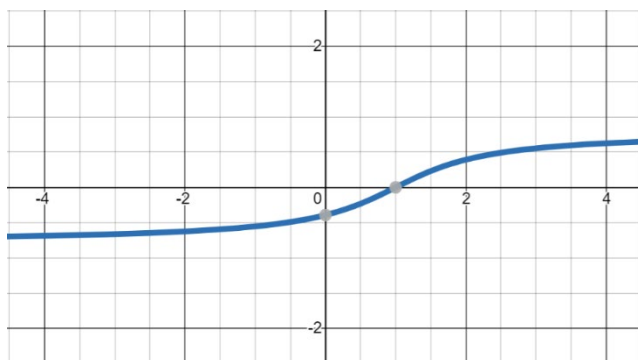
(A)



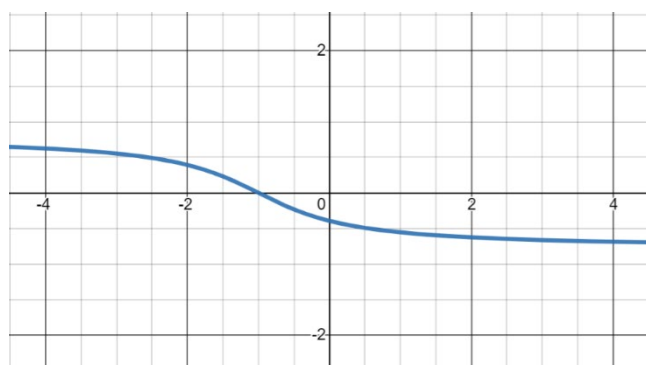
(B)



(C)



(D)



- 5 An equilateral triangle's area is increasing at a rate of  $4 \text{ cm}^2/\text{s}$ . What is the increasing rate of the side length, when the side length is  $\sqrt{3} \text{ cm}$ ?

- (A)  $\frac{4}{3} \text{ cm/s}$
- (B)  $3\sqrt{3} \text{ cm/s}$
- (C)  $\frac{8}{3} \text{ cm/s}$
- (D)  $\frac{4\sqrt{3}}{3} \text{ cm/s}$

**END OF SECTION I**

## Section II

43 Marks

Allow about 81 minutes for this section

Answer questions 6 - 8 in separate booklets.

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Question 6

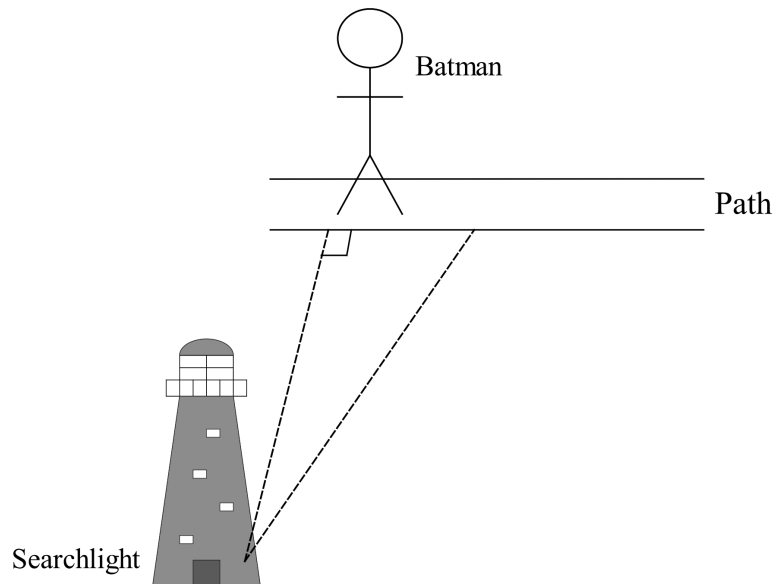
Start a new booklet

14 Marks

- (a) Consider the curve  $y = 2 \cos^{-1}\left(\frac{x}{2}\right)$ .
- (i) State the domain and range. 2
- (ii) Hence or otherwise, sketch the curve on a pair of axes. 2
- (b) Calculate the area enclosed by the curves  $y = x^2 - 4x + 4$  and  $y = -x^2 + 8x - 12$ . 3
- (c) Write  $\cos\left(2 \sin^{-1}\left(\frac{1}{4}\right)\right)$  in the form  $\frac{a}{b}$ , where  $a$  and  $b$  are rational. 2
- (d) Find the cartesian equation for the parametrisation  $x = 4 \cos \theta$  and  $y = 4 \operatorname{cosec} \theta$ . 2

QUESTION 6 CONTINUES ON THE NEXT PAGE

- (e) Batman walks along a straight path at a speed of  $1.5 \text{ m/s}$ . A searchlight is located on the ground 6 metres from the path and is kept focused on Batman. At what rate is the searchlight rotating when Batman is 8 metres from the point on the path closest to the searchlight? 3



