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Student Number

2021 YEAR 12

Mathematics Extension 1

Task 3

7th June, Monday

Q	Marks
MC	/2
3	/10
4	/14
5	/15
Total	/41

General

Instructions:

- Reading time – 5 minutes
- Working time – 55 minutes
- Write using blue or black pen
- NESA approved calculators may be used
- Show relevant mathematical reasoning and/or calculations
- No white-out may be used

Total Marks:

41

Section I - 2 marks

- Allow about 3 minutes for this section

Section II - 39 marks

Allow about 52 minutes for this section

This question paper must not be removed from the examination room.

This assessment task constitutes 35% of the course.

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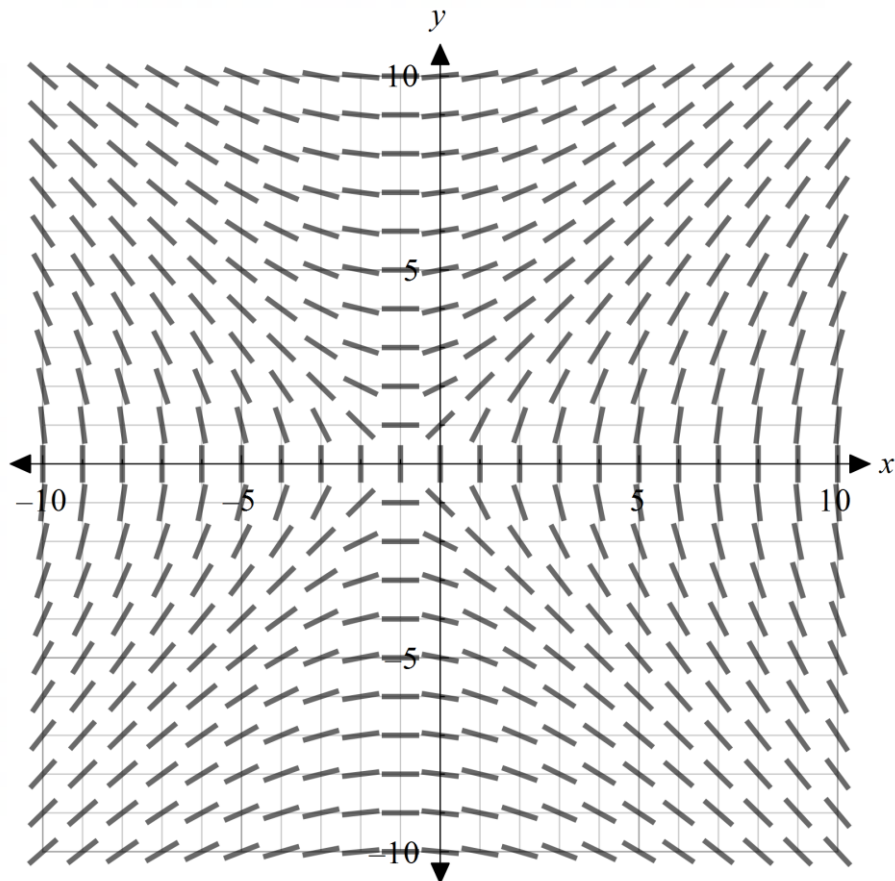
Section I

2 marks

Allow about 3 minutes for this section

Circle the correct answer below.

- 1 Determine which of the differential equations corresponds to the slope field given below.



- (A) $y' = 1 + xy$
- (B) $yy' = 1 + x$
- (C) $xy' = x + 1$
- (D) $y' = \frac{1}{y} + x$

2 The derivative of $\frac{d}{dx} \sin^{-1}\left(\frac{3x}{4}\right)$ is

(A) $\frac{3}{\sqrt{16 - 9x^2}}$

(B) $\frac{3}{4\sqrt{1 - 9x^2}}$

(C) $\frac{9}{\sqrt{16 - 9x^2}}$

(D) $\frac{9}{16\sqrt{1 - x^2}}$

Section II

40 marks

Attempt Questions 3–5

Allow about 52 minutes for this section

Answer each question in a NEW booklet. Extra writing booklets are available.

For questions in Section II, your response should include relevant mathematical reasoning and/or calculations.

Question 3 (10 marks) Answer each question in a NEW booklet.

(a) Confirm that $y = \frac{1}{18}x^3 + 4x^{-3}$ is a solution to $xy'' + 4y' = x^2$. 2