**END OF QUESTION 6**

- (a) The temperature of Mr Chia's hot chocolate is originally  $85^{\circ}\text{C}$ , the temperature of the Maths Staffroom where Mr Chia is waiting for the hot chocolate to cool is  $20^{\circ}\text{C}$ . Mr Chia knows that his hot chocolate will cool to a nice  $65^{\circ}\text{C}$  in 8 minutes. The rate of cooling of the hot chocolate is proportional to the difference between the current hot chocolate temperature and the room temperature. That is,  $\frac{dT}{dt} = -k(T - 20)$  where  $T$  is the temperature of the hot chocolate after  $t$  minutes.
- (i) Show that  $T = 20 + Ae^{-kt}$  is a solution to the differential equation. 1
- (ii) Mr Chia wants his hot chocolate to be a little warmer today. Find when the temperature of the hot chocolate has dropped to  $70^{\circ}\text{C}$ .  
Give your answer correct to the nearest **second**. 3
- (b) Find the exact value of  $\tan\left[\cos^{-1}\left(\frac{12}{13}\right) + \sin^{-1}\left(\frac{7}{25}\right)\right]$  3
- (c) Air is being pumped into a spherical balloon so that its volume increases at a rate of  $100\text{ cm}^3/\text{s}$ . How fast is the radius of the balloon increasing when the diameter is 50 cm? 3  
Give your answer in exact form.

**QUESTION 7 CONTINUES ON THE NEXT PAGE**

- (d) Consider the parametrisation  $x = 2 \cos \theta$  and  $y = 2 \sin \theta$ .
- (i) Show the cartesian equation is  $x^2 + y^2 = 4$  **1**
- (ii) Sketch the curves  $y = -\frac{1}{2}x^2 + 2$  and  $x^2 + y^2 = 4$  on the same set of axes. **2**
- (iii) Hence or otherwise, find the area of the region above the  $x$ -axis bounded by the curve  $y = -\frac{1}{2}x^2 + 2$  and the curve produced by the parametric equations  $x = 2 \cos \theta$  and  $y = 2 \sin \theta$ . **2**

**END OF QUESTION 7**



- (a) Solve the following pair of simultaneous equations.

3

$$\sin^{-1} x + 3 \cos^{-1} y = \frac{29}{12} \pi$$

$$3 \sin^{-1} x - 2 \cos^{-1} y = -\pi$$

- (b) Gravel is being dumped from a conveyor belt at a rate of  $6 \text{ m}^3/\text{min}$ , and its coarseness is such that it forms a pile in the shape of a cone whose base diameter and height are always equal. The surface area of a cone is  $A = \pi r^2 + \pi r l$ , where  $r$  is the radius of the base and  $l$  is the slope length.

- (i) Find the rate at which the height is changing when the height is 9 m.

3

- (ii) The company needs to spray the pile with water to ensure that dust particles do not enter the atmosphere.

Find the rate at which the exposed surface area of the pile, is increasing, when the height is 9 m.

3

**QUESTION 8 CONTINUES ON THE NEXT PAGE**

- (c) Fire sprinklers in a building can be set off when the smoke detector is initiated.

The water for the sprinklers flow from a rain water tank on the building's roof. The tank is initially full. When the sprinklers are initiated the tank is drained so that at time  $t$  hours the volume of water in the tank  $V$ , in kilolitres, is given by:

$$V = -e^t \sin t + 10$$

The water tank will stop flowing water to the sprinklers when it reaches a minimum volume stated by the building's safety regulations.

At this time, the building uses water from the local area to continue the sprinklers if needed and the water tank begins filling up from the local water supply to a specific level that will allow it to continue to collect rain for its future use.

- (i) Show that the first time when the water tank hits its base level of water after the sprinklers have been initiated is after  $\frac{3\pi}{4}$  hours. This is when the building begins to use the local areas water supply. 2
- (ii) After the initial time, when is the tank emptying at its fastest rate? 3

**END OF QUESTION 8**

**END OF ASSESSMENT**

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**2021**

Examination  
Assessment Task 2

Section I – Multiple Choice

## Mathematics Extension 1

- |                 |          |   |                       |   |                       |   |                       |   |                       |
|-----------------|----------|---|-----------------------|---|-----------------------|---|-----------------------|---|-----------------------|
| <b>Question</b> | <b>1</b> | A | <input type="radio"/> | B | <input type="radio"/> | C | <input type="radio"/> | D | <input type="radio"/> |
|                 | <b>2</b> | A | <input type="radio"/> | B | <input type="radio"/> | C | <input type="radio"/> | D | <input type="radio"/> |
|                 | <b>3</b> | A | <input type="radio"/> | B | <input type="radio"/> | C | <input type="radio"/> | D | <input type="radio"/> |
|                 | <b>4</b> | A | <input type="radio"/> | B | <input type="radio"/> | C | <input type="radio"/> | D | <input type="radio"/> |
|                 | <b>5</b> | A | <input type="radio"/> | B | <input type="radio"/> | C | <input type="radio"/> | D | <input type="radio"/> |