(b) Express $\sqrt{3} \sin x + 3 \cos x$ in the form $R \sin(x + \alpha)$. 1

(c) Find all possible solutions for the following 2
 $\sec^2 x - 2 \tan x = 0$

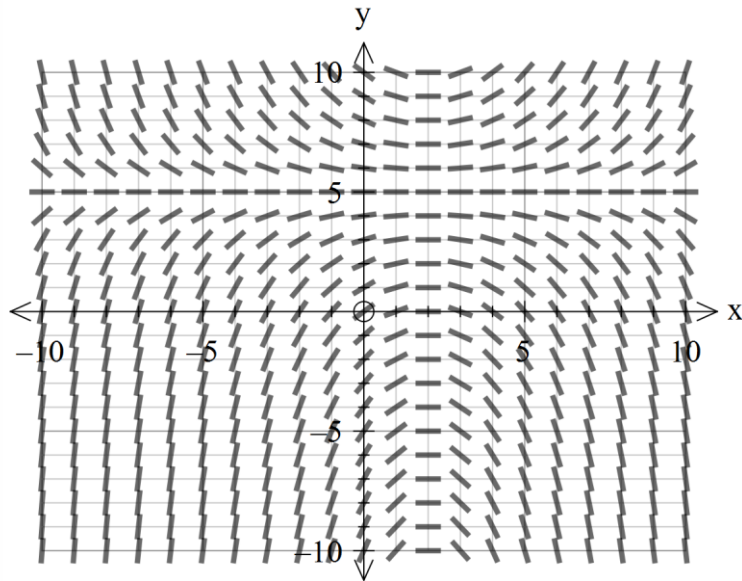
(d) Solve 3
 $y' = 6xy + 2y$

(e) Evaluate 2
 $\int \tan^2 x \, dx$

Question 4 (14 marks) Answer each question in a NEW booklet.

- (a) Sketch the solution curve that satisfies $y(-5) = 2$

1



- (b) Evaluate

2

$$\int \frac{1}{x(1 + (\ln x)^2)} dx$$

- (c) Prove by Mathematical Induction that $4^{n+1} + 5^{2n-1}$ is divisible by 7 for all integers, $n \geq 1$.

3

- (d) Solve the following equation for $x \in [0, 2\pi]$

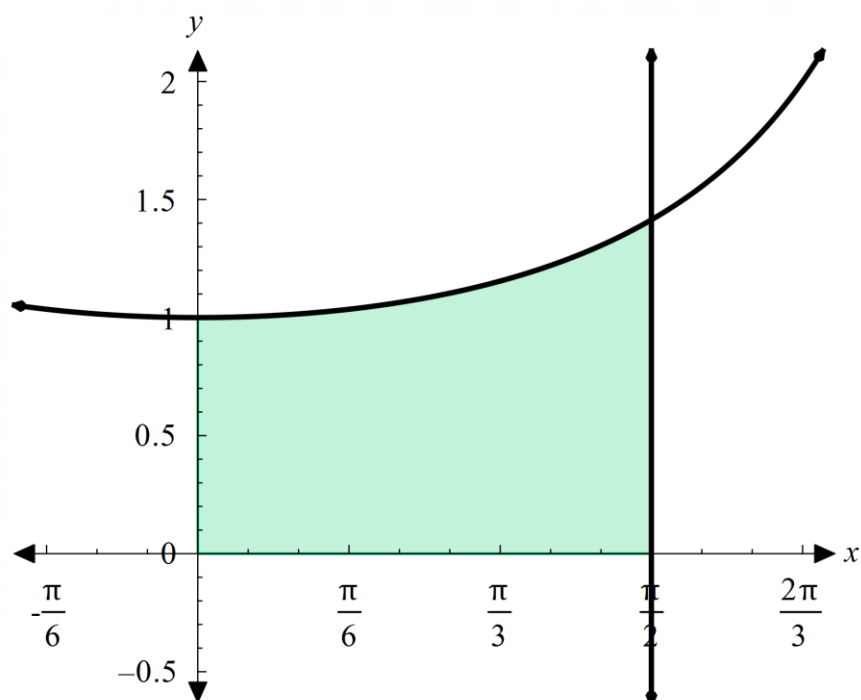
3

$$3 \sin x - 2 \cos x = 2$$

- (e) Evaluate $\int_0^1 x^3 (x^2 + 1)^4 dx$ using the substitution $u = x^2 + 1$.

2

- (f) The following figure shows the region bounded by the y -axis, the x -axis, the line $x = \frac{\pi}{2}$ and the curve $y = \sec \frac{x}{2}$. 3



Find the exact volume of the solid formed by rotating this region about the x -axis.

Question 5 (15 marks) Answer each question in a NEW booklet.

- (a) Solve the following, given that $y(0) = 3$ 3

$$\frac{dy}{dx}(x^2 + 1) = yx$$

- (b) Evaluate $\int_0^3 \sqrt{9 - x^2} dx$ using the substitution $x = 3 \sin \theta$. 4

- (c) Prove that $5^n \geq 3n + 7$ for all positive integers greater than 1. 4

- (d) Sometime Monday morning, it started snowing in Vladivostok and the depth of snow, h , increases at a constant rate. At noon, a snowplough starts to plough the main street of Vladivostok, and ploughs in a straight line at a rate inversely proportional to the depth of snow on the ground. Let x be the distance cleared by the snowplough, and it is observed that the plough covers exactly twice as much ground between noon and 1 pm as it does between 1 pm and 2 pm. It is snowing at a constant rate throughout.

- (i) Show that $\frac{dx}{dt} = \frac{k}{t}$ where t is the time, in hours, since it started snowing. 1

- (ii) What time did it start snowing? 3